Primordial bubbles from quadratic gravity

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A toy model of inflation with a first order phase transition built on a nonminimal generalization of quadratic gravity effectively implements a two field inflation and copiously spurs bubbles *before* the end of the slow roll. In particular, the phase transition may be brought to completion quickly enough to leave an observable signature at the large scales. We identify analytically and numerically the parameter space region capable of fitting the observed galaxy correlation function, while passing the microwave background constraints. Thus, astronomical observations can yield information upon the parameters of fundamental physics.

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I. INTRODUCTION

Inflation with a first order phase transition [1] [for short, first order inflation (FOI)] is now reviving the appeal of Guth's old inflation [2], accelerated expansion and bubble production in one and the same process. The bubbles produced during the phase transition may be the *deus ex machina* for the formation of large scale structure (LSS) [3,4], since they can provide the large scale power that cold dark matter (CDM) models seem to lack. In particular, a bubble geometry can reproduce observed features in the galaxy distribution such as the galaxy correlation function (GCF) [5] and the higher-order moments [6]. However, a warning must be issued at once [7] for the deep "scars" big bubbles leave in the cosmic microwave background (CMB)—a real "challenge to model builders." Indeed, it has been shown [7,8] that the existing variants of FOI do not produce an astrophysically useful spectrum of bubbles, because, if the bubbles are to satisfy the CMB constraints, then they cannot have an impact on the LSS. In this paper we propose a toy model of FOI which overcomes this difficulty.

Models of FOI were motivated by the graceful exit difficulty [9]. In fact, since the phase transition is completed approximately if and when [1]

$$Q = \frac{4\pi\Gamma}{9H^4} \tag{1}$$

first grows to order unity (one bubble produced per horizon four volume), where Γ is the tunneling rate defined later and $H = \dot{a}/a = \dot{\alpha}$ is the Hubble parameter of a spatially flat Friedmann-Robertson-Walker (FRW) metric of scale factor $a(t) = a(0)\exp\alpha(t)$, all one has to do is to increase the numerator or to decrease the denominator (or both) in Q.

Extended inflation (EI) [10] which slows down the expansion of the background to a power law by changing the underlying gravity to Jordan-Brans-Dicke type, with its difficulties and remedies [11], remains a paradigm of the latter option. The other way out, by making Γ increase with expansion, is achieved by two field inflation [12], where one field does the quantum tunneling and the other does the slow rolling under an *ad hoc* chosen potential. In this paper, we will implement this very mechanism of a time-dependent Γ by assuming instead that the underlying gravity is not Einsteinian, but carries also the quadratic corrections in the Ricci curvature R to the Lagrangian, suggested by quantum and superstring [13] theories and appropriate to the early Universe (see, however, Ref. [14]). One has then a fourth order gravity (FOG): this theory has many attractive features among which the existence of nonsingular solutions and the fact that canonical general relativity (GR) is its low energy limit.

The slow rolling field is now the Starobinsky [15] *scalaron*, the potential of which, in the conformal frame, is not chosen *ad hoc*, but dictated by the field equations.

The presentation of our model is organized as follows. First, given that our matter content is in the form of a scalar field ψ, we choose a coupling of ψ to R^2 that carves two channels of different energy in the conformal potential. This allows for bubble production in a first stage, and for pure slow rolling subsequently. As a consequence, we show that under FOG *the phase transition is indeed quickly completed, yielding a signature at a well-defined, large and tunable scale* in the present Universe. Second, we evaluate analytically and numerically the tunneling rate in the slow-rolling, thin-wall limit, applying the canonical technique of Coleman [16]. Third, exploiting the relation between the instant of nucleation of a bubble and its comoving size, we convert the tunneling rate to a bubble spectrum, and approximate the latter as a power law, so as to focus on two parameters only. Finally, we determine the values of the parameters which pass the CMB constraints on large and small scales and fit the GCF, following the results of Ref. [5] and, for such values of the parameters, we display numerical bubble spectra.

Let us observe that the bubble physical size expands overcomovingly [17] as $t^{4/5}$ (i.e., the comoving radius goes as $a^{1/5}$) in the matter-dominated era, so that the volume contained inside bubbles was much smaller in the past. A volume fraction of, say, 50% today, necessary to produce significant structure, was less than 1% at decoupling, and even smaller at the equivalence. It is this relatively small perturbation that allows the microwave tests to be passed. For the same reason, it is also very likely that other cosmological constraints, such as the yields of an inhomogeneous nucleosynthesis (see, e.g., Ref. [18]), the primordial black holes production (e.g., [19]), the gravitational wave generation (e.g., [20]), leave a consistent window in the parameter space of our scenario.

