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Conformally invariant boundary conditions for dilaton gravity

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Quantum mechanical boundary conditions along a timelike line, corresponding to the origin in radial coordinates, in two-dimensional dilaton gravity coupled to N matter fields, are considered. Conformal invariance and vacuum stability severely constrain the possibilities. The simplest choice found corresponds to a nonlinear Liouville-type boundary interaction. The scattering of low-energy matter off the boundary can be computed perturbatively. It is found that weak incident pulses induce damped oscillations at the boundary while large incident pulses produce black holes. The response of the boundary to such pulses is semiclassically characterized by a second order, nonlinear ordinary differential equation which is analyzed numerically.

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I. INTRODUCTION

Two-dimensional dilaton gravity coupled to conformal matter provides a simple framework for studying the puzzles of black hole formation and evaporation. In some models, a fairly complete understanding of the leading term in a $1/N$ expansion (where N is the number of matter fields) of the quantum theory has been obtained [1–5]. Below a certain energy threshold, black holes are not formed (at this level of approximation). Above the threshold, black holes form and evaporate, and information is irretrievably lost into spacelike singularities.

The relevance of these results to the black hole information puzzle can be legitimately questioned on at least two grounds. First, despite the fact that it contains black holes and Hawking evaporation, the theory may not faithfully model the four-dimensional phenomena. Physics in two dimensions is certainly rife with peculiarities. Second, it is not understood, even in principle, how to compute the subleading corrections to the leading large-N behavior. Even a formal definition of the exact quantum theory has not been given for these models. Thus it cannot be claimed that any fully self-consistent model with information loss exists. Clearly it is important to establish whether or not this is the case.

One of the main issues at stake here is the nature of the boundary conditions imposed along a timelike line which lead to the reflection of below-threshold incoming energy [4]. Such a boundary is required in order that the two-dimensional theory faithfully models the desired four-dimensional physics. The two dimensions correspond to time and the half-line $r > 0$. The boundary corresponds to the origin of the four-dimensional, spherically symmetric spacetime. A below-threshold pulse which is reflected off of the boundary in the two-dimensional model corresponds to a low-energy four-dimensional S wave which passes through the origin without gravitationally collapsing.

The boundary conditions are highly constrained by the following consistency conditions.

(1) *Conformal Invariance.* General covariance and energy conservation require that the boundary conditions are conformally invariant.

(2) *Vacuum Compatibility.* The linear dilaton vacuum, or a close cousin, must be compatible with the boundary conditions. If the nature of the vacuum is greatly altered, the model cannot be used to study black hole physics.

(3) *Vacuum Stability.* The boundary conditions should ensure that the vacuum is stable under small perturbations.

In this paper we build on earlier work [4, 6–8] and analyze the problem semiclassically. A solution is found only when $N > 24$ (but is not necessarily "large"). In the large-N limit it agrees with that found previously by Chung and Verlinde [7]. The solution involves an exponential Liouville-type boundary interaction. There is one free parameter Y_0 which governs the strength of the boundary interaction, but this parameter cannot be dialed to turn off the interaction without encountering instabilities. The result of throwing low-energy pulses at the boundary can be computed perturbatively. For a range of Y_0 and $N > 24$, the incoming pulse excites damped oscillations of the boundary.[1] For values of Y_0 outside this range, incoming pulses excite exponentially growing oscillations and vacuum instability. When $N < 24$, there is no stable range of Y_0. (Although al-

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[1]Depending on the gauge condition, either the boundary curve itself or fields at the boundary oscillate.

ternate boundary conditions, which give stable dynamics for $N < 24$, may well exist.) The interesting special case $N = 24$ will be described elsewhere [9] and has been previously discussed from a somewhat different perspective in [6, 8].

For the string theorists in the audience, we note that the question of consistent boundary conditions can be viewed as a problem in open string theory. In that language, our boundary conditions correspond to an open string tachyon condensate.

All the results of this paper are based on a semiclassical approximation. Operationally it is clear how to implement this approximation. It is much less clear when the approximation is reliable. It is generally justified only at large N, so our strongest results pertain to the large-N limit. At finite N the approximation is still justified for the computation of certain quantities in coherent semiclassical states, and may generally provide insight into the structure of the theory. However, we wish to caution the reader that its reliability is much more limited at finite N. While we do believe that exactly conformally invariant boundary conditions of the type we describe exist at finite N, the methods of this paper are insufficient to establish that. A fully quantum treatment may be possible when $N = 24$, where substantial simplifications occur.

In Sec. II we establish our notation and review the transformation of the bulk theory of two-dimensional dilaton gravity to a soluble conformal field theory [2, 3, 11]. We also show that black holes in these theories evaporate semiclassically at a rate proportional to N, even when $N < 24$, as desired. (The literature contains conflicting claims on this point.) In Sec. III our boundary conditions are presented and analyzed, and in Sec. IV we consider their large-N limit. Section V analyzes the short-distance limit of the theory, in which the bulk cosmological constant can be ignored. This limit is of special interest because the full theory, including the exponential boundary interaction, is semiclassically (and possibly exactly) soluble.

The present work on low-energy scattering is a prerequisite to, but still a long way from, a fully consistent quantum description of black hole formation and evaporation as approximately described at large N. In particular we do not address the issue of how the black hole disappears after shrinking to the Planck size. This black hole end point is in general out of causal contact with the point where the collapsing matter arrives at the origin (as discussed in Sec. III) and so cannot be affected by any boundary conditions we impose there. We leave this vexing issue for future work [10].

II. THE BULK THEORY

In this section we describe the bulk conformal field theory of two-dimensional dilaton gravity for arbitrary N, deferring the issue of boundary conditions to the next section. Our aim is mainly to refresh the reader's memory and fix conventions, but we also present some new material on the rate of Hawking evaporation of large mass black holes. The reader is referred to [5] for more thorough reviews of the subject.

