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CKM mixings in an E_6-induced standard model extension and in the minimal supersymmetric standard model

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The number of mixing angles and phases in the two popular extensions of the standard model (SM), the E_6-induced SM extension and the minimal supersymmetric standard model with soft symmetry-breaking terms, is discussed. It is found that two CP-violating phases appear in the minimal supersymmetric SM even for the simplest case of one family.

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About 20 years ago Kobayashi and Maskawa [1] showed that the CP violation can be naturally introduced into the quark sector of the standard model (SM) if the number of families is three or more. As is well known, the number of the observable mixing angles N_θ and the number of complex phases N_ϕ for the n-generation standard model are given by

$$N_\theta = \frac{n(n-1)}{2}, \quad N_\phi = \frac{(n-1)(n-2)}{2}. \qquad (1)$$

On the other hand the various extensions of the SM are widely discussed in the literature. Among them there are models with asymmetrical number of up- and down-type quarks, the most popular example of which is the E_6-induced standard model extension [2].

Supersymmetry is another widely discussed and very promising extension of the SM [3]. Our purpose in this Brief Report is to investigate the Cabibbo-Kobayashi-Maskawa-(CKM)-type mixings in these two extended models.

Let us consider the quark sector of the $SU(3)\times SU(2)\times U(1)$ model with l up quarks and m down quarks whose left components form n SU(2) doublets ($l, m \geq n$). In this case we arrive at the following expressions for the numbers of observable mixing angles and phases:

$$N_\theta = \frac{n(n-1)}{2} + n(l+m-2n), \qquad (2a)$$

$$N_\phi = \frac{(n-1)(n-2)}{2} + (n-1)(l+m-2n). \qquad (2b)$$

Let us illustrate the main points in driving these formulas by considering N_θ in detail. The diagonalization of the mass matrices of the up and down quarks leads in general to

$$N_\theta^0 = \frac{l(l-1)}{2} + \frac{m(m-1)}{2}.$$

Since the rotations within up singlets and down singlets are not observable one must subtract $(l-n)(l-n-1)/2$ and $(m-n)(m-n-1)/2$ parameters from N_θ^0. As to the mixings among the quarks belonging to SU(2) doublets, they are unobservable in photon, Z boson, and gluon interactions. Thus in this sector $2n(n-1)/2$ parameters must be subtracted too. But $W^\pm$ interactions make half of them reappear. So we end up the equation for N_θ given in (2a). For the n-generation E_6-induced model one has $m = 2l = 2n$, and the above formulas become $N_\theta = (n/2)(3n-1)$ and $N_\phi[(n-1)/2](3n-2)$, which give $N_\theta = 12$ and $N_\phi = 7$ in the case of three fermion families, to be compared with $N_\theta = 3$ and $N_\phi = 1$ values for the SM with three families.

We now look at the minimal supersymmetric extension of the standard model. As a first stage let us consider the special case in which one neglects the gaugino interactions. This corresponds to very heavy gauginos ($m_{\tilde{G}} \gg 1$ TeV). Therefore we have effectively the SM with the usual quark sector ($l = m = n$) and an additional squark sector ($l' = m' = 2n$). Since we ignore the intersection between quark and squark sectors we can immediately write down the relevant numbers as

$$N_\theta = n(3n-1), \quad N_\phi = (n-1)(3n-2), \qquad (3)$$

and consequently for $n = 3$ one has $N_\theta = 24$ and $N_\phi = 14$.

The realistic minimal supersymmetric SM (MSSM) contains the quark-squark-gaugino interactions, which restore all the mixing angles and phases that have been absorbed in the interactions with the usual gauge bosons. Therefore, we arrive at the results below:

$$N_\theta = n(5n-3), \quad N_\phi = n(5n-3). \qquad (4)$$

As can be seen from the last expressions, the minimal supersymmetric standard model with three families contains $N_\theta = 36$ observable angles and $N_\phi = 36$ observable phases due to the quark-squark sectors.

The interesting point is that the MSSM includes two observable complex phases even in the simplest case of one family. Indeed according to Eq. (4) we have $N_\theta = N_\phi = 2$ for $n = 1$. In fact, these CP-violating phases were pointed out before [4] in connection with the discussion of the neutron electric dipole moment. Let us consider this point in more detail. As is well known there

