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Effects of transverse polarization on the unitarization of WZ scattering amplitudes

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The invariant-mass distributions for the production of W^-Z^0 (W^+Z^0) via gauge-boson fusion at pp colliders of energies 16 and 40 TeV are calculated using the effective-W approximation supplemented by K-matrix unitarization. Included in the unitarization is the effect of transverse degrees of freedom on the production of $W_L^-Z_L^0$ $(W_L^+Z_L^0)$ gauge bosons. These results are compared with the corresponding results for W^-Z^0 (W^+Z^0) production via $q\bar{q}$ annihilation.

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I. INTRODUCTION

The fermion–gauge-boson sector of the standard model for the electroweak interaction has been well tested. Virtually all predictions for processes in this sector are consistent with the experiment. The situation, however, is not as satisfactory with respect to the interactions among the gauge bosons of the theory, $W^\pm$, Z^0, and γ. There continues to be a spirited discussion concerning the precise form of the triple gauge-boson coupling [1], and the properties of gauge-boson scattering amplitudes [2].

In this paper we study the effects of unitarity on the $J=0$, $I=2$ and the $J=1$, $I=1$ amplitudes for the process $W_L^\pm Z_L^0 \to W_L^\pm Z_L^0$. Recently, Atkinson, Harada, and Sanda [3] and Dicus and Repko [4] studied resonance effects in the $J=1$, $I=1$ channel of $W_L^+W_L^- \to W_L^+W_L^-$ using Padé unitarization. We view the $W^\pm Z^0$ channels as better candidates for observing $J=1$ phenomena since, from an experimental point of view, the $W^\pm Z^0$ states can be more readily reconstructed than W^+W^- states. In addition, the $q\bar{q}$ background for the $W^\pm Z^0$ channels is not as severe as the corresponding background for W^+W^- channel. Our explicit calculation shows that the $q\bar{q}$ background for either the W^-Z^0 or the W^+Z^0 channel is about one-tenth of that of W^+W^- channel. The $W^\pm Z^0$ channels have also been studied by Bagger *et al.* [5], employing a wide variety of models and by Chanowitz and Kilgore [6]. Our treatment of unitarity effects differs from that of Refs. [5,6] in that we include the effect of transversely polarized W's and Z's on the K-matrix unitarized longitudinal amplitudes. In addition, we estimate the effect of a resonance in the $J=1$ partial wave by using the Padé unitarized standard model amplitude rather than explicitly introducing a vector resonance.

In the next section, we present the invariant-mass distributions for WZ production via WZ fusion and $q\bar{q}$ annihilation. The following section contains an outline of the K-matrix unitarization scheme for WZ scattering including the effects of transverse W's and Z's, and in Sec. IV we discuss the effect of unitarization on WZ production.

II. INVARIANT-MASS DISTRIBUTIONS

In this section, we calculate the invariant-mass distribution for the production of W^-Z^0 (W^+Z^0), using the effective-W approximation [7]. A rapidity cut of 1.5 is imposed on both the final W and Z. In addition to this cut, we impose a minimum transverse momentum cut $P_\perp > (P_\perp)_{\rm min} = 30$ GeV on the final W and Z. This cut does not affect the $W^\pm Z^0$ pairs with invariant-mass m_{WZ} greater than about 400 GeV. For the quark number distribution functions, we use the Botts *et al.* (CTEQ) [8] structure functions. Our calculations show that the CTEQ structure functions yield cross sections which are about 15% to 20% larger than Set 2 of the Eichten *et al.* (EHLQ) [9] structure functions.

