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Proton-proton elastic scattering: Landshoff contributions in the diquark model

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Independent multiple scattering ("Landshoff") contributions to proton-proton elastic scattering at wide angles are calculated in the quark-diquark model. Results confirm previous observations on the magnitude of these contributions. The use of the quark-diquark model extends the applicability of perturbative QCD calculations down to lower values of momentum transfer substantially.

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I. INTRODUCTION

Despite tiny cross sections and corresponding difficulties for experimentalists, exclusive processes are the natural approach to study the composite character of hadrons. For large momentum transfer the wave functions deeply penetrate each other without producing a torrent of secondary particles in the final state. Thus compositeness is probed without destroying the observed configuration.

Perturbative quantum chromodynamics (PQCD) in the framework of the hard scattering picture (HSP) [1–3] is the generally accepted theory to describe exclusive processes at large momentum transfer. Factorization of long- and short-range physics, the basic assumption of the HSP, is reflected in the fact that exclusive quantities are expressed as convolutions of process-independent distribution amplitudes (DA's) with a perturbative, hard amplitude for the scattering of nearly collinear constituents.

The applicability of PQCD at intermediate momentum transfer of a few GeV, where experimental data are available, is a matter of passionate controversy [4,5]. The overall momentum transfer in the process has to be shared among the constituents in order to align them suitably for subsequent hadronization into the final state. Consequently, the corresponding strong coupling in parts of the process may become too large for reasonable use of perturbative methods. In particular, this is the case when the momentum of a hadron is unequally shared among its constituents.

Hard elastic proton-proton scattering will certainly be a cornerstone of investigations on the hadronic structure. Unfortunately, the relative complexity of the scattering of composite objects off each other, even if one only takes into account the valence quark Fock states and diagrams on the Born level, has prevented the complete cross sections from being calculated up to now. The complexity is revealed in the huge number of diagrams to be calculated ($\simeq$ in the order of 100 000), as well as in the occurrence of pinch singularities, which are closely related to the existence of independent, multiple scattering ("Landshoff" [6]) contributions.

In a novel treatment of the Landshoff mechanism in elastic proton-proton scattering Botts and Sterman [7,8] pointed out the need for taking into account transverse momenta in the HSP, which have been neglected before. The role of transverse momenta in the Landshoff mechanism is manifold: The energy dependence of the cross section is understandable when one takes into consideration the scaling behavior of momentum components transverse to the scattering plane. Also, as has been shown in [7], the way transverse momenta are dealt with is decisive in deriving a factorized formula for the scattering amplitude. Soft, gluonic ("Sudakov" [9]) corrections have been resummed by the use of renormalization group techniques. Here, the transverse separation between constituents (i.e., the conjugate variable of the transverse momentum) acts as an infrared cutoff and provides the finiteness of the results of loop integrations. The resulting Sudakov factor leads to a suppression of the scattering amplitude and, thus, affects the probability for a proton to contribute to elastic scattering, depending on the transverse separation between the constituents of the proton.

The work of Botts and Sterman [7] on the Landshoff mechanism initiated an approach [10,11] which is addressed to refuting the above-mentioned criticism [4,5] of the applicability of perturbative methods by modifying and improving on the HSP. The basic idea, which has been demonstrated for the calculation of electromagnetic form factors in [10,11], is to take into account transverse momentum flow through the hard scattering amplitude. Dangerous soft integration regions, where the validity of perturbative formulas becomes doubtful, are damped by the Sudakov corrections. Thus, self-consistency of the perturbative calculation is achieved in the modified HSP even for momentum transfers as low as a few GeV. Here, self-consistency is meant in the sense that the bulk of the results is derived with reasonably small values for the strong coupling. Additionally, the nonperturbative, intrinsic transverse structure turns out to be important, as it strengthens the suppression of soft regions. On the other hand, it provides a substantially smaller pertubative result [12,13].

In the course of these developments the role of transverse momenta in hard scattering processes, and correspondingly the transverse structure of hadrons, has received a lot of attention [7,8,10,11,14–17]. In particular, Sotiropoulos and Sterman [18] have discussed the proton-proton elastic scattering near the forward direction in two recent articles.

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In the present paper the Landshoff contributions of proton-proton elastic scattering at wide angles are calculated within a model in which the proton is considered as a quark-diquark system. The treatment of two correlated quarks as an effective diquark is a possibility to cope with nonperturbative effects still present in the kinematic range of interest. A systematic study of photon-proton reactions has been carried out in the quark-diquark model: form factors in the spacelike and timelike region, real and virtual Compton scattering, two-photon annihilations into proton-antiproton pairs, as well as the photoproduction of mesons [19–21].

The motivation for the present investigation in the quark-diquark model is the hope for an improvement of the applicability of the HSP down to lower energies, compared to observations made by Botts [8] in the pure quark picture. On the other hand, the reduction of complexity (two constituents instead of three to deal with in the valence Fock states) results in a technical simplification, which, though not dramatic for the Landshoff contributions, might become decisive for future attempts to calculate all HSP diagrams (several 100s of diagrams instead of several 100 000s).

The paper is organized as follows. In Sec. II the elements of the quark-diquark model necessary for the present calculation are briefly introduced. The mechanism of independent scatterings is envisaged in Sec. III, where gluonic Sudakov corrections in the context of the quark-diquark model are discussed and a factorized formula for helicity amplitudes for the Landshoff contribution to elastic proton-proton scattering at wide angles is given. Section IV contains a discussion of numerical results. Conclusions are given in Sec. V.

II. THE QUARK-DIQUARK MODEL

The basic assumption of the diquark model is the clustering of two of the three valence quarks in a baryon on an intermediate energy scale, which allows us to describe these two quarks, including correlation effects, as an effective particle, the diquark. Hence some nonperturbative effects still present on this intermediate scale are taken into account. The coupling of spin-1/2 and flavor (isospin-1/2) wave functions of two quarks leads to scalar and vector diquark wave functions. The symmetry of proton wave functions requires the spin and the flavor parts of the diquark wave functions to have the same symmetry. In this paper only the scalar sector of the model is considered, which is known to give rise to the bulk of numerical results. The vector sector is esssential for spin effects, but may be negligible for cross section results. The quark-scalar diquark (S) Fock state contribution to the proton state as a function of the usual longitudinal momentum fraction x and transverse momentum $\mathbf{k}_\perp$ of the quark with respect to the proton's momentum P is

$$|P,\lambda\rangle_{Sq} = \int \frac{dx\, d^2k_\perp}{16\pi^3\sqrt{x\,x'}}\, \Psi_S(x,\mathbf{k}_\perp) \times |S(x',\mathbf{k}'_\perp)\, u_\lambda(x,\mathbf{k}_\perp)\rangle \ , \qquad (1)$$

where $x' = 1 - x$ is the longitudinal momentum fraction of the diquark and $\mathbf{k}'_\perp$ its transverse momentum.