The model presented here bears a strong resemblance to the "scale-invariant" EI [21] and with our work of Ref. [22], with regard to the tunability of the epoch of bubble production. In neither case, however, was the spectrum of tunneling-induced inhomogeneities calculated and confronted with astrophysical observations. We earlier introduced the coupling to the R^2 term [23] to solve a fine-tuning problem in a doubly inflationary scenario [24].

II. A NEW BUBBLE PRODUCTION MECHANISM

Let us first briefly review why the primordial bubbles in the current models of EI cannot trigger structure formation. If Γ is the bubble nucleation rate and $V_{\rm FV}$ is the fraction of volume in false vacuum at the time t, the number of bubbles nucleated during the interval dt is

$$\frac{dn_B}{dt} = \Gamma V_{\rm FV}\,. \qquad (2)$$

Since large bubbles, the ones of interest here, are nucleated far before the phase transition ends, we may assume that almost all of the Universe still sits in the false vacuum state, $V_{\rm FV} = a^3$. A bubble of comoving size L will cross out the horizon when $L \approx (aH)^{-1}$. In what follows, the notation L refers to the comoving size of bubbles, while the notation R is left to indicate the actual present size of bubbles *after* the overcomoving expansion. Let us remark that we will express all lengths as comoving lengths so that their physical and comoving sizes coincide at the present time; when we say that a bubble was g times smaller at decoupling we refer to its comoving size: its physical size was clearly $gz_{\rm dec}$ times smaller. Assuming a generic power law accelerated expansion $a \sim t^n$ with $n \gg 1$, one has $dL/dt \sim -1/a$ so that Eq. (2) becomes

$$\frac{dn_B}{dL} \sim -\Gamma L^{-4-4/n}\,, \qquad (3)$$

to first order in $1/n$.

Assuming Γ slowly varying with time, Eq. (3) can be integrated to give the EI power spectrum

$$n_B = (L_{\rm max}/L)^p\,, \qquad p = 3 + 4/n\,, \qquad (4)$$

where the normalization constant $L_{\rm max}$ can be explicitly calculated in terms of the nucleation rate, which in turn depends on the potential parameters of the specific model. However, the very condition that the phase transition be completed at a given time t_e requires that by that time the volume contained in all the bubbles previously nucleated be of the order of the horizon volume. To an order-of-unity factor this condition is

$$\int_{L_h}^{L_e} \frac{dn_B}{dL} L^3 dL = L_h^3\,, \qquad (5)$$

where $L_h = 2H_0^{-1} = 6000h^{-1}$Mpc, and L_e is the size of the bubbles nucleated at the phase transition end. Let $N_{\rm PT} = \ln(L_h/L_e) \gg 1$ be the duration in e-foldings of the phase transition.

This fixes $L_{\rm max}$ as

$$L_{\rm max} = L_h \exp\left[(3-p)N_{\rm PT}/p\right]\,. \qquad (6)$$

With this normalization, Eq. (4) provides the EI bubble spectrum (neglecting the overcomoving expansion). In all the current models of EI one has $N_{\rm PT} \sim 60$: in fact, the phase transition ends when the inflation ends. The reason why the EI spectrum does not work for making reasonable structure is that with $N_{\rm PT} \sim 60$ the scale $L_{\rm max}$ is vanishingly small for p even slightly larger than 3. A detailed comparison with the observational constraints shows indeed that a spectrum with $L_{\rm max}$ as in Eq. (6) is far outside the acceptable range of parameters ([8]; see also below). However, Eq. (6) also indicates the way out: making $N_{\rm PT}$ a free parameter we may hope to generate some acceptable spectrum. For $N_{\rm PT} \sim 10$, in fact, we have $L_{\rm max}$ of the order of the large scale astrophysical structures for reasonable values of the spectral index p. To obtain $N_{\rm PT}$ smaller than its canonical value 60, all we need to do is to produce a phase transition *shorter* than the inflation itself. This is provided in a fairly simple way by our model.

