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Thermodynamics of relativistic fermions with Chern-Simons coupling

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We study the thermodynamics of the relativistic quantum field theory of massive fermions in three space-time dimensions coupled to an Abelian Maxwell-Chern-Simons gauge field. We evaluate the specific heat at finite temperature and density and find that the variation with the statistical angle is consistent with the nonrelativistic ideas on generalized statistics.

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I. INTRODUCTION

Statistics plays a central role in physics, and marks one of the main parting points between the classical and quantum domains. In conventional phenomena, taking place in $3+1$ dimensions, the invariance under the interchange of identical particles forces the wave function to be either symmetric or antisymmetric. The spin-statistics theorem in relativistic quantum field theory then ties this symmetry to the spin, and all fundamental excitations are classified as either fermions or bosons. Through the spin the fermionic or bosonic character can manifest itself even in one- or few-particle systems. However, it is in the thermodynamics of a system that a symmetric or antisymmetric wave function has a dramatic effect.

In (2+1)-dimensional phenomena new possibilities open up, and excitations of generalized statistics interpolating between fermions and bosons may occur. The role of these *anyons* [1] in the quantum Hall effect is fairly well established [2], and they may be relevant also in the superconductivity of some materials at high temperature [3]. Of course, in these and other possible phenomenological applications in two-dimensional condensed matter systems, anyons are just low-energy descriptions of the effective behavior of particles and interactions which, although restricted to two dimensions, exist in three-dimensional space and conform to the conventional boson-fermion classification.

However, the same reasons which make anyons possible at low energies, pose a challenging question. It is certainly of great theoretical interest to establish the extent to which anyonic excitations can exist as fundamental objects in a truly (2+1)-dimensional world, just as ordinary fermions or bosons, or if on the contrary, they are always a low-energy illusion. Mathematically, the possibility of generalized statistics has its origin in the topology of the configuration space of a many particle system in 2+1 dimensions. Under the double interchange of two identical particles, $x_1 - x_2 \to x_2 - x_1 \to x_1 - x_2$, the observability of the probability density requires that the wave function transforms as $\psi \to e^{i2\theta}\psi$. In space-time dimensions greater than three this double interchange is just the identity operation, so we are forced to $\theta = 0$ (bosons) or $\theta = \pi$ (fermions). In 2+1 dimensions, however, the winding of the trajectories followed by the particles as they are interchanged may be nontrivial, in which case the operation is not connected to the identity and θ remains unrestricted.

This, however, is only a mathematical possibility. Its realization in a quantum theory depends on nontrivial issues regarding the short distance behavior of the theory. Indeed, the very notion of the linking of two trajectories assumes some kind of hard core repulsive interaction between the particles (accounting for some generalized exclusion principle), and this may or may not be consistent with a formulation from first principles. In a nonrelativistic treatment, the transformation law of the anyonic wave function under particle interchange provides a well-defined meaning for the concept of generalized statistics. In practice this is implemented by minimally coupling the matter fields to an Abelian Chern-Simons gauge field. However, the nonexistence of an ideal anyon gas, analogous to the Bose and the Fermi gases, represents a major obstacle in the understanding of anyonic thermodynamics. A major tool in this respect has been the perturbative expansion in the statistical angle θ, which when done at the bosonic end exhibits the singular nature of the statistical interaction at short distances [4]. Although at the computational level this can be regularized with a repulsive δ-function contact potential, these short distance

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difficulties are beyond the scope of the non-relativistic theory.

The physics of anyons at short distances must be formulated and discussed in the framework of relativistic quantum field theory. This has been done mostly in the canonical formalism [5], where the main difficulty is again the nonexistence of free asymptotic many-particle states. Besides posing technical difficulties which are still controversial [5], this obscures somewhat the concept of generalized statistics in this framework. To a large extent, it has acquired in the literature a rather algebraic meaning, in terms of the phase factor which enters generalized canonical commutation relations, again interpolating between the conventional commutator and the anticommutator for bosons and fermions. But those algebraic relations are rather formal, so long as there is no direct relation with free asymptotic states, and they do not address the relevant short distant issues, so they may or may not turn out to be consistent with a proper regularization procedure.