In Fig. 1, we include all tree-level (unitary gauge) Feynman diagrams needed to calculate the production of

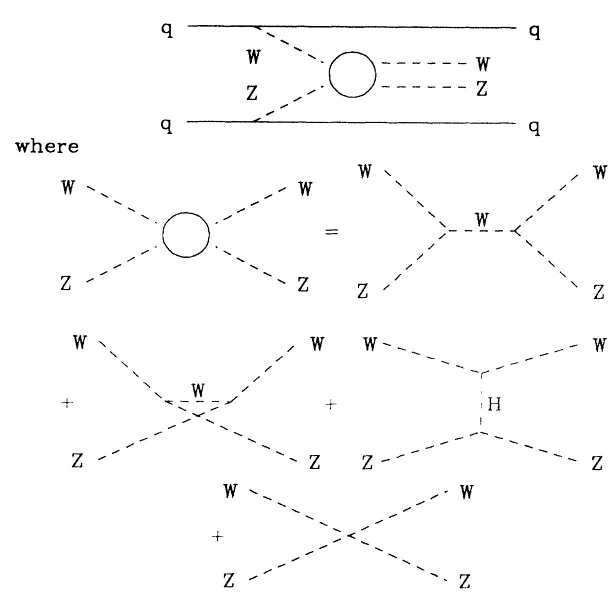

FIG. 1. Feynman graphs for W^-Z^0 (W^+Z^0) production via W^-Z^0 (W^+Z^0) fusion are shown.

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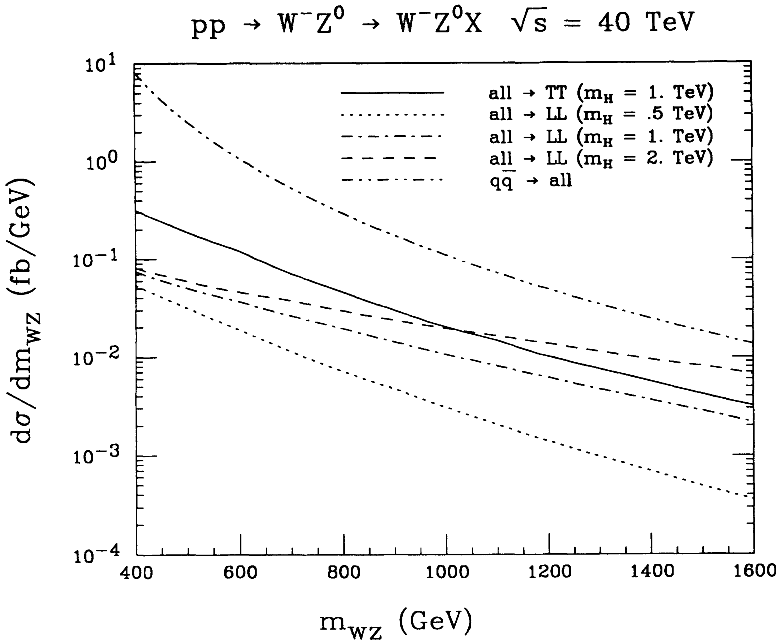

FIG. 2. The invariant-mass distribution for W^-Z^0 production in pp collisions at an energy of 40 TeV and with a rapidity cut of 1.5 is shown. The solid curve is contribution from all polarizations of the initial WZ and transverse final WZ (with Higgs mass of 2 TeV). The dashed, dash-dotted, and dotted curves are the contribution from all polarizations of initial WZ and longitudinal final WZ, for Higgs boson masses of 2.0, 1.0, and 0.5 TeV, respectively. The dashed-double-dotted line is the contribution from $q\bar{q} \to WZ$.

$W^{\pm}Z^0$ pairs via $W^{\pm}Z^0$ fusion in the effective-W approximation. This set of diagrams leads to an amplitude which is well behaved at large values of the invariant-mass m_{WZ} as in the case of W^+W^- scattering [10]. Like the case of W^+W^- [11], in an exact perturbative calculation it is also necessary to include diagrams involving W and Z bremsstrahlung from the quarks. Our amplitudes are calculated numerically and, in the unitary gauge, there are large cancellations involving terms which increase as the square and first power of the center of mass energy. To check the stability of our calculations at large m_{WZ} we also used a nonlinear gauge [12]. In this nonlinear gauge in addition to the diagrams of Fig. 1 there are two diagrams that involve the s- and u-channel exchange of charged Goldstone bosons. The numerical results in the two cases are virtually identical.

