Problem of time and quantum black holes

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We discuss the derivation of the so-called semiclassical equations for both minisuperspace and dilaton gravity. We find that there is no systematic derivation of a semiclassical theory in which quantum mechanics is formulated in a space-time that is a solution of Einstein's equations, with the expectation value of the matter stress tensor on the right-hand side. The issues involved are related to the well-known problems associated with the interpretation of the Wheeler-DeWitt equation in quantum gravity, including the problem of time. We explore the de Broglie–Bohm interpretation of quantum mechanics (and field theory) as a way of spontaneously breaking general covariance, and thereby give meaning to the equations that many authors have been using to analyze black hole evaporation. We comment on the implications for the "information loss" problem.

I. INTRODUCTION

It is usually assumed that the problem of time in quantum gravity is irrelevant to the questions associated with black hole radiation. In particular it is thought that an S matrix (or super scattering operator \$) can be defined and that the whole issue can be discussed within the orthodox interpretation of quantum mechanics (with classical apparatus and a classical observer). The implicit assumption seems to be that the analyses need not, in principle, be different from that of a particle physics scattering experiment.

However, a notion of time evolution is essential to the definition of a scattering (or superscattering) operator, and the question of whether information is lost or not is well posed only within that context. It is also well known that in the usual formulation of quantum gravity [with Dirac or Becchi-Rouet-Stora-Tyutin (BRST) quantization] there is no time evolution. As discussed in detail in the reviews [1] there are serious problems with all attempts to resolve this question. Nevertheless, it is often asserted that as far as the black hole problem is concerned this is not an issue. The reasons are not often explicitly stated, but one or the other of the following assumptions are implicit in most analyses of the problem: (a) Hawking's original calculation [2] is semiclassical, and one should be able to derive a notion of time in this approximation; (b) the black hole space-time is open and asymptotically flat. In this case there is a nonvanishing Hamiltonian associated with spatial infinity. This should enable one to define a notion of time evolution and an S (or \$) matrix.

Now if the semiclassical theory is internally self-consistent one may ignore the problem of time and leave the issue of how to derive the theory from quantum gravity to future work. However, the semiclassical theory is nonlinear (since it involves the expectation value of the matter stress tensor) and thus violates the superposition principle. Thus it is unclear how one could use standard S-matrix ideas. Furthermore, it has become in-

creasingly clear that the issue of information loss cannot be resolved one way or the other without some understanding of what happens at short distance scales[1] (such as distances within a Planck length of the horizon). This is clearly the regime of quantum gravity. Since there is no well-defined theory at these scales[2] (in four dimensions), one can make assumptions about this regime in accordance with ones prejudices. However, there is still a serious problem of principle involved here. Whatever the final version of quantum gravity is like, as long as the requirement of general covariance is imposed, the quantum theory would seem to have the problem of time. The question that arises then is whether the semiclassical analyses can emerge from the quantum theory in some limit, at least for wave functions (such as that of a coherent state) that are amenable to a classical interpretation.

There are arguments in the literature that attempt to derive semiclassical physics, and in particular the (functional) Schrödinger equation, from the Wheeler-DeWitt (WD) equation [6]. These are sometimes used to justify the statement that the semiclassical treatment of the black hole problem can be derived, in the large Planck mass limit, from quantum gravity. A careful examination of these arguments, however, reveals that there is no *systematic* way in which the equations of semiclassical black hole physics can be derived. In fact, the usual argument does give a Schrödinger equation, but in a background, which is a solution of the vacuum Einstein equations; i.e., it is independent of the (quantum) state of matter. Both in the black hole case and in cosmological applications, however, the geometry is supposed to be determined by (the expectation value of) the matter stress tensor, so

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[1] See for example [3–5].

[2] String theory may be the answer, but at present there is no real understanding of anything beyond perturbation theory around flat backgrounds.

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the usual argument is clearly inadequate. Since the original work on this qeustion, there have been attempts to remedy this problem by various means. In the first part of this paper we will argue that none of these are satisfactory.

We will first discuss the issue in minisuperspace. The conceptual problem that we face is already manifest at this level. However, the recent development of two-dimensional dilaton-gravity field theories, which exhibit classical dynamical black hole solutions [Callen, Giddings, Harvey, and Strominger (CGHS), Ref. [7]] gives us a much more interesting toy model within which these questions can be investigated. In fact, the work of Refs. [8–10] established that there is a class of quantum CGHS theories that are exactly solvable (quantum) conformal field theories (CFT's) in two dimensions. These theories are well-defined quantum gravity theories, which have nontrivial (dynamical black hole) solutions in the classical limit. Thus these models are ideal laboratories for the study of the passage from the exact quantum to the semiclassical physics of black holes. Some comments on the issues involved have already been published in [10]. In that paper it was pointed out that the usual (exact) quantization of the theory [which in this case is a two-dimensional (2D) conformal field theory] completely obscures the semiclassical picture. In this paper we elaborate further on this and, in particular, address two new questions. The first is whether there is an alternative to Dirac quantization (such as light cone gauge), which can solve the problem. Our conclusion is that it cannot. The second is whether the fact that we have an open system can resolve the problem as in (b) above. Here too the conclusion is negative. The boundary Hamiltonian is irrelevant for the local dynamics of the quantum field theory and hence has no effect at all on the problem of time.

A possible resolution of these issues is to give up the superposition principle for quantum gravity. In fact, as is well known (see for example [1]), in the attempts to derive the semiclassical equations from the Wheeler-DeWitt equation this principle is effectively abandoned. The argument may be made that the wave function of the universe is unique and one does not have the usual reasons for superposing states, which are valid only for the quantum mechanics of subsystems. Indeed the wave function of the universe as it evolves would give rise to the experimental situations, which behave like superpositions of different states of subsystems. At the fundamental level there may be no need to satisfy the superposition principle. If one accepts this argument then one loses the need to impose the WD equation (as a linear constraint). Of course it is still necessary to recover Einstein's equations in the classical limit. This may be done by imposing the constraint as an expectation value. In other words, the constraint operators do not annihilate the physical states. Instead, the only permissible states (i.e., states with the correct classical limit, giving only geometries allowed by Einstein's theory) are those in which the constraint operators have an expectation value of zero. Since this is a nonlinear condition, a linear superposition of such states is, in general, not a physical state.

Such an ansatz would constitute a spontaneous breakdown of general covariance. In [10] it was argued that this may be what one needs in order to have the standard semiclassical picture. The semiclassical equations that have been used hitherto in all the interesting applications of quantum gravity (e.g., see [11]) are then completely justified, if at the quantum level one takes a coherent state. In the case of the solvable quantum gravity theory [8–10] (defined in the above fashion) that emerges from the classical CGHS model, we see in fact that the semiclassical analysis is exact. However, this is true only for those theories in which there is no boundary in field space (see the third paper of [8] and [10]). In theories with a boundary [12] (if they are well defined) this is not so, but the semiclassical equations will emerge in the large N limit in just the way that one expects in ordinary (nongenerally covariant) quantum mechanics. This is to be contrasted with the fact that if the usual linear constraint equations are used, then there is no way of recovering the semiclassical limit in the large N limit.

