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Perturbative QCD fragmentation functions for production of P-wave charm and bottom mesons

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We calculate the leading order QCD fragmentation functions for the production of P-wave charmed b mesons. Long-distance effects are factored into two nonperturbative parameters: the derivative of the radial wave function at the origin and a second parameter related to the probability for a $(\bar{b}c)$ heavy quark pair that is produced in a color-octet S-wave state to form a color-singlet P-wave bound state. The four $2P$ states and those $3P$ states which lie below the BD flavor threshold eventually all decay into the $1S$ ground state B_c through hadronic cascades or by emitting photons. The total fragmentation probabilities for production of the $1S$ ground state B_c from the cascades of the $2P$ and $3P$ states are about 1.7×10^{-4} and 2.3×10^{-4}, respectively. Thus the direct production of the P-wave states via fragmentation may account for a significant fraction of the inclusive production rate of the B_c at large transverse momentum in high energy colliders. Our analytic results for the P-wave fragmentation functions disagree with those obtained earlier in the literature. Our results also cover the P-wave heavy quarkonium case in the equal mass limit.

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I. INTRODUCTION

Much progress has been made in the past several years to improve our theoretical understanding of the physics of a heavy hadron containing a single heavy quark (charm or bottom quark). This is mainly due to the discovery of the powerful heavy quark spin-flavor symmetry [1] in quantum chromodynamics (QCD) and the development of the heavy quark effective theory [2]. Heavy hadrons containing both the heavy bottom and charm quarks are also of interest. Of particular importance is the $(\bar{b}c)$ meson family. Since the top quark is very heavy and will decay rapidly into W^+b before hadronization, the $(\bar{b}c)$ system might be the only physical system containing two heavy flavors of different masses. The techniques of heavy quark effective theory cannot be applied here because the charm quark is not sufficiently light. However, since this family is intermediate between the $c\bar{c}$ charmonium and the $b\bar{b}$ bottomonium systems, many properties (mass spectrum, transition and decay rates, lifetimes, etc.) of this physical system can be studied by using potential models and QCD sum rules [3, 4].

Recently, it has been pointed out [5] that the dominant production mechanism for these yet unobserved $(\bar{b}c)$ bound states with large transverse momentum at a high energy process is fragmentation, in which an energetic $\bar{b}$ antiquark is first produced at large transverse momentum by a short-distance hard process and subsequently fragments into various $(\bar{b}c)$ bound states. Furthermore, the process-independent fragmentation functions describing these phenomena are shown to be calculable at the heavy quark mass scale using perturbative QCD. In particular, the fragmentation functions for the $\bar{b}$ antiquark to split into the two S-wave states $B_c(n^1S_0)$ and $B_c^*(n^3S_1)$ were calculated [5, 6] to leading order in both α_s and v, where v is the typical relative velocity of the charm quark inside the meson. Including the $1S$ and $2S$ states that lie below the BD flavor threshold ($M_{\rm threshold} = M_D + M_B \approx 7.1$ GeV), a lower bound on the inclusive branching fraction for the production of the B_c ground state from Z^0 decay has been estimated to be about 2.3×10^{-4} [5], which might be too small for this particle to be observed at the CERN e^+e^- collider LEP with the present luminosity. Production of the B_c meson at the hadron colliders via the direct $\bar{b}$ antiquark and induced gluon fragmentation has also been studied in Ref. [7]. About 20 000 B_c with transverse momentum $p_T > 10$ GeV are expected to be produced at the Fermilab Tevatron with an integrated luminosity of 25 pb^{-1}. A clean signature of B_c would be the observation of three charged leptons coming from the same secondary vertex due to the decay $B_c^+ \to J/\psi + \bar{l}'\nu_{l'}$ followed by $J/\psi \to l\bar{l}$. The combined branching fraction for these decays is expected to be about 0.2% [8]. One expects about 40 of these distinctive events at the Tevatron. Thus, unless LEP can increase its luminosity by an order of magnitude or so in the near future, the best place to look for the charmed b meson will be at the Tevatron.

The mass spectrum of the $(\bar{b}c)$ mesons has been obtained by Eichten and Quigg [3] using potential model calculations. Since, unlike the quarkonium system, the $\bar{b}$ and c quarks do not annihilate into gluons, the excited states of the $(\bar{b}c)$ mesons cannot decay directly into light hadrons. States that lie above the BD threshold will strongly decay into a pair of B and D mesons. States that lie below the BD threshold will decay eventually to the ground state B_c either by hadronic cascades or by emitting photons. In order to get a more reliable theoretical prediction for the inclusive production rate of the B_c meson at large transverse momentum, one must include all higher orbitally excited states of the $(\bar{b}c)$ mesons that lie below the BD threshold. Below the BD flavor threshold, there are two sets of S-wave states (the $1S$ and $2S$ states), as many as two complete sets of P-wave states (the $2P$ and perhaps some or all $3P$ states), and one set of D-wave states (the $3D$ states).

In this paper, we report the results of the calculation of the fragmentation functions for $\bar{b}$ antiquark splittings into the four P-wave $(\bar{b}c)$ mesons—the ${}^1P_1$ and ${}^3P_J$ $(J=0,1,2)$ states. In Sec. II, we write down new factorization formulas for the P-wave fragmentation functions that are valid to all orders in strong coupling constant α_s and to leading order in v, where v is the typical relative velocity of the heavy quarks inside the $(\bar{b}c)$ meson. Long-distance effects are factored into two nonperturbative parameters: the derivative of the radial wave function at the origin, and a second parameter related to the probability for the $(\bar{b}c)$ pair that is produced in a color-octet S-wave state to form a color-singlet P-wave bound state. In Sec. III, we calculate the perturbative coefficients of the fragmentation functions to leading order in α_s. Infrared logarithmically divergent terms that appeared at the next-to-leading order in α_s are isolated and absorbed into the second nonperturbative parameter. Mixing effects between the ${}^1P_1$ and ${}^3P_1$ states in the fragmentation functions are also calculated. In Sec. IV, we take the heavy quark limit $m_b/m_c \to \infty$, and show that our fragmentation functions exhibit the heavy quark spin symmetry while earlier results obtained by Chen [9] do not satisfy this important property. In Sec. V, we discuss a crossing relation between the fragmentation function $D_{i\to H}(z)$ for parton i splittings into hadron H and the distribution function $f_{i/H}(x)$ of finding parton i inside hadron H. In Sec. VI, we consider the equal mass limit corresponding to the case of P-wave heavy quarkonium. Numerical results and conclusions are made in Sec. VII.

II. NEW FACTORIZATION FORMULAS FOR P-WAVE FRAGMENTATION FUNCTIONS

Recently Bodwin, Braaten, and Lepage [10] have reformulated the perturbative calculations of the inclusive annihilation rates of heavy quarkonium using the framework of nonrelativistic QCD (NRQCD) [10, 11]. This work points out that the usual factorization assumption that all long-distance effects can be absorbed into the radial wave functions at the origin and their derivatives is not correct when higher order QCD corrections and/or relativistic corrections are taken into account. Infrared divergences that arise in these higher order calculations spoil factorization. To obtain sensible perturbative results that are free from infrared divergences, one must use new factorization formulas that involve a double expansion in the strong coupling constant α_s and in the typical velocity v of the heavy quark inside the quarkonium [10]. The easiest way to organize the expansion in v in these theoretical calculations is to use NRQCD [10, 11].

As in the quarkonium case [12], the heavy quarks inside the $(\bar{b}c)$ meson are moving nonrelativistically and separated at a typical distance of order $1/(mv)$, where $v \ll 1$ is the typical relative velocity of the heavy quarks and $m = m_b m_c/(m_b+m_c)$ is the reduced mass. The calculation of the perturbative fragmentation function is based on separating short-distance perturbative effects involving the scale of order $1/m$ from long-distance nonperturbative effects involving scales of order $1/(mv)$ or larger. To leading order in v, two distinct mechanisms that contribute to the P-wave fragmentation functions have been identified and are referred to as the *color-singlet mechanism* and *color-octet mechanism*. The *color-singlet mechanism* is the production of a $(\bar{b}c)$ pair in a color-singlet ${}^1P_1$ or ${}^3P_J$ state with separation of order $1/m$ in the $(\bar{b}c)$ meson rest frame. The subsequent formation of the P-wave $(\bar{b}c)$ bound state is a long-distance nonperturbative process with a probability of order $v^2(mv/m)^3 \sim v^5$, where the first factor v^2 arises from the derivative of the wave function of the P-wave state and the second factor $(mv/m)^3$ is a volume factor [10, 13]. The *color-octet mechanism* is the production of the $(\bar{b}c)$ pair in a color-octet ${}^1S_0$ or ${}^3S_1$ state with a separation of order $1/m$. The subsequent formation of the P-wave $(\bar{b}c)$ bound state can proceed either through the dominant $|\bar{b}c\rangle$ component or through the small $|\bar{b}cg\rangle$ component of the wave function. In the first case, the $(\bar{b}c)$ pair must radiate a soft gluon to make a transition to the color-singlet P-wave bound state. In the second case, a soft gluon must combine with the $(\bar{b}c)$ pair to form the color-singlet P-wave bound state. In both cases, the probability is of order v^5, with a factor of v^3 coming from the volume factor and an additional factor of v^2 coming either from the probability of radiating a soft gluon or from the small probability of the $|\bar{b}cg\rangle$ component of the wave function [10, 13]. Since the *color-singlet mechanism* and *color-octet mechanism* contribute to the P-wave fragmentation functions at the same order in v, they must both be included for a consistent calculation.