Our results for the production of W^-Z^0 are summarized in Figs. 2 and 3. The cross sections for the production of W^+Z^0 appear in Tables I and II. As expected, there is no pronounced peak for the energies around the Higgs boson mass due to the lack of s-channel Higgs boson exchange in the $W^{\pm}Z^0$ fusion. The contributions from the transversely polarized W's are as important as the longitudinal contributions, if not more so. From Figs.

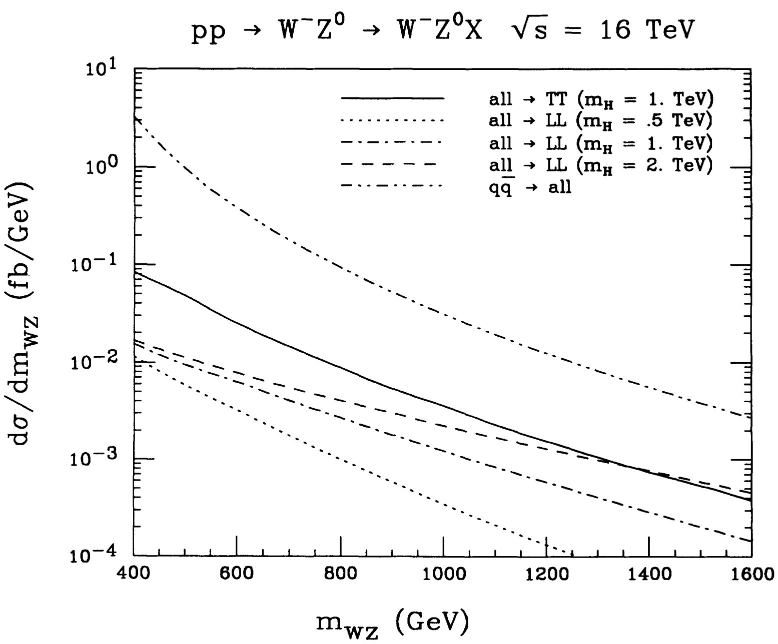

FIG. 3. Same as Fig. 2 except for a collider energy of 16 TeV.

TABLE I. Integrated cross section in femtobarns for W^-Z (W^+Z) production in pp collisions at $\sqrt{s}=40$ TeV with $m_{WZ}\le 3$ TeV and a rapidity cut of 1.5 on both the W and Z.

Process	m_H	$m_{WZ}>0.4$	$m_{WZ}>0.8$	$m_{WZ}>1.2$	$m_{WZ}>1.6$	$m_{WZ}>2.0$
all $\to$ TT	1	67 (79)	13 (17)	3.7 (5.3)	1.3 (2.0)	0.55 (0.74)
all $\to$ LL	0.5	11 (12)	1.8 (2.4)	0.43 (0.68)	0.13 (0.22)	0.044 (0.082)
all $\to$ LL	1	23 (28)	7.0 (10)	2.4 (3.8)	0.91 (1.5)	0.35 (0.62)
all $\to$ LL	2	35 (44)	16 (23)	7.5 (12)	3.6 (6.2)	1.7 (3.2)
$q\bar{q}\to$ all		833 (908)	68 (83)	16 (22)	5.3 (8.2)	2.0 (3.4)
LL $\to$ LL (K-M)	2	34 (46)	16 (24)	8.3 (14)	4.6 (8.0)	2.5 (4.6)
LL $\to$ LL (Padé)	2	37 (53)	19 (30)	11 (19)	7.2 (13)	4.8 (9.0)

2 and 3, the cross section for the production of a transverse final W^-Z^0 pair (including all polarizations of the initial W^-Z^0) is insensitive to the Higgs boson mass m_H for the range 0.5 TeV $\le m_H \le$ 2 TeV. It is evident from these figures that, in the case of $m_H = 0.5$ TeV, the longitudinal polarization is about 15% of the total contribution. When m_H is 1 TeV, the transverse polarization is still dominant. For $m_H = 2$ TeV, the longitudinal contribution begins to compete with the transverse polarization, and, at about $m_{WZ} = 1000$ GeV, it exceeds the transverse contribution.