This derivation of semiclassical physics involves abandoning the linearity of quantum mechanics at the fundamental level. While this may be justified in so far as one takes the point of view that there is a unique wave function of the universe, we believe that it is worthwhile exploring an alternative. To this end we study the de Broglie–Bohm interpretation of quantum mechanics.[3] In this interpretation there are definite orbits for the fields of the theory that are guided by the quantum wave functional. The latter also gives the possible distribution of the initial values of such fields. In the context of quantum gravity, picking an orbit means picking a particular space-time and is tantamount to spontaneously breaking general covariance. The equations for these orbits are, in fact, the semiclassical equations corrected by certain higher-order terms. We believe that this interpretation gives a very natural way of understanding the emergence of semiclassical physics. However, it must be stressed that what one gets is not what is usually used in quantum gravity (which is, in fact, more properly called the Born-Oppenheimer approximation), but the standard $O(\hbar)$ expansion. In other words, one has (semi)classical equations explicitly modified by quantum corrections.

In the next section of this paper we discuss the arguments for the emergence of semiclassical physics from the WD equations, and conclude that there is no systematic derivation of the former. In the third section we discuss this issue in the CGHS theory and come to the same conclusion. In the fourth section we show that both reduced phase-space quantization, and the existence of a boundary Hamiltonian, are irrelevant to the questions that we are addressing. In the fifth section we present a possible resolution of these issues, and in the concluding section we summarize our arguments.

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[3] For a comprehensive review see Ref. [13].

II. THE PROBLEM OF TIME AND THE SEMICLASSICAL APPROXIMATION IN MINISUPERSPACE

In this section we will review the derivation the semiclassical limit of the Wheeler-DeWitt equation for the example of minisuperspace [6]. We consider a homogeneous, isotropic space-time with the metric

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 d\Omega_3^2 ,$$

where the lapse function N and the scale factor a only depend upon t, and $d\Omega_3^2$ is a metric on three space. The action for gravity coupled to a homogeneous scalar field $\phi(t)$ is

$$S = \int dt \left[-a^3 M^2 \left\{ \frac{1}{2N} \left(\frac{\dot{a}}{a} \right)^2 + N V_G(a) \right\} + a^3 \left\{ \frac{\dot{\phi}^2}{2N} - N V_m(\phi) \right\} \right] ,$$

where

$$V_G(a) = -\tfrac{1}{2} k a^{-2} + \tfrac{1}{6} \Lambda ,$$

and M is the Planck mass. The equations of motion from this action (in the $N = 1$ gauge) are

$$\delta N : \quad -M^2 \left(\frac{a \dot{a}^2}{2} - a^3 V_G(a) \right) + a^3 \left(\frac{\dot{\phi}^2}{2} + V_m(\phi) \right) = 0 , \tag{2.1}$$

$$\delta a : \quad M^2 \left\{ \frac{d}{dt}(a\dot{a}) - \frac{\dot{a}^2}{2} - \partial_a(a^3 V_G(a)) \right\}$$
$$+ 3a^2 \left(\frac{\dot{\phi}^2}{2} - V_m(\phi) \right) = 0 , \tag{2.2}$$

$$\delta\phi : \quad \frac{d}{dt}(a^3 \dot{\phi}) + a^3 \partial_\phi V_m(\phi) = 0 . \tag{2.3}$$

The first equation is the constraint that the total energy of the system is zero.

In terms of the Hamiltonian for this system

$$H = -\frac{1}{2aM^2} p_a^2 + a^3 M^2 V_G(a) + \frac{1}{2a^3} p_\phi^2 + a^3 V_m(\phi) \tag{2.4}$$

with

$$p_a = -M^2 a \dot{a}, \quad p_\phi = a^3 \dot{\phi} , \tag{2.5}$$

Eq. (2.1) is the (secondary) constraint

$$H \approx 0 .$$

Upon Dirac quantization of this system one has the physical state condition on the "wave function for the Universe:"

$$\hat{H}|\Psi\rangle = 0 . \tag{2.6}$$

This constraint[4] implies

$$i\hbar \frac{\partial}{\partial t}|\Psi\rangle = \hat{H}|\Psi\rangle = 0 . \tag{2.7}$$

That is, the wave function of the Universe is a stationary state with zero energy, and there is no Schrödinger evolution of the physical states. It also implies that any dynamical variable must commute with the Hamiltonian, on the space of physical states, and thus also be time independent.

Is it possible to formulate a theory with Schrödinger time evolution in the approximation that $M \gg \langle H_m \rangle$, where H_m is the matter Hamiltonian? In the Schrödinger representation the constraint equation is the WD equation

$$\hat{H}\Psi(a,\phi) = \left\{ \frac{\hbar^2}{2M^2} \frac{1}{a^2} \partial_a(a\partial_a) - \frac{\hbar^2}{2a^3} \partial_\phi^2 \right. $$
$$\left. + a^3(M^2 V_G + V_m) \right\} \Psi(a,\phi) = 0 .$$

In the above we have resolved the ordering ambiguity in the p_a term by requiring that the Hamiltonian be Hermitian in the inner product:

$$\langle \Psi | \Phi \rangle = \int da \, d\phi \, a^2 \Psi^*(a,\phi) \Phi(a,\phi) .$$

We now write

$$\Psi(a,\phi) = R(a,\phi) e^{iS(a,\phi)/\hbar}$$

where R and S are real functions. Substituting in the WD equation we find equations for the real and imaginary parts:

$$-\frac{1}{2aM^2}(\partial_a S)^2 + \frac{1}{2a^3}(\partial_\phi S)^2 + a^3[M^2 V_G + V_m]$$
$$+ \frac{\hbar^2}{2a^3 R} \left[\frac{1}{M^2}(a\partial_a)^2 R - \partial_\phi^2 R \right] = 0 , \tag{2.8}$$

$$\partial_\phi(R^2 \partial_\phi S) - \frac{1}{M^2} a \partial_a(R^2 a \partial_a S) = 0 . \tag{2.9}$$

Expanding in powers of M,

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[4] Equation (2.6) is the statement that the wave function Ψ is invariant under time reparametrizations. In the full theory the constraints are a reflection of general covariance at the quantum level.

$$S(a, \phi) = M^2 S_{-1}(a, \phi) + S_0(a, \phi) + M^{-2} S_1(a, \phi) \cdots ,$$

$$R(a, \phi) = R_0(a, \phi) + M^{-2} R_1(a, \phi) + \cdots ,$$

we find the equations

$$O(M^4): \quad 0 = \partial_\phi S_{-1} \tag{2.10}$$

$$O(M^2): \quad 0 = -\frac{1}{2a}(\partial_a S_{-1})^2 + a^3 V_G . \tag{2.11}$$

Defining

$$R_0(a, \phi) = \frac{1}{\sqrt{a \partial_a S_{-1}(a)}} r_0(a, \phi)$$

and

$$\chi(a, \phi) = r_0(a, \phi) e^{i S_0(a,\phi)/\hbar}$$

we have

$$\Psi(a, \phi) = \frac{e^{i M^2 S_{-1}(a)/\hbar}}{\sqrt{a \partial_a S_{-1}(a)}} [\chi(a, \phi) + O(M^{-2})] ,$$

where from Eq. (2.10) S_{-1} only depends upon a. The $O(M^0)$ equation can now be written as

$$i\hbar \frac{\partial \chi(a, \phi)}{\partial T} = H_m \chi(a, \phi) , \tag{2.12}$$

where

$$\frac{\partial}{\partial T} \equiv -\frac{1}{a}(\partial_a S_{-1}) \frac{\partial}{\partial a} \tag{2.13}$$

and

$$H_m \equiv -\frac{\hbar^2}{2a^3} \partial_\phi^2 + a^3 V_m . \tag{2.14}$$

From this we have

$$\frac{\partial a}{\partial T} = \dot{a} = -\frac{1}{a} \partial_a S_{-1}(a) , \tag{2.15}$$

thus defining a definite orbit for the scale factor.

