Beyond S, T, and U

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The contribution to precision electroweak measurements due to TeV physics which couples primarily to the $W^{\pm}$ and Z bosons may be parametrized in terms of the three "oblique correction" parameters, S, T, and U. We extend this parametrization to physics at much lower energies, $\gtrsim 100$ GeV, and show that in this more general case neutral-current experiments are sensitive to only two additional parameters. A third new parameter enters into the $W^{\pm}$ width.

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I. INTRODUCTION

The standard electroweak theory has recently come of age, with experiments now probing its predictions with sufficient accuracy to test its radiative corrections in some detail. Besides providing a detailed test of the model, these precision measurements are also very useful for the constraints they impose on any potential new physics that might exist at energies higher than those that have been hitherto experimentally explored.

A particularly interesting class of new physics that is constrained by these measurements consists of models which satisfy the following three criteria. (1) The electroweak gauge group must be $SU_L(2)\times U_Y(1)$, with no new electroweak gauge bosons apart from the photon, the $W^{\pm}$ and the Z; (2) the couplings of the new physics to light fermions are suppressed compared to its couplings to the gauge bosons; (3) the intrinsic scale, M, of the new physics is large in comparison with M_W and M_Z.

These criteria are particularly interesting principally for two reasons. First, they imply that the contributions of new physics (i.e., the "oblique" corrections [1]) to low-energy observables may be completely described by three parameters, denoted S, T, and U in Ref. [2]. This allows these models to be meaningfully constrained as a group by fitting for these parameters once and for all using the presently available precision electroweak data [3,4]. Second, they include a large class of well-motivated theories, such as technicolor models, models with extra generations, multi-Higgs-boson models, etc. [5].

It is the purpose of this paper to extend this analysis to theories which satisfy the first two of the above criteria, but not the third, i.e., the situation in which the new physics is comparatively light. (Even the second criterion can be discarded under certain circumstances, as we will discuss.) Depending on the properties of the hypothetical new particles, they could have masses as light as $M \lesssim 100$ GeV and yet still have escaped direct detection at the CERN Large Electron-Position Collider (LEP) or at the Fermilab Tevatron. In this case they are best constrained through their loop contributions to precision electroweak measurements. We present here a formalism which may be used to do so. The key point is when the new physics is light, corrections to precision measurements of $O(\alpha M_Z^2/M^2)$, which are negligible when $M \gg M_Z$, must now be taken into account.

A similar situation occurs when calculating the loop contributions of effective, nonrenormalizable operators to vacuum polarizations. The size of these operators is typically of $O(M_Z^2/M^2)$ (or smaller), and hence lead to corrections of $O(\alpha M_Z^2/M^2)$. (Of course, these corrections are large [i.e., $O(\alpha)$] only if the nonrenormalizable operators are ~ 1 (i.e., $M \simeq M_Z$), so that this situation is a variation on the above theme of light new physics.) This is the case that must be considered when using loop effects to constrain anomalous three-gauge-boson interactions, as has been done in Refs. [6] and [7]. As we show in more detail elsewhere [8], the usual analysis in terms of S, T, and U [7] does *not* apply, requiring our more general procedure.

We show that the advantages of the existing STU formalism survive, and that the implications of any such model for neutral-current data may in practice be parametrized in terms of *four* parameters: Peskin's and Takeuchi's S and T, as well as two additional ones, which we call V and X. If the mass, width, and low-energy couplings of the $W^{\pm}$ boson are also included, only *two* more parameters are required: the quantity U of Ref. [2], and one new variable, W. For practical applications W often need not be considered, since it only arises in absolute measurements of the $W^{\pm}$ widths, but cancels in its branching ratios.

Although this economy in the number of parameters required to parametrize the data is similar to the economy that was found for physics at very high scales, its origins here are very different. For physics at very high

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scales only three parameters are possible *a priori,* since in this case the large value for M permits an effective-Lagrangian description in which only the first few lowest-dimension effective interactions need be considered. The same is not true in the present case where, due to the fact that the new degrees of freedom can be comparatively light, one must take into account a very large number of terms. The relatively few parameters which do arise in this case reflect the fact that, at present, precision measurements are made at only a very few scales, $q^2 \approx 0$ and $q^2 = M_Z^2$ or M_W^2. As a result the number of independent probes of new physics is limited by the few scales at which this physics is sampled with sufficient precision. This will certainly change in the future, such as at LEP 200, once other scales become available for more detailed scrutiny.