A. Quantization

Classical dilaton gravity is described by the action

$$S_{\text{classical}} = \frac{1}{2\pi}\int d^2\sigma\sqrt{-g}e^{-2\phi} \times \left[R + 4(\nabla\phi)^2 + 4\lambda^2 - \frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right]. \quad (1)$$

In the conformal gauge,

$$ds^2 = -e^{2\rho}d\sigma^+ d\sigma^-, \quad (2)$$

where $\sigma^\pm = \sigma^0 \pm \sigma^1$, the action becomes

$$S_{\text{classical}} = \frac{1}{\pi}\int d^2\sigma\left[-2\partial_+ e^{-2\phi}\partial_-(\rho-\phi) + \lambda^2 e^{2(\rho-\phi)} + \frac{1}{2}\sum_{i=1}^{N}\partial_+ f_i\partial_- f_i\right]. \quad (3)$$

At the one-loop level, the action (3) acquires a well-known correction

$$-\frac{N}{12\pi}\int d^2\sigma\partial_+\rho\partial_-\rho \quad (4)$$

from the functional measure for the matter fields. The accompanying correction to the stress tensor will be given below. A similar correction to (1) arises from the ρ, ϕ, and ghost measures [12]:

$$\frac{2}{\pi}\int d^2\sigma\partial_+(\rho-\phi)\partial_-(\rho-\phi). \quad (5)$$

The combination $\rho - \phi$ (rather than ρ) appears in (5) because the natural metric for the ρ, ϕ, and ghost fields is $ds^2 = -e^{2\rho-2\phi}d\sigma^+d\sigma^-$, rather than (2). This is the metric which appears in the kinetic term for ρ and ϕ in the classical action (3). We will also see that this choice of measure is required in order that black holes Hawking evaporate at a rate proportional to N, rather than $N-24$.

The choice (2) of conformal gauge leaves unfixed a group of residual coordinate transformations which is isomorphic to the conformal group. Thus in order to maintain coordinate invariance of the quantum theory, one must ensure that conformal invariance of (3), regarded as a theory of the $N+2$ bosons f_i, ρ, and ϕ, is preserved by the quantization procedure. The sum of the classical action (3) plus the one-loop corrections (4) and (5) is not conformally invariant to all orders in the loop expansion parameter $e^{2\phi}$ (although it is conformally invariant to leading order in the $1/N$ expansion). To remedy this we must add counterterms at each order. Conformal invariance does not uniquely fix these counterterms. Different choices lead to inequivalent but in general qualitatively similar theories. It behooves us to choose the counterterms so as to simplify theory as much as possible. As discussed in [3, 2, 11], a judicious choice leads to a soluble conformal field theory. Let[2]

[2]A separate treatment will be given for $N = 24$ ($\gamma = 0$) in [9].

$$\gamma = \frac{N-24}{12}\,,$$
$$Y = -\frac{1}{\gamma}\int d\phi\sqrt{4e^{-4\phi} - 4(\gamma+2)e^{-2\phi} + 2(\gamma+2)}\,,$$
$$X = \rho + \frac{e^{-2\phi} + 2\phi}{\gamma}\,. \tag{6}$$

The action is then given by

$$S = \frac{1}{\pi}\int d^2\sigma\left[-\gamma\partial_+ X\partial_- X + \gamma\partial_+ Y\partial_- Y + \lambda^2 e^{2(X-Y)} + \frac{1}{2}\sum_{i=1}^{N}\partial_+ f_i\partial_- f_i\right]. \tag{7}$$

Substitution of (6) into this action leads back to the original action (3) [corrected by (4) and (5)], plus additional potential terms which are subleading in the loop expansion parameter $e^{2\phi}$. These corrections ensure exact conformal invariance.

The gravitational and matter parts of the stress tensor are given by[3]

$$T^g_{++} = \gamma\left(\partial_+ Y\partial_+ Y - \partial_+ X\partial_+ X + \partial_+^2 X\right)\,,$$
$$T^f_{++} = \frac{1}{2}\sum_{i=1}^{N}\partial_+ f\partial_+ f\,, \tag{8}$$

along with a similar expression for T^g_{--}, T^f_{--}. Together they generate a $c = 26$ Virasoro algebra.

The theory defined by (7) is almost as simple as a free field theory. Indeed, in Sec. V we show how it can be transformed into one. The null combination $(X - Y)$ which appears in the exponential in (7) does not have a singular operator product with itself, and a superselection rule prevents the exponential interaction term from generating corrections to the free operator product expansions (OPE's). For these reasons the theory is substantially simpler than the Liouville theory, which it superficially resembles. For example, the quantum effective action can be (formally) exactly computed and is the same as the original action (7) [4]. However, these simplifications evaporate to a large extent when the boundary is introduced, as we shall see in Sec. III.

It is instructive to consider the semiclassical solutions to this theory. By this we mean solutions of (7), rather than of the original classical action (1). Thus our semiclassical solutions will contain terms of arbitrary order in $e^{2\phi}$, and know about Hawking radiation and black hole evaporation. The semiclassical equations of motion have a two-parameter family of static solutions labeled by the constants μ and F:

$$\gamma X = e^{2\lambda\sigma} + \left(F + \frac{\gamma}{2}\right)\lambda\sigma + \frac{\mu}{\lambda},$$
$$Y = X - \lambda\sigma\,, \tag{9}$$

where $\sigma = \frac{1}{2}(\sigma^+ - \sigma^-)$. We have expressed the solutions here in the "σ gauge," generally defined by the condition $Y = X - \lambda\sigma$, in which the static field configurations are independent of the timelike coordinate. For all these solutions $\phi \to -\lambda\sigma$ and $\rho \to 0$ asymptotically. The vacuum corresponds to $\mu = 0$ and $F = -1$ [11]:

$$\gamma X = e^{2\lambda\sigma} + \left(\frac{\gamma}{2} - 1\right)\lambda\sigma,$$
$$Y = X - \lambda\sigma\,. \tag{10}$$

The gravitational stress tensor takes the value $T^g_{++} = \frac{\lambda^2}{2}$ in the vacuum configuration (10). The nonvanishing right-hand side of this equation cancels (in sigma coordinates) against the ghost stress tensor, $T^{\rm gh}_{++} = 2(\partial_+(\rho - \phi))^2$, which is included in T^g_{++} in the left-hand side of the equation. Equivalently, the ghost vacuum is annihilated by annihilation operators defined in the (Kruskal) coordinates in which $\rho = \phi$ asymptotically.[4] Although this seems the most natural choice, *a priori*, other choices of the ghost vacuum state might be considered. However, we shall see in the next subsection that black holes evaporate at a rate proportional to N only if $F = -1$.

B. Black hole formation and evaporation

Consider matter incident on the vacuum (10) from $\mathcal{I}^-$ characterized by some given energy profile $T^f_{++}(\sigma^+)$. The semiclassical gravitational field is a solution of the constraints

$$T^g_{++} = -T^f_{++} - \frac{F\lambda^2}{2},$$
$$T^g_{--} = -\frac{F\lambda^2}{2}. \tag{11}$$

This is asymptotic to the vacuum (10) only when $F = -1$, but for the moment we consider arbitrary values of F in order to see why $F = -1$ is indeed the correct choice. The solution of these equations is

$$\gamma X = e^{2\lambda\sigma} - \frac{1}{\lambda}e^{\lambda\sigma^+}P_+(\sigma^+) + \frac{1}{\lambda}M(\sigma^+) + \left(F + \frac{\gamma}{2}\right)\lambda\sigma\,,$$
$$Y = X - \lambda\sigma\,, \tag{12}$$

where

$$M(\sigma^+) = \int_{-\infty}^{\sigma^+} T^f_{++}(v^+)dv^+\,,$$
$$P_+(\sigma^+) = \int_{-\infty}^{\sigma^+} e^{-\lambda v^+}T^f_{++}(v^+)dv^+. \tag{13}$$

The Bondi mass measured on $\mathcal{I}^+$ is given by

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[3]Our normalization follows prevalent dilaton gravity conventions, which unfortunately differ from prevalent conformal field theory conventions.