Our physics (in Planck units) is entirely contained in the Lagrangian density $\mathcal{L}$, the sum of a gravity contribution

$$\mathcal{L}_{\rm grav} = -R + \frac{R^2}{6M^2W(\psi)}\,, \qquad (7)$$

where R is the Ricci scalar (not to be confused with the bubble size), and a canonical matter contribution

$$\mathcal{L}_{\rm mat} = 16\pi\left(\frac{1}{2}\psi_{;\mu}\psi^{;\mu} - V(\psi)\right)\,. \qquad (8)$$

The former generalizes the canonical $\mathcal{L}_{\rm grav}$ of quadratic gravity [15] with the inclusion of the scalar coupling $W(\psi)$ in the quadratic term.

With this generalization, the mass M of the Starobinsky scalaron is replaced by a reduced mass

$$M_{\rm eff}(\psi) = MW^{1/2}(\psi). \qquad (9)$$

For M and $M_{\rm eff}$ there are (i) upper limits, of the order of 10^{-5}, from the lack of large scale anisotropy in the cosmic microwave background [15,25] and (ii) lower limits [26] from the Yukawa corrections on scales smaller than $M_{\rm eff}^{-1}$ or M^{-1} to the Newtonian potential of a point mass, neither of which is a problem. The action

$$\int d^4x\sqrt{-g}(\mathcal{L}_{\rm grav} + \mathcal{L}_{\rm mat}) \qquad (10)$$

can be recast in a (maybe more familiar) Brans-Dicke form plus a quadratic correction performing the coordinate transformation $x'^{\mu} = W^{-1/4}x^{\mu}$ and the field redefinition $\psi' = W^{1/4}\psi$.

However, in view of the "roadblock" [14] canonical couplings cause to a satisfactory inflation, it may be worth to trade a variable G for a harmlessly variable M_{eff}.

For our FRW metric, the Ricci scalar $R = -6(\dot{H} + 2H^2)$ can be given by an approximate solution [15], under the slow roll ($|\ddot{R}| \ll H|\dot{R}|$) assumption and for the vacuum case: the latter can be immediately generalized to the present case for ψ =const. By taking $\alpha \equiv \ln(a/a_{\text{in}})$, one finds

$$\alpha = \frac{R - R_{\text{in}}}{4M_{\text{eff}}^2}\,, \tag{11}$$

where the subscript "in" refers to the beginning of the last N_T useful e-foldings; whenever necessary for the actual computations we set $N_T = 60$, without prejudice to generality.

The theory (7) can be conformally transformed into a canonical GR [27] with the new metric

$$\tilde{g}_{\alpha\beta} = e^{2\omega}g_{\alpha\beta}\,, \quad e^{2\omega} = \left|\frac{\partial\mathcal{L}}{\partial R}\right| = 1 - \frac{R}{3M_{\text{eff}}^2}\,. \tag{12}$$

In the slow-rolling approximation, useful relations link ω, H, and the number $N = N_T - \alpha$ of e-foldings to the end of inflation:

$$\frac{4}{3}N = (e^{2\omega} - 1) = \frac{4H^2}{M_{\text{eff}}^2}\,. \tag{13}$$

Correspondingly, $H_{\text{in}} = M_{\text{eff}}(N_T/3)^{1/2}$. From the Lagrangian (7) we obtain then Einstein gravity with two scalar fields ψ and ω, coupled by a potential given by

$$U(\psi,\omega) = e^{-4\omega}\left[V(\psi) + \frac{3M^2}{32\pi}W(\psi)(1 - e^{2\omega})^2\right]\,. \tag{14}$$

In the rescaled Lagrangian, the dimensionless field ω acquires a canonical (up to a numeric factor) kinetic term, while the kinetic term for ψ is multiplied by the factor $e^{-2\omega}$. The expression (14) shows the different roles of the two potentials: $W(\psi)$ rules the early FOG evolution when ω and ψ are large while $V(\psi)$ comes in later as $\omega \to 0$ and GR is recovered. We need to impose two conditions on $W(\psi)$ and $V(\psi)$: that a phase transition be possible, and that at some given instant, *while the expansion is still inflationary*, the barrier between vacuum states vanishes, so that the phase transition comes to an end. The minimal ansatz is then a quartic for $W(\psi)$ and a quadratic for $V(\psi)$:

$$W(\psi) = 1 + \frac{8\lambda}{\psi_0^4}\psi^2(\psi - \psi_0)^2\,, \quad V(\psi) = \frac{1}{2}m^2\psi^2\,. \tag{15}$$