Our results reduce to the previous analyses in the limit that the new physics is heavy. Since our treatment applies to both large and comparatively small values for M, we are able to more quantitatively identify the boundaries of applicability of the previous description. We do so here by explicitly working through an example, in which we take the new physics to consist of a degenerate doublet of new heavy fermions. This example can be regarded as an "existence proof" of new physics for which the new parameters V, W, and X must be taken into account. We also present this example as a sample of the type of diagnostic calculation that will become necessary should a deviation from standard physics ever be detected in the future, using these precision experiments. In this happy event, a comparison between the sizes of the new parameters V, W, and X relative to the size of S and T can be used to infer the mass scale that is associated with the underlying new physics.

II. "OBLIQUE" CORRECTIONS

We start with the observation that, at present, precision electroweak measurements exclusively involve the two-particle scattering of light fermions. Given that the new physics is too heavy to be directly produced in these experiments, there are three ways for it to indirectly contribute. It can contribute to (a) the propagation of the gauge bosons that can be exchanged by the fermions, (b) the three-point fermion-boson couplings, and (c) the four-point direct fermion-fermion interactions (or "box"-diagram corrections).

The importance of criteria (1) and (2) above is that when these are satisfied then only corrections of type (a)—the so-called "oblique" corrections [1]—are dominant. In fact, even criterion (2) is not absolutely necessary—if all new gauge-fermion and four-fermion vertices are such that the fermions appear only through linear combinations of the total standard model currents, then we may use the freedom to perform field redefinitions to put all of the new physics into the vacuum polarizations. (This freedom to recast the effective Lagrangian is exploited in Ref. [8].) Under these circumstances the complete impact of any new heavy degrees of freedom arises through their contributions to the gauge-boson vacuum polarizations, $\Pi_{ab}^{\mu\nu}(q) = \Pi_{ab}(q^2) g^{\mu\nu} + (q^\mu q^\nu \text{terms})$, with $a,b = \gamma, W^\pm, Z$. We therefore supplement the standard model (SM) by adding a new-physics contribution to the gauge-boson vacuum polarizations:

$$\Pi_{ab}(q^2) = \Pi_{ab}^{\rm SM}(q^2) + \delta\Pi_{ab}(q^2) \ . \tag{1}$$

The first term on the right-hand side represents the SM contribution, including all appropriate radiative corrections, while all new-physics effects are contained in the second term.

In Refs. [2–4], it was assumed that the $\delta\Pi_{ab}(q^2)$ are due to new physics at a very high mass scale, and are thus well described by a Taylor expansion to linear order in q^2: $\delta\Pi_{ab}(q^2) \approx A_{ab} + B_{ab} q^2$ [9]. Under this assumption it is straightforward to determine the number of independent parameters required to describe all new-physics effects. The reasoning goes as follows. There are eight quantities describing the new physics: $A_{ab} = \delta\Pi_{ab}(0)$ and $B_{ab} = \delta\Pi'_{ab}(0)$, with the pair (ab) taking the four independent values: $(ab) = (\gamma\gamma)$, $(Z\gamma)$, (ZZ), and (WW). (The prime denotes differentiation with respect to q^2: $\delta\Pi' \equiv d\delta\Pi/dq^2$.) Two of these, $\delta\Pi_{\gamma\gamma}(0)$ and $\delta\Pi_{z\gamma}(0)$, are automatically zero by gauge invariance. Three linear combinations of the remaining six quantities can be eliminated when the three input parameters—say, α, M_Z, and G_F—are renormalized. Thus, all new-physics effects can be described by three combinations of the $\delta\Pi$'s, denoted S, T, and U in Ref. [2] (the precise definitions of these parameters are given later).

In the more general case [10] we cannot assume any specific form for $\delta\Pi_{ab}(q^2)$, and hence the expressions for the vacuum polarizations contain an infinite number of new, unknown parameters which can all potentially enter into the physical observables. This would appear to render any analysis for such new-physics effects virtually useless. Our main purpose in this section is to show that in fact only three additional parameters arise beyond the three that were introduced by Peskin and Takeuchi. We also compute the dependence of a number of well-measured quantities on these "extended" oblique parameters.