Since one expects that the statistics obeyed by the fundamental excitations will have direct consequences in the thermodynamic properties, a possible way around these difficulties is to use the functional integral formulation to study the theory at finite temperature, thus avoiding such difficult issues as the lack of free asymptotic states or the exact kinematical meaning of generalized statistics in the relativistic theory. Here we adopt that point of view, and report our first results for some thermodynamic properties of massive fermions in three spacetime dimensions coupled to an Abelian Maxwell-Chern-Simons field. In particular, we evaluate the specific heat at finite temperature and density for different values of the statistical angle θ. The calculation is done to leading order in the conventional loop expansion, complemented with a low momentum approximation which allows for further simplifications.

We should admit from the start a limitation of our approach. At least within the loop expansion, the theory is not properly defined unless the Maxwell term is included in the pure gauge action. (Note, however, that a Maxwell term is generated anyhow by the quantum fluctuations of the matter fields, even if absent at the classical level.) As it happens, that term is also expected to conceal the effect of the generalized statistic induced by the Chern-Simons term, at least at short distances [6]. Thus, we do not expect to see in our results a very notorious shift, say, from fermionic to bosonic properties as we vary the statistical angle θ. To complicate matters further, in 2+1 dimensions there is no Bose-Einstein condensation, so the specific heat does not exhibit a dramatic difference between fermions and bosons, as we are used to in $3+1$ dimensions. Nevertheless, our results do show that the dependence of the specific heat in the statistical angle is consistent with a smooth transit from a fermionic to a bosonic behavior.

II. THE THEORY AT ZERO CHEMICAL POTENTIAL

We consider massive fermions in three Euclidean dimensions coupled to an Abelian Maxwell-Chern-Simons gauge field with a Lagrangian

$$\mathcal{L} = \bar{\psi}(i\not{\partial} + m)\psi + \frac{1}{4}F_{\mu\nu}^2 - i\frac{e^2}{8\theta}\epsilon_{\mu\nu\lambda}F_{\mu\nu}A_\lambda + e\bar{\psi}\not{A}\psi, \quad (1)$$

where e^2 has dimensions of mass and the statistical angle θ is a dimensionless parameter. Our choice of γ matrices is $\gamma_i = i\sigma_i$, with σ_i the usual Pauli matrices.

In order to establish conventions, describe our treatment and review previous results, we shall evaluate first the partition function at finite temperature and zero chemical potential. In the following section we introduce a nonvanishing chemical potential to discuss the thermodynamics of the system at finite fermion density.

We carry out the finite temperature calculations as usual, compactifying the (Euclidean) time variable into the range $0 \le \tau \le \beta = 1/T$ (in our units, $\hbar = c = k = 1$). Then, the functional integral defining the partition function should be computed using periodic (antiperiodic) boundary conditions (in time) for bosons (fermions). The partition function is then defined as

$$\mathcal{Z} = \mathcal{N}(\beta) \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu \, \exp\left(-S[A_\mu]\right), \quad (2)$$

with

$$S[A_\mu] = \int_0^\beta d\tau \int d^2x \, \mathcal{L} \equiv \int_\beta d^3x \, \mathcal{L}. \quad (3)$$

The relevance of the normalization factor $\mathcal{N}(\beta)$ will be discussed below. Integrating out the fermions one has

$$\mathcal{Z} = \mathcal{N}(\beta)\det{}_{(-)}(i\not{\partial} + m) \times \int \mathcal{D}A_\mu \, \exp\left[-\left(S[A_\mu] + S_q[A_\mu]\right)\right], \quad (4)$$

where the subindex $(-)$ means that the determinant has to be evaluated using antiperiodic boundary conditions, and

$$S_q[A_\mu] = -\ln\det{}_{(-)}\left(1 + \frac{e\not{A}}{i\not{\partial} + m}\right) \quad (5)$$

is the contribution of the fermionic quantum fluctuations to the effective action. To one-loop order this is

$$S_q[A_\mu] = \frac{e^2}{2}\mathrm{tr}\left(\frac{\not{A}}{i\not{\partial} + m}\right)^2. \quad (6)$$