Separation of the short-distance physics from the long-distance effects discussed in the previous paragraph requires the introduction of an arbitrary factorization scale Λ in the range $mv \ll \Lambda \ll m$. The fragmentation functions for the P-wave $(\bar{b}c)$ mesons satisfy factorization formulas that involve this arbitrary scale. To leading order in v^2 and to all order in α_s, the factorization formulas for the fragmentation functions of $\bar{b} \to \bar{b}c(n^1P_1)$ and $\bar{b} \to \bar{b}c(n^3P_J)$ at a scale μ_0 near the heavy quark mass scale consist of two terms:

$$D_{\bar{b}\to\bar{b}c(n^1P_1)}(z,\mu_0) = \frac{H_{1(\bar{b}c)}(n)}{m} D^{(1)}_{\bar{b}\to\bar{b}c({}^1P_1)}(z,\Lambda) + 3\frac{H'_{8(\bar{b}c)}(\Lambda)}{m} D^{(8)}_{\bar{b}\to\bar{b}c({}^1S_0)}(z) , \quad (1)$$

$$D_{\bar{b}\to\bar{b}c(n^3P_J)}(z,\mu_0) = \frac{H_{1(\bar{b}c)}(n)}{m} D^{(1)}_{\bar{b}\to\bar{b}c({}^3P_J)}(z,\Lambda) + (2J+1)\frac{H'_{8(\bar{b}c)}(\Lambda)}{m} D^{(8)}_{\bar{b}\to\bar{b}c({}^3S_1)}(z) , \quad (2)$$

where H_1 and $H'_8(\Lambda)$ are nonperturbative long-distance factors associated with the *color-singlet* and *color-octet mechanism*, respectively, m is the reduced mass, and $n = 2, 3, \ldots$, labels the principal quantum number of the P-wave states. The short-distance factors $D^{(1)}(z,\Lambda)$ and $D^{(8)}(z)$ can be calculated perturbatively as power series in α_s. They are proportional to the fragmentation functions for a $\bar{b}$ antiquark to split into a $(\bar{b}c)$ pair with vanishing relative momentum and definite color-spin-orbital quantum numbers: color-singlet ${}^1P_1$ or ${}^3P_J$ state for $D^{(1)}$ and color-octet ${}^1S_0$ or ${}^3S_1$ state for $D^{(8)}$. Note that in these factorization formulas, the only dependence on Λ is in $D^{(1)}$ and H_8'. As in the quarkonium case, the nonperturbative parameters H_1 and H_8' can be rigorously defined as matrix elements of four-quark operators in nonrelativistic QCD divided by appropriate powers of the heavy quark masses [10]. Their dependence on Λ is governed by a renormalization group equation whose coefficient can be calculated using nonrelativistic QCD [10]. We adopt the same definitions for these matrix elements as in the charmonium case discussed in Ref. [10], except that explicit factors of m_c are replaced by $2m$. To order α_s, $H_{1(\bar{b}c)}(n)$ is scale invariant and is related to the derivative of the nonrelativistic radial wave function at the origin for the P-wave $(\bar{b}c)$ bound states:

$$H_{1(\bar{b}c)}(n) \approx \frac{9}{2\pi}\frac{|R'_{nP}(0)|^2}{(2m)^4}\left[1+\mathcal{O}(v^2)\right] , \tag{3}$$

while $H'_{8(\bar{b}c)}$ satisfies [10, 14]

$$\Lambda\frac{d}{d\Lambda}H'_{8(\bar{b}c)}(\Lambda) = \frac{16}{27\pi}\alpha_s(\Lambda)H_{1(\bar{b}c)} , \tag{4}$$

with the solution

$$H'_{8(\bar{b}c)}(\Lambda) = H'_{8(\bar{b}c)}(\Lambda_0) + \frac{16}{27\beta_0}\ln\left(\frac{\alpha_s(\Lambda_0)}{\alpha_s(\Lambda)}\right) H_{1(\bar{b}c)} , \tag{5}$$

where $\beta_0 = 9/2$ is the first coefficient in the β function for QCD with three flavors of light quarks. If the factorization scale Λ is chosen to be much less than the reduced mass m, the above equation can be used to sum up large logarithms of m/Λ. To avoid large logarithms in the perturbative coefficients $D^{(1)}$ in Eqs. (1) and (2), one can choose Λ on the order of m. Since $|R'_{nP}(0)|^2$ is of order v^5 [10], from Eqs. (3) and (5), we see that the *color-singlet* and *color-octet* contributions in the P-wave fragmentation functions are both of order v^5. Following Ref. [5], the initial scale μ_0 for the P-wave fragmentation functions in Eqs. (1) and (2) can be chosen to be $(m_b + 2m_c)$—the minimal virtuality of the fragmenting $\bar{b}$ antiquark. Fragmentation functions at a higher scale can be obtained by solving the Altarelli-Parisi evolution equation with Eqs. (1) and (2) as the boundary conditions. These boundary conditions will be determined in the next section.

III. FRAGMENTATION FUNCTIONS FOR $\bar{b} \to P$-WAVE $(\bar{b}c)$ MESONS

We now embark on the calculation of the coefficient $D^{(1)}(z,\Lambda)$ in the color-singlet contribution to the fragmentation function. Let us first briefly review the method that was introduced in Refs. [5, 15] of calculating the heavy quark fragmentation functions using perturbative QCD. We refer to Ref. [5] for more details. Let $\mathcal{M}$ denote the amplitude for a high energy source (symbolically denoted by $\bar{\Gamma}$) to create a $\bar{b}^*(q) \to H(p) + \bar{c}(p')$ with total four-momentum $q = p + p'$ as illustrated in Fig. 1. Here H denotes a $(\bar{b}c)$ bound state. The leading order diagram of Fig. 1 involves creating a $(c\bar{c})$ pair from the vacuum. Let $\mathcal{M}_0$ denote the amplitude for the same source to create an on-shell $\bar{b}$ antiquark with the same three-momentum $\mathbf{q}$. Then the fragmentation function for $\bar{b} \to H$ is given by [5]

$$D_{\bar{b}\to H}(z) = \frac{1}{16\pi^2}\int ds\,\theta\left(s - \frac{(m_b+m_c)^2}{z} - \frac{m_c^2}{1-z}\right) \times \lim_{q_0/m_{b,c}\to\infty}\frac{|\mathcal{M}|^2}{|\mathcal{M}_0|^2} , \tag{6}$$

where $s = q^2$ is the virtuality of the fragmenting $\bar{b}$ antiquark. In a frame where the virtual $\bar{b}$ antiquark has four-momentum $q = (q_0, 0, 0, q_3)$, the longitudinal momentum fraction of H is $z = (p_0 + p_3)/(q_0 + q_3)$ and its transverse momentum is $\mathbf{p}_\perp = (p_1, p_2)$. The relation between the transverse momentum and the virtuality is given by

$$\mathbf{p}_\perp^2 = z(1-z)\left(s - \frac{M^2}{z} - \frac{r^2M^2}{1-z}\right) , \tag{7}$$

where we have defined $M = m_b + m_c$, $r = m_c/M$, and $\bar{r} = 1 - r = m_b/M$. The tree-level matrix element squared $|\mathcal{M}_0|^2$ is simply given by $3\mathrm{Tr}(\Gamma\bar{\Gamma}\not{q})$ as $q_0/m_b \to \infty$. The matrix element $\mathcal{M}$ for producing a bound state $H(p)$ consisting of collinear $\bar{b}$ and c quarks with relative momentum k (Fig. 1) can be expressed as a Dirac trace involving the spinor factor $v(\bar{r}p+k)\bar{u}(rp-k)$. To project out the $(\bar{b}c)$ bound state in the nonrelativistic approximation, this spinor factor is replaced by the projection operator

$$v(\bar{r}p+k)\bar{u}(rp-k) \to \sqrt{m_b+m_c}\left(\frac{\bar{r}\not{p}+\not{k}-m_b}{2m_b}\right) \times O\left(\frac{r\not{p}-\not{k}+m_c}{2m_c}\right) , \tag{8}$$

with $O = \gamma_5$ or $-\not{\epsilon}^*(p,S_z)$ for the spin singlet or spin triplet state, respectively, where $\epsilon(p,S_z)$ is the polarization vector associated with the spin triplet state. In addition to this spinor factor, the gluon propagator in Fig. 1 also depends on the relative momentum k. In the axial gauge associated with the four-vector n^μ, the gluon

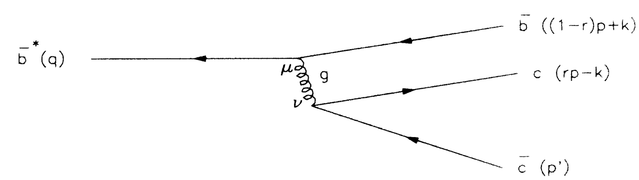

FIG. 1. Feynman diagram for $\bar{b}^* \to \bar{b}c\bar{c}$ which contributes to the fragmentation of $\bar{b}$ into P-wave $(\bar{b}c)$ bound states. The outgoing momenta are $(1-r)p+k$, $rp-k$, and p' for the $\bar{b}$, c, and $\bar{c}$, respectively. k is the relative momentum of the $\bar{b}$ and c.