This carves in (14) two parallel channels of different heights, separated by a peak (PK) at $\psi_{\text{PK}} = \psi_0/2$. The degeneracy of $W(\psi)$ in $\psi = 0$ and $\psi = \psi_0$ is indeed removed by $V(\psi)$; the true vacuum (TV) channel remains at $\psi_{\text{TV}} = 0$, while the false vacuum (FV) channel is slightly displaced from ψ_0. If ω_{end}, see below, is where the barrier between the channels ends, the inequality $\omega > \omega_{\text{end}}$ must be satisfied during bubble nucleation; furthermore, since we work in the thin wall approximation, $U_{\text{FV}} \ll U_{\text{PK}}$, a slightly stronger inequality $\omega > \omega_{\text{thw}}$ is required, where $2\omega_{\text{thw}} \approx 2\omega_{\text{end}} \approx \ln(1 + \beta)$ and

$$\beta = \frac{m}{M}\sqrt{\frac{32\pi\psi_0^2}{3\lambda}}\,, \tag{16}$$

is dimensionless.

In practice, we work with $\beta \ll 1$ and, during the phase transition, $\omega \approx 2$. Two comments are now in order: (i) given that there is one absolute minimum at $\omega = \psi = 0$, the final true vacuum, the ansatz of (15) may still generate an unwanted secondary minimum along the FV channel; care is taken to avoid this occurrence; (ii) the classical motion is not a double inflation in the usual sense, one slow roll for each field, but a sequence of two slow rolls for the same field, ω, down the FV channel first and then, after the end of the phase transition, down the TV channel.

We must now tackle Q. Although our gravity is complex, basic physics [16] still applies in the conformal frame: hence,

$$\tilde{\Gamma} = \mathcal{M}^4\exp(-S_E)\,, \tag{17}$$

where $\mathcal{M}$ is of the order of the energy of the spontaneous symmetry breaking and S_E is the Euclidean action:

$$S_E = \int\sqrt{-g}d^4x\left(\frac{1}{2}e^{-2\omega}\psi_{;\mu}\psi^{;\mu} + U(\psi,\omega)\right) \tag{18}$$

(neglecting the kinetic energy of ω). To obtain a canonical kinetic term, we rescale the coordinates [1] as $x^{\mu} = e^{-\omega}\hat{x}^{\mu}$, so that, *in the new coordinates*

$$S_E = e^{-4\omega}\int\sqrt{-g}d^4\hat{x}\left(\frac{1}{2}\psi_{;\mu}\psi^{;\mu} + U\right) = e^{-4\omega}\hat{S}_E(\psi)\,, \tag{19}$$

where finally $\hat{S}_E$ is canonical.

To evaluate $\hat{S}_E$, we observe that the potential (14) is in the form employed by Coleman in Ref. [16], i.e., a quartic degenerate potential to which a symmetry-breaking term is added, except that the coefficients are ω dependent. Because of the slow-rolling approximation, the ω dependence transforms in a weak time dependence that we neglect. In particular, in the large ω approximation in which we will work, Eq. (14) simplifies considerably and only the symmetry-breaking term keeps the ω dependence: by writing

$$U(\psi,\omega) \approx \Lambda W(\psi) + V(\psi)e^{-4\omega}\,, \tag{20}$$

where

$$\Lambda = \frac{3M^2}{32\pi}\,,$$

we may express the Coleman result as

$$\hat{S} = \frac{27\pi^2}{2\epsilon^3}\left[\int_0^{\psi_0} d\psi\sqrt{2\Lambda(W(\psi)-1)}\right]^4 , \quad (21)$$

in the thin wall limit and to the lowest order in the smallness parameter, the energy density difference $\epsilon \equiv (U_{\rm FV} - U_{\rm TV}) = m^2\psi_0^2/2\exp(-4\omega)$. In our model, this gives (gravitational corrections being small)

$$S_E \approx Ee^{8\omega} , \quad (22)$$

where

$$E = \frac{64\pi^2}{3\beta^4}\left(\frac{\psi_0}{m}\right)^2 . \quad (23)$$

The result, essentially due to the scalaron physics and not to the specific coupling $W(\psi)$, can be recast by use of (13) in another useful way:

$$S_E \approx \left(\frac{N}{N_1}\right)^4 , \quad E \approx \left(\frac{3}{4N_1}\right)^4 , \quad (24)$$

where the subscript "1" marks the scale where $S_E = 1$. Since spinodal decomposition [7] occurs for $S_E < 1$, requiring that $N > N_1(\gg 1)$ gives us confidence that we are dealing with real bubbles and not with generic perturbations.