When computing the trace, the antiperiodic boundary conditions are implemented in momentum space replacing integrals over p^0 by sums over the discrete Matsubara frequencies:

$$p_n^0 = \frac{(2n+1)\pi}{\beta}. \quad (7)$$

The one-loop result for S_q in Eq. (6) can be evaluated in closed form [7]. However, to simplify the numerical analysis, here we shall work to leading order in an expansion in powers of the momenta. Then, following Ref. [8] one finds

$$S_q[A_\mu] = h(\beta)\int_\beta d^3x \left(\frac{1}{6}F_{\mu\nu}^2 - \frac{im}{2}\epsilon_{\mu\nu\lambda}F_{\mu\nu}A_\lambda\right), \quad (8)$$

where the function $h(\beta)$ is given by

$$h(\beta) = \frac{e^2}{8m\pi}\tanh(m\beta/2). \quad (9)$$

Then, in this approximation the partition function is given by a Gaussian integral over the gauge field, times the partition function of free massive fermions:

$$\mathcal{Z} = \mathcal{N}(\beta)\det{}_{(-)}(i\not{\partial} + m)\int \mathcal{D}A_\mu \, \exp\left(-S_{\text{eff}}[A_\mu]\right),$$

where the effective action $S_{\text{eff}} = S + S_q$ is given by

$$S_{\text{eff}}[A_\mu] = \frac{1}{4}\int_\beta d^3x \left\{\left(1 + \frac{2}{3}h(\beta)\right)F_{\mu\nu}^2 - i\left(\frac{e^2}{2\theta} + 2mh(\beta)\right)\epsilon_{\mu\nu\lambda}F_{\mu\nu}A_\lambda\right\}. \quad (10)$$

From the coefficient of the Maxwell term, one sees that, as usual, the loop expansion requires $\alpha = e^2/4\pi m \ll 1$. Similarly, from the coefficient of the Chern-Simons term one would conclude that in this case the loop expansion also requires $\theta \ll 2\pi$. Yet, from the experience gained with three-dimensional fermions at zero temperature [9,10], one could hope that this restriction over θ may not really apply. This, however, is controversial at finite temperature [11].

As discussed in [12], at finite temperature one has to take into account the contribution from the Faddeev-Popov determinant arising from gauge fixing, even if the theory is Abelian. We shall choose the Lorentz gauge ($\partial_\mu A_\mu = 0$) for which the Faddeev-Popov determinant is given by

$$\Delta_{\text{FP}} = \det{}_{(+)}(-\partial^2) \quad (11)$$

where now the (+) subindex indicates that the determinant has to be evaluated using periodic boundary conditions [12]. Evaluating the contribution to the determinants coming from the space-time indices and keeping track of all temperature-dependent factors, one obtains

$$\mathcal{Z} = \det{}_R(i\not{\partial} + m)\det{}_R^{-1/2}(-\partial^2 + M^2), \quad (12)$$

where M^2 is defined by

$$M^2 = \left[\frac{e^2/\theta + 4mh(\beta)}{2 + (4/3)h(\beta)}\right]^2, \quad (13)$$

and the subindex R denotes that one must take the finite part of the determinants: temperature-dependent divergences coming from these determinants and from the normalization factor $\mathcal{N}(\beta)$ cancel each other [12]. Also, following [13], in Eq. (12) we have omitted an overall divergent factor which does not contribute to the thermodynamic properties of the system.

It is interesting to note that the partition function (12) corresponds to an effective theory with a free massive fermion and a free boson with a mass M which is related to the Chern-Simons topological mass. This mass acquires a temperature dependence which arises from the fermion determinant. It should be stressed that all dependence on the statistical angle θ occurs through that mass.

Determinants are easily evaluated. Standard finite temperature methods [12] lead to

$$\ln\det{}_R(i\not{\partial} + m) = V\int \frac{d^2p}{(2\pi)^2}\ln(1 + e^{-\beta\sqrt{p^2+m^2}}),$$

$$\ln\det{}_R(-\partial^2 + M^2) = 2V\int \frac{d^2p}{(2\pi)^2}\ln(1 - e^{-\beta\sqrt{p^2+M^2}}) + 2\beta V\Delta\Omega_R, \quad (14)$$

where $\Delta\Omega_R$ is the finite contribution to the bosonic determinant coming from the "zero-point energy:"

$$\Delta\Omega_R \equiv \frac{1}{2}\int \frac{d^2p}{(2\pi)^2}\sqrt{p^2 + M^2}\bigg|_R = -\frac{1}{12\pi}M^3. \quad (15)$$