III. *K*-MATRIX UNITARIZATION

The results of the previous section illustrate how the scattering of longitudinally polarized W^-Z^0 pairs increases as the Higgs boson mass m_H increases [13,14]. This increase eventually violates unitarity and any attempt to estimate the yield of strongly interacting W^-Z^0 pairs must rely on some form of unitarization. Moreover, the transverse degrees of freedom dominate the cross section for $m_H < 1$ TeV, and it is reasonable to ask what effect they have on the unitarization of the longitudinal amplitude. To address this question, we consider the K-matrix unitarization scheme, including all allowed terms in the 9×9 matrix of helicity amplitudes for a given partial wave. We find that it is possible to obtain the unitarized longitudinal partial wave amplitudes *including the corrections due to channels with nonzero helicity* essentially exactly. As we shall see, unitarity corrections to the other helicity amplitudes can be obtained in a similar manner, provided the one-loop correction to the particular amplitude is known. However, channels with transversely polarized W's and Z's are insensitive to substantial changes in m_H, and therefore unitarity corrections to such channels are negligible in the cross sections that we present.

In the following, a helicity amplitude is expressed as $a_{\lambda_1\lambda_2\lambda_3\lambda_4}$, where λ_1 and λ_2 denote the helicities of the initial W^- and Z^0, while λ_3 and λ_4 are the helicities of the final W^- and Z^0. All the λ_i take the values $0,\pm 1$. The matrix A of perturbative helicity amplitudes is labeled as

$$A=\begin{pmatrix} a_{----} & a_{---0} & a_{---+} & \cdots & a_{--++} \\ a_{-0--} & a_{-0-0} & a_{-0-+} & \cdots & a_{-0++} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{00--} & a_{00-0} & a_{00-+} & \cdots & a_{00++} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{++--} & a_{++-0} & a_{++-+} & \cdots & a_{++++} \end{pmatrix}. \qquad (1)$$

We examine the implications of unitarity on both the weak isospin $I=2$, $J=0$ amplitudes $a^0_{\lambda_1\lambda_2\lambda_3\lambda_4}$, and the $I=1$, $J=1$ amplitudes $a^1_{\lambda_1\lambda_2\lambda_3\lambda_4}$.

In the K-matrix [15–18] unitarization scheme, the unitarized matrix A^K_J of amplitudes for a given angular momentum J is expressed in terms of the corresponding matrix of perturbative amplitudes A_J, as

$$A^K_J=\frac{A_J}{1-iA_J}, \qquad (2)$$

where $A_J=A_J^\dagger$. This equation can be written

$$\langle m|A^K|n\rangle=\sum_j\langle m|A|j\rangle\langle j|(1-iA)^{-1}|n\rangle, \qquad (3)$$

where the subscript J has been omitted. Using the definitions

TABLE II. Integrated cross section in femtobarns for W^-Z (W^+Z) production in pp collisions at $\sqrt{s}=16$ TeV with $m_{WZ}\le 3$ TeV and a rapidity cut of 1.5 on both the W and Z.

Process	m_H	$m_{WZ}>0.4$	$m_{WZ}>0.8$	$m_{WZ}>1.2$	$m_{WZ}>1.6$	$m_{WZ}>2.0$
all $\to$ TT	1	15 (20)	2.1 (3.4)	0.46 (0.83)	0.13 (0.27)	0.043 (0.099)
all $\to$ LL	0.5	1.8 (2.9)	0.20 (0.38)	0.032 (0.068)	0.007 (0.015)	0.002 (0.004)
all $\to$ LL	1	3.5 (5.5)	0.71 (1.3)	0.17 (0.36)	0.045 (0.10)	0.013 (0.031)
all $\to$ LL	2	4.9 (7.9)	1.4 (2.8)	0.49 (1.1)	0.18 (0.41)	0.063 (0.15)
$q\bar{q}\to$ all		305 (392)	18 (32)	3.3 (7.6)	0.85 (2.4)	0.25 (0.85)
LL $\to$ LL (K-M)	2	4.5 (7.6)	1.3 (2.7)	0.51 (1.2)	0.21 (0.54)	0.085 (0.23)
LL $\to$ LL (Padé)	2	4.7 (8.0)	1.5 (3.1)	0.62 (1.5)	0.30 (0.77)	0.17 (0.40)