We have therefore succeeded in obtaining a Schrödinger equation for the evolution of the matter wave function. However, the Hamilton-Jacobi equation (2.11) that determines the metric (i.e., a) is equivalent to the *vacuum Einstein equation*. The usual semiclassical equation

$$G_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle$$

would have, instead of Eq. (2.11), resulted in an equation of the form

$$-\frac{1}{2a}(\partial_a S_{-1})^2 + a^3 V_G = -\frac{1}{M^2} \langle \chi | H_m | \chi \rangle . \tag{2.16}$$

What one needs in order to justify the usual semiclassical picture is a derivation of the Schrödinger equation in a background determined by the above equation. In order to see whether this is possible we will use (2.16) in the WD equation, and determine the conditions under which the resulting equation for the matter wave function is consistent with the Schrödinger equation.

Let us therefore write Ψ as

$$\Psi(a, \phi) = \exp\left(\frac{1}{\hbar} M^2 S_{-1}(a)\right) \tilde{\chi}(a, \phi)$$

$$= \exp\left(\frac{1}{\hbar} M^2 S_{-1}(a)\right) f(a) \chi(a, \phi)$$

where $S_{-1}(a)$ is defined to be a ϕ-independent $(\partial_\phi S_{-1} = 0)$ solution of (2.16). Substituting the above expression in the WD equation gives

$$\frac{i\hbar}{a} \partial_a S_{-1} \partial_a \tilde{\chi} - \frac{M^2}{2a}(\partial_a S_{-1})^2 \tilde{\chi} + a^3 M^2 V_G \tilde{\chi} + \frac{i\hbar}{2a^3} \tilde{\chi}(a \partial_a)^2 S_{-1} + \frac{\hbar^2}{2M^2 a^3}(a \partial_a)^2 \tilde{\chi} + H_m \tilde{\chi} = 0 .$$

Using (2.16) and dividing by f we obtain

$$[H_m - \langle H_m \rangle]\chi + i\hbar \left[\frac{1}{a^3} \partial_a S_{-1} - \frac{i\hbar}{M^2 a^3} \frac{\partial_\alpha f}{f}\right] \partial_\alpha \chi + \frac{\hbar^2}{2M^2 a^3} \partial_\alpha^2 \chi$$

$$+ i\hbar \left[\frac{1}{a^3} \partial_\alpha S_{-1} \frac{\partial_\alpha f}{f} + \frac{1}{2a^3} \partial_\alpha^2 S_{-1} - \frac{i\hbar}{2Ma^3} \frac{\partial_\alpha^2 f}{f}\right] \chi = 0 \tag{2.17}$$

with

$$\partial_\alpha \equiv a \partial_a .$$

The function $f(a)$ will be defined by requiring that the last term in brackets is zero:

$$\frac{1}{a^3} \partial_\alpha S_{-1} \frac{\partial_\alpha f}{f} + \frac{1}{2a^3} \partial_\alpha^2 S_{-1} - \frac{i\hbar}{2Ma^3} \frac{\partial_\alpha^2 f}{f} = 0 .$$

One now defines the time derivative as before,

$$\partial_T \equiv -\frac{1}{a}\partial_a S_{-1}\partial_a = -\frac{1}{a^3}\partial_\alpha S_{-1}\partial_\alpha \ ,$$

so that (2.17) becomes

$$[H_m - \langle H_m\rangle]\chi - i\hbar\frac{\partial\chi}{\partial T} + \frac{\hbar^2}{2M^2 a^3}\partial_\alpha^2\chi + \frac{\hbar^2}{M^2 a^3}\frac{\partial_\alpha f}{f} = 0 \ .$$

$$(2.18)$$

We now assume the Schrödinger-like equation

$$[H_m - \langle H_m\rangle]\chi = i\hbar\frac{\partial\chi}{\partial T} \ , \qquad (2.19)$$

and ask whether the remaining terms in Eq. (2.18) are small compared with the terms in this equation.[5]

For simplicity let us assume that $V_G = 0$ (e.g., late universe with $\Lambda = 0$). This implies, from Eq. (2.16),

$$\partial_\alpha S_{-1} = \frac{a^{3/2}\sqrt{2\langle H_m\rangle}}{M} \ .$$

If we also assume that a is large, we have

$$\partial_\alpha S_{-1} = \frac{a^{3/2}\sqrt{2\langle H_m\rangle}}{M} \to \frac{a^3\sqrt{2\langle V_m\rangle}}{M} \ .$$

After some calculation one finds that the Schrödinger-like equation (2.19) is consistent in the above sense, provided

$$M\sqrt{\langle H_m\rangle}a^{3/2} \underset{a\to\infty}{\sim} M\sqrt{\langle V_m\rangle}a^3 \gg 1 \ .$$

There are several discussions in the literature that are similar to the above. However, we feel that it was necessary to go over this in some detail to demonstrate that the derivation of semiclassical physics is far from clear cut. It is not by any means a systematic approximation from the exact theory even in this highly simplified model.

III. WHEELER-DEWITT EQUATION IN 2D QUANTUM DILATON GRAVITY

The recent revival of interest in the problem of quantum radiation from black holes is almost entirely due to the construction of a two-dimensional (2D) model of dilaton gravity coupled to matter by Callan, Giddings, Harvey, and Strominger [7]. The hope was that in a context where the intractable ultraviolet problems of four-dimensional gravity were absent, the conceptual issues associated with Hawking radiation could be addressed and resolved. Notwithstanding claims to the contrary (e.g., Strominger and Hawking [14]), we believe that even

within this highly simplified context the problem has not been resolved. Our aim here is to demonstrate clearly that this is so precisely because the question is not well posed. Indeed we believe that the role of the CGHS model is that it gives a well-defined context within which we can show precisely what problems arise in formulating the question.

The model is defined by the action

$$S = \frac{1}{4\pi}\int d^2\sigma\sqrt{-g}\left[e^{-2\phi}[R + 4(\nabla\phi)^2 + 4\lambda^2]\right.$$
$$\left. - \frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right] \ . \qquad (3.1)$$

In the above, in addition to the metric g and dilaton ϕ fields, there are N conformal scalar fields f_i. The quantum theory may be defined by the path integral after gauge fixing to the conformal gauge $g_{\alpha\beta} = e^{2\rho}\hat{g}_{\alpha\beta}$, where $\hat{g}$ is a fiducial metric with unit determinant (say). The independence of the functional integral from the fiducial metric then leads to the requirement that the conformal gauge fixed theory with the translationally invariant measured be a conformal field theory (CFT) [8]. This may then be written in terms of the Liouville-like action [8,9]

$$S = \frac{1}{4\pi}\int d^2\sigma\left[\mp\partial_+X\partial_-X \pm \partial_+Y\partial_-Y\right.$$
$$\left. + \sum_{i=1}^{N}\partial_+f_i\partial_-f_i + 2\lambda^2 e^{\mp\sqrt{2/|\kappa|}(X\mp Y)}\right] \ .$$

$$(3.2)$$

Here the upper and lower signs correspond to $\kappa < 0$ and $\kappa > 0$, respectively, and $\kappa = (N-24)/6$, N being the number of matter fields.[6] The field variables in the above are related to the original variables ϕ and ρ that occur in the CGHS action, gauge fixed to the conformal gauge $(g_{\alpha\beta} = e^{2\rho}\eta_{\alpha\beta})$, through the relations

$$Y = \sqrt{2\kappa}\left[\rho + \kappa^{-1}e^{-2\phi} - \frac{2}{\kappa}\int d\phi\, e^{-2\phi}\bar{h}(\phi)\right] \ , \quad (3.3)$$

$$X = 2\sqrt{2/\kappa}\int d\phi\, P(\phi) \ , \qquad (3.4)$$

where

$$P(\phi) = e^{-2\phi}[(1+\bar{h})^2 - \kappa e^{2\phi}(1+h)]^{1/2} \ . \qquad (3.5)$$

In (3.3) and (3.4), the functions $h(\phi)$ and $\bar{h}(\phi)$ parametrize quantum (measure) corrections that may arise when transforming to the translationally invariant

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[5]Note that we can redefine $\chi = \chi_s \exp\{(1/\hbar)\int^t\langle H_m\rangle dt\}$ and obtain the Schrödinger equation for χ_s. In this case the nonlinear character of the equation is hidden in the phase redefinition (see Brout and Venturi in reference [6]).