propagator is given by

$$D_F^{\mu\nu}(l) = \frac{1}{l^2}\left[- g^{\mu\nu} + \frac{n^\mu l^\nu + n^\nu l^\mu}{n\cdot l} + \frac{n^2 l^\mu l^\nu}{(n\cdot l)^2}\right] , \qquad (9)$$

where $l = q-\bar{r}p-k$. The axial gauge vector can be chosen to be $n^\mu = (1, -\mathbf{p}/|\mathbf{p}|)$ so that Fig. 1 is the dominant diagram and hence factorization is manifest. By using the equations of motion for the $\bar{b}$ and c quarks, one can show that terms that are proportional to l^ν in the gluon propagator do not contribute to the matrix element $\mathcal{M}$. For the P-wave states, one needs to expand the above projection operator and the gluon propagator appearing in the matrix element $\mathcal{M}$ to first order in the relative momentum k. After some manipulations by using the standard covariant methods [16], the matrix element for the n^1P_1 state can be written in the form

$$\mathcal{M}(n^1P_1) = \delta_{ij}\frac{1}{3r^2\bar{r}}\frac{g^2 R'_{nP}(0)}{\sqrt{4\pi M}}\frac{1}{(s-\bar{r}^2M^2)^3} \times \epsilon^*_\alpha(p,L_z)\bar{\Gamma}V^\alpha\gamma_5 v(p') , \qquad (10)$$

where i and j are the fundamental color indices, g is the strong coupling constant, and $\epsilon_\alpha(p,L_z)$ is the polarization vector of the $^1P_1$ state. The vertex factor V^α is given by

$$V^\alpha = 8\bar{r}q^\alpha(\not{q}+\bar{r}M)(\not{p}-2M) - (s-\bar{r}^2M^2)\left[2(1-2r)(\not{q}+\bar{r}M)\gamma^\alpha - \frac{4\bar{r}}{n\cdot(q-\bar{r}p)}q^\alpha(\not{p}+M)\not{n}\right]$$
$$-\frac{(s-\bar{r}^2M^2)^2}{n\cdot(q-\bar{r}p)}\left[\frac{1}{M}(\not{p}+(1-2r)M)\gamma^\alpha - 2r\bar{r}\frac{1}{n\cdot(q-\bar{r}p)}n^\alpha(\not{p}+M)\right]\not{n} . \qquad (11)$$

Similarly, the matrix elements $\mathcal{M}(n^3P_J)$ for the three spin triplet states can be written in the compact form

$$\mathcal{M}(n^3P_J) = \delta_{ij}\frac{1}{3r^2\bar{r}}\frac{g^2R'_{nP}(0)}{\sqrt{4\pi M}}\frac{1}{(s-\bar{r}^2M^2)^3}\Psi_{\alpha\beta}(J,J_z)\bar{\Gamma}V^{\alpha\beta}v(p') , \qquad (12)$$

with

$$\Psi_{\alpha\beta}(J,J_z) = \sum_{L_z,S_z}\langle 1,L_z;1,S_z|J,J_z\rangle\epsilon^*_\alpha(p,L_z)\epsilon^*_\beta(p,S_z) \qquad (13)$$

$$= \begin{cases} \frac{1}{\sqrt{3}}(g_{\alpha\beta} - \frac{p_\alpha p_\beta}{M^2}) & \text{for } J=0, \\ \frac{i}{\sqrt{2}}\frac{1}{M}\epsilon_{\alpha\beta\xi\eta}p^\xi\epsilon^{\eta *}(p,J_z) & \text{for } J=1, \\ \epsilon_{\alpha\beta}(p,J_z) & \text{for } J=2, \end{cases} \qquad (14)$$

where $\epsilon_\alpha(p,L_z)$ and $\epsilon_\beta(p,S_z)$ are the polarization vectors in the orbital and spin space of the ^3P_J states, respectively; and $\epsilon_\alpha(p,J_z)$ and $\epsilon_{\alpha\beta}(p,J_z)$ are the helicity wave functions of the total spin $J=1$ and $J=2$ states, respectively. Our conventions are $g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ and $\epsilon^{0123} = +1$. The vertex tensor $V^{\alpha\beta}$ in Eq. (12) is given by

$$V^{\alpha\beta} = 8\bar{r}Mq^\alpha(\not{q}+\bar{r}M)\gamma^\beta$$
$$+(s-\bar{r}^2M^2)\left[\frac{1}{M}(\not{q}+\bar{r}M)\left(2g^{\alpha\beta}[(1-2r)\not{p}-2M] + [\gamma^\alpha,\gamma^\beta]\not{p}\right) + \frac{4\bar{r}}{n\cdot(q-\bar{r}p)}q^\alpha(\not{p}+M)\gamma^\beta\not{n}\right]$$
$$+\frac{(s-\bar{r}^2M^2)^2}{n\cdot(q-\bar{r}p)}\left[\frac{1}{M}\left(g^{\alpha\beta}[(1-2r)\not{p}+M] - \frac{1}{2}[\not{p}+(1-2r)M][\gamma^\alpha,\gamma^\beta]\right) + 2r\bar{r}\frac{1}{n\cdot(q-\bar{r}p)}n^\alpha(\not{p}+M)\gamma^\beta\right]\not{n} . \qquad (15)$$

The general procedure for extracting the P-wave fragmentation functions from the above matrix elements is the same as in the case of the S-wave [5] except the intermediate steps are much more involved. We will skip over these tedium and present only the final results. From Eq. (6), we can define generalized fragmentation functions $D^{(1)}(z,s)$ that depend on both the longitudinal momentum fraction z and the virtuality s according to

$$D^{(1)}(z) = \int ds\,\theta\left(s - \frac{M^2}{z} - \frac{r^2M^2}{1-z}\right)D^{(1)}(z,s) . \qquad (16)$$

For the $^1P_1$ state, we have

$$D^{(1)}_{\bar{b}\to\bar{b}c(^1P_1)}(z,s) = \frac{32\alpha_s^2(2m_c)}{81}\frac{r\bar{r}^3}{(1-\bar{r}z)^4}\sum_{n=0}^{3}\frac{f_nM^{8-2n}}{(s-\bar{r}^2M^2)^{5-n}} , \qquad (17)$$

with

$$f_0 = 64r^2\bar{r}^3(1-\bar{r}z)^4 , \tag{18}$$
$$f_1 = 8r\bar{r}(1-\bar{r}z)^3[3-2r-2r^2-2\bar{r}(2+4r-r^2)z+\bar{r}^2(1-2r)z^2] , \tag{19}$$
$$f_2 = -(1-\bar{r}z)^2[2(1-2r+4r^2)-(3-42r+64r^2-16r^3)z-2r\bar{r}(23-14r-4r^2)z^2+\bar{r}^2(1+12r)(1-2r)z^3] , \tag{20}$$
$$f_3 = (1-z)[1-2(1-2r)z+(3-2r+2r^2)z^2-2\bar{r}(2+r-2r^2)z^3+\bar{r}^3(2+r^2)z^4] . \tag{21}$$

Integrating Eq. (17) over s as in Eq. (16), we obtain the fragmentation function $D^{(1)}(z)$ for the color-singlet contribution in the $^1P_1$ state:

$$D^{(1)}_{\bar{b}\to\bar{b}c(^1P_1)}(z) = \frac{16\alpha_s^2(2m_c)}{243}\frac{r\bar{r}^3z(1-z)^2}{(1-\bar{r}z)^8}\{6-6(4r^2-8r+5)z$$
$$+(32r^4-96r^3+250r^2-210r+69)z^2+8\bar{r}(4r^4+12r^3-48r^2+37r-12)z^3$$
$$+2\bar{r}^2(16r^4+161r^2-114r+42)z^4-6\bar{r}^3(4r^3+28r^2-15r+7)z^5+\bar{r}^4(46r^2-14r+9)z^6\} . \tag{22}$$