Consistently, the density contrast of the bubble-like inhomogeneities is $|\delta\rho/\rho| \approx (U_{\rm PK} - U_{\rm TV})/U_{\rm PK} \approx \beta^{-2}$ for $\beta > 1$ and ≈ 1 for $\beta < 1$; therefore, the very condition that we are in the thin wall limit, equivalent to $\beta < 1$, also guarantees that the bubbles are strong underdensities. While the smallest bubbles will rapidly thermalize through collisions and matter infall, the biggest bubbles will remain essentially void until the present, driving early structure formation. Turning now back to the physical frame, after taking into account the conformal rescaling of four volume [1] $\Gamma = e^{4\omega}\tilde{\Gamma}$, and expressing H by the slow roll approximation (13), we have finally

$$Q(N) = \exp\left(\frac{N_0^4 - N^4}{N_1^4}\right) ,$$
$$\left(\frac{\mathcal{M}}{M_{\rm eff}}\right)^4 = \frac{9}{64\pi}\exp\left[\left(\frac{N_0}{N_1}\right)^4\right] \quad (25)$$

where we have introduced a new parameter N_0 to mark the end of the phase transition $[Q(N_0) = 1]$, and the beginning of the pure slow-rolling stage. It is the freedom to choose N_0 that will allow for a successful production of astrophysically acceptable bubbles. An estimate of N_0 is equivalent to an estimate of $\mathcal{M}$, another instructive example of the deep links between fundamental physics and LSS provided by inflation. Obviously, the constraint $N_0 > N_1$ must be satisfied. To make contact with the astronomical intuition, the comoving scales of interest $L(N)$ are first read as a function of the N that applies when they cross out the horizon:

$$H(N)L(N) = H_{\rm in}L_h\exp(N - N_T) . \quad (26)$$

Subsequently, they are overcomovingly [17] expanded into bubble radii $R(N) = f(z)L(N)$, where $f(z) = z^{1/5}$ for the comoving scales $> 13h^{-1}$ Mpc that reenter the horizon after the equivalence redshift $z_{\rm eq} \approx 24\,000$, and $f(z) = z_{\rm eq}^{1/5} \approx 7.45$ for all the smaller scales. To fix the ideas, if $N_T = 60$ and $L_h = 6000h^{-1}$ Mpc is the present horizon, then $N = 57$ corresponds to $300h^{-1}$ Mpc, and $N = 50$ corresponds to a fraction of a Mpc.

If the tunneling process keeps going for $N \to 0$, the situation will be similar to that of EI where there are too many small bubbles (nucleated at the end of inflation) and too few big ones.

III. RESULTS AND DISCUSSION

The bubble spectrum can be evaluated by the knowledge of the rate at which bubbles are generated [9], as previously done for the EI. Since we want quantitative results, we keep now all the relevant factors. The number of bubbles nucleated in the interval dt is

$$\frac{dn_B}{dt} = \Gamma a^3 V_{\rm in}\exp\left[-\frac{4\pi}{3}\int_0^t dt'\Gamma(t')a^3(t')\left(\int_{t'}^t \frac{d\tau}{a(\tau)}\right)^3\right], \quad (27)$$

where $V_{\rm in}$ is the horizon volume at $N = N_T$, $V_{\rm in} = 4\pi/3H_{\rm in}^3$, and where the exponential factor accounts for the fraction of space which remains in the false vacuum. For exact results we must resort to the numerical evaluation of the rate equation along with the numerical integration of the field equations and of the generalized Klein-Gordon equation for ψ.