The reasoning which demonstrates that a total of six parameters suffice to describe *all* new-physics oblique corrections is similar to that shown above for the case of S, T, and U. As before, the agreement between the data and the SM predictions, including radiative corrections, implies that the $\delta\Pi_{ab}(q^2)$ cannot be larger than at most $\sim 1\%$ of the size of their tree-level SM counterparts. It follows that we may simply perturb in the oblique corrections and stop at linear order. The key observation is that current precision measurements sample the vacuum polarizations only at a few values of q^2: $q^2 \approx 0$ and $q^2 = M_W^2$ or M_Z^2. Therefore we expect only the parameters $\delta\Pi_{\gamma\gamma}(q^2)/q^2$ $(q^2 = 0, M_Z^2)$, $\delta\Pi_{z\gamma}(q^2)/q^2$ $(q^2 = 0, M_Z^2)$, $\delta\Pi_{WW}(q^2)$ $(q^2 = 0, M_W^2)$, and $\delta\Pi_{zz}(q^2)$ $(q^2 = 0, M_Z^2)$ to appear in any well-measured observables. [As before, since $\delta\Pi_{\gamma\gamma}(0)$ and $\delta\Pi_{z\gamma}(0)$ are zero by gauge invariance, $\delta\Pi_{\gamma\gamma}(q^2)/q^2$ and $\delta\Pi_{z\gamma}(q^2)/q^2$ are well defined at $q^2 = 0$.] In addition, for those observables related to the decay of an on-shell W or Z, the parameters $\delta\Pi'_{WW}(M_W^2)$, and

$\delta\Pi'_{ZZ}(M_Z^2)$ will also appear (these correspond simply to wave function renormalizations). Of these ten parameters, three combinations can be eliminated by renormalizing the input parameters. Finally, the parameter $\delta\Pi_{\gamma\gamma}(M_Z^2)$ corresponds physically to new-physics effects in the photon propagator at the Z peak. Since the SM photon contributions at the Z resonance are already suppressed by the $O(\alpha)$ ratio $\Gamma_Z/M_Z \sim 0.03$, $\delta\Pi_{\gamma\gamma}(M_Z^2)$ constitutes a correction to a correction and can be ignored. Thus, we deduce that only six combinations of the $\delta\Pi$'s will appear in observables.

With this knowledge in hand we now compute explicitly the experimental implications of the $\delta\Pi_{ab}(q^2)$. SM radiative corrections to any new-physics contribution may also be ignored to within the accuracy we require. We may therefore simply work to tree level in the oblique corrections, $\delta\Pi_{ab}$, and then add the result to the corresponding SM contribution, including potential radiative corrections.

A. Shifting the standard-model couplings

There are two distinct ways in which the $\delta\Pi_{ab}(q^2)$ can enter into predictions for any particular observable. Besides contributing directly to predictions for the quantities of interest, they can also change the numerical values that are inferred from experiment for the various SM electroweak parameters, such as for the electric charge, e, $s_w \equiv \sin\theta_w$, etc. This change then shifts the SM prediction for all other quantities. We first compute this shift.

The standard electroweak interactions are parametrized by three variables, (in addition to other parameters, like fermion masses, which do not concern us here) which we denote as $\tilde{e}$, $\tilde{s}_w$, and $\tilde{m}_Z$. (The tildes on these parameters are a reminder that, due to the presence of new physics, they do not take their usual values.) The values for these are fixed by comparing SM predictions to the three best-measured observables, which we take to be (i) the fine-structure constant, α, as measured in low-energy electron scattering, (ii) Fermi's constant, G_F, as measured in muon decay, and (iii) the Z boson mass, M_Z, as measured at LEP. In order to calculate the change that the new physics implies for these parameters we must compute the contribution of the $\delta\Pi_{ab}$'s to these quantities.

We therefore calculate the new-physics corrections to low-energy electron scattering and to muon decay, from which α and G_F are extracted, working to tree level in the oblique corrections. The shift in the Z boson mass follows from its definition in terms of the pole of the propagator. This leads to the following expressions:

$$\begin{aligned}
\alpha &= \alpha_{\rm SM}(\tilde{e})[1+\delta\hat{\Pi}_{\gamma\gamma}(0)] , \\
G_F &= (G_F)_{\rm SM}(\tilde{e},\tilde{s}_w,\tilde{m}_Z)\left[1-\frac{\delta\Pi_{WW}(0)}{M_W^2}\right] , \qquad (2)\\
M_Z^2 &= (M_Z^2)_{\rm SM}(\tilde{e},\tilde{s}_w,\tilde{m}_Z)\left[1+\frac{\delta\Pi_{ZZ}(M_Z^2)}{M_Z^2}\right] .
\end{aligned}$$

Here and below, we use the notation $\delta\hat{\Pi}_{ab}(q^2)\equiv\delta\Pi_{ab}(q^2)/q^2$. We define the "standard" parameters by equating the left-hand sides of these expressions to their SM formulae. For example, we define e by requiring

$$4\pi\alpha = 4\pi\alpha_{\rm SM}(e) \equiv e^2 + (\text{loops}) ,$$

$$G_F = e^2/(4\sqrt{2}s_w^2c_w^2m_Z^2) + (\text{loops}) ,$$

etc. This leads to the following expressions:

$$\begin{aligned}
\tilde{e} &= e[1-\tfrac{1}{2}\delta\hat{\Pi}_{\gamma\gamma}(0)] , \\
\tilde{s}_w^2 &= s_w^2\left[1-\frac{c_w^2}{c_w^2-s_w^2}\left[\delta\hat{\Pi}_{\gamma\gamma}(0)-\frac{\delta\Pi_{ZZ}(M_Z^2)}{M_Z^2}+\frac{\delta\Pi_{WW}(0)}{M_W^2}\right]\right] , \qquad (3)\\
\tilde{m}_Z^2 &= m_Z^2\left[1-\frac{\delta\Pi_{ZZ}(M_Z^2)}{M_Z^2}\right] .
\end{aligned}$$