The dynamical content of the model is invoked by treating the diquark as an elementary particle with corresponding Feynman rules for the propagator of a scalar diquark and the gluon diquark vertex as

$$\begin{aligned} S \text{ propagator}: &\quad \frac{i}{p^2 - m_S{}^2 + i\varepsilon}\delta_{ij}, \\ SgS \text{ vertex}: &\quad i\, g_s\, t^a\, (p_1 + p_2)_\mu \ , \end{aligned} \qquad (2)$$

respectively, where $g_s = \sqrt{4\pi\alpha_s}$ is the QCD coupling, i, j are color indices, and $t^a = \lambda^a/2$ are the Gell-Mann color matrices. Diquarks are in an antitriplet color state, as is necessary to form a color neutral baryon out of a diquark and a single colored quark. The composite nature of the diquarks is taken into account by the introduction of phenomenological vertex functions which may be parametrized by

$$F_S(Q^2) = \delta_S \left(\frac{Q_S{}^2}{Q_S{}^2 + Q^2}\right),$$
$$\delta_S = \begin{cases} 1 & \text{for} \quad Q^2 < Q_S^2, \\ \alpha_s(Q^2)/\alpha_s(Q_S^2) & \text{for} \quad Q^2 \geq Q_S^2, \end{cases} \qquad (3)$$

where Q^2 is the modulus of the squared momentum of the gluon entering the vertex. This form is chosen to ensure that the diquark model evolves into the pure quark HSP in the limit $Q^2 \to \infty$.

III. LANDSHOFF CONTRIBUTIONS TO *pp* ELASTIC SCATTERING

A. The Landshoff mechanism

Independent scatterings in exclusive processes occur when pairs of constituents accidently scatter by the same angle. In this case the momentum transfer has not to be distributed in the hadrons any further in order to guarantee nearly collinear outgoing constituents, which are able to hadronize again; the outgoing constituents are already suitably aligned by chance in this special kinematical situation. This results in a lower minimal number of gluons to be exchanged as compared to the minimal number of gluons necessary in a general HSP diagram. Consequently, with increasing energy Landshoff contributions do not decrease according to "dimensional counting" rules [22], but a bit slower.

This so-called Landshoff effect may be explained by the observation that components of momenta transverse to the scattering plane exhibit a different scale dependence as compared to the other components. The reason for this behavior is that independent scatterings can be spatially separated in the direction transverse to the scattering plane. On the contrary, with respect to directions in the scattering plane the independent hard scatterings are restricted to take place in a small region, the extension of which is inversely proportional to the center of mass energy, i.e., proportional to $1/Q$.

The scaling behavior may be illustrated by considering the kinematics of a Landshoff process. In the quark-diquark model two types of diagrams contribute as indicated in Fig. 1. Figure 1(a) corresponds to an independent quark-quark and diquark-diquark scattering, whereas Fig. 1(b) shows two independent quark-diquark scatterings. The kinematics of Fig. 1(a) will be discussed in the following; the kinematics of Fig. 1(b) can be inferred from the former by substitutions. Neglecting masses and choosing the scattering plane to be the (z-x) plane, the momenta in the hadronic center of mass system are given by

$$\begin{aligned} &P_1 = (Q, 0, 0, Q), \qquad P_2 = (Q, 0, 0, -Q), \\ &P_3 = (Q, Q\sin\theta, 0, Q\cos\theta), \\ &P_4 = (Q, -Q\sin\theta, 0, -Q\cos\theta)\ . \end{aligned} \tag{4}$$

The internal quark momenta [see Fig. 1(a)] may be parametrized as

$$\begin{aligned} \mathbf{p}_1 &= x_1\mathbf{P}_1 + \mathbf{k}_1 \quad \Rightarrow\ p_1 = (x_1Q + \sigma_1/Q, k_{1x}, k_{1y}, x_1Q), \\ \mathbf{p}_2 &= x_2\mathbf{P}_2 + \mathbf{k}_2 \quad \Rightarrow\ p_2 = (x_2Q + \sigma_2/Q, k_{2x}, k_{2y}, -x_2Q), \\ \mathbf{p}_3 &= x_3\mathbf{P}_3 + \mathbf{k}_3 \quad \Rightarrow\ p_3 = (x_3Q + \sigma_3/Q, x_3Q\sin\theta + k_{3x}, k_{3y}, x_3Q\cos\theta + k_{3z}), \\ \mathbf{p}_4 &= x_4\mathbf{P}_4 + \mathbf{k}_4 \quad \Rightarrow\ p_4 = (x_4Q + \sigma_4/Q, -x_4Q\sin\theta + k_{4x}, k_{4y}, -x_4Q\cos\theta + k_{4z})\ . \end{aligned} \tag{5}$$

The internal momenta of the diquarks, p_i', have an analogous form. In the energy components extra terms, σ_i/Q, are included to allow for "on-shellness" of the quarks. Assuming all transverse momenta (and, therefore, all σ_i induced in the energy components by the transverse momenta) to be small compared to Q, the four-momentum conservation for the quark-quark scattering reads

$$\begin{aligned} &\delta^{(4)}(p_1 + p_2 - p_3 - p_4) \\ &\quad \simeq \frac{\delta(x_1 - x_3)\,\delta(x_2 - x_4)\,\delta(x_1 - x_2)}{2Q^3\sin\theta} \\ &\quad\quad \times\delta(k_{1y} + k_{2y} - k_{3y} - k_{4y})\ . \end{aligned} \tag{6}$$

All longitudinal momentum fractions x_i involved in the quark-quark scattering are constrained to be equal. This is characteristic for the special kinematic situation in the Landshoff mechanism. The usual, hadronic Mandelstam variables take the values $s = 4Q^2$ and $t = -2Q^2(1 - \cos\theta)$. Their partonic counterparts are approximated by $\hat{s} \simeq x^2 s; \hat{t} \simeq x^2 t; \hat{s}' \simeq x'^2 s; \hat{t}' \simeq x'^2 t$. Hence, both partonic scattering angles are equal to the scattering angle of the hadronic process and, therefore, aligned constituents remain aligned during the scattering process. The power of Q in the denominator of Eq. (6) is determined by the scaling behavior of the energy component and the two momentum components in the scattering plane. The momentum conservation in the direction transverse to the scattering plane (here in the y direction) is independent of the Q scale, as can be seen from Eq. (6).

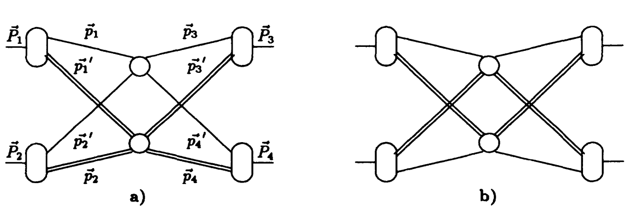

FIG. 1. Types of diagrams contributing to elastic proton-proton scattering at wide angles via the Landshoff mechanism. Protons are considered as quark-diquark systems; double lines indicate the diquarks.