[6]Note that this definition has the opposite sign to the κ defined in [8]. Also, henceforth we will assume $\kappa > 0$.

measure (see the third paper of [8] for details). The statement that the quantum theory has to be independent of the fiducial metric (set equal to η in the above) implies that this gauge fixed theory is a CFT. The above solution to this condition was obtained by considering only the leading terms of the β function equations, but it can be shown, in the cases where P has no zeros,[7] that the Liouville-like theory is an exact solution to the conformal invariance conditions.

The above considerations mean that there are two classes of quantum CGHS models: (a) those for which P has a zero so that the integration range for the X, Y fields cannot be extended over the whole real line; (b) those for which P has no zeros and are exact CFT's. In the case of theories of class (a), one may get semiclassical physics that looks like spherically symmetric collapse and evaporation in four dimensions by imposing reflection boundary conditions where the boundary is timelike [12]. There are, however, two problems with this. First, at the semiclassical level the model does not have a lower bound to the Arnowitt-Deser-Misner (ADM) energy [15]. This is reflected in the fact that there is a pulse of negative energy, the so-called thunderpop [12], just before complete evaporation of the black hole. Second, the model (because of the boundary in the functional integral) is probably not an exact CFT, and hence it is unlikely that it is a true representation of the original quantum CGHS theory. We will nevertheless make a few speculative remarks about it in the next section.

Class (b) gives us an exactly solvable theory of quantum 2D dilaton gravity. However, it does not give us the picture of four-dimensional spherically symmetric black hole evaporation that many authors have been looking for. The space-time is conformal to two-dimensional Minkowski space, and there is no black hole singularity.[8] Nevertheless, as stressed in the work of Ref. [10], it gives us a theoretical laboratory in which to study the emergence of semiclassical physics from the exact quantum theory. In [10] we attempted to do this starting from the Fock space formulation of the theory. Our conclusion was that the only way to recover the semiclassical physics of the theory was to abandon the standard procedure of quantization in which the physical state condition was imposed on the states, and instead only require that the expectation value in physical states of the constraint operators be zero. Although we have not found any problem with this approach, it is more satisfying to get the semiclassical equations, while preserving the superposition principle for the quantum gravity wave function. To this end we discuss the Schrödinger wave functional approach to the problem with the constraints being implemented through the WD equations for dilaton gravity.

The stress tensor for two-dimensional dilaton gravity can be written as

$$T_{\pm\pm} = \tfrac{1}{2}(\partial_\pm X \partial_\pm X - \partial_\pm Y \partial_\pm Y)$$

$$+ \left(\frac{\kappa}{2}\right)^{1/2} \partial_\pm^2 Y + \tfrac{1}{2}\sum_{i=1}^{N} \partial_\pm f_i \partial_\pm f_i$$

$$T_{+-} = -\left(\frac{\kappa}{2}\right)^{1/2} \partial_+ \partial_- Y - \lambda^2 e^{\sqrt{2/\kappa}(X+Y)} .$$

Defining

$$\zeta_+ = \left(\frac{2}{\kappa}\right)^{1/2} (X+Y) + \ln\frac{2}{\kappa}, \quad \zeta_- = \left(\frac{2}{\kappa}\right)^{1/2} (X-Y)$$

the components are

$$T_{\pm\pm} = \frac{\kappa}{4}[\partial_\pm \zeta_+ \partial_\pm \zeta_- + \partial_\pm^2(\zeta_+ - \zeta_-)] + \frac{1}{2}\sum_{i=1}^{N} \partial_\pm f_i \partial_\pm f_i$$

$$T_{+-} = -\frac{\kappa}{4}\partial_+ \partial_-(\zeta_+ - \zeta_-) - \frac{\kappa}{2}\lambda^2 e^{\zeta_+} .$$

The Hamiltonian density is

$$\begin{aligned} T_{00} &= T_{++} + T_{--} + 2T_{+-} \\ &= \frac{\kappa}{8}[\dot\zeta_+ \dot\zeta_- + \zeta_+' \zeta_-' + 2(\zeta_+'' - \zeta_-'') - 8\lambda^2 e^{\zeta_+}] \\ &\quad + \frac{1}{4}\sum_{i=1}^{N}[\dot f_i^2 + f_i'^2] , \end{aligned}$$

where the overdot and prime denote differentiation with respect to the space-time coordinates τ and σ, respectively.

Now using the canonical momenta

$$\Pi_\pm = \frac{\kappa}{16\pi}\dot\zeta_\mp, \quad \Pi_{f_i} = \frac{1}{4\pi}\dot f_i \qquad (3.6)$$

we obtain

$$\begin{aligned} T_{00} &= \frac{32\pi^2}{\kappa}\Pi_+ \Pi_- + \frac{\kappa}{8}[\zeta_+' \zeta_-' + 2(\zeta_+'' - \zeta_-'') - 8\lambda^2 e^{\zeta_+}] \\ &\quad + \frac{1}{4}\sum_{i=1}^{N}[f_i'^2 + 16\pi^2 \Pi_{f_i}^2] . \end{aligned} \qquad (3.7)$$

We also have the momentum density

$$T_{01} = 2\pi\left[\zeta_-' \Pi_- + \zeta_+' \Pi_+ \right.$$

$$\left. + 2\frac{\partial}{\partial\sigma}(\Pi_- - \Pi_+) + \sum_{i=1}^{N} f_i' \Pi_{f_i}\right] . \qquad (3.8)$$

We now quantize using $\Pi_u \to (1/i)(\delta/\delta u)$ and obtain the corresponding operators $\hat T_{00}$ and $\hat T_{01}$. The Wheeler-DeWitt equation is thus[9]

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[7]This implies some restrictions on the possible quantum corrections, but there is a large class that satisfies these conditions.

[8]See the third reference in [8].