[4]There is a potential conflict here with the no ghost theorem which we have not addressed. See Ref. [6] for a discussion of this theorem in two-dimensional dilaton gravity.

$$m(y^-) = 2e^{\lambda(y^+-y^-)}(\lambda\,\delta\rho + \partial_+\delta\phi - \partial_-\delta\phi)\,. \quad (14)$$

This expression is evaluated in asymptotically inertial coordinates $y^\pm$ in which $\rho \to 0$, and $\delta\phi$ and $\delta\rho$ are the deviations of ϕ and ρ from their vacuum values. The inertial coordinates $y^\pm$ at $\mathcal{I}^+$ are

$$y^+ = \sigma^+,$$
$$y^- = \sigma^- - \frac{1}{\lambda}\ln\left(1 - \frac{1}{\lambda}\bar{P}_+ e^{\lambda\sigma^-}\right), \quad (15)$$

where $\bar{P}_+ \equiv P_+(\infty)$. To leading order in $e^{-\lambda\sigma^+}$, one finds

$$\delta\phi = -\frac{\delta Y}{2Y} = \delta\rho\,, \quad (16)$$

where

$$\gamma\delta Y = \frac{\bar{M}}{\lambda} - \frac{1}{4}(\gamma - 2F)\ln\left(1 + \frac{1}{\lambda}\bar{P}_+ e^{\lambda y^-}\right), \quad (17)$$

with $\bar{M} \equiv M(\infty)$. The Bondi mass is then given by

$$m = \bar{M} - \frac{\lambda}{2}\left(\frac{N}{24} - 1 - F\right) \times \left[\ln\left(1 + \frac{1}{\lambda}\bar{P}_+ e^{\lambda y^-}\right) + \frac{\bar{P}_+}{\bar{P}_+ + \lambda e^{-\lambda y^-}}\right]. \quad (18)$$

The rate of change of the mass is evidently proportional to N if and only if $F = -1$.

At early retarded times ($y^- \to -\infty$) the mass (18) decays at the rate predicted by naive semiclassical reasoning which ignores back reaction. At late times ($y^- \to +\infty$), however, a disaster occurs: the mass plummets to minus infinity. This is not so surprising because μ in Eq. (9) can be arbitrarily negative and the system has no ground state. The "vacuum" (10) is unstable under arbitrarily small perturbations in the context of the bulk theory considered so far. We shall see that this disaster is averted for small $\bar{P}_+$ when appropriate boundary conditions are imposed.

III. BOUNDARY CONDITIONS

In the bulk theory of the preceding section, the range of values taken by X and Y is unrestricted. For $\gamma > 0$ this is unphysical because taking Y below a certain minimum value corresponds, in this case, to complex values of the original dilaton field ϕ [4]. When two-dimensional dilaton gravity is derived by spherical reduction from four dimensions, this corresponds to transverse two-spheres of negative area. The bothersome negative energy configurations are also characterized by large regions in which Y is below this minimum. For $\gamma < 0$ the field redefinition (6) is nondegenerate for all values of Y but the bulk theory has nevertheless the same negative energy problem.

One can try to remedy these problems by simply restricting the range of Y so that the original fields ρ and ϕ are real. However, the resulting functional integral no longer defines a conventional quantum field theory, and in particular does not correspond to any easily identifiable conformal field theory.

An alternate procedure, which does lead to a conformal field theory, is to impose boundary conditions along a timelike line σ = const at or near the "origin." Incoming fields will then be reflected at this line to outgoing fields. In the physical region to the right of this line, the semiclassical vacuum values of X and Y will correspond to real values of ρ and ϕ. Strong quantum fluctuations, or large incoming pulses, will still produce low values of Y, but this does not preclude the perturbative construction of a low-energy S matrix for asymptotic observers.

One might hope to also effect a cure of the negative energy instability of the $\gamma < 0$ theory by imposing a boundary condition at some value of Y, even if there is no natural minimum value in that case. As we shall see, however, the solutions exhibit a qualitatively different behavior for $\gamma < 0$ and we have not found any boundary conditions which stabilize that theory.

A. Conformal invariance and vacuum compatibility

It is imperative that the boundary conditions respect conformal invariance. Otherwise the theory is not generally covariant and cannot be regarded as a theory of gravity. Boundary conformal invariance is equivalent to [13]

$$T_{++}(0,\tau) = T_{--}(0,\tau), \quad (19)$$

where T is the total stress tensor for all fields. If either Dirichlet or Neumann boundary conditions are imposed on the matter fields f_i,

$$\partial_+ f_i(0,\tau) \pm \partial_- f_i(0,\tau) = 0, \quad (20)$$

then the boundary condition (19) implies

$$\partial_+^2 X - \partial_+ X\partial_+ X + \partial_+ Y\partial_+ Y = \partial_-^2 X - \partial_- X\partial_- X + \partial_- Y\partial_- Y. \quad (21)$$

This equation is *semiclassically* solved by either

$$\partial_+ X - \partial_- X = \lambda F(Y)\, e^X,$$
$$\partial_+ Y - \partial_- Y = -\lambda\frac{\partial F}{\delta Y}\, e^X, \quad (22)$$

where $F(Y)$ is an arbitrary function,[5] or

$$\partial_+ X - \partial_- X = \lambda A e^{X - Y_0},$$
$$Y = Y_0, \quad (23)$$

where A and Y_0 are constants. In the following we will focus on (23). The imposition of these boundary conditions transforms the relatively trivial bulk theory to a highly nonlinear, interacting theory.

Compatibility of these boundary conditions with the vacuum solution (10) constrains the parameters A and

[5]In the language of string theory this corresponds to an open string tachyon, $T = F(Y)e^X$, which satisfies the classical β-function condition, $\nabla\Phi \cdot \nabla T = 8T$.