The energy dependence of the hadronic scattering amplitude for the Landshoff process may now be summed up as

$$M_{fi} \sim Q^{-3}\ F_S^2(Q^2) \quad \text{(modulo logs)}\ . \tag{7}$$

The factor Q^{-3} originates from the momentum conservation for the quark-quark scattering, Eq. (6). The momentum conservation for the diquark-diquark scattering has not to be considered separately, because it is automatically implied due to the overall (hadronic) momentum conservation, if that for the quark-quark scattering holds. The second factor, $F_S^2(Q^2)$, stems from the diquark-diquark scattering, whereas the quark-quark scattering only depends on the scattering angle. The hadronic wave functions depend only logarithmically on the energy scale. Consequently, the Landshoff contributions to the differential cross section for elastic proton-proton scattering in the quark-diquark description behave as

$$\frac{d\sigma}{dt} = f(s/t)\cdot s^{-5}\cdot F_S^4(Q^2) \longrightarrow f(s/t)\cdot s^{-9} \quad \text{for}\quad s\to\infty\ . \tag{8}$$

This has to be compared with the predictions from the dimensional counting rules: Inserting one additional hard gluon in a Landshoff diagram converts it into a HSP diagram. Then, there are no longer two separate momentum conservations, and the Q^{-3} dependence caused by the δ function is dropped. But instead, the additional elements of the Feynman diagram give rise to a factor Q^{-4}. (The insertion of a virtual gluon between two quark lines, for example, introduces two quark propagators, two quark-gluon vertices, and the gluon propagator, behaving asymptotically like $\{Q^{-1}\}^2$, $\{Q^0\}^2$, and $\{Q^{-2}\}^1$, respectively.) Thus, the amplitude for a general HSP diagram behaves as $Q^{-4}F_S^2(Q^2)$, which asymptotically leads to an s^{-10} behavior for the differential cross section.

B. Sudakov corrections

The leading radiative corrections to elastic proton-proton scattering are similar in form to vertex loop corrections. For the case of QED Sudakov [9] has shown that the coincidence of "soft" and "collinear" divergences in vertex corrections typically leads to double logarithmic terms. Infrared divergencies are regularized in these calculations by allowing for small virtualities of external fermion lines. A similar form has been derived for QCD vertex corrections [23], where the non-Abelian character of QCD is reflected in the appearance of $\ln[\ln(q^2/m^2)]$ terms. Higher order of loop corrections may be taken into account by exponentiating single-loop results [24].

In the quark-diquark picture, Sudakov corrections to proton-proton scattering are very similar to the corrections in the pion-pion case, due to the fact that diquarks carry the same color as antiquarks. In Fig. 2 two types of gluonic corrections are indicated. In axial gauge leading logarithms are given by corrections of type I, which may be factorized into the wave functions. Corrections of type II, which are nonfactorizable, result in nonleading logarithms.

A single-loop calculation in leading logarithm approximation has been carried out for gluonic corrections to the proton wave function in the quark-diquark model. Exponentiating the result to account for higher loops (but not for nonleading logarithms) leads to a suppression factor

$$\exp\left[-S(x,b,Q)\right] = \exp\left[-s(x,b,Q) - s(1-x,b,Q)\right] \quad (9)$$

with

$$s(x,b,Q) = \frac{C_F}{2\beta_1}\left\{ \ln\left(\frac{x\sqrt{2}Q}{\Lambda_{\rm QCD}}\right) \ln\left[\frac{\ln\left(x\sqrt{2}Q/\Lambda_{\rm QCD}\right)}{-\ln\left(b\Lambda_{\rm QCD}\right)}\right] - \ln\left(\frac{x\sqrt{2}Q}{\Lambda_{\rm QCD}}\right) - \ln(b\Lambda_{\rm QCD})\right\}, \quad (10)$$

where $C_F = 4/3$ is the color factor and $\beta_1 = (11 - 2/3 n_f)/4$. Throughout this paper $n_f = 3$ and $\Lambda_{\rm QCD} = 200\,{\rm MeV}$ is used. The result in Eq. (9) and Eq. (10) is equal to the correction for a pion wave function and, hence, confirms that Sudakov corrections depend on color and not on spin.

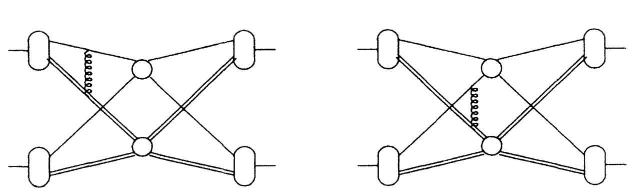

FIG. 2. One-loop gluonic corrections to the diagrams of Fig. 1. Type I corrections may be factorized into the wave functions. Type II corrections are nonfactorizable.

An essential point in Eq. (10) is the appearance of the "impact parameter" b, which acts as an infrared cutoff. The physically intuitive picture is that the proton is viewed as a color dipole, formed by a quark and a diquark. Therefore the momentum range of soft gluons contributing to the corrections is limited: The upper limit is given by the large component of the quark (diquark) momentum, i.e., $x\sqrt{2}Q$ or $x'\sqrt{2}Q$, respectively. Harder gluons are considered as higher order corrections to the hard scatterings and not as a part of the soft Sudakov corrections. The lower limit is induced by the inverse of the transverse separation of the color charges, $1/b$. Gluons with wavelengths larger than the dipole parameter b effectively experience a color neutral object and decouple from the proton. The larger the range of momenta between these two limits for a given configuration, the stronger is the suppression by the Sudakov factor. For a very small transverse separation the infrared limit $1/b$ is close to the upper limit; there is no suppression. A larger value of b results in a strong suppression. In Fig. 3 the Sudakov factor Eq. (9) is displayed for a given value of $x = 0.5$ and different values of Q. Clearly the suppression tends to force b to zero for increasing Q.

Although the tendency of the Sudakov corrections to keep colored constituents together is somehow similar to the effect of confinement, it should be emphasized that Eq. (9) and Eq. (10) are entirely perturbative. The Sudakov factor describes the fact that the probability for a scattering process to take place in the exclusive channel is decreasing with increasing spatial separation. It should not be mixed up with the nonperturbative effect

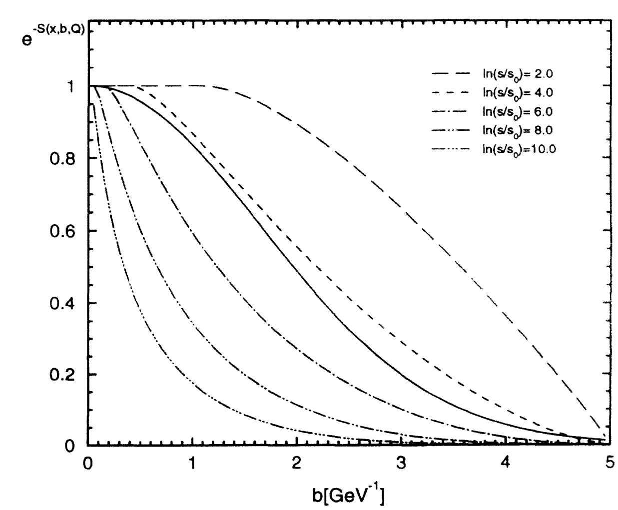

FIG. 3. Sudakov factor of Eq. (9) for $x = 0.5$ and different values of $\ln(s/s_0)$ with $s_0 \equiv 1\,{\rm GeV}^2$ (dashed and dashed-dotted lines). For comparison the Gaussian $\exp\left(-b^2/4\beta^2\right)$ caused by the intrinsic transverse momentum dependence is also shown (solid line).

of confinement.