Similarly, for the ^3P_J states, we have

$$D^{(1)}_{\bar{b}\to\bar{b}c(^3P_J)}(z,s) = \frac{32\alpha_s^2(2m_c)}{243}\frac{r\bar{r}^3}{(1-\bar{r}z)^4}\sum_{n=0}^{3}\frac{f_n^{(J)}M^{8-2n}}{(s-\bar{r}^2M^2)^{5-n}} , \tag{23}$$

with

$$f_0^{(0)} = 64r^2\bar{r}^3(1-\bar{r}z)^4 , \tag{24}$$
$$f_1^{(0)} = 8r\bar{r}(1-\bar{r}z)^3[1-18r+14r^2-2\bar{r}(1-2r+7r^2)z+\bar{r}^2(1+2r)z^2] , \tag{25}$$
$$f_2^{(0)} = -(1-\bar{r}z)^2[2(1-4r)(1+6r-4r^2)-(5+14r-8r^2+80r^3-64r^4)z$$
$$+2\bar{r}(2+9r+18r^2-28r^3-16r^4)z^2-\bar{r}^2(1+6r+16r^2-32r^3)z^3] , \tag{26}$$
$$f_3^{(0)} = (1-z)[1-4r-(1-4r)(1-2r)z-r\bar{r}(3-4r)z^2]^2 , \tag{27}$$
$$f_0^{(1)} = 192r^2\bar{r}^3(1-\bar{r}z)^4 , \tag{28}$$
$$f_1^{(1)} = 24r\bar{r}(1-\bar{r}z)^3[2(1-r-r^2)-\bar{r}(3+10r-2r^2)z+\bar{r}^2z^2] , \tag{29}$$
$$f_2^{(1)} = -6(1-\bar{r}z)^2[2(1+2r)-(5-2r+6r^2)z+2\bar{r}(2-3r-4r^2)z^2-\bar{r}^2(1-2r+2r^2)z^3] , \tag{30}$$
$$f_3^{(1)} = 6(1-z)[1-2(1-2r)z+(1-4r)(1-2r)z^2+2r\bar{r}(1-2r)z^3+r^2\bar{r}^2z^4] , \tag{31}$$

and

$$f_0^{(2)} = 320r^2\bar{r}^3(1-\bar{r}z)^4 , \tag{32}$$
$$f_1^{(2)} = 8r\bar{r}^2(1-\bar{r}z)^3[2(4+13r)-(1+70r-26r^2)z-\bar{r}(7+8r)z^2] , \tag{33}$$
$$f_2^{(2)} = -4\bar{r}^2(1-\bar{r}z)^2[4(1+4r)-(7+12r-32r^2)z+2(1+13r-26r^2+8r^3)z^2+(1-30r-5r^2+4r^3)z^3] , \tag{34}$$
$$f_3^{(2)} = 4\bar{r}^2(1-z)[2-4(1-2r)z+(5-8r+12r^2)z^2-2(1-2r)(3+2r^2)z^3+(3-12r+12r^2+2r^4)z^4] . \tag{35}$$

Integrating Eq. (23) over s as in Eq. (16), we obtain

$$D^{(1)}_{\bar{b}\to\bar{b}c(^3P_0)}(z) = \frac{16\alpha_s^2(2m_c)}{729}\frac{r\bar{r}^3z(1-z)^2}{(1-\bar{r}z)^8}\{6(4r-1)^2+6(4r-1)(20r^2-16r+5)z$$
$$+(832r^4-1456r^3+1058r^2-362r+63)z^2-8\bar{r}(100r^4-184r^3+118r^2-22r+9)z^3$$
$$+2\bar{r}^2(416r^4-776r^3+369r^2+42r+24)z^4-2\bar{r}^3(240r^4-516r^3+232r^2+59r+9)z^5$$
$$+\bar{r}^4(96r^4-240r^3+134r^2+34r+3)z^6\} , \tag{36}$$
$$D^{(1)}_{\bar{b}\to\bar{b}c(^3P_1)}(z) = \frac{32\alpha_s^2(2m_c)}{243}\frac{r\bar{r}^3z(1-z)^2}{(1-\bar{r}z)^8}\{6+6(4r-5)z$$
$$+(16r^4+64r^2-98r+63)z^2+8\bar{r}(2r^4+2r^3-13r^2+11r-9)z^3$$
$$+2\bar{r}^2(8r^4-16r^3+47r^2-18r+24)z^4+2\bar{r}^3(8r^3-24r^2+r-9)z^5+\bar{r}^4(12r^2+2r+3)z^6\} , \tag{37}$$

and

$$D^{(1)}_{\bar{b}\to\bar{b}c(^3P_2)}(z) = \frac{64\alpha_s^2(2m_c)}{729}\frac{r\bar{r}^5z(1-z)^2}{(1-\bar{r}z)^8}\{12+12(2r-5)z+(92r^2-76r+135)z^2+4(10r^3-54r^2+31r-45)z^3$$
$$+2(46r^4-16r^3+123r^2-78r+75)z^4-4\bar{r}(6r^4+9r^3+40r^2-13r+18)z^5$$
$$+\bar{r}^2(12r^4-12r^3+55r^2-10r+15)z^6\} . \tag{38}$$

As in Ref. [5], the scale of the strong coupling constant is chosen to be $2m_c$—the minimal energy of the exchange gluon. The results of these fragmentation functions for the P-wave states disagrees with those given in Ref. [9].

Integrating Eqs. (22) and (36)–(38) over z, we obtain the fragmentation probabilities $P^{(1)} = (H_1/m)\int_0^1 dz\, D^{(1)}(z)$ of the color-singlet contribution for $\bar{b}$ fragments into $\bar{b}c(n^{2S+1}P_J)$:

$$P^{(1)}_{\bar{b}\to\bar{b}c(n^1P_1)} = \frac{16\alpha_s^2(2m_c)}{243}\frac{H_{1(\bar{b}c)}(n)}{m}\left[3\frac{r\ln r}{\bar{r}^3}(10r^4+50r^3+91r^2+20r+7)\right.$$
$$\left. -\frac{1}{35\bar{r}^2}(24r^6-256r^5-2083r^4-9538r^3-5758r^2-907r-172)\right], \quad (39)$$

$$P^{(1)}_{\bar{b}\to\bar{b}c(n^3P_0)} = \frac{16\alpha_s^2(2m_c)}{729}\frac{H_{1(\bar{b}c)}(n)}{m}\left[3\frac{r\ln r}{\bar{r}^3}(32r^6+48r^5-398r^4+194r^3+337r^2+32r+1)\right.$$
$$\left. +\frac{1}{35\bar{r}^2}(9944r^6-17384r^5-34289r^4+56116r^3+11036r^2+347r+60)\right], \quad (40)$$

$$P^{(1)}_{\bar{b}\to\bar{b}c(n^3P_1)} = \frac{32\alpha_s^2(2m_c)}{243}\frac{H_{1(\bar{b}c)}(n)}{m}\left[3\frac{r\ln r}{\bar{r}^3}(20r^4+46r^3+31r^2+8r+1)\right.$$
$$\left. -\frac{1}{35\bar{r}^2}(12r^6-268r^5-4461r^4-4006r^3-2326r^2-23r-58)\right], \quad (41)$$

and

$$P^{(1)}_{\bar{b}\to\bar{b}c(n^3P_2)} = \frac{64\alpha_s^2(2m_c)}{729}\frac{H_{1(\bar{b}c)}(n)}{m}\left[3\frac{r\ln r}{\bar{r}^3}(4r^6+36r^5+5r^4-20r^3+140r^2+4r+11)\right.$$
$$\left. +\frac{1}{35\bar{r}^2}(1468r^6+5570r^5-7345r^4+12080r^3+6130r^2+712r+285)\right]. \quad (42)$$

For a given principal quantum number n, the ${}^1P_1$ state and the ${}^3P_1$ state, constructed in the LS coupling scheme, are mixed in general to give rise to the physical states 1^+ and $1^{+'}$ defined as

$$|1^{+'}\rangle = \cos\theta|{}^1P_1\rangle - \sin\theta|{}^3P_1\rangle, \quad (43)$$
$$|1^{+}\rangle = \sin\theta|{}^1P_1\rangle + \cos\theta|{}^3P_1\rangle. \quad (44)$$

Thus, in general, we have

$$D^{(1)}_{\bar{b}\to\bar{b}c(1^{+'})}(z) = \cos^2\theta D^{(1)}_{\bar{b}\to\bar{b}c({}^1P_1)}(z) + \sin^2\theta D^{(1)}_{\bar{b}\to\bar{b}c({}^3P_1)}(z) - \sin\theta\cos\theta D^{(1)}_{\text{mix}}(z), \quad (45)$$
$$D^{(1)}_{\bar{b}\to\bar{b}c(1^{+})}(z) = \sin^2\theta D^{(1)}_{\bar{b}\to\bar{b}c({}^1P_1)}(z) + \cos^2\theta D^{(1)}_{\bar{b}\to\bar{b}c({}^3P_1)}(z) + \sin\theta\cos\theta D^{(1)}_{\text{mix}}(z). \quad (46)$$

Similar equations hold for the corresponding generalized fragmentation functions $D^{(1)}(z,s)$ as well. With the matrix elements of $\mathcal{M}(n^1P_1)$ and $\mathcal{M}(n^3P_1)$ given by Eqs. (10) and (11) and (12)–(15), respectively, we can calculate their interferences and obtain the mixing term

$$D^{(1)}_{\text{mix}}(z,s) = \frac{64\sqrt{2}\alpha_s^2(2m_c)}{81}\frac{r\bar{r}^3}{(1-\bar{r}z)^3}\sum_{n=0}^{2}\frac{f_n^{\text{mix}}M^{6-2n}}{(s-\bar{r}^2M^2)^{4-n}}, \quad (47)$$

with

$$f_0^{\text{mix}} = -8r\bar{r}(1-\bar{r}z)^3[2-3r-\bar{r}(1+r)z], \quad (48)$$
$$f_1^{\text{mix}} = (1-\bar{r}z)[2(1+2r^2)-(5-24r+36r^2-8r^3)z+2\bar{r}(2-15r+10r^2-2r^3)z^2-\bar{r}^2(1-8r-4r^2)z^3], \quad (49)$$
$$f_2^{\text{mix}} = -(1-z)[1-(1-3r)z+r(2+r)z^2+\bar{r}r^2z^3]. \quad (50)$$