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are several contributions to the masses of squarks and to obtain the squark mass eigenstates one should introduce the following unitary matrices:

$$\begin{pmatrix} \tilde{u}_L \\ \tilde{u}_R \end{pmatrix} = \begin{pmatrix} \cos\alpha e^{i\phi_1} & \sin\alpha e^{i\phi_2} \\ -\sin\alpha e^{i\phi_3} & \cos\alpha e^{i\phi_4} \end{pmatrix} \begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{pmatrix}, \quad (5a)$$

$$\begin{pmatrix} \tilde{d}_L \\ \tilde{d}_R \end{pmatrix} = \begin{pmatrix} \cos\beta e^{i\Psi_1} & \sin\beta e^{i\Psi_2} \\ -\sin\beta e^{i\Psi_3} & \cos\beta e^{i\Psi_4} \end{pmatrix} \begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{pmatrix}. \quad (5b)$$

Here $\tilde{u}_{1,2}$ and $\tilde{d}_{1,2}$ are squark mass eigenstates and due to the unitarity $\phi_1+\phi_4=\phi_2+\phi_3$ and $\psi_1+\psi_4=\psi_2+\psi_3$ hold. The interaction Lagrangian containing the squarks is [3]

$$L = L_{\tilde{q}\tilde{q}V} + L_{\tilde{q}\tilde{q}g} + L_{q\tilde{q}\tilde{\chi}^\pm} + L_{\tilde{q}\tilde{q}VV} + L_{\tilde{q}\tilde{q}Vg} + L_{\tilde{q}\tilde{q}H} + L_{q\tilde{q}\tilde{\chi}^0} + L_{q\tilde{q}\tilde{g}}, \quad (6)$$

where V represents gauge bosons $(W^\pm, Z)$ and $g, \tilde{g}, \tilde{\chi}^\pm, \tilde{\chi}^0$ represent gluons, gluinos, charginos, and neutralinos, respectively. Finally H stands for the physical Higgs particles.

The mixings of L-type and R-type squarks given in (5) should be inserted into each term of the above interaction Lagrangian. In the squark-squark-gauge boson part the phase angles can be absorbed into the squark fields through the redefinitions such as $\tilde{u}_1 \to e^{-i\phi_1}\tilde{u}_1$, $\tilde{u}_2 \to e^{-i\phi_2}\tilde{u}$, $\tilde{d}_1 \to e^{-i\psi_1}\tilde{d}$, and $\tilde{d}_2 \to e^{-i\psi_2}\tilde{d}$. The same situation is valid for all the other terms of the Lagrangian except the quark-squark-neutralino and quark-squark-gluino interactions. In these terms the two phases $\delta_1 = \phi_1 - \phi_3$ and $\delta_2 = \psi_1 - \psi_3$ reappear. The number of the physical mixing angles is obviously again two: namely, α and β.

We have examined the mixing angles and phases in the two most popular extensions of the standard model, the E_6-induced SM and the minimal supersymmetric SM with the soft supersymmetry-breaking terms. It is found that two observable CP violating complex phases appear in MSSM even for the simplest case of single family taking into account only quark and squark interactions. For the realistic MSSM with three generations the numbers of the physical angles and phases are $N_\theta = 36$ and $N_\phi = 36$, respectively. Furthermore, the diagonalization of the neutralino and the chargino mass matrices actually brings the possibility of introducing extra angles and phases. Finally, a similar analysis can be repeated for the lepton-slepton sectors, which will introduce more angles and phases. This huge number of observable mixing angles and CP violating phases might be seen as an indication of the fact that the supersymmetry (SUSY) should be realized at a more fundamental, say preonic level.

Other considerations [5] on the mechanism of the generation of squark masses in realistic SUSY theory and possible mixings between different families support the above argument. Because in a supersymmetric theory additional box diagrams appear during the calculation of the K_L-K_S transition, to achieve agreement with the experiment one should have a SUSY (Glashow-Iliopoulos-Maiani) cancellation, which requires $\tilde{m}_c^2 - \tilde{m}_u^2 \approx m_c^2$. This looks unnatural, since experimentally $\tilde{m}_u^2, \tilde{m}_c^2 >$ (100 GeV)^2 and these squark mass terms are introduced into the Lagrangian by hand in MSSM with soft SUSY-breaking mechanism. However it might still be possible to construct models in which squark mass degeneracy may appear natural, e.g., the $N=1$ supergravity version with a hidden sector singlet doing the SUSY breaking, and all squarks are degenerate at the Planck scale but the mass splitting at the weak scale is within the experimental constraints. Furthermore, it is important to ensure the absence of flavor-changing neutral gaugino interactions, which otherwise would make disastrously large contributions to the K_1-K_2 mass difference. So this also leads to still more stringent conditions on the degeneracy of squarks belonging to different generations. There are no reasonable arguments for small values of mixing angles and mass degeneration in MSSM. Finally, in our opinion SUSY must be realized on the preonic level; therefore, the aim should be constructing a realistic supersymmetric preonic model and extracting its consequences at lepton-quark level.

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