Resummation techniques based on renormalization group equations have been developed to take into account leading as well as nonleading logarithms to all orders [25,26,7]. Working in a phenomenological model like the quark-diquark model, it seems reasonable to take a pragmatic point of view: Only the exponentiated, leading logarithmic corrections of Eq. (9) and Eq. (10) are considered in the present calculations. Tacitly it is assumed that the neglect of nonleading corrections is an acceptable approximation, as is indicated by the results of resummations in the pure quark picture.

Mueller [27] and Botts and Sterman [7] have shown, for the cases of pion-pion and proton-proton scattering in the pure quark picture, that gluonic Sudakov corrections to the Landshoff contributions shift the power of the asymptotic behavior near to the dimensional counting expectation. Their arguments can readily be transferred to the present case of proton-proton scattering viewed in the diquark model: Assuming, for the moment, that the only b dependence of the amplitude is contained in the Sudakov factors Eq. (9) of the four proton wave functions, the integration over the b space can be estimated by insertion of Eq. (10) and the use of a saddle-point approximation in the form

$$\int_0^\infty db\ \exp\left[-4\,S(x,b,Q)\right] \simeq \frac{\sqrt{2\pi}}{\Lambda_{\rm QCD}}\,\frac{\sqrt{c}}{1+c}\,\sqrt{\ln\left(\sqrt{2xx'}Q/\Lambda_{\rm QCD}\right)} \times\left(\frac{\sqrt{2xx'}Q}{\Lambda_{\rm QCD}}\right)^{-c\,\ln\left(\frac{1+c}{c}\right)}, \quad (11)$$

where

$$c=\frac{4\,C_F}{\beta_1}=\frac{64}{27},\qquad b_{\rm SP}=\frac{1}{\Lambda_{\rm QCD}}\left(\frac{\sqrt{2xx'}Q}{\Lambda_{\rm QCD}}\right)^{-\frac{c}{1+c}}. \quad (12)$$

Thus the leading power of Q, induced in the amplitude by the Sudakov corrections, is given by $-c\ \ln\,(1+1/c) = -0.83$. Consequently the power of s in Eq. (8) for the differential cross section is changed to -9.83, which will be not distinguishable experimentally from a power -10 in the foreseeable future.

C. Hadronic helicity amplitudes

Using Eq. (1) for the proton helicity states, a matrix element for the hadronic process reads

$$\langle P_3P_4|T|P_2P_1\rangle=\int\prod_{i=1}^{4}\frac{dx_i\,d^2k_{\perp i}}{16\pi^3\sqrt{x_ix_i'}}\,\Psi_S^*(\mathbf{p}_3)\Psi_S^*(\mathbf{p}_4)\Psi_S(\mathbf{p}_2)\Psi_S(\mathbf{p}_1) \times\left\{\langle q_3q_4|\hat{T}|q_1q_2\rangle\langle S_3S_4|\hat{T}'|S_2S_1\rangle+\langle q_3S_4|\hat{T}|q_2S_1\rangle\langle S_3q_4|\hat{T}'|S_2q_1\rangle\right\}, \quad (13)$$

where $\hat{T}$ and $\hat{T}'$ denote the two partonic transition matrices of the independent scatterings. S_i and q_i symbolize the ith scalar diquark and the ith quark, respectively. The momenta of hadrons, P_i, quarks, p_i, and diquarks, p_i', are defined as indicated in Eq. (4) and Eq. (5). Recalling the relation between transition matrix elements and Feynman amplitudes $T_{fi}=i\,(2\pi)^4\,\delta^{(4)}\,(\mathbf{P}_f-\mathbf{P}_i)\ M_{fi}$ and the fact that momentum conservation for each of the independent partonic scatterings implies the overall hadronic momentum conservation, Eq. (6) leads to

$$M_{fi}(s,t)=\frac{i}{(2\pi)^8}\,\frac{1}{2^5\,Q^3\,\sin\vartheta}\int_0^1\frac{dx}{x^2x'^2}\int\prod_{i=1}^{4}d^2k_{\perp i}\ \delta\left(k_{1y}+k_{2y}-k_{3y}-k_{4y}\right) \times\Big\{\Psi_S^*(x,\mathbf{k}_{\perp 4})\Psi_S^*(x,\mathbf{k}_{\perp 3})\ \hat{M}_{qq;qq}(x,\hat{s},\hat{t})\hat{M}'_{SS;SS}\ (x,\hat{s}',\hat{t}')\ \Psi_S(x,\mathbf{k}_{\perp 2})\Psi_S(x,\mathbf{k}_{\perp 1}) +\Psi_S^*(x',-\mathbf{k}_{\perp 4})\Psi_S^*(x,\mathbf{k}_{\perp 3})\ \hat{M}_{qS;qS}(x,\hat{s},\hat{t})\hat{M}'_{Sq;Sq}\ (x,\hat{s}',\hat{t}')\ \Psi_S(x',-\mathbf{k}_{\perp 2})\Psi_S(x,\mathbf{k}_{\perp 1})\Big\}, \quad (14)$$

where $\hat{M}$ and $\hat{M}'$ denote the partonic amplitudes. Following the basic idea of Botts and Sterman [7], a factorized formula can be derived, when the remaining δ function in Eq. (14), which is caused by the conservation of momentum components transverse to the scattering plane, is expressed by its Fourier transform

$$\delta\left(k_{1y}+k_{2y}-k_{3y}-k_{4y}\right)=\frac{1}{2\pi}\int_{-\infty}^{\infty}db\ e^{i\,b\cdot(k_{1y}+k_{2y}-k_{3y}-k_{4y})}\,. \quad (15)$$

The terms $e^{i\cdot k_{iy}}$ may be reabsorbed together with momentum integrations over the k_{iy} by the definition of wave functions in the form

$$\tilde{\Psi}_S(x, k_{ix}, b) \equiv \int \frac{dk_{iy}}{2\pi}\, \Psi_S(x, \mathbf{k}_{\perp,i}) e^{-i\, b\cdot k_{iy}} \ . \tag{16}$$

These wave functions $\tilde{\Psi}_S(x, k_{ix}, b)$ are the Fourier transforms of the old ones $\Psi_S(x, \mathbf{k}_{\perp,i})$ with respect to the y component. Equation (16) defines the parameter b to be the conjugate variable to the transverse momentum k_{iy}. Hence, as was mentioned above, b may be associated with the separation of quark and diquark in the proton in the y direction. The effects of soft gluonic Sudakov corrections are taken into account in leading logarithm approximation at this stage of the calculation by multiplying the wave functions $\tilde{\Psi}(x, k_{ix}, b)$ with the exponential factor $\exp[S(x, b, Q)]$ of Eq. (9).