[9]Several authors have discussed the WD equation for dilaton gravity (see Ref. [22] for example), but none of them have focused on the problem that we are addressing.

$$\hat{T}_{00}\Psi[\zeta_+, \zeta_-, f_i] = 0 , \qquad (3.9)$$

and we also have the spatial diffeomorphism constraint

$$\hat{T}_{01}\Psi[\zeta_+, \zeta_-, f_i] = 0 . \qquad (3.10)$$

Writing Ψ in the form

$$\Psi = R[\zeta_+, \zeta_-, f_i] \exp\{iS[\zeta_+, \zeta_-, f_i]\}$$

we obtain, for the real part of $\hat{T}_{00}\Psi = 0$,

$$\frac{32\pi^2}{\kappa} \frac{\delta S}{\delta\zeta_+} \frac{\delta S}{\delta\zeta_-} + \kappa V[\zeta_+, \zeta_-] + V_m[f']$$

$$+ Q + 4\pi^2 \sum_{i=1}^{N} \left(\frac{\delta S}{\delta f_i}\right)^2 = 0 \quad (3.11)$$

with

$$V[\zeta_+, \zeta_-] = \tfrac{1}{8}[\zeta_+'\zeta_-' + 2(\zeta_+'' - \zeta_-'') - 8\lambda^2 e^{\zeta_+}] ,$$

$$V_m[f'] = \tfrac{1}{4} \sum_{i=1}^{N} f_i'^2 ,$$

$$Q = -\frac{32\pi^2}{\kappa} \frac{1}{R} \frac{\delta^2 R}{\delta\zeta_+\delta\zeta_-} - \frac{4\pi^2}{R} \sum_{i=1}^{N} \frac{\delta^2 R}{\delta f_i^2} .$$

The imaginary part of $\hat{T}_{00}\Psi = 0$ gives

$$\sum_{i=1}^{N} \frac{\delta}{\delta f_i}\left(R^2 \frac{\delta S}{\delta f_i}\right)$$

$$+ \frac{4}{\kappa}\left[\frac{\delta}{\delta\zeta_+}\left(R^2 \frac{\delta S}{\delta\zeta_-}\right) + \frac{\delta}{\delta\zeta_-}\left(R^2 \frac{\delta S}{\delta\zeta_+}\right)\right] = 0 . \quad (3.12)$$

For the real and imaginary parts of $\hat{T}_{01}\Psi = 0$ we have

$$\left(\zeta_-' + 2\frac{\partial}{\partial\sigma}\right)\frac{\delta R}{\delta\zeta_-} + \left(\zeta_+' - 2\frac{\partial}{\partial\sigma}\right)\frac{\delta R}{\delta\zeta_+} + \sum_{i=1}^{N} f_i' \frac{\delta R}{\delta f_i} = 0 ,$$

$$(3.13)$$

$$\left(\zeta_-' + 2\frac{\partial}{\partial\sigma}\right)\frac{\delta S}{\delta\zeta_-} + \left(\zeta_+' - 2\frac{\partial}{\partial\sigma}\right)\frac{\delta S}{\delta\zeta_+} + \sum_{i=1}^{N} f_i' \frac{\delta S}{\delta f_i} = 0 ,$$

$$(3.14)$$

respectively. The $O(M^{-1})$ expansion in the minisuperspace example is replaced here by an expansion in κ^{-1}:

$$S[\zeta_+, \zeta_-, f_i] = \kappa S_{-1}[\zeta_+, \zeta_-, f_i]$$
$$+ S_0[\zeta_+, \zeta_-, f_i] + \frac{1}{\kappa}S_1[\zeta_+, \zeta_-, f_i] + \cdots ,$$

$$R[\zeta_+, \zeta_-, f_i] = R_0[\zeta_+, \zeta_-, f_i] + \frac{1}{\kappa}R_1[\zeta_+, \zeta_-, f_i] + \cdots .$$

One finds

$$O(\kappa^2): \quad 0 = \frac{\delta S_{-1}}{\delta f_i} , \qquad (3.15)$$

$$O(\kappa): \quad 0 = 32\pi^2 \frac{\delta S_{-1}}{\delta\zeta_+} \frac{\delta S_{-1}}{\delta\zeta_-}$$
$$+ \tfrac{1}{8}[\zeta_+'\zeta_-' + 2(\zeta_+'' - \zeta_-'') - 8\lambda^2 e^{\zeta_+}] . \quad (3.16)$$

As in the minisuperspace example one finds that largest component of S is independent of the matter fields, and the metric is determined by the vacuum Einstein equation [Eq. (3.16)].

For the order 1 equations we find

real : $\quad V_m[f'] + 4\pi^2 \sum_{i=1}^{N} \left(\frac{\delta S_0}{\delta f_i}\right)^2 + 32\pi^2 \left[\frac{\delta S_{-1}}{\delta\zeta_+} \frac{\delta S_0}{\delta\zeta_-} + \frac{\delta S_{-1}}{\delta\zeta_-} \frac{\delta S_0}{\delta\zeta_+}\right] - 4\pi^2 \sum_{i=1}^{N} \frac{1}{R_0} \frac{\delta^2 R_0}{\delta f_i^2} = 0 ,$

imaginary : $\quad \sum_{i=1}^{N} \frac{\delta}{\delta f_i}\left[R_0^2 \frac{\delta S_0}{\delta f_i}\right] + 4\left[\frac{\delta}{\delta\zeta_+}\left(R_0^2 \frac{\delta S_{-1}}{\delta\zeta_-}\right) + \frac{\delta}{\delta\zeta_-}\left(R_0^2 \frac{\delta S_{-1}}{\delta\zeta_+}\right)\right] = 0 .$

One now defines a (local) time by

$$\frac{\delta}{\delta T} = 32\pi^2 \left\{\frac{\delta S_{-1}}{\delta\zeta_+} \frac{\delta}{\delta\zeta_-} + \frac{\delta S_{-1}}{\delta\zeta_-} \frac{\delta}{\delta\zeta_+}\right\} . \qquad (3.17)$$

The real equation becomes

$$\frac{\delta S_0}{\delta T} = -V_m[f'] - 4\pi^2 \sum_{i=1}^{N} \left(\frac{\delta S_0}{\delta f_i}\right)^2$$

$$+ 4\pi^2 \sum_{i=1}^{N} \frac{1}{R_0} \frac{\delta^2 R_0}{\delta f_i^2} .$$

Now given a function g (independent of the $\{f_i\}$) satisfying

$$\frac{1}{g}\frac{\delta g}{\delta T} = 32\pi^2 \frac{\delta^2 S_{-1}}{\delta \zeta_+ \delta \zeta_-} ,$$

and defining r_0,

$$r_0 = R_0 g ,$$

the imaginary equation becomes the continuity equation

$$\frac{\delta r_0}{\delta T} = -\frac{4\pi^2}{r_0}\sum_{i=1}^{N}\frac{\delta}{\delta f_i}\left(r_0^2 \frac{\delta S_0}{\delta f_i}\right) .$$

Writing

$$\chi = r_0 e^{iS_0}$$

we have the Schrödinger equation

$$i\frac{\delta \chi}{\delta T} = \mathcal{H}_m \chi$$

with

$$\mathcal{H}_m = -4\pi^2 \sum_{i=1}^{N}\frac{\delta^2}{\delta f_i^2} + V_m[f'] .$$

Thus again we have the Schrödinger equation for matter, in a background determined by the vacuum Einstein equation.

