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Eikonal diagrams in multiparton semihard interactions: Nonlogarithmic terms

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As a complement to the analysis previously presented about the leading behavior of the subset of the eikonal diagrams in the three-body partonic interaction, we add a few remarks on terms next to leading in $\ln s$.

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The large number of semihard rescatterings of high energy partons, which has been foreseen in production of minijets on nuclear targets [1], is a reason for interest in three-body parton interactions. In a recent paper [2] we have approached a few aspects of the problem of three-body parton interaction in the high-s fixed-t region. The most general case of three-body parton interaction has a very complicated structure in QCD [3]. Among the different components of the three-body amplitude with vacuum quantum number exchange, gauge invariant and with a definite color-group factor, one may identify a term whose space-time factor is given, at the lowest order in the coupling constant, by the set of Feynman diagrams at the same order in the Abelian theory. The set of Feynman diagrams which contribute to this term of the three-body parton amplitude is the set of graphs where the interaction of the projectile parton with each of the target partons is represented by a box diagram or by a crossed box diagram. The order in the coupling constant is g^8 and there are altogether 24 diagrams. In Ref. [2] we have worked out the leading behavior of all the dominant cuts of the corresponding term in the three-body parton amplitude and we have shown explicitly how the Abramovskii, Gribov, and Kancheli cutting rules [4] are satisfied.

The graphs which we have considered are indeed typical of an Abelian theory, and, in the limit $s \to \infty$, $t/s \to 0$, the leading behavior is obtained by the eikonal approximation [5]. In order to implement the eikonal approximation [6], the longitudinal variables are integrated while the transverse variables are kept fixed and, for the coupling at the vertices, only the leading contribution of the convective current is taken into account. The singularities to be considered for an eikonal line are only those in the Glauber region, namely, the singularities with values for the longitudinal light-cone variables of order q_t^2/s, with q_t a transverse momentum loop variable. A novelty to be pointed out when considering a three-body interaction is that the longitudinal variables to be integrated include a further one in addition to the loop integration variables. More explicitly, in the case of interest, a projectile parton with large light-cone momentum p_+ hits two target partons with large light-cone components $(k_1-Q)_-$ and $(k_2+Q)_-$ yielding, as a final result of the interaction, three partons with momenta p_+, $(k_1)_-$, and $(k_2)_-$, the light-cone components p_-, $(k_1)_+$, $(k_2)_+$, Q_+, and all transverse components being negligible. While the force which bounds partons in a hadron allows Q_- to vary within a range of order $\sqrt{s}$, one may explicitly verify at the lowest order in the coupling constant that the three-body parton amplitude has singularities in Q_- inside the Glauber region. In the case of the three-body interaction the eikonal approximation is implemented by treating the integration on Q_- in the same way as the longitudinal loop integration variables.

Within the eikonal approximation all the leading contributions of the dominant cuts of the three-body parton amplitude have been obtained in Ref. [2]. The eikonal approximation allows one also to work out, graph by graph, the leading terms in $\ln s$. One can, in fact, divide the graphs into three different groups according to the behavior at large s: $(\ln s)^2$, $\ln s$, and no $\ln s$. All graphs with a $\ln s$ factor are irreducible, while reducible graphs do not produce any $\ln s$ factor. A graph is called reducible when it may be disconnected by removal of one line. Reducible graphs do not produce any $\ln s$ factor as a consequence of the integration on Q_-. When the integration on Q_- is treated as the other loop integration variables, namely, it is extended from $-\infty$ to $+\infty$, in each reducible graph Glauber singularities can be avoided by contour deformation in such a way that the contribution of reducible graphs is exactly zero.

To gain a better insight into the meaning of the eikonal approximation with respect to the analysis presented in Ref. [2], we find it useful to add here a few remarks on the effect of a more careful treatment of the integration domain, which gives rise to next to leading $\ln s$ corrections to the eikonal approximation. The correction term is obviously most important for reducible diagrams, which within the eikonal approximation are zero. A reducible diagram here is constructed by linking two box (or crossed box) diagrams. The simple box (or crossed box) behaves as a $\ln s$ at large s. For each reducible diagram the effect of integrating on Q_- is to degrade to a constant the $(\ln s)^2$ behavior at large s, which corresponds to the product of the $\ln s$ behavior of two box diagrams. On the other hand, the behavior at large s of each irreducible diagram is not modified by the finite integration limits of Q_-. The eikonal approximation therefore gives correctly, graph by graph, all leading $\ln s$ terms. It does not give correctly, graph by graph, the constant term, which, however, for a reducible diagram is the leading one.