The insertion of Eq. (15) and the use of the definition Eq. (16) leads to the factorized formula

$$\begin{aligned} M_{fi}(s,t) = &\frac{i}{(2\pi)^5}\, \frac{1}{2^5\, Q^3\, \sin\theta} \int_0^1 \frac{dx}{x^2 x'^2} \int db \int \sum_{i=1}^{4} dk_{ix} \ \ \exp\left[-4\, S(x,b,Q)\right] \\ &\times \Big\{ \tilde{\Psi}^*_S(x, k_{4x}, b) \tilde{\Psi}^*_S(x, k_{3x}, b)\, \hat{M}_{qq;qq}(x, \hat{s}, \hat{t}) \hat{M}'_{SS;SS}(x, \hat{s}', \hat{t}')\, \tilde{\Psi}_S(x, k_{2x}, b) \tilde{\Psi}_S(x, k_{1x}, b) \\ &\quad + \tilde{\Psi}^*_S(x', k_{4x}, b) \tilde{\Psi}^*_S(x, k_{3x}, b)\, \hat{M}_{qS;qS}(x, \hat{s}, \hat{t}) \hat{M}'_{Sq;Sq}(x, \hat{s}', \hat{t}')\, \tilde{\Psi}_S(x', k_{2x}, b) \tilde{\Psi}_S(x, k_{1x}, b) \Big\} \end{aligned} \tag{17}$$

for the helicity amplitude. Note that it is the inclusion of transverse momenta in the calculation which provides the key to deriving the factorized formula. This is based on the simple fact that the Fourier transform of a convolution integral factorizes. Furthermore, the gluonic corrections are treated such that they are described by exponential factors to the wave functions, which do not destroy the factorization.[1]

To perform the integrations over transverse momenta k_{xi} and k_{yi}, the latter contained in the definition of the $\tilde{\Psi}_S(x, k_{ix}, b)$, an ansatz for the wave functions has to be made. Here, the choice

$$\Psi_S(x, \mathbf{k}_\perp) = f_S\ \phi_S(x)\ \Sigma(x, \mathbf{k}_\perp) \tag{18}$$

is used where

$$\Sigma(x, \mathbf{k}_\perp) = 16\pi^2\, \beta^2\, g(x)\ \exp\left[-g(x)\, \beta^2\, k_\perp^2\right]$$

and

$$g(x) = 1 \quad \text{or} \quad 1/xx' \ . \tag{19}$$

The transverse momentum dependence is modeled as a simple Gaussian, where the case $g(x) = 1$ assumes factorization of longitudinal and transverse degrees of freedom and the case $g(x) = 1/(xx')$ is inspired by harmonic oscillator wave functions transformed to the light cone, which have been proposed to describe meson wave functions [28]. Correspondingly, two types of DA's are used in the form

$$\phi_A(x) = N_A\, xx'^3\, \exp\left[-\beta^2 \left(\frac{m_q^2}{x} + \frac{m_S^2}{x'}\right)\right] \quad \text{for} \quad g(x) = 1/xx' \ , \tag{20a}$$

$$\phi_B(x) = N_B\, xx'^3 \ \text{for} \quad g(x) = 1 \ . \tag{20b}$$

The polynomial $\sim xx'^3$ is the equivalent to the asymptotic DA $\sim x_1x_2x_3$ in the pure quark picture and is related to the latter by integration over one degree of freedom. The values of N_A and N_B are fixed by the normalization condition $\int_0^1 \phi(x)dx = 1$. The Gaussian ansatz for the $k_\perp$ dependence models the unknown, intrinsic (nonperturbative) transverse structure of the proton. Note that fixing the oscillator parameter β with a phenomenological input, like the root mean square (rms) of the transverse momentum $\langle {k_\perp}^2\rangle^{1/2}$, introduces a hadronization or confinement scale.

With the ansatz of Eq. (19) the transverse momentum integrations lead to

$$\begin{aligned} M_{fi}(s,t) = &\frac{i\, 4\pi^3}{Q^3\, \sin\theta}\, {f_S}^4 \int_0^1 \frac{dx}{x^2 x'^2} \\ &\times \int db\ \phi^4(x)\ \hat{M}(x, \hat{s}, \hat{t})\ \hat{M}'(x, \hat{s}', \hat{t}') \\ &\times \exp\left[-\frac{b^2}{g(x)\beta^2}\right] \exp\left[-4\, S(x,b,Q)\right] \ . \end{aligned} \tag{21}$$

Equation (21) displays the two exponential suppression factors brought about by the intrinsic, nonperturbative transverse structure and by the perturbative Su-

[1] As was shown in [7] the factorized form holds even for loop corrections, which cannot be written as an exponential multiplying the wave functions (type II in Fig. 2), when a suitable "soft approximation" is used.

dakov corrections. Obviously, the borderline between both effects will not be clear cut in nature. Nevertheless, Eq. (21) indicates the point of view adopted in the present paper: The perturbative formula for the Sudakov factor is taken literally even in regions where perturbative calculations are known to become invalid, i.e., $1/b$ as low as $\Lambda_{\rm QCD}$. The intrinsic transverse structure, represented by the Gaussian, gives a weight function for the probability of finding transverse distances in a proton. Configurations with large b values, corresponding to the soft regions mentioned above (i.e., $1/b \to \Lambda_{\rm QCD}$), have a tiny probability to be found. Hence the error induced in the calculation by retaining incorrectly the perturbative Sudakov formula in the very soft region is expected to be small.

It is instructive to take a closer look at the interplay of both exponentials in Eq. (21). In Fig. 3 the Gaussian $\exp\left(-b^2/4\beta^2\right)$ [i.e., $g(x) = 1$] for the value $\beta^2 = 1.389$ GeV^{-2} (see Sec. IV) and the Sudakov factor $\exp\left[-S(x = 0.5, b, Q)\right]$, the latter for different values of $\ln(s/s_0)$ with $s_0 \equiv 1$ GeV^2, are shown for comparison. Clearly for large values of $\ln(s/s_0)$ the Sudakov factor dominates the product of the two exponentials. Thus, the asymptotic behavior of the cross section as estimated by the saddle-point approximation Eq. (11), i.e., $d\sigma/dt \sim s^{-9.83}$, is not affected by the additional intrinsic transverse structure. However, in the region of $\ln(s/s_0)$ smaller than $\simeq 5$ GeV^2 the Gaussian dominates the product of both exponentials. Hence, taking into account the intrinsic transverse structure, e.g., in the form of a Q-independent Gaussian as in the present work, damps the Q dependence implied by the Sudakov factor at least in the region of presently available data.

The transition from an energy region where the transverse structure is dominated by the nonperturbative intrinsic momentum dependence to a region where it is dominated by Sudakov corrections has been discussed recently in [18], too. There, for the proton-proton elastic scattering near the forward direction, the transition is supposed to show up in the differential cross section as a transition from a t^{-8} behavior to a t^{-10} behavior.