In order to see whether, as in the minisuperspace example, one can at least give a heuristic argument to justify the usual semiclassical equations we proceed as follows. Define

$$\Psi[\zeta_+, \zeta_-, f_i] = F[\zeta_+, \zeta_-]\chi[\zeta_+, \zeta_-, f_i]$$
$$\times \exp\{i\kappa S_{-1}[\zeta_+, \zeta_-]\} .$$

Substitution in the WD Eq. (3.9) gives [using Eq. (3.17)]

$$\left(\hat{T}_{00}^m + 32\kappa\pi^2 \frac{\delta S_{-1}}{\delta \zeta_+}\frac{\delta S_{-1}}{\delta \zeta_-} + \kappa V_G\right)F\chi - i\frac{\delta}{\delta T}(F\chi) - \frac{32\pi^2}{\kappa}\frac{\delta^2}{\delta \zeta_+ \delta \zeta_-}(F\chi) - 32i\pi^2 F\chi \frac{\delta^2 S_{-1}}{\delta \zeta_+ \delta \zeta_-} = 0 \qquad (3.18)$$

with

$$\hat{T}_{00}^m \equiv \sum_{i=1}^{N}\left\{-4\pi^2 \frac{\delta^2}{\delta f_i^2} + \frac{1}{4}f_i'^2\right\} .$$

We now take S_{-1} to be an f_i independent solution of the semiclassical equation

$$32\kappa\pi^2 \frac{\delta S_{-1}}{\delta \zeta_+}\frac{\delta S_{-1}}{\delta \zeta_-} + \kappa V_G = -\langle\hat{T}_{00}^m\rangle , \qquad (3.19)$$

and Eq. (3.18) becomes

$$(\hat{T}_{00}^m - \langle\hat{T}_{00}^m\rangle)\chi - i\frac{\delta \chi}{\delta T} - 32i\pi^2 \left(\frac{\delta^2 S_{-1}}{\delta \zeta_+ \delta \zeta_-} - \frac{i}{\kappa}\frac{1}{F}\frac{\delta^2 F}{\delta \zeta_+ \delta \zeta_-} + \frac{1}{32\pi^2}\frac{1}{F}\frac{\delta F}{\delta T}\right)\chi$$

$$-\frac{32\pi^2}{\kappa}\left(\frac{1}{F}\frac{\delta F}{\delta \zeta_+}\frac{\delta \chi}{\delta \zeta_-} + \frac{1}{F}\frac{\delta F}{\delta \zeta_-}\frac{\delta \chi}{\delta \zeta_+} + \frac{\delta^2 \chi}{\delta \zeta_+ \delta \zeta_-}\right) = 0 . \qquad (3.20)$$

In this case the functional F will be defined by requiring that the coefficient of χ in the third term be zero:

$$\frac{\delta^2 S_{-1}}{\delta \zeta_+ \delta \zeta_-} - \frac{i}{\kappa}\frac{1}{F}\frac{\delta^2 F}{\delta \zeta_+ \delta \zeta_-} + \frac{1}{32\pi^2}\frac{1}{F}\frac{\delta F}{\delta T} = 0 .$$

We now want to determine under what conditions Eq. (3.20) reduces to the Schrödinger-like equation

$$(\hat{T}_{00}^m - \langle\hat{T}_{00}^m\rangle)\chi = i\frac{\delta \chi}{\delta T} .$$

In particular, note that the extra terms

$$-\frac{32\pi^2}{\kappa}\left(\frac{1}{F}\frac{\delta F}{\delta \zeta_+}\frac{\delta \chi}{\delta \zeta_-} + \frac{1}{F}\frac{\delta F}{\delta \zeta_-}\frac{\delta \chi}{\delta \zeta_+} + \frac{\delta^2 \chi}{\delta \zeta_+ \delta \zeta_-}\right)$$

will become small as $\kappa(N)$ gets large. However, in the same limit our semiclassical equation (3.19) will approach the vacuum Einstein equation and the usual formulation cannot be recovered.

One might ask whether the required equations can be obtained if $\langle\hat{T}^m\rangle$ scales with κ so that the left-hand side of (3.19) is of the same order in the expansion as the right-hand side. However, then we have no justification for separating (3.20) as the $O(\kappa^0)$ terms. In other words if the large κ approximation is to be used it must be done systematically and that just leads us to the previous results, i.e., the Schrödinger equation in the vacuum background. As far as we can see there is no alternative argument (as in the mini-superspace case) either.

IV. OTHER POSSIBILITIES

A. Reduced phase-space quantization (light cone gauge)

An alternative to Dirac quantization is to first solve the classical constraints and then quantize the physical dynamical variables. In this case one has an intrinsic (local) time in terms of which the wave function(al) of the physical variables satisfies a local functional Schrödinger equation. It may then be thought that this gives us a definition of time in terms of which the S (or $) matrix can be defined. However, as discussed in some detail by Kuchař [1], this method does not give a resolution to the problem of time: it is beset with the same difficulties as the Dirac quantization method, in addition to suffering from problems such as the multiple choice question stemming from the nonuniqueness in the choice of the time variable. Also the Hamiltonian that is obtained from solving the classical constraints is in general nonlocal.[10] More importantly, we cannot find any way of reproducing the semiclassical physics of black hole collapse and Hawking radiation starting from this version of the exact quantum theory.

It should also be stressed at this point that the mere existence of a Hermitian reduced (local) Hamiltonian is not at all a solution of the problem posed by Hawking. First, since the theory does not admit the usual semiclassical picture it is not clear how the problem can even be posed. Second, although a Hermitian Hamiltonian will lead to unitary evolution, the point is that according to the semiclassical picture part of the state has gone into the black hole and cannot be reconstructed by an asymptotic observer outside the hole. This is the origin of information loss according to Hawking [2]. Any refutation of Hawking's claim should at least first reproduce this semiclassical picture in the appropriate regime.

B. Boundary Hamiltonian

It is well known that the classical Hamiltonian for a space-time that is asymptotically flat (such as that of a black hole) has a boundary contribution. Since the bulk Hamiltonian is weakly zero in a generally covariant theory, the total energy (ADM mass) of such configurations is given by the value of this boundary term. It has been shown by Regge and Teitelboim [16] that this term is necessary in order to cancel a boundary term that arises in the derivation of Hamilton's equations for the system.

In a local quantum field theory, however, such a boundary Hamiltonian cannot play any role whatsoever. Indeed, microcausality requires that all local fields will commute with such a boundary Hamiltonian and it is irrelevant to the derivation of the Heisenberg equations. The time evolution of the quantum field theory does not depend on such a boundary Hamiltonian. To put it another way one must define the theory with an infrared cutoff (or by smearing with test functions, which fall off rapidly at spatial infinity). Thus there will be no boundary contributions to the energy. One should not expect the S matrix (or $ matrix) to be defined by some quantum analogue of the ADM energy. The latter seems to have meaning only within the (semi)classical context.

V. DE BROGLIE–BOHM INTERPRETATION

When one applies quantum mechanics to the universe as a whole, the usual pragmatic interpretation, which separates the world into classical observing system and quantum systems, is clearly untenable. This has indeed been a serious conceptual barrier (quite distinct from the technical problems of quantum gravity) to understanding quantum gravity. One possibility is to adopt the so-called Everett (or many worlds) interpretation. However, it is not at all clear that this interpretation provides us with an explanation of why it is that experimental results have definite values.[11] The other possibility (recommended to cosmologists and by implication practitioners of quantum gravity by Bell [17]) is the de Broglie–Bohm (dBB) interpretation [13].

We have argued in this paper (and in [10]) that none of the usual arguments for deriving the semiclassical picture from an exact formulation of quantum gravity are valid. As we have seen the problem is essentialy the problem of time that has been discussed extensively in the literature on quantum gravity [1]. The dBB picture gives a natural way of getting parametric time in quantum gravity [19,13], and it is the only way that we have found where one can establish the validity of the physical picture of the semiclassical calculations. After reviewing very briefly the dBB formulation of one-particle quantum mechanics, we will discuss the dBB interpretation of quantum minisuperspace. Next, we will work out the dilaton gravity case and discuss under what conditions the semiclassical calculations might be valid.

Substituting the form $\psi(x,t) = R(x,t)\exp[iS(x,t)]$ into the Schrödinger equation, with R and S real functions, one has from the real and imaginary equations:

$$\frac{\partial S}{\partial t} + \frac{(\partial_x S)^2}{2m} + V + Q = 0 , \tag{5.1}$$

$$\frac{\partial R^2}{\partial t} + \partial_x(R^2 \partial_x S) = 0 \tag{5.2}$$

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[10]Though in the case of dilaton gravity it is possible to have a local Hamiltonian.