In our notation m_Z denotes the standard model parameter, as opposed to the physical quantity, M_Z. We make this distinction since these can differ, depending on the renormalization scheme used to perform the SM radiative corrections.

The prediction for any other observable, A, may now be written $A = A_{\rm SM}(\tilde{e},\tilde{s}_w,\tilde{m}_Z)+\delta A$, where the first term is the SM prediction, and where the second term is the "direct" contribution of the new oblique corrections to the observable in question. In order to take advantage of the most precise radiatively corrected SM calculations, it is then useful to reexpress A using Eqs. (3), as $A = A_{\rm SM}(e,s_w,m_Z)+\delta A'$, where $A_{\rm SM}(e,s_w,m_Z)$ takes the same numerical value as it does in the standard model in the absence of new physics, and where $\delta A'$ contains all new-physics corrections, including the shifts in the electroweak parameters and the "direct" contributions.

B. Low-energy observables: S, T, and U

The contributions to observables from the new physics are now easily computed. We concentrate first on low-energy observables, for which we may take $q^2 \approx 0$. These include deep-inelastic neutrino scattering, atomic parity violation experiments, etc.

For example, low-energy measurement of parity-violating asymmetries, such as A_{LR} and $A_{\rm FB}$ in electron scattering, can be used to define an effective value for $\sin^2\theta_w$. This is given by

$$(s_w^2)_{\rm eff}(q^2\approx 0)=\tilde{s}_2^2-s_w c_w \delta\hat{\Pi}_{Z\gamma}(0)$$
$$=s_w^2\left[1-\frac{c_w^2}{c_w^2-s_w^2}\left[\delta\hat{\Pi}_{\gamma\gamma}(0)-\frac{\delta\Pi_{ZZ}(M_Z^2)}{M_Z^2}+\frac{\delta\Pi_{WW}(0)}{M_W^2}\right]-\frac{c_w}{s_w}\delta\hat{\Pi}_{Z\gamma}(0)\right] . \tag{4}$$

Clearly the effects of the new physics on any such observable are given by the induced shifts in the SM parameters, as well as a linear combination of the various $\delta\Pi_{ab}$'s evaluated at $q^2\approx 0$. As a result they never probe the oblique corrections beyond linear order in their expansions in powers of q^2, and so they are completely described in terms of the three Peskin-Takeuchi parameters, S, T, and U:

$$\frac{\alpha S}{4s_w^2c_w^2}=\left[\frac{\delta\Pi_{ZZ}(M_Z^2)-\delta\Pi_{ZZ}(0)}{M_Z^2}\right]-\frac{(c_w^2-s_w^2)}{s_w c_w}\delta\hat{\Pi}_{Z\gamma}(0)-\delta\hat{\Pi}_{\gamma\gamma}(0) ,$$
$$\alpha T=\frac{\delta\Pi_{WW}(0)}{M_W^2}-\frac{\delta\Pi_{ZZ}(0)}{M_Z^2} , \tag{5}$$
$$\frac{\alpha U}{4s_w^2}=\left[\frac{\delta\Pi_{WW}(M_W^2)-\delta\Pi_{WW}(0)}{M_W^2}\right]-c_w^2\left[\frac{\delta\Pi_{ZZ}(M_Z^2)-\delta\Pi_{ZZ}(0)}{M_Z^2}\right]-s_w^2\delta\hat{\Pi}_{\gamma\gamma}(0)-2s_wc_w\delta\hat{\Pi}_{Z\gamma}(0) .$$

These definitions are deliberately cast in a way that does *not* assume that the $\delta\Pi_{ab}(q^2)$ are linear functions of q^2. Note that although these expressions resemble the formulation of S, T, and U given in Ref. [3], the analysis of these authors only applies when $\delta\Pi_{ab}(q^2)=A_{ab}+B_{ab}q^2$. With these definitions Eq. (4) takes the usual form:

$$\frac{(s_w^2)_{\rm eff}(q^2\approx 0)}{(s_w^2)_{\rm SM}}=1+\frac{\alpha S}{4s_w^2(c_w^2-s_w^2)}-\frac{c_w^2\alpha T}{c_w^2-s_w^2} . \tag{6}$$

Any other low-energy observable may be analyzed in a similar fashion.