Using the Feynman rules of the quark-diquark model the helicity amplitudes can be calculated. Only three of them are nonzero and get contributions from four diagrams of the type shown in Fig. 1(a) and two diagrams of the type shown in Fig. 1(b):

$$M^{\rm hadr}_{\{\lambda\}}(s,t) = \frac{i\,4^4\pi^5 f_S{}^4}{9\,Q^3\sin\theta}\int_0^1 \frac{dx}{x^2x'^2}\int_{-\infty}^{\infty} db\,\exp\left[-\frac{b^2}{g(x)\beta^2}\right]\exp\left[-4\,S(x,b,Q)\right]\alpha_s(x^2t)\alpha_s(x'^2t)$$
$$\times\left\{\phi^4(x)\,F_S^2(x'^2\,t)P^{(i)}_{\{\lambda\}}(s,t) - 2\,\phi^2(x)\phi^2(x')\,F_S(x^2t)\,F_S(x'^2t)P^{(ii)}_{\{\lambda\}}(s,t)\right\}\,, \tag{22}$$

where $\{\lambda\}$ denote the three sets of helicities $(++,++)$, $(+-,+-)$, and $(-+,+-)$. The expressions $P^{(i)}_{\{\Lambda\}}$ and $P^{(ii)}_{\{\Lambda\}}$ are explicitly given by

$$P^{(i)}_{++,++}(s,t) = \left(\frac{s(s-u)}{t^2} + \frac{s(s-t)}{u^2} - \frac{s^2}{ut}\right); P^{(ii)}_{++,++}(s,t) = \left(\frac{su}{t^2} - \frac{st}{u^2}\right),$$
$$P^{(i)}_{+-,+-}(s,t) = \left(\frac{1}{3}\frac{u(s-t)}{ut} - \frac{u(s-u)}{t^2}\right); P^{(ii)}_{+-,+-}(s,t) = \left(\frac{su}{t^2}\right),$$
$$P^{(i)}_{-+,+-}(s,t) = \left(\frac{t(s-t)}{u^2} - \frac{1}{3}\frac{t(s-u)}{ut}\right); P^{(ii)}_{-+,+-}(s,t) = \left(\frac{st}{u^2}\right)\,. \tag{23}$$

Using these results the differential cross section

$$\left.\frac{d\sigma^{pp\to pp}}{dt}\right|_{\substack{\rm Ldsh\\ \rm diquark}}(s,t) = \frac{1}{16\pi}\,\frac{1}{s(s-4m_p{}^2)}\,\frac{1}{4}\left\{|M_{++,++}|^2 + |M_{+-,+-}|^2 + |M_{+-,-+}|^2\right\} \tag{24}$$

has been calulated for a scattering angle of 90°.

IV. NUMERICAL RESULTS

Parameters for the quark-diquark wave functions are taken from [21]: $Q_S = 3.22\,\text{GeV}^2$ and $\beta^2 = 0.247\,\text{GeV}^{-2}$ and $1.389\,\text{GeV}^{-2}$ for wave functions (20a) and (20b), respectively. These values for the oscillator parameter correspond to a rms transverse momentum $\langle k_\perp{}^2\rangle^{1/2}$ of 600 MeV. The value for $f_S = 73.85$ MeV has been fixed by fits to the data of electromagnetic form factors of the nucleons [21]; for the quark and the diquark the following constituent masses are used: $m_q = 330$ MeV and $m_S = 580$ MeV.

Soft end-point regions of integration over longitudinal momenta ($x \to 0$ or 1) with corresponding singularities in the strong couplings and in the gluon propagators are avoided by the introduction of a cutoff parameter C and the condition

$$\xi \geq C\frac{\Lambda}{\sqrt{2}Q} \qquad \text{for } \xi = x, x'\,. \tag{25}$$

Independence from the cutoff serves as an indication for the range of applicability of the formalism. In the region of small b values, $b \leq 1/\sqrt{2}xQ$, the Sudakov factor $e^{-S(x,b,Q)}$ is set to unity, which is its value at $b = 1/\sqrt{2}xQ$.

Results for Landshoff contributions to the differential cross sections at 90° obtained with the wave function from Eqs. (18), (19), and (20a) are shown in Fig. 4. The dimensionless quantity $R(s) = d\sigma/dt|_{90^\circ} \times 10^{-8} s^{10} s_0^{-8}$ is plotted, which should become constant according to the dimensional counting rules. Results obtained with wave function (20b) are very similar in shape and magnitude to those in Fig. 4. They are smaller by a few percent and the independence of the cutoff is shifted a bit to higher $\ln(s/s_0)$. Since differences are really tiny, no extra figure for this case is shown.

The rise of the curves at lower values of $\ln(s/s_0)$ is caused by the Q dependence of the cutoff prescription and, more important, by the behavior of the phenomenological vertex functions $F_S(Q^2)$, which have not yet reached their asymptotic Q^{-2} behavior. The position of the maximum is predominantly determined by the value of the diquark parameter Q_S. The decrease of the curves is induced by the behavior of the strong coupling; the cross section is proportional to α_s^4.

A comment about the experimental data should be made here: The data reveal roughly the expected scaling behavior with s^{-10}, modified by (as it seems to be) an oscillation. It has been suggested [29] that the data indeed do not show the beginning of an oscillation, but rather a two-peak structure, with the second peak caused by diquark correlations in the proton. The present results confirm the existence of a bump, even roughly peaked in the energy region of interest. However, the shape and magnitude of this bump disfavor this explanation for the structure of the data.