$$\langle m|A^K|n\rangle = a^K_{mn} , \tag{4}$$

$$\langle m|A|j\rangle = a_{mj} , \tag{5}$$

$$\langle j|(1-iA)^{-1}|n\rangle = \frac{\mathrm{cof}_{nj}}{\det(1-iA)} , \tag{6}$$

where cof_{nj} denotes the cofactor of $(1-iA)_{nj}$, we have

$$a^K_{mn} = \frac{\sum_j a_{mj}\mathrm{cof}_{nj}}{\det(1-iA)} . \tag{7}$$

Our objective is to obtain an expression for a^K_{mn} which agrees with the perturbative result to order α_W^2 and includes unitarity corrections to this order. To accomplish this, recall that, to order α_W^2, the matrix elements of the matrix A_J of Eq. (2) are of the form

$$\begin{aligned}(A_J)_{mn} &= a_{mn} \\ &= a^{(1)}_{mn} + \mathrm{Re}(a^{(2)}_{mn}) ,\end{aligned} \tag{8}$$

with $a^{(1)}_{mn}$ denoting the Born term and $a^{(2)}_{mn}$ the one-loop correction. To order α_W, the cofactor and determinant can be expanded as

$$\mathrm{cof}_{nj} = \delta_{nj}(1 - i\sum_k a^{(1)}_{kk}) + ia^{(1)}_{nj} , \tag{9}$$

$$\det(1-iA) = (1 - i\sum_k a^{(1)}_{kk}) , \tag{10}$$

which results in the expression

$$a^K_{mn} = \frac{a_{mn} - ia^{(1)}_{mn}\sum_j a^{(1)}_{jj} + i\sum_j a^{(1)}_{mj}a^{(1)}_{jn}}{1 - i\sum_j a^{(1)}_{jj}} . \tag{11}$$

To order α_W^2, a^K_{mn} reads

$$a^K_{mn} = a^{(1)}_{mn} + \mathrm{Re}(a^{(2)}_{mn}) + i\sum_j a^{(1)}_{mj}a^{(1)}_{jn} , \tag{12}$$

which is equal to the sum of Born amplitude plus the real and imaginary part of its one-loop correction. Futhermore, the matrix A^K is unitary apart from corrections of order α_W^3. It is also clear that the one-loop correction to the matrix element a_{mn} is sufficient to determine the unitarized matrix element a^K_{mn} to order α_W^2. The application of Eq. (11) to the $J = 0$ and $J = 1$ partial waves of WZ scattering is discussed below.

A. J=0

For $J = 0$, the initial W^- and Z^0 have the same helicity as do the final W^- and Z^0. In this case, the matrix A_J has the form

$$A_0 = \begin{pmatrix} a^0_{++++} & a^0_{++00} & a^0_{++--} \\ a^0_{++00} & a^0_{0000} & a^0_{++00} \\ a^0_{++--} & a^0_{++00} & a^0_{++++} \end{pmatrix} , \tag{13}$$

where we have used the relations

$$\begin{aligned} a_{\lambda_1\lambda_2\lambda_3\lambda_4} &= a_{-\lambda_1-\lambda_2-\lambda_3-\lambda_4} , \\ a_{\lambda_1\lambda_2\lambda_3\lambda_4} &= a_{\lambda_3\lambda_4\lambda_1\lambda_2} . \end{aligned} \tag{14}$$