Y_0. For a given value of A, the X boundary condition is satisfied for the vacuum solution (10) only along the timelike line

$$\lambda\sigma = \omega_0,$$
$$\gamma A = 2e^{\omega_0} + \frac{(\gamma-2)}{2}e^{-\omega_0}, \tag{24}$$

along which

$$\gamma Y = e^{2\omega_0} - \frac{(\gamma+2)}{2}\omega_0. \tag{25}$$

A and Y_0 are, therefore, determined by the single free parameter[6] ω_0. For a given value of A and $\gamma > 2$, there are two values of ω_0 consistent with (24). However, we shall see that at most one is stable. Note also that there is a nonzero minimum value of A:

$$A_{\rm min} = \frac{2\sqrt{\gamma-2}}{\gamma}, \tag{26}$$

for $\gamma > 2$. Thus it is not possible in this case to analyze the theory perturbatively in the strength of the boundary interaction.

The boundary conditions (22) and (23) are only semiclassical solutions of the operator condition (19) for conformal invariance. To verify that they imply the reflection condition (19) on the stress tensor to leading order, X and Y are treated like c-number fields. The boundary conditions can presumably be modified order by order in the loop expansion of (7) in order to maintain (19).

Of course, the semiclassical approximation is not always reliable. Corrections to the semiclassical approximation can be systematically suppressed by taking γ to be large, as will be discussed in Sec. IV. However, even when γ is not large, the semiclassical approximation can be good for certain quantities or certain states. For example, we expect the semiclassical approximation to be good for calculating the radiation rate of a large black hole for any value of γ. We also expect that a necessary condition for the semiclassical approximation (23) to the boundary conditions to be good is that the boundary is in a weak-coupling, large-radius (i.e., large Y) region. It follows immediately from (25) that it is always possible to arrange that this is the case by adjusting the free parameter A in (23) to be very large.

B. Dynamical boundary curve

A boundary condition imposed at $\sigma = \text{const}$ restricts the left and right conformal invariance to a diagonal subgroup. Separate left and right invariance can be regained, however, at the price of allowing the boundary to follow a general trajectory, described by the equation $x_B^-(x^+) = x^-$. In this case constancy of Y along the boundary implies

$$\frac{1}{u}\partial_+ Y + u\,\partial_- Y = 0, \tag{27}$$

where

$$u(x^+) \equiv \left(\frac{\partial x_B^-}{\partial x^+}\right)^{1/2}, \tag{28}$$

and $(\frac{1}{u}, u)$ is the tangent vector to the boundary curve. The Neumann or Dirichlet conditions on the matter fields become

$$\frac{1}{u}\partial_+ f_i \pm u\,\partial_- f_i = 0\,. \tag{29}$$

The general form of the X boundary conditions follows readily from the observation that (23) is equivalent to the geometric condition that the extrinsic curvature of the boundary curve in the metric $ds^2 = -e^{2X}dx^+dx^-$ is constant. For a general curve the first equation of (23) becomes

$$\frac{1}{u}\partial_+ X - u\,\partial_- X = \lambda A e^{X-Y_0} + \frac{1}{u^2}\partial_+ u. \tag{30}$$

The boundary conditions (27) and (30) can be used to relate the components T^g_{++} and T^g_{--} of the gravitational stress tensor along the boundary. Since (30) holds everywhere along the boundary, a new identity can be obtained by acting on both sides with the operator $\frac{1}{u}\partial_+ + u\partial_-$ which generates translations along the boundary. One finds

$$\frac{1}{u^2}(\partial_+^2 X - \partial_+ X\partial_+ X) = u^2(\partial_-^2 X - \partial_- X\partial_- X) - \frac{1}{u}\partial_+^2\left(\frac{1}{u}\right). \tag{31}$$

Taken together with the Y boundary condition (27) this implies that

$$\frac{1}{u^2}T^g_{++} = u^2 T^g_{--} - \frac{\gamma}{u}\partial_+^2\left(\frac{1}{u}\right). \tag{32}$$

The last "Schwinger" term is well known in studies of moving mirrors [14]. It vanishes in "straight-line" gauges for which u is constant. This Schwinger term was omitted in the reflecting boundary conditions discussed in [4]. The boundary conditions of [4] can apparently not be derived as the semiclassical limit of any conventional quantum mechanical boundary conditions.[7] They nevertheless give rise to a stable evolution which conserves semiclassical energy.

A differential equation for the boundary curve can be easily derived in Kruskal gauge which is defined by the condition

$$X(x^+,x^-) = Y(x^+,x^-)\,, \tag{33}$$

where Kruskal and sigma coordinates are related by

$$\lambda x^\pm = \pm e^{\pm\lambda\sigma^\pm}. \tag{34}$$

[6]This constraint could be relaxed by considering $\mu \neq 0$ solutions in (9), but we have not explored this possibility.

[7]This observation was made in collaboration with S. Trivedi.

Adding (27) and (30) we then find

$$\frac{2}{u}\partial_+ Y = \lambda A + \frac{1}{u^2}\partial_+ u\,. \tag{35}$$

Multiplying by u and acting on both sides with $(\partial_+ + u^2\partial_-)$ (i.e., differentiating along the boundary), one obtains

$$2\partial_+^2 Y + 2u^2\partial_-\partial_+ Y = \lambda A\partial_+ u + \partial_+^2 \ln u\,. \tag{36}$$

In Kruskal gauge the general solution (12) becomes

$$Y = -\frac{1}{\gamma}\left(\lambda^2 x^+ x^- + x^+ P_+ - \frac{M}{\lambda} + \frac{\gamma+2}{4}\ln(-\lambda^2 x^+ x^-)\right). \tag{37}$$

The derivatives of Y are then

$$\partial_+ Y = -\frac{1}{\gamma}\left(\lambda^2 x^- + P_+ + \frac{\gamma+2}{4x^+}\right),$$
$$\partial_+^2 Y = -\frac{1}{\gamma}\left(T^f_{++} - \frac{\gamma+2}{4x^{+2}}\right),$$
$$\partial_-\partial_+ Y = -\frac{\lambda^2}{\gamma}. \tag{38}$$

Substituting these relations into (36) leads to

$$\partial_+^2 \ln u + \lambda A\partial_+ u + \frac{2\lambda^2 u^2}{\gamma} - \frac{\gamma+2}{2\gamma x^{+2}} = -\frac{2}{\gamma}T^f_{++}. \tag{39}$$

An alternate form of this equation is obtained by defining

$$\omega(\sigma^+) = \lambda\sigma^+ + \ln u\,. \tag{40}$$

ω is a useful variable because, unlike u, it is a constant in the vacuum. The resulting equation

$$\omega'' + k(\omega)\lambda\omega' + \frac{\partial V(\omega)}{\partial\omega}\lambda^2 = -\frac{2}{\gamma}T^f_{++}\,, \tag{41}$$

can be interpreted in terms of a particle moving in a potential subject to a driving force and a nonlinear damping force, where $k(\omega) = Ae^\omega - 1$, $\frac{\partial V(\omega)}{\partial\omega} = \frac{2}{\gamma}e^{2\omega} - Ae^\omega + \frac{\gamma-2}{2\gamma}$, and the primes denote differentiation with respect to σ^+. Damping arises because boundary energy can be dissipated into (or absorbed from) the rest of the spacetime. Note that the damping becomes negative for sufficiently negative ω. The potential is

$$V = \frac{1}{\gamma}e^{2\omega} - Ae^\omega + \frac{\gamma-2}{2\gamma}\omega. \tag{42}$$