The thermodynamic potential is defined as

$$\Omega = -\frac{1}{\beta V}\ln \mathcal{Z}, \quad (16)$$

so that in the present case we have

$$\Omega = -\frac{2}{\beta}\int \frac{d^2p}{(2\pi)^2}\ln(1 + e^{-\beta\sqrt{p^2+m^2}}) + \frac{1}{\beta}\int \frac{d^2p}{(2\pi)^2}\ln(1 - e^{-\beta\sqrt{p^2+M^2}}). \quad (17)$$

It should be stressed that we have not included in the thermodynamic potential the zero-point energy contribution $\Delta\Omega_R$, which is eliminated by an appropriate normal ordering [14].

Now, using $C_v = -\beta^2\,\partial^2(\beta\Omega)/\partial\beta^2$, we get

$$C_v = \frac{1}{4\pi\beta^2}\int_{\beta m}^\infty dx\,\frac{x^3}{\cosh^2(x/2)} + \frac{1}{8\pi\beta^2}\int_{\beta M}^\infty dx\,\frac{x^3}{\sinh^2(x/2)} + \frac{1}{\pi}\frac{\partial M}{\partial\beta}\left(\frac{\beta^2M^2}{\exp(\beta M) - 1}\right) + \frac{1}{2\pi}\frac{\partial^2 M}{\partial\beta^2}\left\{\beta^2 M\ln[1 - \exp(-\beta M)]\right\} + \frac{1}{2\pi}\left(\frac{\partial M}{\partial\beta}\right)^2\left(-\beta^3 M + \beta^2\ln[1 - \exp(-\beta M)] - \frac{\beta^3 M}{\exp(-\beta M) - 1}\right). \quad (18)$$

Except for being at zero chemical potential, this result was the goal of our calculation. It is not usual in field theory to look at purely thermodynamic functions such as C_v. But, as this calculation has illustrated, for our purposes is important since it gives direct information about the quantum excitations of the system, without need of facing difficult issues related to the asymptotic states. In the next section we extend this calculation to incorporate a finite fermionic density.

III. THE THEORY AT NONZERO CHEMICAL POTENTIAL

To describe the system at finite fermion density we introduce a chemical potential μ, so the Lagrangian in Eq. (1) becomes

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ + m - i\gamma_0\mu)\psi + \frac{1}{4}F_{\mu\nu}^2 - i\frac{e^2}{8\theta}\epsilon_{\mu\nu\lambda}F_{\mu\nu}A_\lambda + e\bar{\psi}A\!\!\!/\psi. \tag{19}$$

The contribution of the fermionic quantum excitations to the effective action is now

$$S_q[A_\mu] = -\ln\det{}_{(-)}\left(1 + \frac{eA\!\!\!/}{i\partial\!\!\!/ + m - i\gamma_0\mu}\right). \tag{20}$$

Comparing this with Eq. (8) we see that the presence of the chemical potential μ amounts to a shift of the Matsubara frequencies p_n^0 in Eq. (7) to

$$p_n^0 = \frac{(2n+1)\pi}{\beta} - i\mu. \tag{21}$$

Then, repeating the previous calculation with these shifted frequencies, a tedious but straightforward calculation leads to the effective action

$$S_{\text{eff}}[A_\mu] = \frac{1}{4}\int_\beta d^3x \left\{ \left[1 + \frac{2}{3}g(\beta,\mu)\right] F_{\mu\nu}^2 - i\left(\frac{e^2}{2\theta} + 2mg(\beta,\mu)\right)\epsilon_{\mu\nu\lambda}F_{\mu\nu}A_\lambda \right\}, \tag{22}$$

where now

$$g(\beta,\mu) = \frac{e^2}{16\pi m}\left[\tanh\left(\frac{\beta}{2}(m-\mu)\right) + \tanh\left(\frac{\beta}{2}(m+\mu)\right)\right]. \tag{23}$$

This result holds, again, to leading order in a low momentum expansion. Moreover, we have neglected in this expression Lorentz noninvariant terms which are of order $\beta\exp(-\beta m)$. These terms are negligible for βm sufficiently big, which is anyway assumed by the low momentum approximation.