However, one relevant feature of the solution can be guessed at analytically. We have shown in Eq. (4) that the bubble spectrum in EI is provided by a power law. It turns out that in our case too it is possible to approximate the spectrum with a similar power law around any convenient bubble radius. To do this, we first change variable in Eq. (27) from the nucleation epoch t to the scale L in horizon crossing at t, by use of the relation $dL/dt \approx -H_{\rm in}L_h/a$ valid during slow roll. This gives

$$dn_B/dL = -3L_h^3 e^I QL^{-4} , \quad (28)$$

where I is the argument of the exponential in (27). We can approximate Q for $N \sim N_x$ as

$$Q(N) \approx Q_x\exp[s(N_x - N)] , \quad (29)$$

where $Q_x = \exp[(N_0^4 - N_x^4)/N_1^4]$ and $s = 4N_x^3/N_1^4$. In terms of the bubble comoving size L, since $L = L_h e^{N-N_T}$ (assuming H is slowly varying during the slow roll), we obtain

$$Q(L) \approx Q_x(L/L_x)^{-s} , \quad (30)$$

where L_x is the comoving bubble scale enucleated at N_x. Substituting (30) in (28), we obtain

$$dn_B/dL = -3L_h^3 e^I Q_x L_x^s L^{-4-s} . \quad (31)$$

As before, we now approximate e^I to unity, as we consider the tunneling far from the completion of the transition (we will later check this against the numerical integration).

It follows that

$$n_B = \left[\frac{3L_h^3 Q_x L_x^s}{3+s}\right] L^{-3-s} = (L_{\max}/L)^p \,, \qquad (32)$$

so that

$$p = 3 + s = 3 + 4(N_x^3/N_1^4) \,, \qquad (33)$$

$$L_{\max} = \left[3L_h^3 Q_x L_x^{p-3}/p\right]^{1/p} . \qquad (34)$$

$L_{\max}$ is the normalization of the bubble spectrum in a horizon volume $V_h = 4\pi L_h^3/3$ and in comoving scales.

Equation (34) compares directly to Eq. (6). For N_x close to N_0, in fact, we can approximate $Q_x \approx \exp[(p-3)(N_0 - N_x)]$. Then N_x disappears from the expression for $L_{\max}$ and we obtain

$$L_{\max} \approx L_h \exp[(3-p)(N_T - N_0)/p] \,, \qquad (35)$$

where $N_T - N_0$ is indeed the duration of the phase transition, now a free parameter. However, what we need is actually the spectrum of bubbles in an observable volume V_s at present, including the effect of the overcomoving expansion.

We use $V_s = (500h^{-1}\text{ Mpc})^3$ in order to compare with the observational constraints of Ref. [5]. We need then to multiply $L_{\max}$ by the post-equivalence amplification factor $f = z_{\text{eq}}^{1/5}$ and by the volume reduction factor $\tau^{1/p}$, where $\tau = V_s/V_h \approx 1.4 \times 10^{-4}$. Around a scale of, say, $R_x = 30$ Mpc (corresponding to a comoving scale $L_x \approx 4$ Mpc), we determine finally the power law normalization

$$R_{\max} = z_{\text{eq}}^{1/5} \tau^{1/p} L_h \left[\frac{3Q_x}{p}\left(\frac{L_x}{L_h}\right)^{(p-3)}\right]^{1/p} . \qquad (36)$$

We underline that the two observables $R_{\max}$ and p depend only on N_0 and N_1 (for fixed N_x), while there are four microphysics parameters, ψ_0, λ, m, and M (the fifth, $\mathcal{M}$, being in principle derivable from them). We exploit this large freedom to satisfy the constraints on the potential mentioned in the previous section.

We are now in a position to compare our bubble spectrum with the EI bubble spectrum (4) and with the constraints on the large scale structure and on the CMB. We already worked out the constraints on spectra of primordial bubbles in Ref. [5] so we only review and update the results here.

Concerning the large scale structure, it has since long been known [3] that a geometry of bubbles can reproduce several features of the large scale matter distribution. We have shown in Ref. [5] (see also Ref. [6]) that the observed GCF is fitted by a model of bubbles drawn from a power-law spectrum $n_B = (R_{\max}/R)^p$ provided