Integrating Eq. (47) over s as in Eq. (16), we obtain

$$D^{(1)}_{\text{mix}}(z) = -\frac{32\sqrt{2}\alpha_s^2(2m_c)}{243}\frac{r\bar{r}^3z(1-z)^2}{(1-\bar{r}z)^6}\{6-6(2r^2-4r+3)z$$
$$+(24r^3+52r^2-52r+21)z^2+2\bar{r}(14r^3-6r^2+15r-6)z^3-\bar{r}^2(2r^2+8r-3)z^4\}. \quad (51)$$

The fragmentation probability of the mixing term is given by

$$P^{(1)}_{\rm mix} = \frac{H_{1(\bar{b}c)}(n)}{m}\int_0^1 dz D^{(1)}_{\rm mix}(z)$$
$$= -\frac{32\sqrt{2}\alpha_s^2(2m_c)}{243}\frac{H_{1(\bar{b}c)}(n)}{m}\left[3\frac{r\ln r}{\bar{r}^3}(18r^4+28r^3-11r^2-2r+1) + \frac{1}{5\bar{r}^2}(24r^5+587r^4-18r^3-88r^2-3r+8)\right] . \quad (52)$$

We next turn to the calculation of the color-octet coefficient $D^{(8)}(z)$ to leading order in α_s. Notice that to leading order in α_s, all the color-singlet contributions obtained above are free from infrared divergences. All the long-distance effects can be factored and absorbed into the derivative of the wave function, or equivalently H_1 to leading order in v^2. The color-singlet coefficients $D^{(1)}(z)$ do not depend on the factorization scale Λ to leading order in α_s. Beyond leading order, infrared singularities that appear in the color-singlet contribution arise from the radiation of a soft gluon in the process $\bar{b}^* \to (\bar{b}c)\bar{c}g$. Imagine we are actually doing such a next-to-leading order calculation of $D^{(1)}(z)$ with an energy cutoff Λ ($mv \ll \Lambda \ll m$) imposed on the radiated soft gluon. This cutoff will allow us to isolate the infrared logarithmic divergent terms in this higher order calculation and absorb them into the nonperturbative parameter H_8' associated with the *color-octet mechanism.* However, there is a short cut to achieve the same goal by following the same method as in Ref. [12]. One can calculate the fragmentation functions for the processes $\bar{b}^* \to \bar{b}c(^1S_0,\underline{8})$ and $\bar{b}^* \to \bar{b}c(^3S_1,\underline{8})$, where the $(\bar{b}c)$ pair is in the appropriate color-octet S-wave state. The calculations of these fragmentation functions are identical to the color-singlet S-wave case [5], except one has to replace the S-wave wave function $R_S(0)$ by a fictitious "color-octet S-wave wave function at the origin" $R_8(0)$, and the color factor C_F^2 by $C_F/6$ where $C_F = 4/3$ for color $SU(3)$. $R_8(0)$ is related to the nonperturbative parameter $H'_{8(\bar{b}c)}(\Lambda)$ by

$$H'_{8(\bar{b}c)}(\Lambda) = \frac{2}{3\pi}\frac{|R_8(0)|^2}{(2m)^2} . \quad (53)$$

Using this trick, we can easily extract the coefficients of the color-octet contributions $D^{(8)}(z)$. Analogous to Eq. (16) of the color-singlet case, we define $D^{(8)}(z,s)$ as

$$D^{(8)}(z) = \int ds\, \theta\left(s - \frac{M^2}{z} - \frac{r^2M^2}{1-z}\right) D^{(8)}(z,s) , \quad (54)$$

where

$$D^{(8)}(z,s) = \frac{\alpha_s^2(2m_c)}{81}\frac{r\bar{r}^3}{(1-\bar{r}z)^2}\sum_{n=0}^{2}\frac{g_n M^{6-2n}}{(s-\bar{r}^2M^2)^{4-n}} . \quad (55)$$

For the $^1S_0$ state, we have

$$g_{0(^1S_0)} = -12r\bar{r}(1-\bar{r}z)^2 , \quad (56)$$
$$g_{1(^1S_0)} = -3(1-\bar{r}z)[2(1-2r)-(3-4r+4r^2)z + \bar{r}(1-2r)z^2] , \quad (57)$$
$$g_{2(^1S_0)} = 3(1-z)(1+rz)^2 . \quad (58)$$

Similarly, for the $^3S_1$ state, we have

$$g_{0(^3S_1)} = -12r\bar{r}(1-\bar{r}z)^2 , \quad (59)$$
$$g_{1(^3S_1)} = -(1-\bar{r}z)[2(1+2r)-(1+12r-4r^2)z - \bar{r}(1+2r)z^2] , \quad (60)$$
$$g_{2(^3S_1)} = (1-z)[1+2rz+(2+r^2)z^2] . \quad (61)$$

Integrating Eq. (55) over s as in Eq. (54), we obtain the color-octet contributions to the fragmentation functions

$$D^{(8)}_{\bar{b}\to\bar{b}c(^1S_0)}(z) = \frac{\alpha_s^2(2m_c)}{162}\frac{r\bar{r}^3z(1-z)^2}{(1-\bar{r}z)^6}[6-18(1-2r)z+(21-74r+68r^2)z^2 - 2\bar{r}(6-19r+18r^2)z^3+3\bar{r}^2(1-2r+2r^2)z^4], \quad (62)$$

and

$$D^{(8)}_{\bar{b}\to\bar{b}c(^3S_1)}(z) = \frac{\alpha_s^2(2m_c)}{162}\frac{r\bar{r}^3z(1-z)^2}{(1-\bar{r}z)^6}[2-2(3-2r)z+3(3-2r+4r^2)z^2 - 2\bar{r}(4-r+2r^2)z^3+\bar{r}^2(3-2r+2r^2)z^4] . \quad (63)$$

Integrating Eqs. (62) and (63) over z, we obtain the fragmentation probabilities $P^{(8)} = (H_8'/m)\int_0^1 dz\, D^{(8)}(z)$ for the color-octet contributions:

$$P^{(8)}_{\bar{b}\to\bar{b}c(^1S_0)} = \frac{\alpha_s^2(2m_c)}{54}\frac{H'_{8(\bar{b}c)}(\Lambda)}{m}\left[\frac{r\ln r}{\bar{r}^3}(1+8r+r^2-6r^3+2r^4) + \frac{1}{15\bar{r}^2}(8+13r+228r^2-212r^3+53r^4)\right] , \quad (64)$$

and

$$P^{(8)}_{\bar b\to\bar bc(^3S_1)} = \frac{\alpha_s^2(2m_c)}{162}\frac{H'_{8(\bar bc)}(\Lambda)}{m}\left[\frac{r\ln r}{\bar r^3}(7-4r+3r^2+10r^3+2r^4)+\frac{1}{15\bar r^2}(24+109r-126r^2+174r^3+89r^4)\right] . \tag{65}$$

To avoid large logarithms of m/Λ appearing in the higher order calculation of the color-singlet coefficients $D^{(1)}(z)$, one should choose $\Lambda \sim m$ in the matrix element $H'_{8(\bar bc)}(\Lambda)$.

Combining both contributions from the *color-singlet* and *color-octet mechanisms*, and including the mixing effects in the $^1P_1$ and $^3P_1$ states, the total fragmentation probabilities are given by

$$P_{\bar b\to\bar bc(n^3P_0)} = P^{(1)}_{\bar b\to\bar bc(n^3P_0)} + P^{(8)}_{\bar b\to\bar bc(^3S_1)} , \tag{66}$$

$$P_{\bar b\to\bar bc(n\,1^{+'})} = \cos^2\theta P^{(1)}_{\bar b\to\bar bc(n^1P_1)} + \sin^2\theta P^{(1)}_{\bar b\to\bar bc(n^3P_1)} - \sin\theta\cos\theta P^{(1)}_{\rm mix} + 3P^{(8)}_{\bar b\to\bar bc(^1S_0)} , \tag{67}$$

$$P_{\bar b\to\bar bc(n\,1^{+})} = \sin^2\theta P^{(1)}_{\bar b\to\bar bc(n^1P_1)} + \cos^2\theta P^{(1)}_{\bar b\to\bar bc(n^3P_1)} + \sin\theta\cos\theta P^{(1)}_{\rm mix} + 3P^{(8)}_{\bar b\to\bar bc(^3S_1)} , \tag{68}$$

$$P_{\bar b\to\bar bc(n^3P_2)} = P^{(1)}_{\bar b\to\bar bc(n^3P_2)} + 5P^{(8)}_{\bar b\to\bar bc(^3S_1)} . \tag{69}$$

As in the S-wave case [5], under the Altarelli-Parisi evolution to a higher scale, the shapes of these P-wave fragmentation functions are softened while the fragmentation probabilities remain unchanged.