For $\gamma > 0$, in the vacuum (10) we have $T^f_{++} = 0$ and the particle sits at the local minimum of the potential. The value ω_0 of ω at the minimum is

$$e^{\omega_0} = \frac{\gamma}{4}\left(A + \sqrt{A^2 - A^2_{\text{min}}}\right), \tag{43}$$

[where A_{min} is defined in Eq. (26)] in agreement with (24). For $\gamma < 0$, the potential goes to minus infinity for large negative or positive ω, and there is no local minimum. Substituting $\omega = \omega_0$ into (40), one finds

$$\frac{\partial x_B^-}{\partial x^+} = u^2 = \frac{e^{2\omega_0}}{\lambda^2 x^{+2}}\,, \tag{44}$$

or

$$-\lambda^2 x^+ x_B^- = e^{2\omega_0}\,. \tag{45}$$

The vacuum boundary curve is thus a hyperbola in Kruskal coordinates, which means it is a straight line located at $\lambda\sigma = \omega_0$ in sigma coordinates.

C. Vacuum stability

It does not appear possible to solve analytically for the boundary trajectory for a general incoming pulse but one can easily obtain the leading order in a perturbation expansion in the strength of the incoming pulse. Linearizing (41) around the vacuum solution one finds

$$\gamma\hat\omega'' + b\lambda\hat\omega' + (b+2)\lambda^2\hat\omega = -2T^f_{++}\,, \tag{46}$$

where $\hat\omega = \omega - \omega_0$ and

$$b = 2e^{2\omega_0} - \frac{\gamma+2}{2}\,. \tag{47}$$

The general solution to the corresponding homogeneous equation is given by

$$\hat\omega = C_+ e^{\alpha_+\sigma^+} + C_- e^{\alpha_-\sigma^+}\,, \tag{48}$$

where

$$\alpha_\pm = \frac{\lambda}{2\gamma}\left(-b \pm \sqrt{b^2 - 4\gamma b - 8\gamma}\right). \tag{49}$$

Stability of the vacuum under small perturbations requires that both solutions in (48) be exponentially damped. It is straightforward to show that this is possible only for $\gamma > 0$. Even for $\gamma > 0$, the behavior of the solutions depends on the choice of A (or equivalently ω_0). In this case, stability requires $b > 0$. This condition can be understood by noting that $b = \frac{\gamma}{\lambda}\partial_\sigma Y(\omega_0)$ when Y is in the vacuum configuration (10) and, therefore, both b and the slope of Y change sign at a minimum value Y_{min} of Y. In other words, the boundary conditions can only stabilize the system when the boundary is placed on the physical side of Y_{min}.

The condition $b > 0$ can also be translated into a restriction on A, which is

$$A > \frac{2}{\sqrt{\gamma+2}}. \tag{50}$$

For $\gamma > 2$, this condition is a consequence of $A > A_{\text{min}}$ so all $\gamma > 2$ vacuum-compatible boundary conditions are stable. When $0 < \gamma < 2$, this is a new restriction. Apparently (for $\gamma \neq 0$) $A = 0$ is never a stable boundary conditon, and one cannot perturb in A.

For $\gamma < 0$ (i.e., $N < 24$) the perturbations grow exponentially for any value of A, and we know of no stable boundary conditions. The case $\gamma = 0 (N = 24)$ will be discussed in a separate publication [9].

While u settles back to its vacuum values after the incident perturbation is reflected, $x_B^-(x^+)$ undergoes a constant shift. This can be directly seen from Eq. (35):

$$\frac{2}{u}\partial_+ Y = \lambda A + \frac{1}{u^2}\partial_+ u. \tag{51}$$

The right-hand side goes to a constant, but u itself vanishes as $x^+ \to \infty$. This implies that asymptotically

$$\lambda^2 x_B^- + \bar{P}_+ = -\frac{e^{2\omega_0}}{x^+}, \tag{52}$$

so that x_B^- goes to $-\bar{P}_+/\lambda^2$. The boundary curve (52) corresponds to a coordinate transformation of the original vacuum.

Our boundary conditions were constructed so as to be consistent with the conformal invariance of the bulk theory, which in particular means that energy is conserved when the fields are reflected from the boundary. Conformal invariance at $\mathcal{I}^+$ ($\mathcal{I}^-$) ensures that the change in the Bondi mass equals the outgoing (incoming) energy flux. However, conformal invariance does not guarantee that the total incoming and outgoing energies are equal: energy could get stuck on the boundary. To see that this does not happen for sufficiently small incoming pulses, consider the following function defined in the Kruskal gauge:

$$m(x^+,x^-) = \frac{\gamma^2}{\lambda}\partial_+ Y \partial_- Y + \gamma\lambda Y + \frac{(\gamma+2)\lambda}{4}\left[\ln(\gamma Y) - 2\right] + \frac{B\lambda}{Y}, \tag{53}$$

where the constant B is given by

$$B = \frac{1}{4}e^{-2\omega_0} b^2 Y_0 - \gamma Y_0^2 - \frac{(\gamma+2)}{4} Y_0 \left[\ln(\gamma Y_0) - 2\right]. \tag{54}$$

B is chosen so that m vanishes on the boundary if u takes its vacuum form $e^{\omega_0}/\lambda x^+$, while the constant terms in (53) have been chosen so that m vanishes asymptotically in the vacuum. For solutions of the equations of motion, which correspond to matter energy incident on the vacuum, it is straightforward to establish that $m(x^+,x^-)$ has the properties

$$\lim_{x^\pm \to \pm\infty} m(x^+,x^-) = m(x^\mp), \tag{55}$$

where $m(x^\mp)$ is the Bondi mass defined in Eq. (14). Asymptotically the function $m(x^+,x^-)$ thus provides an alternate definition of the Bondi mass.

We will now evaluate $m(x^+,x^-)$ along the boundary curve and verify that it vanishes before and well after all the incoming matter is reflected. In Kruskal coordinates the boundary conditions (27) and (30) imply that along the boundary $Y = Y_0$ and

$$\partial_+ Y \partial_- Y = -\frac{1}{4}\left(\lambda A + \frac{1}{u^2}\partial_+ u\right)^2. \tag{56}$$

We have already seen that u settles back to its vacuum form after a small pulse is reflected from the boundary. In fact, the right-hand side of (56) goes to the same constant in the two limits $x^+ \to 0$ and $x^+ \to \infty$ and the boundary energy thus vanishes at future timelike infinity. Since the total energy is conserved, this implies that the incoming and outgoing energy are equal.