C. Observables at $q^2=M_{W,Z}^2$: beyond S, T, and U

We take as our first example the mass of the $W^\pm$, which is given in the SM by $(M_W^2)_{\rm SM}(\tilde{e},\tilde{s}_w,\tilde{m}_Z)=\tilde{m}_Z^2\tilde{c}_w^2+$(loop corrections). The oblique corrections change this to

$$M_W^2=(M_W^2)_{\rm SM}(\tilde{e},\tilde{s}_w,\tilde{m}_Z)\left[1+\frac{\delta\Pi_{WW}(M_W^2)}{M_W^2}\right] . \tag{7}$$

Shifting to the "standard" parameters then gives

$$M_W^2=(M_W^2)_{\rm SM}(e,s_w,m_Z)\left[1-\frac{c_w^2}{c_w^2-s_w^2}\frac{\delta\Pi_{ZZ}(M_Z^2)}{M_Z^2}+\frac{s_w^2}{c_w^2-s_w^2}\left[\delta\hat{\Pi}_{\gamma\gamma}(0)+\frac{\delta\Pi_{WW}(0)}{M_W^2}\right]+\frac{\delta\Pi_{WW}(M_W^2)}{M_W^2}\right] .$$
$$=(M_W^2)_{\rm SM}(e,s_w,m_Z)\left[1-\frac{\alpha S}{2(c_w^2-s_w^2)}+\frac{c_w^2\alpha T}{(c_w^2-s_w^2)}+\frac{\alpha U}{4s_w^2}\right] . \tag{8}$$

A slightly different analysis is required when considering the widths of the W and Z, since care must be taken to include the proper wave function renormalization corrections, $\delta\Pi'_{WW}(M_W^2)$ and $\delta\Pi'_{ZZ}(M_Z^2)$. For example, the $W^\pm$ width is obtained by multiplying the lowest-order SM result by the renormalization factor, $Z_W=1+\delta\Pi'_{WW}(M_W^2)$, that arises due to the use of the fully summed propagator. As a result

$$\delta\Gamma(W\to e\nu)=\frac{\tilde{e}^2M_W}{48\pi\tilde{s}_w^2}\delta\Pi'_{WW}(M_W^2) . \tag{9}$$

Note that we use the physical mass, M_W, in this expression since this is what appears in the phase space integration. Transforming to "standard" parameters leads to

$$\frac{\Gamma(W\to{\rm all})}{\Gamma_{\rm SM}(W\to{\rm all})}=\frac{\Gamma(W\to e\nu)}{\Gamma_{\rm SM}(W\to e\nu)}$$
$$=1+\frac{1}{c_w^2-s_w^2}\left[s_w^2\delta\hat{\Pi}_{\gamma\gamma}(0)+c_w^2\left[\frac{\delta\Pi_{WW}(0)}{M_W^2}-\frac{\delta\Pi_{ZZ}(M_Z^2)}{M_Z^2}\right]\right]+\delta\Pi'_{WW}(M_W^2) . \tag{10}$$

The Z width into neutrinos may be computed in an identical way. Proceeding along precisely the same lines as for the $W^{\pm}$ width, and using $Z_Z = 1 + \delta\Pi'_{ZZ}(M_Z^2)$, gives the following result:

$$\frac{\Gamma(Z\to\nu\bar{\nu})}{\Gamma_{SM}(Z\to\nu\bar{\nu})} = 1 - \frac{\delta\Pi_{ZZ}(M_Z^2)}{M_Z^2} + \frac{\delta\Pi_{WW}(0)}{M_W^2} + \delta\Pi'_{ZZ}(M_Z^2) . \quad (11)$$

A new feature enters into the calculation of the Z-boson widths into charged-particle final states, since these receive contributions from $\delta\Pi_{Z\gamma}(M_Z^2)$. The result is a contribution to the effective value for s_W^2 that is measured in A_{LR}, A_{FB} at the Z resonance. We find

$$\frac{(s_w^2)_{\rm eff}(q^2\approx M_Z^2)}{(s_w^2)_{SM}} = \frac{(s_w^2)_{\rm eff}(q^2\approx 0)}{(s_w^2)_{SM}} - \frac{c_w}{s_w}[\delta\hat{\Pi}_{Z\gamma}(M_Z^2) - \delta\hat{\Pi}_{Z\gamma}(0)] . \quad (12)$$