Following the same steps as in Sec. II, for the thermodynamic potential we obtain

$$\Omega = -\frac{1}{\beta}\int\frac{d^2p}{(2\pi)^2}\ln(1 + e^{-\beta(\sqrt{p^2+m^2}+\mu)}) - \frac{1}{\beta}\int\frac{d^2p}{(2\pi)^2}\ln(1 + e^{-\beta(\sqrt{p^2+m^2}-\mu)}) + \frac{1}{\beta}\int\frac{d^2p}{(2\pi)^2}\ln(1 - e^{-\beta\sqrt{p^2+M_{(\mu)}^2}}), \tag{24}$$

where now

$$M_{(\mu)}^2 = \left[\frac{e^2/\theta + 4mg(\beta,\mu)}{2 + (4/3)g(\beta,\mu)}\right]^2. \tag{25}$$

For the specific heat we now find

$$C_v = \frac{1}{8\pi\beta^2}\left[\int_{\beta(m+\mu)}^\infty dx\,\frac{(x-\beta\mu)x^2}{\cosh^2(x/2)} + \int_{\beta(m-\mu)}^\infty dx\,\frac{(x+\beta\mu)x^2}{\cosh^2(x/2)} + \int_{\beta M_{(\mu)}}^\infty dx\,\frac{x^3}{\sinh^2(x/2)}\right] + \frac{1}{\pi}\frac{\partial M_{(\mu)}}{\partial\beta}\left(\frac{\beta^2M_{(\mu)}^2}{\exp(\beta M_{(\mu)})-1}\right) + \frac{1}{2\pi}\frac{\partial^2M_{(\mu)}}{\partial\beta^2}\{\beta^2M_{(\mu)}\ln[1-\exp(-\beta M_{(\mu)})]\} + \frac{1}{2\pi}\left(\frac{\partial M_{(\mu)}}{\partial\beta}\right)^2\left(-\beta^3M_{(\mu)} + \beta^2\ln[1-\exp(-\beta M_{(\mu)})] - \frac{\beta^3M_{(\mu)}}{\exp(-\beta M_{(\mu)})-1}\right). \tag{26}$$

Similarly, for the fermion number density $\rho = -\partial\Omega/\partial\mu$, we get

$$\rho = \int\frac{d^2p}{(2\pi)^2}\left[\frac{1}{1+e^{\beta(\sqrt{p^2+m^2}+\mu)}} - \frac{1}{1+e^{\beta(\sqrt{p^2+m^2}-\mu)}}\right] - \frac{1}{2\pi\beta}M_{(\mu)}\frac{\partial M_{(\mu)}}{\partial\mu}\ln\left[2\sinh\left(\frac{\beta}{2}M_{(\mu)}\right)\right]. \tag{27}$$

This is a constraint equation which must be inverted to determine the chemical potential μ in terms of a given density and temperature. It should be noticed that the first two terms in this expression correspond to the free fermion (relativistic) gas, with both fermions and antifermions contributing to the density. Similarly, in Eq. (26), the first three terms correspond to the contributions to C_v from free fermions and antifermions of mass m and chemical potential $\pm\mu$, and from free bosons of mass $M_{(\mu)}$ without a chemical potential. The numerical analysis of these equations is the subject of the next section.

IV. NUMERICAL RESULTS

In this section we present our numerical results. For given values of the statistical angle θ and the fermionic density ρ, we invert Eq. (27) numerically (with a precision of one part in 10^5), and then substitute the chemical potential into Eq. (26) to obtain the specific heat C_v as a function of the temperature at different values of ρ and θ. Since here both e^2 and m have dimensions of mass, we find it useful to parametrize the theory in terms of the dimensionless variables $\alpha = e^2/4\pi m$, ρ/m^2, βm, and θ.