The most important case to be considered is therefore the case of reducible diagrams. We will use in the following the same notation used in Ref. [2]. It is sufficient to consider the four diagrams in the first line of Fig. 2 (Ref. [2]). The relevant integration for this set of reducible diagrams is expressed as

$$\int \frac{1}{a_1 a_3}\frac{1}{a_4 a_6}\left(\frac{1}{a_2}+\frac{1}{a_2'}\right)\left(\frac{1}{a_5}+\frac{1}{a_5'}\right)\frac{1}{a_7 a_8' a_9}. \qquad (1)$$

By representing the light-cone components of the external momenta p, k_1, k_2, Q and the loop variables q_1, q_2 as

$$\begin{aligned} p &\equiv \sqrt{s}(\alpha, 0, 0),\\ k_{1,2} &\equiv \sqrt{s}(0, \beta_{k_{1,2}}, 0),\\ Q &\equiv \sqrt{s}(0, \beta_Q, 0),\\ q_{1,2} &\equiv \sqrt{s}(\alpha_{1,2}, \beta_{1,2}, \mathbf{q}_{t1,2}/\sqrt{s}),\end{aligned}$$

the explicit expressions for the a_i's in expression (1) are

$$\begin{aligned}
&a_1, \quad (q_2 - Q)^2 = \alpha_2(\beta_2 - \beta_Q)s - q_{2t}^2,\\
&a_2, \quad (k_2 + q_2)^2 = \alpha_2(\beta_2 + \beta_{k_2})s - q_{2t}^2,\\
&a_3, \quad (q_2)^2 = \alpha_2\beta_2 s - q_{2t}^2,\\
&a_2', \quad (k_2 + Q - q_2)^2 = -\alpha_2(\beta_{k_2} - \beta_2 + \beta_Q)s - q_{2t}^2,\\
&a_4, \quad (q_1 + Q)^2 = \alpha_1(\beta_1 + \beta_Q)s - q_{1t}^2,\\
&a_5, \quad (k_1 + q_1)^2 = \alpha_1(\beta_1 + \beta_{k_1})s - q_{1t}^2,\\
&a_6, \quad (q_1)^2 = \alpha_1\beta_1 s - q_{1t}^2,\\
&a_5', \quad (k_1 - Q - q_1)^2 = -\alpha_1(\beta_{k_1} - \beta_1 - \beta_Q)s - q_{1t}^2,\\
&a_7, \quad (p - q_2 + Q)^2 = (\alpha - \alpha_2)(-\beta_2 + \beta_Q)s - q_{2t}^2,\\
&a_9, \quad (p - q_1)^2 = (\alpha - \alpha_1)(-\beta_1)s - q_{1t}^2,\\
&a_8, \quad (p - q_1 - q_2)^2 = (\alpha - \alpha_1 - \alpha_2)(-\beta_1 - \beta_2)s - q_t^2,\\
&a_8', \quad (p + Q)^2 = \alpha\beta_Q s + i\epsilon,
\end{aligned}$$

where $\mathbf{q}_t = \mathbf{q}_{1t} + \mathbf{q}_{2t}$ and q_{it}^2 is defined as $q_{it}^2 = q_{it}^2 - i\epsilon$. The integrations in expression (1) run on $\alpha_1, \alpha_2, \beta_1, \beta_2$, and β_Q. The integrations on β_1, β_2 are performed by taking the residua of the poles $1/a_7$ and $1/a_9$ and force α_1 and α_2 to vary between 0 and 1. The quantity to be integrated on α_1, α_2 is

$$\begin{aligned}\int \frac{1}{q_{1t}^2}\frac{1}{q_{2t}^2}&\left[\frac{1}{\alpha_2(1-\alpha_2)(\beta_{k_2}+\beta_Q)s - q_{2t}^2} + \frac{1}{-\alpha_2(1-\alpha_2)\beta_{k_2}s - q_{2t}^2}\right]\\
&\times\left[\frac{1}{\alpha_1(1-\alpha_1)\beta_{k_1}s - q_{1t}^2} + \frac{1}{-\alpha_1(1-\alpha_1)(\beta_{k_1}-\beta_Q)s - q_{1t}^2}\right]\frac{(1-\alpha_2)^2}{\alpha_2(1-\alpha_2)\beta_Q s - q_{2t}^2}\frac{(1-\alpha_1)^2}{\alpha_1(1-\alpha_1)\beta_Q s - q_{1t}^2}\frac{1}{\beta_Q + i\epsilon}. \qquad (2)\end{aligned}$$