The global behavior of the boundary curves will be analyzed in more detail for the large-N case in Sec. IV.

D. A disaster

We have seen how conformally invariant boundary conditions, imposed at a boundary placed on the weak coupling side of $Y = 0$, ensure that small pulses incoming from $\mathcal{I}^-$ are reflected (in a distorted form) up to $\mathcal{I}^+$. The behavior for large pulses is quite different. Consider a pulse which begins (in the Kruskal gauge) at an initial x_i^+ and has total momentum $\bar{P}_+$. It was seen in Sec. II that, in the absence of a boundary, the mass plunges to minus infinity at a point on $\mathcal{I}^+$, which is located at $x^- = -\bar{P}_+/\lambda^2$. This behavior is potentially changed by the presence of the boundary. The pulse first reaches the boundary at

$$(x^+,x^-) = \left(x_i^+, -\frac{e^{2\omega_0}}{\lambda^2 x_i^+}\right). \tag{57}$$

By causality the behavior on $\mathcal{I}^+$ cannot be affected by boundary reflection prior to $x^- = -e^{2\omega_0}/\lambda^2 x_i^+$. Thus for

$$\bar{P}_+ > e^{2\omega_0}/x_i^+ , \tag{58}$$

the mass still plunges to minus infinity. *No boundary condition at the origin can possibly avert this disaster for sufficiently large incoming momentum* $\bar{P}_+$. Since the causal past of the point $x^- = -\bar{P}_+/\lambda^2$ on $\mathcal{I}^+$ may include only regions of weakly coupled dynamics, this disaster cannot in general be averted by modifications of strongly coupled dynamics. Aversion of this disaster requires fundamentally new input, such as the inclusion of topology-changing processes. Such considerations are beyond the scope of the present work.

It should be noted that this disaster does not necessarily imply a sickness in the X,Y conformal field theory itself: rather it arises in the transcription from the X,Y conformal field theory to a ρ,ϕ theory of dilaton gravity. The point $\lambda^2 x^- = -\bar{P}^+$ is at a finite distance in the fiducial metric used to regulate the X,Y conformal field theory, and the X,Y fields can be continued past this point. The reflected pulse (and the information it carries) eventually comes out back. However, this occurs "after the end of time" as measured by the physical metric $ds^2 = -e^{2\rho}dx^+dx^-$.

IV. THE LARGE-N LIMIT

An interesting special case (considered previously by Chung and Verlinde [7]) of our equations is obtained by taking the limit of a large number of matter fields, i.e., $\gamma \to \infty$. In taking this limit, A, λ^2, Y, $2\tilde{X} = 2X - \ln(N/12)$, and $\tilde{T}^f_{++} = \frac{12}{N}T^f_{++}$ are held fixed. The

$\tilde{X}$ and Y OPE's vanish in this limit and corrections to the semiclassical approximation are systematically suppressed.

The large-N formulas are derived as in the previous sections and we collect a few of them here. The semiclassical action (7) becomes

$$S = \frac{N}{12\pi}\int d^2\sigma\left[-\partial_+\tilde{X}\partial_-\tilde{X} + \partial_+Y\partial_-Y + \lambda^2 e^{2(\tilde{X}-Y)} + \frac{6}{N}\sum_{i=1}^{N}\partial_+f_i\partial_-f_i\right], \tag{59}$$