We have performed numerical computations covering the following domain of parameters:

$$\begin{aligned} \alpha &\in (10^{-2}, 10^{-6}), \\ \frac{\rho}{m^2} &\in (10^{-8}, 10), \\ \theta &\in (10^{-4}, 1), \end{aligned} \tag{28}$$

and in the temperature range

$$\beta m \in (1, 10^2). \tag{29}$$

There are no qualitative differences for the results over the whole domain of parameters. Typical curves for the specific heat as a function of the temperature are presented in Figs. 1 and 2 for $\theta = 1$ and $\theta = 10^{-4}$, respectively, at $\alpha = 9.73\times 10^{-7}$, and $\rho/m^2 = 1.78\times 10^{-8}$. These are values relevant for high-T_c superconductors [15], and correspond to $m = 7.5 \times 10^{10}\,\mathrm{cm}^{-1}$, thrice the value of the bare electron mass, and a fine structure constant in $2+1$ space-time dimensions obtained from that in $3+1$ space-time dimensions dividing by the interplanar spacing.

Although the curves in Figs. 1 and 2 seem almost identical, the specific heat in Fig. 1 ($\theta = 1$) is slightly larger than that in Fig. 2 ($\theta = 10^{-4}$). This is a characteristic feature, which we find to hold in the entire parameter range, although the actual difference depends on the values of α and ρ. We think it is a very interesting result, since it is consistent with the (unimpressive) difference between the free bosonic and fermionic specific heats in $2+1$ dimensions. This is illustrated in more detail in Figs. 3 and 4. In Fig. 3 we present the difference of the

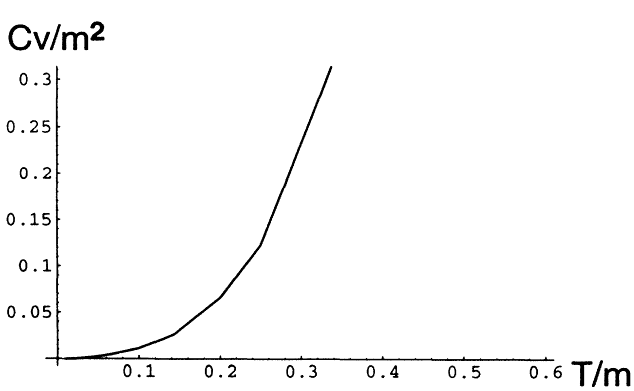

FIG. 1. Specific heat as a function of temperature at $\alpha = 9.73 \times 10^{-7}$, $\rho/m^2 = 1.78 \times 10^{-8}$, and $\theta = 1$.

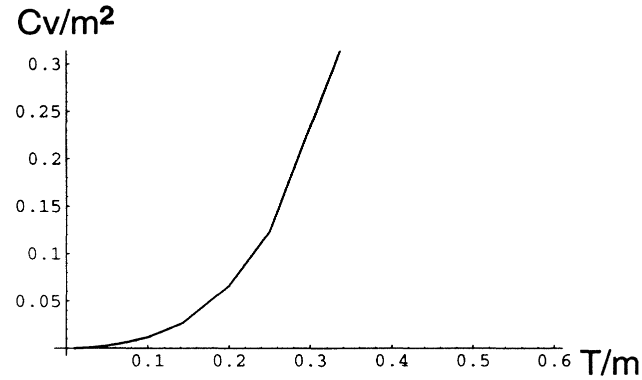

FIG. 2. Specific heat as a function of temperature at $\alpha = 9.73 \times 10^{-7}$, $\rho/m^2 = 1.78 \times 10^{-8}$, and $\theta = 10^{-4}$.

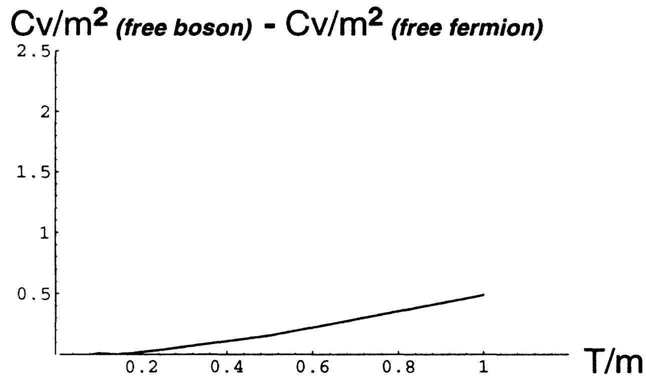

FIG. 3. Difference of the specific heats C_v(bosonic) and C_v(fermionic) for the free relativistic (charged) boson and/or fermion gases, both at $\rho/m^2 = 0.01$.