$$R_{\max}/28h^{-1}\text{ Mpc} = (p/10)^{-1.3} . \qquad (37)$$

For a primordial bubble model to be successful, it is then required that the normalization of the predicted bubble spectrum be close to Eq. (37). For the CMB constraints, we notice that a completely empty bubble of radius L at decoupling produces a Sachs-Wolfe distortion on the microwave temperature of $\Delta T/T \sim L^2/L_d^2$, if L_d denotes the horizon scale at decoupling. In a pixel corresponding to a size of $L_p > L$ at decoupling, a further factor of L^2/L_p^2 smears the signal [8]. Finally, the overcomoving expansion stretches the bubble size by a factor $g \equiv z_{\text{dec}}^{1/5} \approx 4$. There are two main CMB constraints arising from observational upper bounds to such Sachs-Wolfe effect. Full-sky, low-resolution surveys like COBE can detect rare big bubbles as cold spots. On the other hand, a large number of small bubbles can be detected as Poissonian fluctuations in high resolution, small coverage experiments with antenna beam around 1°. Assuming a power-law spectrum like (27), both constraints can be put in form of restrictions on the two parameters p and $R_{\max}$ for large p [5].

For the radii we are concerned with, the Rees-Sciama effect due to bubbles on the line of sight is a minor one [28]. The "large-bubbles" constraint amounts to

$$R_{\max} < R_l(p) \equiv gL_v \left[\frac{\tau(p-1)L_h}{pL_v}\right]^{1/p} , \qquad (38)$$

where $L_v \approx 18h^{-1}$ Mpc is the smallest bubble (at decoupling) that can give an observable signal ($\Delta T/T \approx 10^{-5}$) in a COBE pixel. The "small-bubbles" constraint gives

$$R_{\max} < R_s(p) \equiv g\left(\frac{12\pi L_h \tau}{p\theta_p^2}\right) \times \left\{\frac{\Delta(p-5)L_d^2 L_p^2 [p(L_1+L_t) - L_t]^{1/2}}{[p(p-1)]^{1/2} L_1^{5-p/2}}\right\}^{2/p} , \qquad (39)$$

where θ_p is the beam angular size (we assume 1°) in radians, $\Delta \approx 10^{-5}$ is the observational upper bound on $\Delta T/T$, L_1 is the smallest bubble not completely thermalized at decoupling, and $L_t \approx 7h^{-1}$ Mpc is the last scattering surface thickness. The constraints we use in the following are for reasonable values of L_1 (see Ref. [5]).

The main results are contained in Figs. 1 and 2. In Fig. 1, on the plane $(p, R_{\max})$, we display as a shaded area the parametric region of cosmological interest, i.e., the models which satisfy the CMB constraints (38) and (39), and are close to the curve (37) (and are thus able to fit the GCF). Incidentally, the constraint from the black body spectrum [5], being much less sharp, is omitted here.

It is clear that the EI spectrum (4) (corrected for the overcomoving expansion and reduced to V_s) is far from the acceptable region. On the contrary, it can be seen that the curve $R_{\max}(p)$ given in Eq. (36) crosses the acceptable region for some values of N_0 and N_1: this shows that our model is capable to produce pairs $p, R_{\max}$ which pass the CMB tests and have interesting large scale features. It turns out that $N_0 \in (49,51)$ and $p \in (6,15)$, i.e., $N_1 \in (15,21)$ satisfy all the constraints. As anticipated, a phase transition lasting $60 - N_0 \approx 10$ e-foldings produces a bubble spectrum with interesting astrophysical effects on the large scale structure. If, more qualitatively, one imposes to the model only the minimal requirement to give some significant structure, for instance that 50% of