A different explanation has been suggested some years

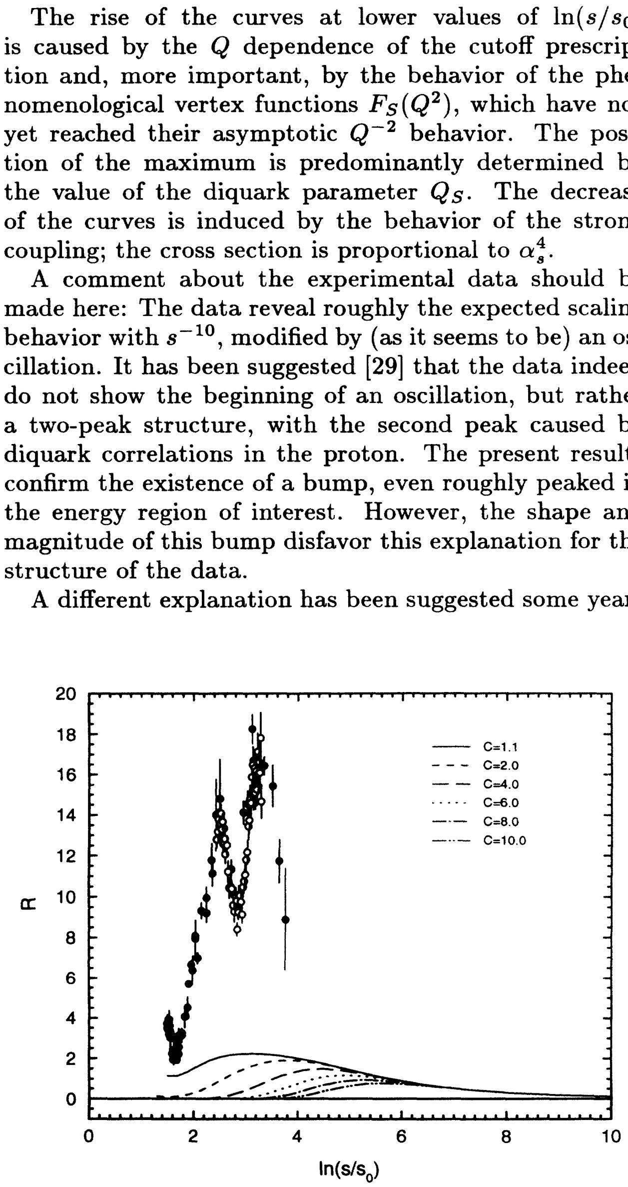

FIG. 4. Elastic proton-proton scattering at 90°. The dimensionless quantity $R(s) = d\sigma/dt|_{90^\circ} \times 10^{-8} s^{10} s_0^{-8}$ obtained with wave function (20a) is plotted against $\ln(s/s_0)$. Data are taken from the data compilation [31].

ago. Ralston and Pire [30] concluded the existence of a phase proportional to $\ln(s/s_0)$ from analyticity properties of gluonic corrections and fitted the coefficients to the data. Botts and Sterman [7] showed that corrections of type II in Fig. 2, neglected in the present paper, cause imaginary parts in the Sudakov functions. However, the resulting phase is proportional to the ratio $\ln(s/s_0)/\ln(b\Lambda)$, which approaches a constant in the region where the saddle-point approximation Eq. (11) is valid, because of the power-law behavior of the saddle-point $b_{SP} \sim s^{-0.35}$. Although the derivation of the phase from corrections of type II in Fig. 2 is beyond the scope of this paper, a comment can be made about this problem: The inclusion of the intrinsic transverse structure, neglected in previous papers, reconciles the s dependence of the phase in the region $\ln(s/s_0) \leq 5$. Here, the Gaussian damps the Sudakov factor and the dominantly contributing b region is almost s independent.

The magnitude of the results in Fig. 4 is roughly by a factor of 10 below the experimental data, but it is definitely not suppressed by many orders of magnitude as has been presumed before [2]. In this sense the result is an independent confirmation of the observations Botts made in the pure quark picture [8]. At $\ln(s/s_0) = 5$ the results in [8] vary in the range $R = 0.07$–9.0 depending on the distribution amplitude chosen and the value of the cutoff. This is in accordance with the range $R = 0.5$–1.5 found in the present calculation in the framework of the quark-diquark model. The largest uncertainty in the magnitude of the results is caused by the normalization of the wave functions. The value $f_S = 73.85\,\mathrm{MeV}$ is taken from fits to the electromagnetic form factors of the nucleons [21]. These have been done without assuming a specified $k_\perp$ dependence, which leads to the freedom to vary f_S in a limited range. Assuming a specified $k_\perp$ dependence, like the Gaussian in the present case, fixes the relation between $f_S, \langle k_\perp{}^2\rangle$, and P_{qS}, the probability to find a proton as a system of a quark and a scalar diquark. The presently used values of $\langle k_\perp{}^2\rangle^{1/2}$ and f_S correspond to $P_{qS} \simeq 1$. Constraining, for example, this probability to be $P_{qS} = 0.5$ would change f_S by a factor of 1/2 and correspondingly change the cross section by a factor of 1/16. These considerations may indicate that lack of knowledge about the nonperturbative wave functions easily induces uncertainties of one order of magnitude.

Comparison to experiment is, somehow, ambiguous because the results show strong cutoff dependence in the region of the data. Independence of the cutoff and therefore applicability of the formalism is reached for $\ln(s/s_0) \simeq 6$ (i.e., $s \simeq 400\,\mathrm{GeV}^2$). This has to be contrasted with a value of $\ln(s/s_0) \simeq 8$ (i.e., $s \simeq 3000\,\mathrm{GeV}^2$) given in [8]. Thus the hope of improving the applicability down to lower values of s by using the quark-diquark model is fulfilled.

A further improvement of applicability will surely be obtained by taking into account also the transverse momenta in the hard scattering amplitudes themselves, which have been neglected up to now. Thus, the strategy of [10,11], developed for form factors, could be adopted; i.e., to characterize soft regions by both small x and small transverse momenta (or large b values). These regions are suppressed by the Sudakov factor. The transfer of this concept to the present case of proton-proton scattering is not straightforward, because the inclusion of transverse momenta in the gluon propagators would destroy the factorized form of Eq. (21). What can safely be done to improve the applicability of the calculation is the replacement of the arguments of the strong coupling in the form

$$\begin{aligned} \alpha_s\left(x^2t\right) &\to \alpha_s\left[\max\left(x^2t, 1/b^2\right)\right], \\ \alpha_s\left(x'^2t\right) &\to \alpha_s\left[\max\left(x'^2t, 1/b^2\right)\right]. \end{aligned} \tag{26}$$

Thus, the largest scale in each independent hard process determines the strength of the coupling. For vanishingly small x, x' the transverse scale of the process takes over. A similar approach has been used by Sotiropoulos and Sterman [18] in the context of elastic proton-proton scattering near the forward direction.

Results with these prescriptions, Eq. (26), are displayed in Fig. 5 in comparison with the curves from Fig. 4. Evidently, they coincide asymptotically. For small values of $\ln(s/s_0)$ the modified version lies a bit below the results obtained with a cutoff $C = 1.1$. This effect is readily explained by noting that the arguments of the coupling have become smaller on the average. It is worth emphasizing that the modified result (i.e., the solid line in Fig. 5) is derived entirely without a cutoff (or $C = 0$). On the contrary, results of calculations without the replacement, Eq. (26), diverge for $C \leq 1$. The reliability of the modified calculation, and thus the effectiveness of Sudakov suppression of soft regions, can be checked by testing the portion which has been obtained with reasonably small values of the strong coupling, say $\alpha_s \leq 0.5$. It turns out that at $\ln(s/s_0) = 1.5$ already 52% and at $\ln(s/s_0) = 3$ even 84% of the full result fulfills this criterion.

Clearly, the simple replacement Eq. (26) does not substitute for a calculation with all the transverse momentum dependence taken into acount in the hard scatterings. But results are quite encouraging that a more complete, still lacking, calculation will render the formalism reliable down to the energy range of the data.

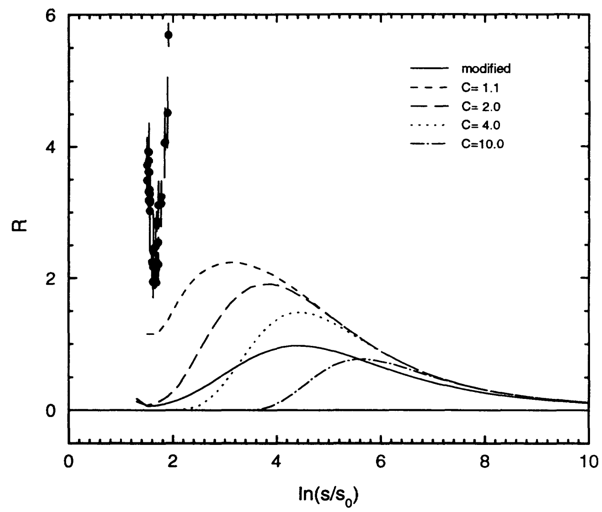

FIG. 5. Comparison of modified calculation, Eq. (26), with the cutoff method (cf. Fig. 4). Both calculations are done with wave function (20a).