Before closing this section, we note that one can also define fragmentation functions that depend on both the longitudinal momentum fraction z and the transverse momentum $|\mathbf{p}_\perp|$. Introducing the dimensionless variable $t = |\mathbf{p}_\perp|/M$ and using Eq. (7) to trade the variable s with t, we can define the generalized fragmentation functions $D(z,t)$ and $D(t)$ according to

$$\int_0^\infty dt\, D(t) = \int_0^\infty dt \int_0^1 dz\, D(z,t) \tag{70}$$

$$= \int ds \int dz\, \theta\left(s - \frac{M^2}{z} - \frac{r^2M^2}{1-z}\right) D(z,s) . \tag{71}$$

Therefore,

$$D(z,t) = \frac{2M^2t}{z(1-z)}D(z,s),$$

$$\text{with } s = M^2\left[\frac{1+t^2}{z} + \frac{r^2+t^2}{1-z}\right] . \tag{72}$$

The above relation holds for the color-singlet and the color-octet contributions. These functions $D(t)$ and $D(z,t)$ are useful for studying the transverse motion of the meson with respect to the jet axis of the fragmenting heavy quark. The expressions of $D(t)$ for the color-octet S-wave contributions can be obtained from Ref. [17]. Analytic results of $D(t)$ for the P-wave states can also be derived but will not be given here.

IV. HEAVY QUARK SYMMETRY

In this section, we temporarily leave the heavy-heavy ($\bar bc$) system and turn our attention to the heavy-light B- or D-meson system. In the limit of $m_Q/\Lambda_{\rm QCD} \to \infty$, both the heavy quark spin $\mathbf{S}_Q$ and the total spin $\mathbf{J}$ of a heavy hadron containing a single heavy quark Q become good quantum numbers. This implies that in the spectroscopy of the hadron containing a single heavy quark Q, the angular momentum of the light degrees of freedom $\mathbf{J}_l = \mathbf{J} - \mathbf{S}_Q$ is also a good quantum number. We refer collectively to all the degrees of freedom in the heavy-light hadron other than the heavy quark as the light degrees of freedom. For the heavy-light ($Q\bar q$) meson, $\mathbf{J}_l = \mathbf{S}_q + \mathbf{L}$ where $\mathbf{S}_q$ is the spin of the light antiquark $\bar q$ and $\mathbf{L}$ is the orbital angular momentum. Thus the hadronic states can be labeled simultaneously by the eigenvalues j and j_l of the total spin $\mathbf{J}$ and the angular momentum of the light degrees of freedom $\mathbf{J}_l$, respectively. In general [18], the spectrum of hadrons containing a single heavy quark Q has, for each j_l, a degenerate doublet with total spins $j_+ = j_l + 1/2$ and $j_- = j_l - 1/2$. (For the case of $j_l = 0$, the total spin must be 1/2.) For P-wave heavy-light mesons, the orbital angular momentum $L = 1$ and j_l can then be either 1/2 or 3/2. Thus $(j_-, j_+) = (0,1)$ and (1,2) for $j_l = 1/2$ and 3/2, respectively. As a result, we expect to have two distinct doublets $(^3P_0, 1^{+'})$ and $(1^+, {}^3P_2)$ in the limit $m_b/m_c \to \infty$, i.e., $r \to 0$. In this limit, the mixing coefficients in Eqs. (43) and (44) can be determined by the Clebsch-Gordan coefficients in the tensor product of a spin 1/2 state and a spin 1 state with the result

$$|1^{+'}\rangle = \sqrt{\frac{1}{3}}|^1P_1\rangle + \sqrt{\frac{2}{3}}|^3P_1\rangle , \tag{73}$$

$$|1^{+}\rangle = -\sqrt{\frac{2}{3}}|^1P_1\rangle + \sqrt{\frac{1}{3}}|^3P_1\rangle ; \tag{74}$$

i.e., we are transforming the states $^1P_1$ and $^3P_1$ in the LS coupling scheme to the states $1^{+'}$ and 1^+ in the jj coupling scheme.

In their discussions of the heavy quark fragmentation functions within the context of heavy quark effective theory, Jaffe and Randall [19] showed that a more natural variable to use is given by

$$y = \frac{\frac{1}{z} - \bar{r}}{r} , \tag{75}$$

rather than the usual fragmentation variable z. Furthermore, these authors also showed that the heavy quark fragmentation function can be expanded as a power series in r:

$$D(y) = \frac{1}{r}a(y) + b(y) + O(r) , \tag{76}$$

where $a(y), b(y)$, etc., are functions of the variable y. The leading term $a(y)$ is constrained by the heavy quark spin-flavor symmetry while all the higher order terms contain spin-flavor symmetry breaking effects. One can recast our results for the P-wave fragmentation functions derived in Sec. III in the above form. By carefully expanding the powers of r and $(1-\bar{r}z)$, the leading contributions to the fragmentation functions are given by

$$D^{(1)}_{\bar{b}\to\bar{b}c(^1P_1)}(y) \to \alpha_s^2 \frac{16}{243r}\frac{(y-1)^2}{y^8} \times(9y^4 - 4y^3 + 40y^2 + 96) , \tag{77}$$

$$D^{(1)}_{\bar{b}\to\bar{b}c(^3P_0)}(y) \to \alpha_s^2 \frac{16}{243r}\frac{(y-1)^2}{y^8}(y^4 - 4y^3 + 8y^2 + 32) , \tag{78}$$

$$D^{(1)}_{\bar{b}\to\bar{b}c(^3P_1)}(y) \to \alpha_s^2 \frac{32}{243r}\frac{(y-1)^2}{y^8}(3y^4 - 4y^3 +16y^2 + 48) , \tag{79}$$

$$D^{(1)}_{\bar{b}\to\bar{b}c(^3P_2)}(y) \to \alpha_s^2 \frac{320}{243r}\frac{(y-1)^2}{y^8}(y^4 + 4y^2 + 8) , \tag{80}$$

$$D^{(1)}_{\rm mix}(y) \to -\alpha_s^2 \frac{32\sqrt{2}}{243r}\frac{(y-1)^2}{y^6}(3y^2 + 4y + 8) . \tag{81}$$

Therefore, in the heavy quark limit, we have

$$D^{(1)}_{\bar{b}\to\bar{b}c(1^{+\prime})}(y) \to \frac{1}{3}D^{(1)}_{\bar{b}\to\bar{b}c(^1P_1)}(y) + \frac{2}{3}D^{(1)}_{\bar{b}\to\bar{b}c(^3P_1)}(y) + \frac{\sqrt{2}}{3}D^{(1)}_{\rm mix}(y) \to \alpha_s^2 \frac{16}{81r}\frac{(y-1)^2}{y^8}(y^4 - 4y^3 + 8y^2 + 32) , \tag{82}$$

$$D^{(1)}_{\bar{b}\to\bar{b}c(1^{+})}(y) \to \frac{2}{3}D^{(1)}_{\bar{b}\to\bar{b}c(^1P_1)}(y) + \frac{1}{3}D^{(1)}_{\bar{b}\to\bar{b}c(^3P_1)}(y) - \frac{\sqrt{2}}{3}D^{(1)}_{\rm mix}(y) \to \alpha_s^2 \frac{64}{81r}\frac{(y-1)^2}{y^8}(y^4 + 4y^2 + 8) . \tag{83}$$

These imply the spin counting ratios

$$\frac{D^{(1)}_{\bar{b}\to\bar{b}c(^3P_0)}(y)}{D^{(1)}_{\bar{b}\to\bar{b}c(1^{+\prime})}(y)} \to \frac{1}{3} , \tag{84}$$

$$\frac{D^{(1)}_{\bar{b}\to\bar{b}c(1^{+})}(y)}{D^{(1)}_{\bar{b}\to\bar{b}c(^3P_2)}(y)} \to \frac{3}{5} , \tag{85}$$

as expected from the heavy quark spin symmetry.

Similarly, as already shown in Ref. [5], the color-octet coefficients $D^{(8)}_{\bar{b}\to\bar{b}c(^1S_0)}(z)$ and $D^{(8)}_{\bar{b}\to\bar{b}c(^3S_1)}(z)$ given by Eqs. (62) and (63), respectively, also satisfy the heavy quark spin symmetry in the limit $r \to 0$. Thus, heavy quark spin symmetry is a powerful tool to check our tedious results obtained in the previous section. We note that the mixing effects and the color-octet contributions were not considered in Ref. [9]. Moreover, the results given there do not have the proper heavy quark limit given by Eq. (76).

Since our P-wave fragmentation functions derived in the previous section are consistent with the general QCD analysis of heavy quark fragmentation functions by Jaffe and Randall [19], they may be useful as phenomenological fragmentation functions for $c \to D^{**}$ and $b \to B^{**}$, with the mass ratio r and the two overall factors $\alpha_s^2 H_1/m$ and $\alpha_s^2 H_8'/m$ treated as free parameters.

V. BRAATEN-LEVIN SPIN COUNTING RULES

The fragmentation function $D_{i\to H}(z)$ for a parton i splitting into a hadron H is related to the distribution function $f_{i/H}(x)$ of finding the parton i inside the hadron H by analytic continuation [20, 21]

$$f_{i/H}(x) = x D_{i\to H}\left(\frac{1}{x}\right) . \tag{86}$$

The results of our perturbative S-wave and P-wave fragmentation functions obtained in Refs. [5, 22] and in this paper allow us to study the perturbative tail of the distribution functions of the heavy quark inside the heavy S-wave and P-wave mesons as well. From our explicit calculations, we see that $f_{i/H}(x)$ has a pole located at $x = \bar{r}$. The pole is cut off by nonperturbative effects related to the formation of bound states of the $(\bar{b}c)$ pair. This pole is of order 6 and 8 for the S-wave and P-wave states, respectively. In the general case of L waves ($L = 0, 1, 2, \ldots$ or S, P, D waves, ...), we expect this pole is of order $6 + 2L$. Therefore, we can expand $f(x)$ as a Laurent series:

$$f(x) = \frac{a_n(r)}{(x-\bar{r})^n} + \text{less singular terms} , \tag{87}$$

with $n = 6 + 2L$ for the general L waves.