The corresponding expression for the unitarized longitudinal amplitude takes the form

$$(a^K_0)_{0000} = \frac{a^{0\,(1)}_{0000} + \mathrm{Re}(a^{0\,(2)}_{0000}) - 2i(a^{0\,(1)}_{0000}\,a^{0\,(1)}_{++++} - (a^{0\,(1)}_{++00})^2)}{1 - i(a^{0\,(1)}_{0000} + 2a^{0\,(1)}_{++++})} . \tag{15}$$

From the form of Eq. (15), it is easy to see that, to order α_W^2, the unitarity corrections to W_LZ_L scattering from transversely polarized W's and Z's are determined by the Born amplitudes for these polarizations. The one-loop corrections to the longitudinal amplitude are available in the work of Dawson and Willenbrock [19] and Veltman and Yndurian [20].

B. J=1

In this case, the initial and final W^-'s and Z^0's can have unequal helicities, except that the combinations $W^-_\pm Z^0_\mp$ are forbidden for either the initial or final state. This leads to a 7×7 matrix A_1 of the form

$$A_1 = \begin{pmatrix} a^1_{++++} & a^1_{+++0} & a^1_{++0+} & a^1_{++00} & a^1_{++0-} & a^1_{++-0} & a^1_{++--} \\ a^1_{+++0} & a^1_{+0+0} & a^1_{+00+} & a^1_{+000} & a^1_{+00-} & a^1_{+0-0} & a^1_{++-0} \\ a^1_{++0+} & a^1_{+00+} & a^1_{0+0+} & a^1_{0+00} & a^1_{0+0-} & a^1_{+00-} & a^1_{++0-} \\ a^1_{++00} & a^1_{+000} & a^1_{0+00} & a^1_{0000} & a^1_{0+00} & a^1_{+000} & a^1_{++00} \\ a^1_{++0-} & a^1_{+00-} & a^1_{0+0-} & a^1_{0+00} & a^1_{0+0+} & a^1_{+00+} & a^1_{++0+} \\ a^1_{++-0} & a^1_{+0-0} & a^1_{+00-} & a^1_{+000} & a^1_{+00+} & a^1_{+0+0} & a^1_{+++0} \\ a^1_{++--} & a^1_{++-0} & a^1_{++0-} & a^1_{++00} & a^1_{++0+} & a^1_{+++0} & a^1_{++++} \end{pmatrix} , \tag{16}$$

where we have used the relations Eqs. (14) . The form of $(a^K_1)_{0000}$ is analogous to $(a^K_0)_{0000}$ of Eq. (15), and, again, we can use the known one-loop corrections to the longitudinal amplitude [19,20] to complete the calculation.

IV. DISCUSSION

A comparison of the Born and unitarized invariant-mass distributions is presented in Figs. 4 and 5. The curves labeled K matrix are obtained from the partial wave decomposition of the scattering amplitude retaining the unitarized $J = 0$ and $J = 1$ contributions computed using Eq. (11) of Sec. III. To verify that this equation is a satisfactory approximation to the exact result, Eq. (7), we used the latter equation to compute $(a^K)_{0000}$ for several values of m_{WZ}. The difference between the two results was negligible.