At this point a simplification occurs, as advertised. All of the new physics contributions combine into compact expressions involving Peskin's and Takeuchi's three parameters, as well as the following three new ones, which we call V, W, and X:

$$\alpha V = \delta\Pi'_{ZZ}(M_Z^2) - \left[\frac{\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)}{M_Z^2}\right] ,$$

$$\alpha W = \delta\Pi'_{WW}(M_W^2) - \left[\frac{\delta\Pi_{WW}(M_W^2) - \delta\Pi_{WW}(0)}{M_W^2}\right] , \quad (13)$$

$$\alpha X = -s_w c_w \left[\frac{\delta\Pi_{Z\gamma}(M_Z^2)}{M_Z^2} - \delta\hat{\Pi}_{Z\gamma}(0)\right] .$$

Note that these expressions would vanish if $\delta\Pi_{ab}(q^2)$ were simply a linear function of q^2.

In terms of these parameters our previous expressions for the $W^{\pm}$ and Z widths, and $(s_w^2)_{\rm eff}(M_Z^2)$, become

$$\frac{\Gamma(W\to{\rm all})}{\Gamma_{SM}(W\to{\rm all})} = 1 - \frac{\alpha S}{2(c_w^2 - s_w^2)} + \frac{c_w^2\alpha T}{(c_w^2 - s_w^2)} + \frac{\alpha U}{4s_w^2} + \alpha W , \quad (14)$$

$$\frac{\Gamma(Z\to\nu\bar{\nu})}{\Gamma_{SM}(Z\to\nu\bar{\nu})} = 1 + \alpha T + \alpha V , \quad (15)$$

$$\frac{(s_w^2)_{\rm eff}(q^2\approx M_Z^2)}{(s_w^2)_{SM}} = 1 + \frac{\alpha S}{4s_w^2(c_w^2 - s_w^2)} - \frac{c_w^2\alpha T}{c_w^2 - s_w^2} + \frac{\alpha X}{s_w^2} . \quad (16)$$

Note that one of these new parameters, W, turns out to appear only in the expression for Γ_W. The Z width into any fermion pairs may be expressed in terms of the remaining two parameters, V and X, where V describes a contribution to the overall normalization of the strength of the interaction, and X acts to shift the effective value of $(s_w^2)_{\rm eff}$.

D. Numerical results

We tabulate the contributions to some precision measurements in Table I. In preparing this table we use the following numerical values in obtaining these results: $\alpha(M_Z) = 1/127.8$, $s_w^2(M_Z) = 0.2323$, and $M_Z = 91.17$ GeV. For the SM predictions we choose the fiducial values, $m_t = 140$ GeV and $m_H = 100$ GeV. When appropriate, our numbers clearly reduce to the results of Ref. [3], from which we also have taken the experimental limits.

Several features come to light on inspection of Table I. First, for neutral-current data at low energies and at the Z resonance, only the four parameters S, T, V, and X arise.[1] Of these, only S and T contribute to low-energy observables for which $q^2 \approx 0$, since V and X appear only

TABLE I. The new physics contributions to various well-measured electroweak observables. We use top-quark and Higgs-boson masses of 140 and 100 GeV for the standard model numbers. Other quantities are as discussed in the text.

Observable	Prediction	Present constraint
Cs parity violation	$Q_W(^{133}_{55}Cs) = -73.20 - 0.8S - 0.005T$	$-71.04\pm1.58\pm0.88$
W Mass	$M_W = 80.20 - 0.29S + 0.45T + 0.34U$ GeV	80.14 ± 0.31 GeV
$\Gamma(W\to{\rm all})$	$\Gamma/\Gamma_{SM} = 1 - 0.0073S + 0.011T + 0.0084U + 0.0078W$	1.02 ± 0.05[a]
$\Gamma(Z\to\nu\bar{\nu})$	$\Gamma/\Gamma_{SM} = 1 + 0.0078T + 0.0078V$	0.992 ± 0.036
$\Gamma(Z\to e^+e^-)$	$\Gamma/\Gamma_{SM} = 1 - 0.0021S + 0.0093T - 0.0044X + 0.0078V$	1.004 ± 0.011
$\Gamma(Z\to{\rm all})$	$\Gamma/\Gamma_{SM} = 1 - 0.0038S + 0.011T - 0.0082X + 0.0078V$	1.002 ± 0.008
Z asymmetries	$(s_w^2)\,{\rm expt}/(s_w^2)_{SM} = 1 + 0.016S - 0.011T + 0.034X$	0.978 ± 0.056
eD asymmetry	$(s_w^2)\,{\rm expt}/(s_w^2)_{SM} = 1 + 0.016S - 0.011T$	0.965 ± 0.086
eC asymmetry	$(s_w^2)\,{\rm expt}/(s_w^2)_{SM} = 1 + 0.016S - 0.011T$	0.86 ± 0.22
$R \equiv \sigma(\nu_\mu e)/\sigma(\bar{\nu}_\mu e)$	$R/R_{SM} = 1 - 0.029S + 0.021T$	0.997 ± 0.11

[a]CDF Collaboration, K. Einsweiler (private communication).