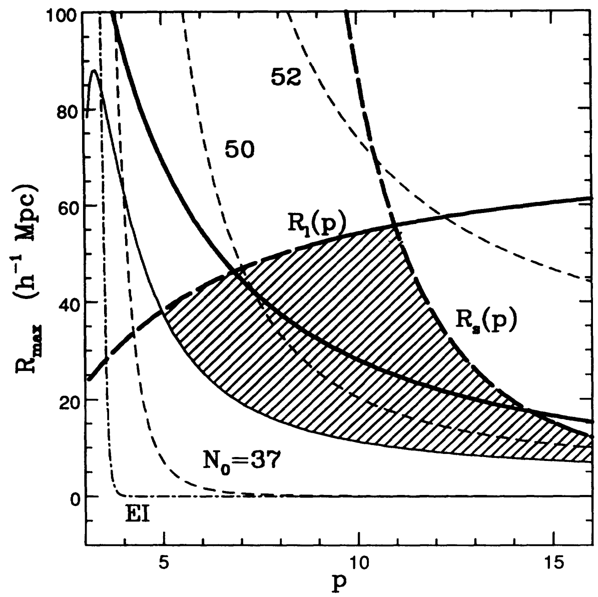

FIG. 1. Models in the shaded region satisfy the observational bounds on the parameter space (p, R_{max}) discussed in the text. We plot (i) as a heavy solid line the curve given by (37); (ii) as a heavy dashed line (labeled R_l) the constraint from COBE isotropy on the large scales ($\sim 10^0$); (iii) as a heavy dashed line (labeled R_s) the constraint from CMB isotropy on the small scales ($\sim 1^0$); (iv) as short-dashed lines the curves $R_{max}(p)$ given by (36) for $N_0 = 37$, 50, and 52 from left to right; (v) $R_{max}(p)$ for the original [4] EI model: the intersection with the GCF curve is way out of the CMB allowed region; and (vi) as a light solid line the lower boundary of the region where at least 50% of the Universe is filled with bubbles larger than 3 h^{-1} Mpc.

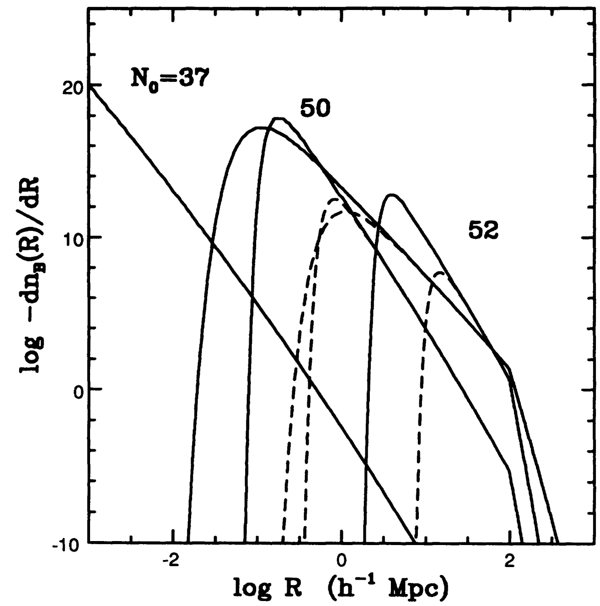

FIG. 2. Bubble spectra, $\ln[-dn_B(R)/dR]$, vs present bubble radius in Mpc: solid lines refer to numerical spectra, broken lines to approximated spectra. We display one valid model, $N_0 = 50$, amidst two invalid ones, $N_0 = 52$ (early enucleation and hence too much power on the large scales) and $N_0 = 37$ (late enucleation and hence too little power on the large scales). For $N_0 = 50$, the steeper slope is obtained for $N_1 = 18$ (which yields $p = 8$), the milder slope for $N_1 = 23$ ($p = 5$).

the space be contained in bubbles of at least 3 h^{-1} Mpc, then the allowed parameter space becomes considerably wider; Fig. 1 shows that EI bubble spectrum (4) does not meet even this milder requirement. In Fig. 2 we give the numerical and analytical bubble spectra. As in (36) above, the normalization contains the two extra factors τ and f. The agreement between solid and broken lines is complete as expected on the large scales, i.e., far from the turnover. The intermediate case, $N_0 = 50$, shows that it is possible to produce bubbles at the right time and, therefore, of the right size.

To conclude, in this paper we have attempted to build a working model of first order inflation with the help of the simplest quadratic corrections to Einstein's gravity: in order to allow for bubble enucleation, we made use of an *ad hoc* quartic coupling of the scalar field to the curvature squared term. We have shown that the advantage of quadratic gravity is to allow for a sufficient period of slow roll after the completion of the phase transition. This overcomes the difficulty extended inflation has in producing useful bubbles, by providing a bubble spectrum capable of having an observable impact on the large scale structure (in fact at the level of the observed galaxy correlation function) and, at the same time, of evading the CMB constraints. A clear prediction of models based on primordial bubbles is that the currently observed large voids should be effectively empty, with the exception of some matter pushed inside by peculiar motions. From an observational point of view, however, the situation is complicated by the fact that inflation by itself produces a spectrum of ordinary, linear, and Gaussian fluctuations with as many underdensities as overdensities. The evolution of the former would eventually lead to a population of large, almost spherical regions not completely emptied at present.

Our model is a version of two-field inflation where at least one of the two potentials, that of the slow rolling field, is built in. Of course, one can design a suitable potential that implements the same mechanism in ordinary gravity.

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