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[11]Recent work by Gell-Mann and Hartle [18], as well as earlier work cited in their work may clarify these issues, but we do not understand these works sufficiently well to apply them to our problem.

with

$$Q = -\frac{\hbar^2}{2m}\frac{\partial_x^2 R}{R}, \quad R = |\psi|^2 . \tag{5.3}$$

In the dBB interpretation one now postulates that the particle moves on a trajectory $X(t)$ with momentum given by the Hamilton-Jacobi formula

$$P_x = m\dot{X} = \partial_x S|_{x=X} .$$

When this is used in Eq. (5.1), it is in the form of the classical Hamilton-Jacobi equation, except that there is an additional "quantum potential" term Q. By differentiating (5.1) one obtains the classical equations of motion corrected by quantum terms coming from the Q term. Once the initial wave function and the initial position of the particle are given, the theory predicts the wave function and particle position at any future time. The wave function itself plays a dual role. First, it affects the particle motion through the Q term in the equations of motion. Second, it gives the distribution of possible initial values of the particle position, and hence as a result of the continuity Eq. (5.3), also the distribution of positions at any future time. The interpretation can be shown to be in agreement with all experimental tests of quantum mechanics. Its great merit is that it avoids the ambiguities and paradoxes associated with the pragmatic Copenhagen interpretation. This includes in particular the dividing line between the classical and quantum realms, and the mysterious (nonunitary) collapse of the wave function. In effect it is an observer-independent realist interpretation, and it seems to us that the dBB formulation, or something along those lines, is essential for the discussion, not only of quantum cosmology, but also of the physics of black holes.

Let us now describe the dBB interpretation in the case of mini-superspace.[12] We rewrite Eq. (2.8) as

$$-\frac{1}{2aM^2}(\partial_a S)^2 + \frac{1}{2a^3}(\partial_\phi S)^2 + a^3[M^2 V_G + V_m + Q] = 0 \tag{5.4}$$

with

$$Q = \frac{\hbar^2}{2a^6}\frac{1}{R}\left[\frac{1}{M^2}(a\partial_a)^2 R - \partial_\phi^2 R\right] . \tag{5.5}$$

Using the canonical momenta defined by Eq. (2.5) we define the trajectories for the scale factor $A(t)$ and the matter field $\Phi(t)$ by

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[12]These equations have already been derived by Vink [19]. However, his interpretation seems to be rather different, since he seems to use them as a step towards the derivation of the usual semiclassical equations (2.16) and (2.19), which in our opinion are untenable except under very special conditions as outlined in Sec. II, and for this purpose we do not think it is necessary to invoke the dBB interpretation.

$$\frac{dA(t)}{dt} = \frac{1}{A(t)M^2}\partial_a S\Big|_{\substack{a=A\\\phi=\Phi}} ,$$

$$\frac{d\Phi(t)}{dt} = \frac{1}{A(t)^3}\partial_\phi S\Big|_{\substack{a=A\\\phi=\Phi}} . \tag{5.6}$$

It should be noted that the equation for the classical trajectory of the scale factor in (2.15) is of exactly the same form as the above equation, except that there S was just the solution to the classical Hamilton-Jacobi equation for a in the absence of matter. Thus time was defined only with respect to some vacuum configuration. On the other hand in the dBB interpretation parametric time arises in the full quantum theory precisely because choosing a particular trajectory amounts to a spontaneous breakdown of general covariance. It is only the full ensemble of trajectories described by the wave functional Ψ that satisfies general covariance.

Substituting Eqs. (5.6) into Eq. (5.4) yields

$$-M^2\left(\frac{A\dot{A}^2}{2} - A^3 V_G\right) + A^3\left(\frac{\dot{\Phi}^2}{2} + V_m + Q\right) = 0 .$$

As $Q \to 0$ we obtain the classical Eq. (2.1).

Differentiating (5.6) with respect to t, and differentiating (5.4) with respect to a, one obtains, after some manipulation,

$$M^2\left\{\partial_t(A\dot{A}) - \frac{\dot{A}^2}{2} - \partial_a(A^3 V_G)\right\}$$

$$+ 3A^2\left(\frac{\dot{\Phi}^2}{2} - V_m\right) = \partial_a(A^3 Q) .$$

Similarly one gets

$$\frac{d}{dt}(A^3\dot{\Phi}) + A^3\partial_\phi V_m = -A^3\partial_\phi Q .$$

In the limit $Q \to 0$ these two equations become the classical equations of motion (2.2) and (2.3), respectively.

The classical equations are obtained in the regime where Q is negligible. It should be noted that although this term is explicitly of $O(\hbar^2)$ this is not sufficient reason to neglect it. Whether one can neglect it or not is a delicate question depending on the form of the wave functional. Even in quantum mechanics there are states that have no classical limit, such as stationary states, for which this term exactly cancels the classical potential term. In quantum cosmology an example of this is the Hartle-Hawking wave function [20], which is real and thus will not allow a classical limit. It also will not have an evolving geometry or matter fields according to the dBB interpretation; it is the analogue of a stationary state and is a superposition of expanding and contracting states. The Vilenkin wave function [21] on the other hand gives an expanding quantum trajectory with a well-defined classical limit.

The dBB interpretation is extended to field theory in a straightforward manner by replacing the trajectory of

the particle by trajectories for the field variables. In 2D dilaton gravity the analogs of Eqs. (5.1) and (5.2) are the WD equations given by (3.11) and (3.12). In addition we also have the constraint equations (3.13) and (3.14). The trajectories are defined using Eq. (3.6):

$$\dot{Z}_+(\sigma,\tau) \equiv \frac{\partial Z_+(\sigma,\tau)}{\partial \tau} = \frac{16\pi}{\kappa}\frac{\delta S}{\delta \zeta_-}\Bigg|_{\substack{\zeta_\pm = Z_\pm \\ f_i = F_i}} \ ,$$

$$\dot{Z}_-(\sigma,\tau) \equiv \frac{\partial Z_-(\sigma,\tau)}{\partial \tau} = \frac{16\pi}{\kappa}\frac{\delta S}{\delta \zeta_+}\Bigg|_{\substack{\zeta_\pm = Z_\pm \\ f_i = F_i}} \ , \qquad (5.7)$$

$$\dot{F}_i(\sigma,\tau) \equiv \frac{\partial F_i(\sigma,\tau)}{\partial \tau} = 4\pi\frac{\delta S}{\delta f_i}\Bigg|_{\substack{\zeta_\pm = Z_\pm \\ f_i = F_i}} \ .$$

These equations *implicitly* define a time parameter τ. As before we get dynamics only for complex wave functionals ($S \neq 0$).