[1]The absence of U in these expressions follows from our choice of three standard-model input observables.

in observables that are defined at $q^2=M_Z^2$. As a result, all of the predictions for low-energy quantities agree with earlier work [3]. In particular, the favoring of negative values for S by the cesium atomic parity violation experiments is not affected by the introduction of the additional parameters.

Next, although the Z results are the most precise, some $W^{\pm}$ properties, such as measurements of M_W, are sufficiently accurate to competitively bound the relative parameters. For these charged-current observables two more parameters enter: the usual quantity U for quantities defined at $q^2 \simeq 0$ (as well as M_W), and the parameter W in the $W^{\pm}$-boson decay widths.

We next apply these results to an illustrative example.

III. TECHNIFERMIONS: AN EXAMPLE

New, massive fermions which carry electroweak quantum numbers, but which do not mix appreciably with ordinary light fermions, furnish a concrete model to which the above reasoning applies. If these fermions are sufficiently heavy, say, $m \gtrsim 1$ TeV, their implications may be summarized as contributions to S, T, and U. We wish to explore here the much lighter mass range, $m \sim$(several hundred GeV), that can still be consistent with such particles not having been detected at current accelerators. Our goal is to show how the formalism just presented can be used to constrain the properties of such particles. As a bonus we can vary the fermion mass, and determine quantitatively at what point the usual three-parameter description becomes sufficiently accurate.

We therefore require the vacuum polarization that is induced by such a collection of fermions. Evaluating the graph of Fig. 1 produces the following result:

$$\delta\Pi_{ab}(q^2)=\frac{1}{2\pi^2}\sum_{ij}\int_0^1 dx\, f_{ab}(q^2,x) \times \ln\left[\frac{m_{ij}^2(x)-q^2x(1-x)}{\mu^2}\right], \tag{17}$$

where $m_{ij}^2(x)\equiv m_i^2(1-x)+m_j^2 x$, and

$$f_{ab}(q^2,x)=\frac{g_L^a g_L^{b*}+g_R^a g_R^{b*}}{2}\left[x(1-x)q^2-\frac{m_{ij}^2(x)}{2}\right] +\frac{g_L^a g_R^{b*}+g_R^a g_L^{b*}}{2}\left[\frac{m_i m_j}{2}\right]. \tag{18}$$

In these expressions, m_i and m_j are the masses of the fermions which circulate in the loop, and g_L^a and g_R^a represent their left- and right-handed couplings to the gauge bosons: $a=\gamma, W^{\pm}, Z$. For a standard-model doublet $g_L^{\gamma}=g_R^{\gamma}=eQ_i$, $g_L^Z=(e/s_w c_w)[T_{3i}-Q_i s_w^2]$, $g_R^Z=(e/s_w c_w)[-Q_i s_w^2]$, $g_L^W=e/\sqrt{2}s_w$, and $g_R^W=0$. We have renormalized $\delta\Pi_{ab}$ using the modified minimal subtraction scheme ($\overline{\mathrm{MS}}$), and μ^2 is the associated renormalization scale.

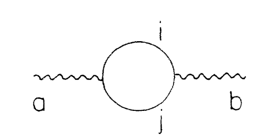

FIG. 1. The Feynman graph through which the heavy fermion doublet contributes to the gauge-boson vacuum polarization.

For simplicity we consider only the specific case of one additional doublet of degenerate leptons, for which $m_i=m_j\equiv m$. Then

$$\delta\Pi_{\gamma\gamma}(q^2)=\frac{e^2q^2}{2\pi^2}\int_0^1 dx\, x(1-x)\ln\left[\frac{m^2-q^2x(1-x)}{\mu^2}\right], \tag{19}$$

$$\delta\Pi_{Z\gamma}(q^2)=\frac{e^2q^2}{2\pi^2 s_w c_w}\left[\frac{1}{4}-s_w^2\right] \times\int_0^1 dx\, x(1-x)\ln\left[\frac{m^2-q^2x(1-x)}{\mu^2}\right], \tag{20}$$

$$\delta\Pi_{WW}(q^2)=\frac{e^2}{8\pi^2 s_w^2}\int_0^1 dx\left[q^2x(1-x)-\frac{m^2}{2}\right] \times\ln\left[\frac{m^2-q^2x(1-x)}{\mu^2}\right], \tag{21}$$

$$\delta\Pi_{ZZ}(q^2)=\frac{e^2}{8\pi^2 s_w^2} \times\int_0^1 dx\left[(1-2s_w^2+4s_w^4)q^2x(1-x)-\frac{m^2}{2}\right] \times\ln\left[\frac{m^2-q^2x(1-x)}{\mu^2}\right]. \tag{22}$$

Using these expressions in the definitions, Eqs. (5) and (13), gives the parameters S through X as functions of the mass of the doublet. The parameter T, which is a measure of custodial symmetry breaking, vanishes since the doublet is degenerate. We plot the behavior of the remainder of these parameters against m in Fig. 2. Note that all dependence on μ cancels in the definitions of S through X.