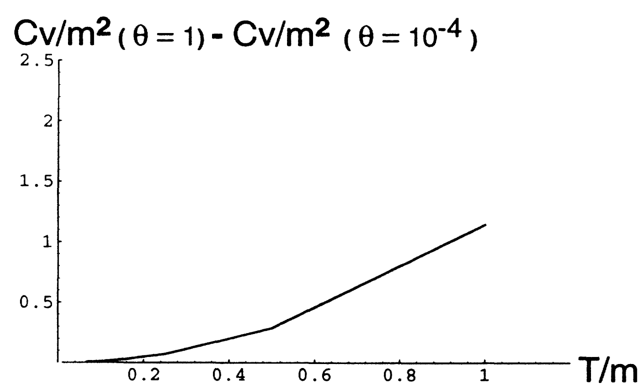

FIG. 4. Difference of the specific heats $C_v(\theta = 1)$ and $C_v(\theta = 10^{-4})$, both at $\alpha = 0.01$ and $\rho/m^2 = 0.01$.

specific heats C_v(bosonic) $-C_v$(fermionic) for the free relativistic (charged) boson and/or fermion gases, both at $\rho/m^2 = 0.01$. This is to be compared with Fig. 4, where we present the difference $C_v(\theta = 1) - C_v(\theta = 10^{-4})$, both at $\alpha = 0.01$ and $\rho/m^2 = 0.01$. Although this is not a proof of a shift from fermionic towards bosonic statistics (whatever that means in this relativistic system), it is certainly consistent with that intuitive picture: since we start from a fermionic system at $\theta = 0$, one expects a transition towards a bosonic behavior as θ is increased, provided the nonrelativistic ideas about Chern-Simons-generalized statistics hold in this case.

In connection with the curves shown in Figs. 1–4, it is worthwhile noticing that their apparent violation of the classical Dulong-Petit law is not so. For a (2+1)-dimensional nonrelativistic classical ideal gas one finds $C_v = \rho$. In our case, however, ρ stands for the Abelian charge density, and therefore is the density of particles minus the density of antiparticles, as shown by the first terms in Eq. (27) in the case of fermions. Hence, one can indeed reach large values of C_v without a corresponding increment of ρ.

We have studied also the fermion density as a function of the chemical potential. We obtained a smooth function which does not exhibit the step structure found in [16] where it is interpreted as a signal of a critical behavior. Note, however, that that reference considers a high temperature range (opposite from ours) and starts from a pure Chern-Simons system, without a Maxwell term.

V. CONCLUSIONS

We have analyzed at finite temperature and density a system of massive fermions in three space-time dimensions coupled to an Abelian Maxwell-Chern-Simons field. In particular, we evaluated the specific heat for different values of the statistical angle θ. This was done to leading order in the loop expansion and in the low momentum approximation. In doing so, we have shown that it is possible to obtain physical information of the system without having to give an explicit answer to such difficult issues as the lack of free asymptotic states or the exact kinematical meaning of generalized statistic in relativistic quantum field theory.

Our main result, the specific heat as a function of temperature and density [Eqs. (26) and (27)], was analyzed in a wide range of parameters within the low-temperature regime. This limitation to low temperatures arises in part from the low momentum expansion, which we here adopted for simplicity but can be avoided [7]. However, to move to higher temperatures one will also have to face the contribution of Lorentz noninvariant terms which here we have discarded. We hope to come back to this point in a future work.

As we ourselves had expected, the specific heat has only a mild dependence on the statistical angle θ. However, this dependence is consistent with what one expects from the nonrelativistic picture of generalized statistics, thus suggesting that those ideas may still be applicable, to some extent, in the relativistic domain.

As we pointed out earlier, a more marked dependence on the statistical angle θ may be expected in a pure Chern-Simons theory without Maxwell term. That, however, will possibly require a nonperturbative treatment. In this respect it may be of interest to consider other thermodynamic functions which may be more sensible to the long distance properties, and therefore more immune to the presence of the Maxwell term.

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