V. CONCLUSIONS

It has been emphasized in this work that transverse momenta are the key to understanding multiple, independent scattering processes with respect to factorization and their scaling behavior. The present calculation of Landshoff contributions to the elastic proton-proton scattering at wide angles in the framework of the quark-diquark model is an independent confirmation of observations made before [8] in the pure quark picture. The magnitude of Landshoff contributions is small, but definitely not suppressed by many orders of magnitude and, therefore, *a priori* not negligible. The main uncertainties in the calculation stem from our incomplete knowledge of nonperturbative wave functions. The applicability of the calculation, as indicated by cutoff independence, is beyond the range of experimental access. It has been shown that the phenomenological quark-diquark model, as additional assumption to the hard scattering picture, improves the range of applicability of the perturbative calculations substantially down to smaller values of s, though still outside the energy region of experimental data.

A way out of this problem has been discussed and numerically tested by taking into account transverse momenta scales, at least in the argument of the strong coupling. The results clearly indicate that an extension of the HSP by taking into account transverse momenta consistently (i.e., also in the hard scattering amplitudes) will lead to self-consistency of the perturbative calculation and, thus, will improve the reliability of the results down to even small values of s.

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[1] S.J. Brodsky and G.R. Farrar, Phys. Rev. D **11**, 1309 (1975).
[2] G.P. Lepage and S.J. Brodsky, Phys. Rev. D **22**, 2157 (1980).
[3] A.V. Efremov and A.V. Radyushkin, Phys. Lett. **94B**, 245 (1980).
[4] N. Isgur and C.H. Llewellyn Smith, Phys. Rev. Lett. **52**, 1080 (1984); Nucl. Phys. **B317**, 526 (1989); Phys. Lett. B **217**, 535 (1989).
[5] F.M. Dittes and A.V. Radyushkin, Sov. J. Nucl. Phys. **34**, 293 (1981); A.V. Radyushkin, in *Proceedings of the European Workshop on Hadronic Physics with Electrons Beyond 10 GeV*, Dourdan, France, 1990, edited by B. Frois and J.F. Mathiot [Nucl. Phys. **A532**, 141c (1991)]; Report No. CEBAF-TH-93-12, 1993 (unpublished).
[6] P.V. Landshoff, Phys. Rev. D **10**, 1024 (1974).
[7] J. Botts and G. Sterman, Nucl. Phys. **B325**, 62 (1989).
[8] J. Botts, Nucl. Phys. **B353**, 20 (1991).
[9] V. Sudakov, Zh. Eksp. Teor. Fiz. **30**, 87 (1956) [Sov. Phys. JETP **3**, 65 (1956)].
[10] H.N. Li and G. Sterman, Nucl. Phys. **B381**, 129 (1992).
[11] H.N. Li, Phys. Rev. D **48**, 4243 (1993).
[12] R. Jakob and P. Kroll, Phys. Lett. B **315**, 463 (1993); **319**, 545(E) (1993).
[13] J. Bolz, R. Jakob, P. Kroll, M. Bergmann, and N.G. Stefanis, Wuppertal Report No. WU-B-94-06, 1994 (unpublished); Report No. WU-B-94-16, 1994 (unpublished).
[14] T. Hyer, Phys. Rev. D **47**, 3875 (1992).
[15] C. Coriano and H.N. Li, Phys. Lett. B **309**, 409 (1993).
[16] A.R. Zhitnitsky, Phys. Lett. B **329**, 493 (1994); Dallas Report No. SMU-HEP-94-01, 1994 (unpublished).
[17] T. Gousset and B. Pire, Saclay Report No. DAPNIA/SPhN-94-09, 1994 (unpublished).
[18] M. Sotiropoulos and G. Sterman, Nucl. Phys. **B419**, 59 (1994); Stony Brook Report No. ITP-SB-93-83, 1993 (unpublished).
[19] P. Kroll, in *Proceedings of the Adriatico Research Conference on "Spin and Dynamics in Nuclear and Particle Physics,"* Trieste, Italy, 1988, edited by A.O. Barut (World Scientific, Singapore, 1990); P. Kroll, M. Schürmann, and W. Schweiger, Z. Phys. A **338**, 339 (1991); **342**, 429 (1992); P. Kroll, M. Schürmann, and W. Schweiger, Int. J. Mod. Phys. A **6**, 4107 (1991).
[20] P. Kroll, T. Pilsner, M. Schürmann, and W. Schweiger, Phys. Lett. B **316**, 546 (1993).
[21] R. Jakob, P. Kroll, M. Schürmann, and W. Schweiger, Z. Phys. A **347**, 109 (1993).
[22] S.J. Brodsky and G.R. Farrar, Phys. Rev. Lett. **31**, 1153 (1973); V.A. Matveev, R.M. Muradyan, and A.V. Tavkhelidze, Lett. Nuovo Cimento **7**, 719 (1973).
[23] J.M. Cornwall and G. Tiktopoulos, Phys. Rev. D **13**, 3370 (1976).
[24] V.V. Belokurov and N.I. Ussyukina, Phys. Lett. **94B**, 251 (1980); H.D. Dahmen and F. Steiner, Z. Phys. C **11**, 247 (1981).
[25] J.C. Collins, Phys. Rev. D **22**, 1478 (1980); J.C. Collins and D.E. Soper, Nucl. Phys. **B193**, 381 (1981); **B194**, 445 (1982).
[26] A. Sen, Phys. Rev. D **24**, 3281 (1981).
[27] A.H. Mueller, Phys. Rep. **73**, 237 (1981).
[28] G.P. Lepage, S.J. Brodsky, T. Huang, and P.B. Mackenzie, in *Particles and Fields—2*, Proceedings of the Banff Summer Institute, Banff, Canada, 1981, edited by A.Z. Capri and A.N. Kamal (Plenum, New York, 1983).
[29] V.V. Anisovich, L.G. Dakhno, V.A. Nikonov, and M.G. Ryskin, Phys. Lett. B **292**, 169 (1992); in *Proceedings of the International Workshop on Quark Cluster Dynamics*, Bad Honnef, Germany, 1992, edited by K. Goeke, P. Kroll, and H.-R. Petry (Springer-Verlag, Berlin, 1993).
[30] J.P. Ralston and B. Pire, Phys. Rev. Lett. **49**, 1605 (1982); B. Pire and J.P. Ralston, Phys. Lett. **117B**, 233 (1982).
[31] J. Bystricki and F. Lehar, *Nucleon-Nucleon Scattering Data* (Fachinformationszentrum, Karlsruhe, 1978).