An interesting result recently obtained by Braaten and Levin [23] states that the r-dependent coefficients $a_n(r)$ satisfy the simple spin counting rules. To be more specific, we have

$$a_6(^1S_0) : a_6(^3S_1) = 1 : 3 , \tag{88}$$

for the S waves, and

$$a_8(^1P_1) : a_8(^3P_0) : a_8(^3P_1) : a_8(^3P_2) = 3 : 1 : 3 : 5 , \tag{89}$$

for the P waves.

From Eqs. (62) and (63) or using the results in Ref. [5], we can calculate the two coefficients $a_6(^1S_0)$ and $a_6(^3S_1)$ for the S-wave states to leading order in α_s (up to an overall common factor of $\alpha_s^2 H_8'/m$):

$$a_6(^1S_0) = \lim_{x\to\bar r}\left[(x-\bar r)^6 x D^{(8)}_{^1S_0}\left(\frac{1}{x}\right)\right] = \frac{4}{81}(r\bar r)^5 \ , \quad (90)$$

$$a_6(^3S_1) = 3\lim_{x\to\bar r}\left[(x-\bar r)^6 x D^{(8)}_{^3S_1}\left(\frac{1}{x}\right)\right] = \frac{4}{27}(r\bar r)^5 \ . \quad (91)$$

Similarly, from Eqs. (22) and (36)–(38), we obtain the coefficients $a_8(^1P_1)$ and $a_8(^3P_J)$ for the P-wave states to leading order in α_s (up to an overall common factor of $\alpha_s^2 H_1/m$):

$$a_8(^1P_1) = \lim_{x\to\bar r}\left[(x-\bar r)^8 x D^{(1)}_{^1P_1}\left(\frac{1}{x}\right)\right] = \frac{512}{81}(r\bar r)^7 \ , \quad (92)$$

$$a_8(^3P_0) = \lim_{x\to\bar r}\left[(x-\bar r)^8 x D^{(1)}_{^3P_0}\left(\frac{1}{x}\right)\right] = \frac{512}{243}(r\bar r)^7 \ , \quad (93)$$

$$a_8(^3P_1) = \lim_{x\to\bar r}\left[(x-\bar r)^8 x D^{(1)}_{^3P_1}\left(\frac{1}{x}\right)\right] = \frac{512}{81}(r\bar r)^7 \ , \quad (94)$$

$$a_8(^3P_2) = \lim_{x\to\bar r}\left[(x-\bar r)^8 x D^{(1)}_{^3P_2}\left(\frac{1}{x}\right)\right] = \frac{2560}{243}(r\bar r)^7 \ . \quad (95)$$

Moreover, from Eq. (51), we have

$$a_8^{\rm mix} = \lim_{x\to\bar r}\left[(x-\bar r)^8 x D^{(1)}_{\rm mix}\left(\frac{1}{x}\right)\right] = 0 \ . \quad (96)$$

Therefore, our explicit results of the S-wave and P-wave fragmentation functions do satisfy the Braaten-Levin spin counting rules. Notice that unlike the spin counting in the heavy quark limit studied in the last section which requires $r\to 0$, the Braaten-Levin spin counting rules hold for arbitrary values of r.

VI. FRAGMENTATION FUNCTIONS FOR $c\to h_c$ AND $c\to\chi_{cJ}$ $(J=0,1,2)$

Our results derived in Secs. II and III can be easily applied to the cases of P-wave charmonium and bottomonium by taking the equal mass limit $m_c = m_b$, i.e., $r=\bar r = 1/2$. Simple formulas can be obtained in this limit and are given below for convenience. To be specific, we will consider the P-wave charmonium case. To leading order in v^2, the fragmentation functions for a charm (or anticharm) quark to fragment into various P-wave charmonium states consist of two terms:

$$D_{c\to h_c}(z,\mu_0) = \frac{H_{1(c\bar c)}}{m_c} D^{(1)}_{c\to c\bar c(^1P_1)}(z,\Lambda) + 3\frac{H'_{8(c\bar c)}(\Lambda)}{m_c} D^{(8)}_{c\to c\bar c(^1S_0)}(z) \ , \quad (97)$$

$$D_{c\to \chi_{cJ}}(z,\mu_0) = \frac{H_{1(c\bar c)}}{m_c} D^{(1)}_{c\to c\bar c(^3P_J)}(z,\Lambda) + (2J+1)\frac{H'_{8(c\bar c)}(\Lambda)}{m_c} D^{(8)}_{c\to c\bar c(^3S_1)}(z) \ , \quad (98)$$

where Λ is a factorization scale within the range $m_c v \ll \Lambda \ll m_c$. $H_{1(c\bar c)}$ is related to the derivative of the radial wave function of the P-wave charmonium, $R'_P(0)$, according to [10, 13]

$$H_{1(c\bar c)} \approx \frac{9}{2\pi}\frac{|R'_P(0)|^2}{m_c^4} \ . \quad (99)$$

$H'_{8(c\bar c)}$ satisfies the renormalization group equation [10, 14]

$$\Lambda\frac{d}{d\Lambda}H'_{8(c\bar c)}(\Lambda) = \frac{16}{27\pi}\alpha_s(\Lambda)H_{1(c\bar c)} \ , \quad (100)$$

with the solution

$$H'_{8(c\bar c)}(\Lambda) = H'_{8(c\bar c)}(\Lambda_0) + \frac{16}{27\beta_0}\ln\left(\frac{\alpha_s(\Lambda_0)}{\alpha_s(\Lambda)}\right) H_{1(c\bar c)} \ , \quad (101)$$

where $\beta_0 = 25/6$ is the first coefficient in the β function for QCD with four light flavors. If the factorization scale Λ is chosen to be much less than the charm quark mass m_c, the above equation can be used to sum up large logarithms of m_c/Λ.

To leading order in α_s, we have

$$D^{(1)}_{c\to c\bar c(^1P_1)}(z) = \frac{16\alpha_s^2(2m_c)}{81}\frac{z(1-z)^2}{(2-z)^8} \times[64-128z+176z^2-160z^3 +140z^4-56z^5+9z^6] \ , \quad (102)$$

$$D^{(1)}_{c\to c\bar c(^3P_0)}(z) = \frac{16\alpha_s^2(2m_c)}{729}\frac{z(1-z)^2}{(2-z)^8} \times[192+384z+528z^2-1376z^3 +1060z^4-376z^5+59z^6] \ , \quad (103)$$

$$D^{(1)}_{c\to c\bar c(^3P_1)}(z) = \frac{64\alpha_s^2(2m_c)}{243}\frac{z(1-z)^2}{(2-z)^8} \times[96-288z+496z^2-408z^3 +202z^4-54z^5+7z^6] \ , \quad (104)$$

$$D^{(1)}_{c\to c\bar c(^3P_2)}(z) = \frac{128\alpha_s^2(2m_c)}{729}\frac{z(1-z)^2}{(2-z)^8} \times[48-192z+480z^2-668z^3 +541z^4-184z^5+23z^6] \ . \quad (105)$$

The corresponding fragmentation probabilities $P^{(1)} = (H_1/m_c)\int_0^1 dz\, D^{(1)}(z)$ are simply given by

$$P^{(1)}_{c\to h_c} = \frac{8\alpha_s^2(2m_c)}{81}\frac{H_{1(c\bar c)}}{m_c}\left[\frac{18107}{35} - 746\ln 2\right] \ , \quad (106)$$

$$P^{(1)}_{c\to \chi_{c0}} = \frac{8\alpha_s^2(2m_c)}{729}\frac{H_{1(c\bar c)}}{m_c}\left[\frac{119617}{35} - 4926\ln 2\right] \ , \quad (107)$$

$$P^{(1)}_{c\to \chi_{c1}} = \frac{64\alpha_s^2(2m_c)}{243}\frac{H_{1(c\bar c)}}{m_c}\left[\frac{1151}{7} - 237\ln 2\right] \ , \quad (108)$$

$$P^{(1)}_{c\to \chi_{c2}} = \frac{32\alpha_s^2(2m_c)}{729}\frac{H_{1(c\bar c)}}{m_c}\left[\frac{54743}{35} - 2256\ln 2\right] \ . \quad (109)$$

The color-octet coefficients can be obtained by making appropriate changes in the radial wave function and color factor to the results of the S-wave fragmentation functions $D_{c\to\eta_c}(z)$ and $D_{c\to J/\psi}(z)$ given in Ref. [22], or simply by setting $r=\bar{r}=1/2$ in Eqs. (62) and (63). The results are

$$D^{(8)}_{c\to c\bar{c}(^1S_0)}(z)=\frac{\alpha_s^2(2m_c)}{162}\frac{z(1-z)^2}{(2-z)^6}\times\left[48+8z^2-8z^3+3z^4\right], \quad (110)$$

$$D^{(8)}_{c\to c\bar{c}(^3S_1)}(z)=\frac{\alpha_s^2(2m_c)}{162}\frac{z(1-z)^2}{(2-z)^6}\times\left[16-32z+72z^2-32z^3+5z^4\right]. \quad (111)$$

The fragmentation probabilities $P^{(8)}=(H_8'/m_c)\int_0^1 dz\, D^{(8)}(z)$ of the color-octet piece are given by

$$P^{(8)}_{c\to c\bar{c}(^1S_0)}=\frac{\alpha_s^2(2m_c)}{54}\frac{H'_{8(c\bar{c})}(\Lambda)}{m_c}\left[\frac{773}{30}-37\ln 2\right], \quad (112)$$

$$P^{(8)}_{c\to c\bar{c}(^3S_1)}=\frac{\alpha_s^2(2m_c)}{162}\frac{H'_{8(c\bar{c})}(\Lambda)}{m_c}\left[\frac{1189}{30}-57\ln 2\right]. \quad (113)$$

To avoid large logarithms of m_c/Λ appearing in the color-singlet contributions, one should set $\Lambda\sim m_c$ in the matrix element $H'_{8(c\bar{c})}(\Lambda)$.