Because of the numerators the region $\alpha_i \approx 0$ dominates over the region $\alpha_i \approx 1$. For $\alpha_i \approx 0$ one may approximate $1 - \alpha_i \approx 1$. Using the scaled variables $\alpha_i s \equiv u_i$ one may write

$$\begin{aligned}\int_0^{w_1}\int_0^{w_2} \frac{1}{s^2 q_{1t}^2 q_{2t}^2}&\left[\frac{1}{(\beta_{k_2}+\beta_Q)u_2 - q_{2t}^2} - \frac{1}{\beta_{k_2}u_2 + q_{2t}^2}\right]\frac{1}{\beta_Q + i\epsilon}\\
&\times\left[\frac{1}{\beta_{k_1}u_1 - q_{1t}^2} - \frac{1}{(\beta_{k_1}-\beta_Q)u_1 + q_{1t}^2}\right]\frac{1}{\beta_Q u_2 - q_{2t}^2}\frac{1}{\beta_Q u_1 - q_{1t}^2} \qquad (3)\end{aligned}$$

with w_i =small number $\times s$. The integrations on u_1, u_2 give as a result

$$\begin{aligned}-\frac{1}{s^2(q_{1t}^2 q_{2t}^2)^2}&\left[\frac{1}{\beta_{k_2}}\ln\frac{(\beta_{k_2}+\beta_Q)w_2 - q_{2t}^2}{\beta_Q w_2 - q_{2t}^2} + \frac{1}{\beta_{k_2}+\beta_Q}\ln\frac{\beta_{k_2}w_2 + q_{2t}^2}{-\beta_Q w_2 + q_{2t}^2}\right]\\
&\times\left[\frac{1}{\beta_{k_1}-\beta_Q}\ln\frac{\beta_{k_1}w_1 - q_{1t}^2}{\beta_Q w_1 - q_{1t}^2} + \frac{1}{\beta_{k_1}}\ln\frac{(\beta_{k_1}-\beta_Q)w_1 + q_{1t}^2}{-\beta_Q w_1 + q_{1t}^2}\right]\frac{1}{\beta_Q + i\epsilon}. \qquad (4)\end{aligned}$$

The large-s limit (actually $w_i \to \infty$) gives no problems if $\beta_Q \neq \beta_{k_1}, -\beta_{k_2}$. Explicitly the limit is

$$\begin{aligned}&-\frac{1}{s^2(q_{1t}^2 q_{2t}^2)^2}\frac{1}{\beta_Q + i\epsilon}\\
&\times\left[\frac{1}{\beta_{k_2}}\ln\frac{\beta_{k_2}+\beta_Q}{\beta_Q + i\epsilon} + \frac{1}{\beta_{k_2}+\beta_Q}\ln\frac{\beta_{k_2}}{-\beta_Q - i\epsilon}\right]\\
&\times\left[\frac{1}{\beta_{k_1}-\beta_Q}\ln\frac{\beta_{k_1}}{\beta_Q + i\epsilon} + \frac{1}{\beta_{k_1}}\ln\frac{\beta_{k_1}-\beta_Q}{-\beta_Q - i\epsilon}\right]. \qquad (5)\end{aligned}$$

For $\beta_Q \to -\beta_{k_2}$ one may write $\beta_Q = -\beta_{k_2} + \chi$ when a singularity might arise and $\beta_Q = -\beta_{k_2}$ in all other cases. One obtains

$$\begin{aligned}&\frac{-1}{s^2(q_{1t}^2 q_{2t}^2)^2}\frac{1}{-\beta_{k_2}}\\
&\quad\times\left[\frac{1}{\beta_{k_2}}\ln\frac{-\chi w_2 + q_{2t}^2}{\beta_{k_2}w_2 + q_{2t}^2} + \frac{1}{\chi}\ln\frac{\beta_{k_2}w_2 + q_{2t}^2}{\beta_{k_2}w_2 + q_{2t}^2 - \chi w_2}\right]\\
&\quad\times\phi(-\beta_{k_2}, \beta_{k_1}), \qquad (6)\end{aligned}$$

which is integrable for $\beta_Q \to -\beta_{k_2}$. The case $\beta_Q \to \beta_{k_1}$ is analogous.