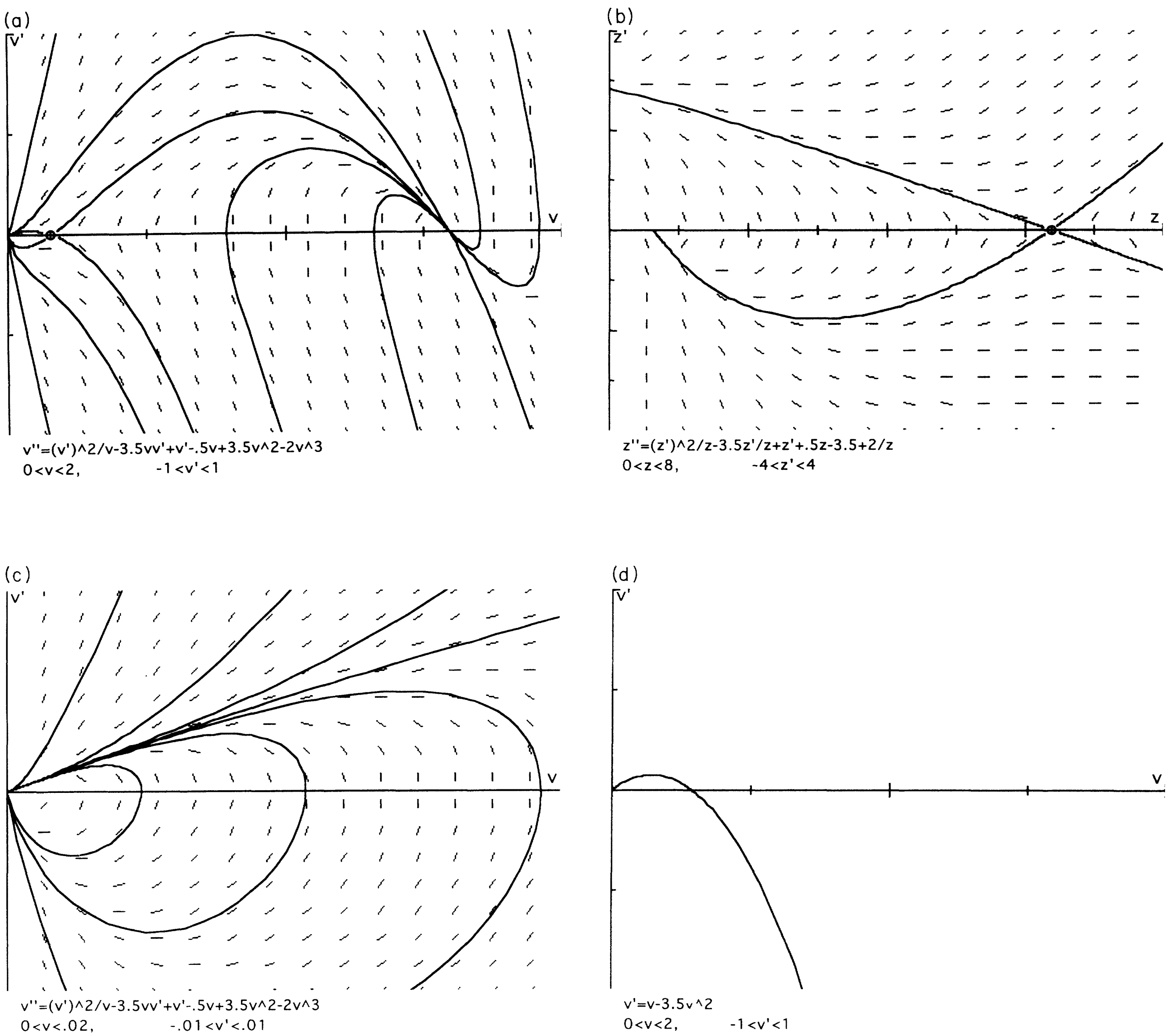

FIG. 1. (a) Phase portrait for overdamped ($A = 3.5$) trajectories. The two trajectories emanating from the saddle point at $v = 0.157$ go to the vacuum at $v = 1.59$ and to the origin. The two trajectories which asymptote to the saddle point divide the phase plane into basins of attraction of the origin and of the vacuum. (b) Saddle trajectories for $A = 3.5$ as a function of $z = 1/v$. The vacuum is located at $z = 0.629$. The basin of attraction of the vacuum for this overdamped case extends to $z = 0$ (infinite v). (c) Enlarged view of Fig. 1(a) near the origin. All trajectories emanating from the origin have an initial positive universal slope $v' = 0.5v$. Approaching trajectories arrive from negative v' with an asymptotically vertical slope. (d) For $A = 3.5$, the boundary enters a black hole and a spacelike line with $Y = Y_0$ will branch off of the timelike boundary at the points along the curve indicated. Comparison with Fig. 1(a) reveals that the curve crosses $v' = 0$ between the saddle and the vacuum.

and the ++ constraint equation can be written

$$\partial_+ Y \partial_+ Y - \partial_+ \tilde{X} \partial_+ \tilde{X} + \partial_+^2 \tilde{X} + \tilde{T}^f_{++} = 0\,. \qquad (60)$$

The solution corresponding to incoming matter (without reflection) is, in the σ gauge,

$$\begin{aligned} Y &= \tilde{X} - \lambda\sigma \\ &= e^{2\lambda\sigma} - \frac{1}{\lambda} e^{\lambda\sigma^+} \tilde{P}_+(\sigma^+) + \frac{1}{\lambda}\tilde{M}(\sigma^+) - \frac{\lambda}{2}\sigma\,, \end{aligned} \qquad (61)$$

where $\tilde{P}_+$ and $\tilde{M}$ are constructed from $\tilde{T}^f_{++}$. The boundary conditions (23) are unchanged. The boundary equation becomes

$$\omega'' + \lambda(Ae^{\omega} - 1)\omega' + \lambda^2 \left(2e^{2\omega} - Ae^{\omega} + \frac{1}{2}\right) = -2\tilde{T}^f_{++}\,. \qquad (62)$$

In the vacuum

$$e^{\omega_0} = \frac{1}{4}\left(A + \sqrt{A^2 - 4}\right). \qquad (63)$$

For $A < 2$ there is no vacuum. The general solution of the linearized equation is given by (48) with exponents:

$$\alpha_\pm = \frac{\lambda}{2}\left(-\tilde{b} \pm \sqrt{\tilde{b}^2 - 4\tilde{b}}\right), \qquad (64)$$

where $\tilde{b} = 2e^{2\omega_0} - \frac{1}{2}$. As before, the boundary curve is stable under small perturbations provided $A > 2$.

We wish to understand the global behavior of solutions to (62). It turns out that Eq. (62) has a fixed point at $\omega = -\infty$ as well as at $\omega = \omega_0$. In order to study this point it is convenient to introduce yet another boundary variable:

$$v = e^{\omega} = e^{\lambda\sigma^+} u. \qquad (65)$$

In terms of v the boundary equation is

$$\begin{aligned} v'' - \frac{1}{v}(v')^2 + \lambda(Av - 1)v' + \lambda^2\left(2v^3 - Av^2 + \frac{v}{2}\right) \\ = -2v\tilde{T}^f_{++}. \end{aligned} \qquad (66)$$

The phase portrait (for $\tilde{T}^f_{++} = 0$) is plotted in Figs. 1(a), 1(b), and 1(c) for the overdamped case $A = 3.5$ and in Fig. 2 for the underdamped case $A = 2.1$. The vacuum ($v = e^{\omega_0}$, $v' = 0$) is the only fully stable fixed point of the equations. However, there is a curious degenerate fixed point at the origin $v = v' = 0$, which is attractive for negative v' and repulsive for positive v'. This unusual behavior arises because of an exact degeneracy of the equations at $v = 0$. (This degeneracy could be lifted by higher-order corrections to our equation.)

Near the origin, the equation is dominated by the terms of degree one in v. This observation leads to the general solution near the origin:

$$v \sim a e^{c e^{\lambda\sigma^+} + \frac{\lambda}{2}\sigma^+}, \qquad (67)$$

where $a > 0$ and c are integration constants. A blowup of the phase portrait near the origin appears in Fig. 1(c). Solutions with $c < 0$ are approaching the degenerate fixed point, while those with $c > 0$ are emerging from it. Emerging trajectories have a universal initial slope of $\frac{\lambda}{2}$, while approaching trajectories have an asymptotically infinite "vertical" slope. It is instructive to estimate the boundary mass along these approaching trajectories. At large N, the mass formula (53) becomes

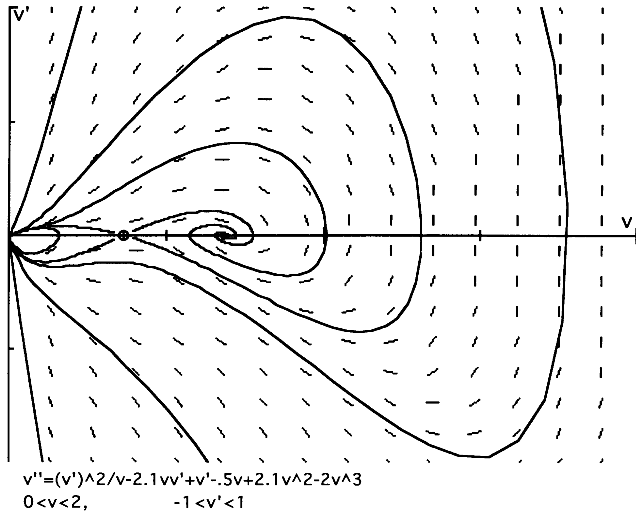

FIG. 2. Phase portrait for the underdamped case $A = 2.1$. The saddle point ($v = 0.365$) and the vacuum ($v = 0.685$) are closer than in Fig. 1(a) due to the smaller value of A. Note that the basin of attraction of the vacuum is (in contrast to the $A = 3.5$ case) now a teardrop-shaped region bounded by the two trajectories which emanate from the origin and asymptote to the saddle.

$$m(x^+, x^-) = \frac{1}{\lambda}\partial_+ Y \partial_- Y + \lambda Y + \frac{\lambda}{4}(\ln Y - 2) + \frac{B\lambda}{Y}\,, \qquad (68)$$

where the constant B is adjusted as in (54) so that m vanishes on the boundary in the vacuum. The formula (56) which gives the variation of the mass along the boundary is unchanged. One finds for $c < 0$ that, on the boundary,

$$m \sim -\frac{c^2\lambda}{4a^2} e^{-2ce^{\lambda\sigma^+} + \lambda\sigma^+}\,, \qquad (69)$$

so that the mass plummets to minus infinity along approaching ($c < 0$) trajectories.