Adding both color-singlet and color-octet contributions, the total fragmentation probabilities for c (or $\bar{c}$) to split into the four P-wave charmonium states are given by

$$P_{c\to h_c}=P^{(1)}_{c\to c\bar{c}(^1P_1)}+3P^{(8)}_{c\to c\bar{c}(^1S_0)}, \quad (114)$$

$$P_{c\to\chi_{cJ}}=P^{(1)}_{c\to c\bar{c}(^3P_J)}+(2J+1)P^{(8)}_{c\to c\bar{c}(^3S_1)}. \quad (115)$$

We note that in the equal mass case, mixings between the $^1P_1$ state and the $^3P_1$ state are not allowed by charge conjugation. Once produced by fragmentation, the P-wave χ_{cJ} states can radiatively decay into J/ψ and therefore contribute to the inclusive J/ψ production cross section.

Extensions of the formulas given in this section to the P-wave states of bottomonium system are straightforward.

VII. DISCUSSIONS

According to the results from potential model calculations [3], the radial wave functions at the origin for the first two sets of P-wave $(\bar{b}c)$ bound states are given by $|R'_{2P}(0)|^2=0.201$ GeV^5 and $|R'_{3P}(0)|^2=0.264$ GeV^5. The corresponding $H_{1(\bar{b}c)}(2)$ and $H_{1(\bar{b}c)}(3)$ are about 10 MeV and 14 MeV, respectively, where we have used $m_c=1.5$ GeV and $m_b=4.9$ GeV. We will choose the factorization scale $\Lambda=m$. In the limit $m\gg\Lambda_0$, the contribution to $H'_{8(\bar{b}c)}$ from the perturbative evolution dominates, and one can estimate $H'_{8(\bar{b}c)}(m)$ by setting $\alpha_s(\Lambda_0)\sim 1$ and neglecting the constant $H'_{8(\bar{b}c)}(\Lambda_0)$ in Eq. (5). Taking $\alpha_s(m)=0.38$, we estimate $H'_{8(\bar{b}c)}(m)\approx 1.3$ MeV and 1.8 MeV for the $2P$ and $3P$ states, respectively. Numerically, the color-octet contributions are small compared with the color-singlet contributions. The mixing angles for the $^1P_1$ and $^3P_1$ states are also obtained in [3] with the results $(\cos\theta,\sin\theta)=(0.999,0.030)$ and $(0.957,0.290)$ for the $2P$ and $3P$ states, respectively. The mixing angles are surprisingly small which implies the states constructed in the LS coupling scheme are actually very close to the physical mass eigenstates. The total fragmentation probabilities for $\bar{b}$ to split into the four $2P$ states are estimated to be

$$P_{\bar{b}\to\bar{b}c(2^3P_0)}\approx 2.3\times10^{-5}, \quad (116)$$

$$P_{\bar{b}\to\bar{b}c(2\,1^{+\prime})}\approx 4.4\times10^{-5}, \quad (117)$$

$$P_{\bar{b}\to\bar{b}c(2\,1^{+})}\approx 4.8\times10^{-5}, \quad (118)$$

$$P_{\bar{b}\to\bar{b}c(2^3P_2)}\approx 5.6\times10^{-5}. \quad (119)$$

Similarly, for the $3P$ states, we obtain

$$P_{\bar{b}\to\bar{b}c(3^3P_0)}\approx 3.0\times10^{-5}, \quad (120)$$

$$P_{\bar{b}\to\bar{b}c(3\,1^{+\prime})}\approx 8.1\times10^{-5}, \quad (121)$$

$$P_{\bar{b}\to\bar{b}c(3\,1^{+})}\approx 4.0\times10^{-5}, \quad (122)$$

$$P_{\bar{b}\to\bar{b}c(3^3P_2)}\approx 7.4\times10^{-5}. \quad (123)$$

Since all the $2P$ states will decay 100% to the $1S$ pseudoscalar ground state B_c, one should add up all the probabilities from Eqs. (116)–(119) to give the total fragmentation probability for the B_c production rate from the cascades of the $2P$ states, which is about 1.7×10^{-4}. This is comparable to the probability 3.8×10^{-4} of the direct fragmentation of $\bar{b}\to B_c$, and is about 10% of the lower bound 1.5×10^{-3} for the probability of $b\to B_c$ including the $1S$ and $2S$ states obtained in Ref. [5]. Similarly, the total probability for the production of B_c from the cascades of the $3P$ states is about 2.3×10^{-4}. Therefore the $2P$ and $3P$ states together account for a significant fraction of the inclusive production rate of the B_c. While the $2P$ states are expected to lie below the BD flavor threshold, the $3P$ states may or may not lie below this threshold. If any one of the four $3P$ states lies above this flavor threshold, it will quickly dissociate into a pair of B and D mesons, and will not contribute to the inclusive B_c production rate. The p_T distributions and the total inclusive production cross sections for these P-wave states with realistic rapidity cut at the $p\bar{p}$ colliders are now under investigation [24]. The four $3D$ states are also expected to lie below the BD threshold [3]. The corresponding D-wave fragmentation functions and probabilities have not yet been calculated.

For the P-wave charmonium case, $H_{1(c\bar{c})}$ has been phenomenologically determined to be ≈ 15 MeV from the light hadronic decay rates of the χ_{c1} and χ_{c2} [13]. We set the factorization scale $\Lambda=m_c$. Following Ref. [12], we will take $H'_{8(c\bar{c})}(m_c)\approx 3$ MeV. Numerically, the color-octet contributions are minuscule. The total fragmentation probabilities for $c \to h_c$ and $c \to \chi_{cJ}$ are estimated to be

$$P_{c\to h_c} \approx 1.8 \times 10^{-5} , \tag{124}$$

$$P_{c\to \chi_{c0}} \approx 2.4 \times 10^{-5} , \tag{125}$$

$$P_{c\to \chi_{c1}} \approx 2.8 \times 10^{-5} , \tag{126}$$

$$P_{c\to \chi_{c2}} \approx 1.1 \times 10^{-5} . \tag{127}$$

The production of χ_{cJ} states contribute to the inclusive production rate of J/ψ through the radiative decay $\chi_{cJ} \to J/\psi + \gamma$. Multiplying the fragmentation probabilities given above by the appropriate branching ratios of 0.7%, 27%, and 14%, we find that the probability of a J/ψ inside a primary charm quark jet coming from the cascades of the χ_{cJ} states is approximately 0.9×10^{-5}. This is about an order of magnitude smaller than the probability 1.2×10^{-4} for the direct fragmentation of $c \to J/\psi$ obtained in Ref. [22], but still larger than the probability 3×10^{-6} for the direct fragmentation of $g \to J/\psi$ obtained in Ref. [15]. The probability of finding a J/ψ inside a primary gluon jet coming from the cascades of the χ_{cJ} states has also been estimated to be about 8×10^{-5} [12]. The inclusive J/ψ production rate at large transverse momentum region in $p\bar{p}$ collisions is dominated by the gluon fragmentation into χ_{cJ} followed by their radiative decays into J/ψ [12]. Fragmentation contributions to the inclusive production cross sections of J/ψ, ψ', and χ_{cJ} at $p\bar{p}$ colliders have been studied extensively in Refs. [25–28].

Fragmentation is the dominant mechanism for production of $(\bar{b}c)$ mesons at large transverse momentum in high energy colliders. In this paper, we have calculated the fragmentation functions for production of the P-wave $(\bar{b}c)$ mesons to leading order in α_s. These P-wave fragmentation functions can be used to calculate the inclusive P-wave $(\bar{b}c)$ mesons production cross sections at large transverse momentum. The production of the P-wave states account for about 20% of the inclusive production rate of the ground state B_c. The D-wave contributions to the inclusive production rate of B_c should be smaller than those of the P waves. Thus these P-wave fragmentation functions, together with the S-wave fragmentation functions obtained in Ref. [5], should allow an accurate calculation of the inclusive production rate of the B_c meson at large transverse momentum in high energy colliders [24].

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