We functionally differentiate (the spatial integral of) (3.11) with respect to $\zeta_+(\sigma)$, and the second equation in (5.7) with respect to time to obtain

$$\frac{\kappa}{8}[\ddot{Z}_-(\sigma) - Z''_-(\sigma)] - \lambda^2\kappa e^{Z_+(\sigma)}$$

$$= -\frac{\delta}{\delta\zeta_+(\sigma)}\int Q(\sigma')d\sigma'\Bigg|_{\substack{\zeta_\pm = Z_\pm \\ f_i = F_i}} \ . \qquad (5.8)$$

In a similar manner we obtain the quantum extensions of the other classical equations of motion:

$$\frac{\kappa}{8}(\ddot{Z}_+ - Z''_+) = -\frac{\delta}{\delta\zeta_-}\int d\sigma' Q(\sigma')\Bigg|_{\substack{\zeta_\pm = Z_\pm \\ f_i = F_i}} \qquad (5.9)$$

and

$$\frac{1}{2}(\ddot{F}_i - F''_i) = -\frac{\delta}{\delta f_i}\int d\sigma' Q(\sigma')\Bigg|_{\substack{\zeta_\pm = Z_\pm \\ f_i = F_i}} \ . \qquad (5.10)$$

In conjunction with the above equations of motion one also has, after substituting (5.7) into (3.11) and (3.14), the de Broglie–Bohm versions of the constraints:

$$\frac{\kappa}{8}[\dot{Z}_+\dot{Z}_- + Z'_+Z'_- + 2(Z''_+ - Z''_-) - 8\lambda^2 e^{Z_+}]$$

$$+\frac{1}{4}\sum_{i=1}^N (F'^2_i + \dot{F}^2_i) = -Q \ , \qquad (5.11)$$

$$\frac{\kappa}{4}\left[\left(Z'_- + 2\frac{\partial}{\partial\sigma}\right)\dot{Z}_+ + \left(Z'_+ - 2\frac{\partial}{\partial\sigma}\right)\dot{Z}_-\right]$$

$$+\sum_{i=1}^N F'_i\dot{F}_i = 0 \ . \qquad (5.12)$$

These are to be compared with the classical equations of motion derived from the action (3.2):

$$\ddot{\zeta}_- - \zeta''_- = 8\lambda^2 e^{\zeta_+} \ ,$$

$$\ddot{\zeta}_+ - \zeta''_+ = 0 \ , \qquad (5.13)$$

$$\ddot{f}_i - f''_i = 0 \ ,$$

and the constraint equations coming from (3.7) and (3.8).

When $Q \to 0$, equations (5.8)–(5.12) reduce to the semiclassical equations discussed in [8,9,12] and many other works.[13] This is just the usual WKB approximation. The question as to what extent the semiclassical equations are valid then becomes one of deciding when and in what regimes the quantum potential term becomes negligible. As pointed out earlier, in general, this cannot be done for any wave functional Ψ. In particular, since the Wheeler-DeWitt equation is real it is always possible to find a real solution and, as in the stationary state of quantum mechanics or the Hartle-Hawking wave function discussed above, there will not be any time evolution. In order to have a dynamic semiclassical picture, one needs a complex wave functional.

Given a complex wave functional one may then ask under what conditions Q is negligible. Unfortunately we have not yet been able to solve the WD equation for the theory in order to decide this issue.[14] We can, however, speculate that, since the theories with no boundary in field space (see Sec. III and [12] and [10]) have effective actions that are the same as the (semi)classical actions, the wave functional is a pure phase so that R =const and Q is zero. On the other hand it is likely that in theories with boundary, as in that described by [12], Q is in fact significant, especially when one approaches the boundary of field space (which in the semiclassical analysis becomes spacelike and hence gives rise to information loss in this analysis). It should also be pointed out that Q incorporates all the Einstein-Podolsky-Rosen- (EPR-) type non-localities of quantum mechanics and may well account for the mechanism by which information is extracted from behind the horizon of the semiclassical picture. It may also completely vitiate the semiclassical picture in that the black hole itself may be washed out. These are questions that are presently under investigation.

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[13]We call these semiclassical rather than classical equations because the definitions of the fields incorporate $O(\kappa\hbar)$ conformal anomaly corrections. It should also be noted that if we start from the WD equation of the original classical theory, then $O(\hbar)$ corrections, which are responsible for Hawking radiation, are missing.

[14]The solutions that exist in the literature [22] do not include the conformal anomaly corrections, and hence it is not clear how to obtain the comparison to the calculations of Hawking radiation.

VI. CONCLUSIONS

Generally covariant quantum field theories are topological theories in so far as all correlation functions are independent of space-time geometry, and the only physical operators are those which commute with the constraints. The theory will thus only contain topological information [23]. The problem of time in quantum gravity is just a manifestation of this fact. It appears then that only the spontaneous breaking of general covariance at the quantum level can restore a geometrical background within which physical processes can be discussed. One of the conclusions of this work is that the work on semiclassical gravity (quantum field theory in curved space-time) has to be interpreted in this way.

In the particular case of black hole formation and evaporation it has been argued that these questions are irrelevant. We find though that this is not the case. First we showed, by a detailed discussion of both minisuperspace and dilaton gravity, that there is no systematic derivation of quantum field theory in a background that is determined by the Einstein equation with a source term given by the expectation value of the matter stress tensor. What one gets from the expansion in inverse powers of the Planck mass or the large N expansion is quantum field theory for the matter sector in a background that is a solution of the vacuum Einstein equations [6]. Thus, for instance, the usual applications of quantum field theory (QFT) in flat space particle physics for energy densities that are small compared to the Planck density are justified. One may argue in the minisuperspace case that, while there is no systematic derivation of the so-called semiclassical equations, it is still possible to show that in the late Universe the functional Schrödinger equation is consistent with that which arises from the Wheeler-DeWitt equation, once a classical cosmological solution (determined by the expectation value of the matter solution) is used. This argument perhaps justifies quantum field theory calculations in cosmological backgrounds. However, for 2D dilaton gravity, the only known field theoretic example that exhibits (at the semiclassical level) the formation and evaporation of black holes, we cannot find such an argument.

We are then left with the problem of justifying the semiclassical equations of dilaton gravity that many authors[15] have used to explore the problems associated with Hawking radiation. The equations in question involve space-time differential equations for c-number fields. In an earlier paper by one of us [10], it was argued that if these are to be interpreted as expectation values of the corresponding equations for quantized fields then one is forced to drop the constraint equation (WD or BRST) as an equation on the states. In other words,

the semiclassical equations cannot emerge as expectation values, in generally covariant physical states, of the corresponding operator equations. One way out suggested in that paper was to abandon the superposition principle at the level of the wave function of the Universe, and to impose the constraint by defining physical states as those in which the constraint is satisfied as an expectation value. Although we do not see any logical flaw in this, in this paper we have examined an alternative that enables us to obtain the required semiclassical physics from the WD equation. This involves the de Broglie–Bohm interpretation of quantum mechanics.

It seems to us that any discussion of quantum gravity must go beyond the pragmatic interpretation that divides the world into classical apparatus and quantum system. This is certainly the case for quantum cosmology, but the arguments presented in this paper indicate that this is so even for the black hole problem, at least if we start from generally covariant physical states. As far as we can see, the only way to get a picture of the evolution of a black hole is to pick a quantum trajectory (in the sense of dBB) from the ensemble of trajectories that may be described by the Schrödinger wave functional. This constitutes a spontaneous breakdown of general covariance. The resulting equations of motion and constraint equations are precisely the semiclassical equations discussed in the literature except that there are correction terms. Given a solution of the WD equation, these additional terms may be evaluated, and to the extent that they are small, one may say that the semiclassical approximation is valid. We have argued that for the dilaton gravity theories with no field space boundary, these correction terms are (probably) absent. However, in this case there is no black hole singularity and no information loss problem.[16] On the other hand, in the theories with boundary, such as that of Russo-Susskind-Thorlacius (RST) [12], it is not at all clear that one should ignore these correction terms. We are presently engaged in finding solutions to the WD equation for such models so that these issues may be further clarified.

ACKNOWLEDGMENTS

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[15]See [24] for reviews.

[16]Information is lost only in the trivial sense that left movers carry information from the right end of space to the left end and do not communicate with the right movers. This is a situation that one may obtain in ordinary flat space quantum mechanics and does not imply nonunitary evolution.

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