The curves in Fig. 2 verify the dominance of the parameter S when m is large. The parameters V, W, and X fall quite quickly to zero, as might be expected. However, for m small (~ 100 GeV), the new-physics parameters V and W are about the same size as S and U. (In Fig. 2, X is much smaller than the other parameters. This can be traced to the prefactor $(\frac{1}{4}-s_w^2)$ multiplying $\delta\Pi_{Z\gamma}(q^2)$ [Eq. (20)]. However, this is a peculiarity of this particular example—one does not expect X to necessarily be smaller than V and W. Similarly, $U \simeq -W$ for this case of one new doublet of degenerate leptons, but this is not expected to be true in general.) This example illustrates that there do indeed exist types of (light) new physics which contribute significantly to the extended oblique parameters V, W, and X. Therefore, if one wishes to use

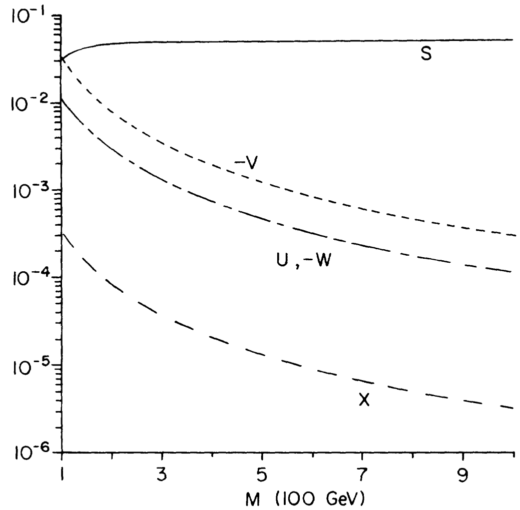

FIG. 2. A plot of the oblique-correction parameters S, U, V, W, and X against the mass of the heavy doublet of degenerate fermions which generate them. The parameter T vanishes identically because the doublet is degenerate.

current precision data to constrain such new physics, the fits should include not only S, T, and U, but also V, W, and X.

IV. CONCLUSIONS

The STU parametrization of oblique electroweak corrections, as presented in Ref. [2], has proven to be a useful tool for constraining new physics from above the electroweak scale. It is useful because it summarizes into a few quantities the implications for precision electroweak experiments of a broad class of interesting models. One of its limitations is that it can only be applied when the scale, M, of new physics is high enough to justify neglecting powers of M_Z^2/M^2. We have extended the analysis to the case where the threshold for new physics is too low to justify this approximation.

We find that even for the case of comparatively light new physics, current precision electroweak measurements are in practice only sensitive to a small number of independent parameters. Precisely three new ones are required, which we call V, W, and X. The dependence on these parameters of many of the well-measured observables is summarized in Table I. Neutral-current data is completely described by four quantities, S, T, V, and X, of which the latter two only contribute to observables defined at the Z resonance. A description of $W^{\pm}$ physics requires both the Peskin-Takeuchi U parameter, and the additional variable W, although W will only be relevant once measurements of the $W^{\pm}$ width considerably improve.

So few parameters suffice because at present precision experiments are confined to light-fermion scattering with four-momentum transfer that is either equal to, or much lower than, M_W or M_Z. As a result, the effects of any oblique correction are felt at only two energy scales, $q^2=0$ and $q^2=M_Z^2$ or M_W^2. This emphasizes the importance of performing precision measurements at other values of q^2, such as will be possible at LEP 200, since these experiments can probe complementary facets of the underlying physics.

We have illustrated how the parameters S through X can arise using a particularly simple example of a degenerate doublet of heavy fermions. This permits us to quantitatively follow the heavy-mass dependence of all six quantities. We verify how S comes to dominate as the doublet mass grows. However, for small masses, the parameters V and W are as large as S and U. (In this particular example, X happens to be considerably smaller than the other parameters; this will not be true in general.) In the future such an analysis could prove useful if a discrepancy between the standard model and these measurements were ever to arise. In this case, the mass scale of the new physics can be inferred from a comparison of the size of the new parameters V, W, and X, and that of S and T.

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