If a shock wave of total mass M is sent at the boundary along $\sigma^+ = 0$, the initial data just after the shock wave is $v(0) = e^{\omega_0}, v'(0) = -2e^{\omega_0}M$, where e^{ω_0} is the vacuum value of v. As can be seen from Figs. 1(b) or Fig. 2, for sufficiently large M, the initial data lie in the basin of attraction of the degenerate fixed point.

The pathological behavior along trajectories heading to the degenerate fixed point is directly associated to black hole formation as follows. By definition, a black hole is a region in which $\partial_+ Y < 0$ [5]. The boundary may at some point enter such a region, and thereafter be in or behind a black hole. When $\partial_+ Y$ crosses zero a spacelike line along which $Y = Y_0$ will branch off from the timelike boundary line along which $Y = Y_0$. At the branching point, both the tangential and normal derivatives of Y along the boundary will vanish. The condition for this to occur follows directly from the boundary condition (35), and in terms of the variable v it reads

$$v'_{cr}(v) = \lambda(v - Av^2). \tag{70}$$

For the special case of shock waves this reduces to the energy threshold $M = 2\lambda(e^{2\omega_0} - \frac{1}{4})$ for an apparent horizon to form before the shock wave reaches the boundary. It is easy to see from Fig. 1(d) that any curve which begins at the vacuum $[v(0) = e^{\omega_0}, v'(0) = 0]$ and asymptotes to the degenerate fixed point at the origin from below necessarily crosses the line $v' = v'_{cr}$. (The converse is not true, as may be seen from the figures: the boundary may enter and leave a region where $\partial_+ Y < 0$ and still settle back to the vacuum.) Thus the runaway behavior lies behind a black hole, and is another form of the "disaster" discussed at the end of Sec. III.

V. THE $\lambda^2 = 0$ LIMIT

In this section we consider the theory obtained by setting $\lambda^2 = 0$ in the action (7). Although black holes are absent in this limit, it is of special interest for several reasons. First, the theory remains semiclassically soluble even after the nonlinear boundary conditions (23) are imposed. The resulting theory is comparable in complexity to Liouville theory, and is of interest in its own right as a nontrivial conformal field theory with boundary interactions. It is similar to a boundary theory which was recently exactly solved in [15, 16] and analogous methods may be applicable here. Second, it arises as an effective short distance theory for (7) at scales short relative to λ^{-1}. Finally, it bears the following direct relationship to the $\lambda^2 \neq 0$ theory. The fields X and Y for nonzero λ^2 can be expressed in terms of free fields x and y,

$$\partial_+\partial_- x = \partial_+\partial_- y = 0, \tag{71}$$

via the relation

$$\begin{aligned} X - Y &= x - y, \\ Y &= y + \frac{\lambda^2}{\gamma}\int d\sigma^+ \int d\sigma^- e^{2(x-y)}. \end{aligned} \tag{72}$$

The gravitational stress tensor (9) becomes simply

$$T^g_{++} = \gamma\left(\partial_+ y\partial_+ y - \partial_+ x\partial_+ x + \partial_+^2 x\right). \tag{73}$$

The OPE's

$$\begin{aligned} \partial_+ x(\sigma^+)\partial_+ x(\sigma^{+'}) &= \frac{-1}{4\gamma(\sigma^+ - \sigma^{+'})^2}, \\ \partial_+ y(\sigma^+)\partial_+ y(\sigma^{+'}) &= \frac{1}{4\gamma(\sigma^+ - \sigma^{+'})^2}, \end{aligned} \tag{74}$$

imply the original OPE's for X and Y when substituted into (72). Thus the field redefinition (72) transforms the bulk $\lambda^2 \neq 0$ theory into the manifestly free $\lambda^2 = 0$ theory.

Now let us apply the boundary condition (23) for $Y_0 = 0$ directly on the free fields $x = x_+ + x_-$ and $y = y_+ + y_-$:

$$\begin{aligned} \partial_+ x - \partial_- x &= \alpha e^x, \\ y &= 0, \end{aligned} \tag{75}$$

where $\alpha = \lambda A$. Note that this is *not* the same as rewriting (23) in terms of the free fields x and y, in which case we would have a rather complex set of boundary conditions for the free fields. It is straightforward to solve (75) for the outgoing fields x_-, y_- in terms of the incoming fields x_+, y_+. The result is

$$\begin{aligned} y_-(\tau) &= -y_+(\tau), \\ x_-(\tau) &= x_+(\tau) - \ln\left(\alpha\int^{\tau} d\tilde{\tau}\, e^{2x_+(\tilde{\tau})}\right). \end{aligned} \tag{76}$$

A convenient gauge is

$$x_+(\omega^+) = \omega^+, \tag{77}$$

and (76) then implies

$$x_-(\omega^-) = -\omega^- - \ln\frac{\alpha}{2}. \tag{78}$$

The constraints further imply that

$$\partial_+ y\partial_+ y = 1 - \frac{1}{\gamma}T^f_{++}, \tag{79}$$

or

$$y_+(\omega^+) = \int^{\omega^+} d\tilde{\omega}^+ \sqrt{1 - \frac{1}{\gamma}T^f_{++}(\tilde{\omega}^+)}. \tag{80}$$

The reflection condition (76) gives

$$y_-(\tau) = -y_+(\tau). \tag{81}$$

Evidently the configuration

$$x_\pm = y_\pm = \pm\omega^\pm \tag{82}$$

is stable under small matter perturbations from $\mathcal{I}^-$. Unfortunately (82) does not quite correspond to the x, y configuration of the vacuum (10) of the $\lambda \neq 0$ theory. The x, y configuration which does correspond to (10) is apparently not stable under perturbations, and thus (76) does not directly translate into stable boundary conditions for the $\lambda \neq 0$ theory.

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