For $m_H = 2$ TeV, the effect of unitarization is to

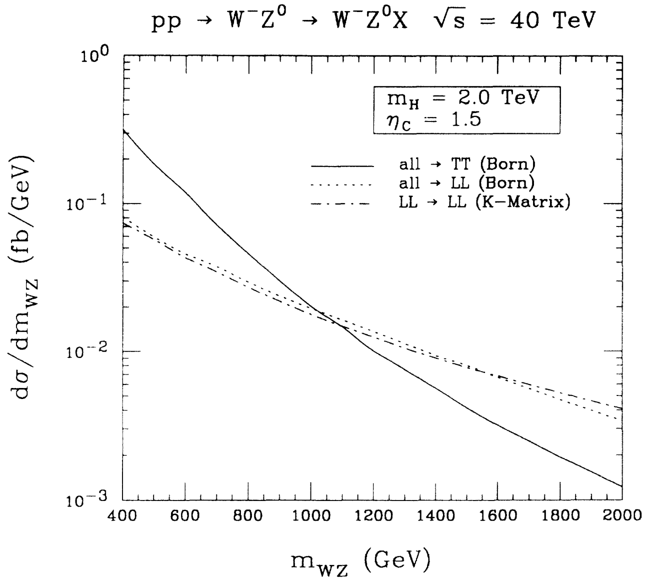

FIG. 4. The invariant-mass distribution resulting from the K-matrix unitarization of the combined $J = 0$ and $J = 1$ partial waves of the $W_L^- Z_L^0$ elastic amplitude (Born plus one-loop) is shown in the dash-dotted curve for $\sqrt{s} = 40$ TeV. For comparison, the solid (dotted) curve is the cross section for $W^- Z^0$ production with all polarizations of initial $W^- Z^0$ and transverse (longitudinal) final $W^- Z^0$.

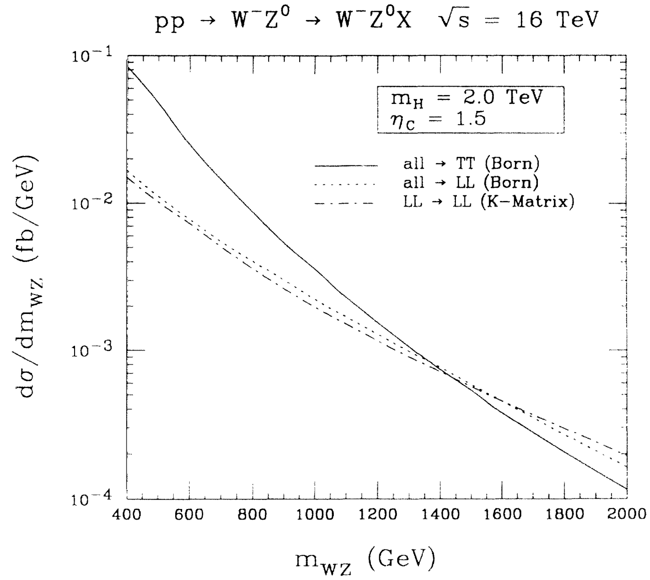

FIG. 5. Same as Fig. 4 except for $\sqrt{s} = 16$ TeV.

slightly reduce the longitudinal cross section at low values of m_{WZ}, while slightly increasing the cross section at high values of m_{WZ}. Increasing the value of m_H to simulate a unitarized, strongly interacting $W_L Z_L$ scattering amplitude tends to further decrease the longitudinal cross section. This can be traced to the fact the $J = 0$ amplitudes $a^{0\,(1)}_{0000}$ and $\mathrm{Re}(a^{0\,(2)}_{0000})$ have opposite signs [21].

To assess the possibility of observing WZ scattering in pp collisions, Tables I and II contain the integrated cross sections for various values of m_{WZ}. We use our K-matrix unitarized amplitude to represent nonresonant WZ scattering and use the Padé unitarized longitudinal amplitude to determine the effect of a 2.5 TeV resonance in the $J = 1$ partial wave [4]. For $\sqrt{s} = 40$ TeV, a simple cut on the invariant-mass m_{WZ} can yield signal (S) to background (B) ratios greater than 1 for $m_{WZ} > 1.6$ TeV. To achieve the observability criterion $S/\sqrt{B} \geq 5$, it is necessary to impose additional cuts on the transverse momentum of the Z and on the leptons resulting from the decay of the W and Z [5,6]. In the case $\sqrt{s} = 16$ TeV, an invariant-mass cut alone is not sufficient to obtain $S/B > 1$. While it is possible to improve this situation using additional cuts [5,6], the task measuring gauge boson scattering and probing the electroweak symmetry breaking mechanism at a 16 TeV collider is very difficult, even given a tenfold increase in luminosity.

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