The contribution of each of the four terms in expression (1) is given by each one of the four terms in expression (4). As one may notice, each of the terms in expression (4) develops a $(\ln s)^2$ when $\beta_Q \to 0$, which would correspond to the product of two box or crossed box diagrams with on-shell initial and final states. Integrating rather on β_Q one obtains a constant for each of the four terms in expression (4). If, moreover, the integration on β_Q is extended from $-\infty$ to $+\infty$ one gets zero.

One may verify that keeping into account the integration limits for β_Q the leading behavior of the irreducible diagrams is not changed, because it is originated by the behavior of the integrand in a small region around $\beta_Q = 0$. As an explicit example one may consider the case where in expression (1) the factor $1/a_8'$ is replaced by $1/a_8$, which corresponds to a topology with two overlapping box diagrams and gives rise to a leading term of order $\ln s$. Because of the pole $1/a_8$ the integrations on β_1 and on β_2 are no longer independent. It is therefore convenient to introduce the sum and the difference of β_1 and β_2. After integrating on the difference one may express the result as a function of $\gamma \equiv \beta_1 + \beta_2$ and of $\delta \equiv \gamma + \beta_Q$:

$$\int \left[\frac{1}{\alpha_2(\beta_{k_2}-\gamma)s+\mathcal{A}_2} - \frac{1}{\alpha_2(\beta_{k_2}+\delta)s-\mathcal{A}_2}\right] \frac{1}{-\alpha_2\delta s+\mathcal{A}_2}\,\frac{1}{-\alpha_2\gamma s+\mathcal{A}_2}$$
$$\times \left[\frac{1}{\alpha_1(\beta_{k_1}+\gamma)s+\mathcal{A}_1} - \frac{1}{\alpha_1(\beta_{k_1}-\delta)s-\mathcal{A}_1}\right] \frac{1}{\alpha_1\delta s+\mathcal{A}_1}\,\frac{1}{\alpha_1\gamma s+\mathcal{A}_1}\,\frac{1}{(1-\alpha_2)\delta s+\mathcal{A}_2-\mathcal{B}}\,\frac{1}{-(1-\alpha_1)\gamma s+\mathcal{A}_1-\mathcal{B}}\,, \tag{7}$$

where

$$\mathcal{A}_i = \alpha_i \frac{q_t^2}{2(1-\alpha_1-\alpha_2)} - q_{it}^2, \quad \mathcal{B} = \frac{q_t^2}{1-\alpha_1-\alpha_2}.$$

Introducing the variables $y_i = \alpha_i s$ the integration domain grows with s. One obtains terms with $\ln s$ only if either a single y_1 or a single y_2 is left in the denominators. This may only happen when both γ and δ are close to zero, which shows that the integration limits for β_Q are irrelevant for the terms proportional to $\ln s$. The same conclusion can be drawn for the "box in the box" graphs, which, term by term, produce $(\ln s)^2$ factors.

Our conclusions are, therefore, that a more careful analysis on the integration domain, namely, the finite integration limits for Q_-, gives rise to a correction term to the eikonal approximation for the three-body partonic interaction discussed in Ref. [2] which affects, graph by graph, the next to leading $\ln s$ terms. For the present discussion the most important case is the one of reducible graphs: they are zero within the eikonal approximation and the correction term, graph by graph, is constant with s. An important remark is, however, that by adding the leading terms of all the nonreducible diagrams ($\sum_{i=9}^{24} \mathcal{M}_i$, using for $\mathcal{M}_i$ the explicit expressions in Ref. [2]), one obtains as a result the eikonal expression, in spite of the fact that the sum does not include all possible orderings of the quanta exchanged by the projectile line. Since the eikonal expression represents also the behavior (constant with s) of the whole sum of diagrams, both reducible and irreducible, one may conclude that the constant term, which is obtained for each reducible diagram separately, as a consequence of the finite integration limits of Q_-, is compensated by a term with opposite sign, which originates analogously from the irreducible diagrams. Similar considerations hold for the different cut amplitudes, which also behave as a constant at large s. Although each different cut diagram receives a constant contribution by the finite integration limits on Q_-, only the constant contribution, originated in the Glauber region and which is taken into account by the eikonal approximation, contributes to the final cut amplitude.

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