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Entropy and baryon number conservation in the deconfinement phase transition

A. Leonidov,[1,*] K. Redlich,[1,2] H. Satz,[1,3] E. Suhonen,[3,4] and G. Weber[1]
[1]*Fakultät für Physik, Universität Bielefeld, D-33501 Bielefeld, Germany*
[2]*Institute for Theoretical Physics, University of Wrocław, PL-50205 Wrocław, Poland*
[3]*Theory Division, CERN, CH-1211 Geneva 23, Switzerland*
[4]*Department of Theoretical Physics, University of Oulu, SF-90570 Oulu, Finland*
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The conservation of entropy and baryon number in the deconfinement phase transition is studied in the framework of the bag model. In the standard construction of the equilibrium phase transition from a quark-gluon plasma into a hadron gas a subsequent dilution and reheating of the system on the phase boundary is necessary to preserve the entropy and baryon number conservation. We propose modifying the bag pressure to depend explicitly on temperature and baryon chemical potential. It is shown that this modification is sufficient to construct a model in agreement with the Gibbs equilibrium criteria for a phase transition, while simultaneously assuring entropy and baryon number conservation on the phase boundary. Within this model the quark-gluon plasma hadronizes at a fixed temperature and chemical potential.

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I. INTRODUCTION

Statistical QCD predicts that strongly interacting matter exists in two different states [1–3]. Lattice gauge theory has shown that at low energy density it behaves as a gas of individual hadrons, whereas in the asymptotic limit of high density it is composed of quarks and gluons. The lattice approach is very successful in the description of thermodynamical properties of strongly interacting matter in the baryon-free environment [4]. The critical behavior of hadronic matter with finite baryon chemical potential, however, can at present only be studied in the framework of phenomenological models [5–8]. Here, a phase transition between the hadron gas and quark-gluon plasma is obtained by construction via the Gibbs criteria for a phase equilibrium. Within the framework of such models, the specific entropy per baryon S/B is discontinuous across the phase transition [7–11]. In the ideal gas approximation, the ratio S/B is almost always larger in a quark-gluon plasma than in a hadron resonance gas, if we compare them at the same temperature and chemical potential—mostly because of the large entropy content in the gluonic sector. The S/B ratio is an observable measured in heavy ion collisions [10,11]. Thus, the behavior of S/B across a phase transition is of importance when verifying experimental signals of quark-gluon plasma formation in heavy ion collisions, in this particularly, strangeness production. We will show, however, that this behavior can crucially depend on the hadronic equation of state.

In any thermodynamic system, the change of state is only possible at increasing or constant entropy. In the standard, bag model equation of state [7–11], the appearance of a discontinuity in the ratio S/B makes the phase transition at fixed temperature T and fixed chemical potential μ irreversible. In the transition from a quark-gluon plasma into a hadron gas, the values of T and μ have to be changed during the hadronization while still observing the Gibbs criteria for the phase equilibrium. The conservation of total entropy and baryon number adjusts the ratio of the plasma to hadronic volume along the critical curve where the values of T and μ are determined by the pressure equality. In the case of vanishing baryon number a deconfinement phase transition appears at a fixed critical transition temperature. The evolution of the system is thus qualitatively different at zero and finite baryon density.

In this work we propose a phenomenological model for deconfinement phase transition which allows this transition to occur at a common temperature and chemical potential and consequently implies the S/B ratio to be continuous across the critical curve. As a basis we use a bag model equation of state for the plasma with T- and μ-dependent bag constant. The hadronic phase is treated as a pion-nucleon gas with hard core repulsion between nucleons.

The paper is organized as follows. In Sec. II we investigate first a simplified model of massless particles. In Sec. III the model of isentropic deconfinement phase transition is constructed and discussed. In Sec. IV we present a discussion of our numerical results.

II. ENTROPY PER BARYON AND DECONFINEMENT

In order to illustrate the behavior of the entropy per baryon S/B across the phase transition, we discuss first a simplified model of deconfinement. The hadron phase

*On leave from P. N. Lebedev Physics Institute, 117924 Leninsky Pr. 53, Moscow, Russia.

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is considered as a gas of noninteracting massless nucleons and pions. Here, the partition function in terms of the baryonic chemical potential μ and the temperature T has the following form:

$$\frac{T}{V}\ln Z_h(\mu,T)=\frac{\pi^2T^4}{9}+\frac{\mu^2T^2}{3}+\frac{\mu^4}{6\pi^2}. \quad (2.1)$$

For the quark-gluon plasma we have the partition function of massless u and d quarks and gluons, with a phenomenological bag constant B_0,

$$\frac{T}{V}\ln Z_q(\mu_q,T)=\frac{37\pi^2T^4}{90}+\mu_q^2T^2+\frac{1}{2\pi^2}\mu_q^4-B_0. \quad (2.2)$$

The analysis of S/B across the critical curve, $\mu_c=\mu_c(T_c)$, obtained from the Gibbs criteria for the phase equilibrium of hadronic matter and the quark-gluon plasma,

$$P_h(\mu_c,T_c^h)=P_q(\mu_c^q,T_c^q) \text{ with } \mu_c=3\mu_c^q; T_c^h=T_c^q\equiv T_c, \quad (2.3)$$

requires in general a numerical investigation. However, in the above model, the appearance of a discontinuous structure of S/B at (μ_c,T_c) can be illustrated in the limit of $\mu_c\to 0$. From Eqs. (2.1) and (2.2) one finds that independently of temperature

$$\lim_{\mu\to 0}[(S/B)_q/(S/B)_h]=222/20. \quad (2.4)$$

Thus, S/B shows a discontinuity at the critical temperature and is larger in the plasma than in the hadron gas phase.

In order to construct a transition at fixed μ and T preserving S/B we propose to modify the quark-gluon plasma equation of state by assuming the bag pressure B_0 in (2.2) to be dependent on temperature and baryon chemical potential. In this case the requirement of $(S/B)_h=(S/B)_q$ along the critical curve (2.3) leads to the following differential equation:

$$\frac{\frac{74\pi^2}{45}T^3+2(\frac{\mu}{3})^2T-\frac{\partial B(\mu,T)}{\partial T}}{\frac{2}{3}\left(\frac{\mu}{3}\right)T^2+\frac{2}{3\pi^2}\left(\frac{\mu}{3}\right)^3-\frac{\partial B(\mu,T)}{\partial\mu}}=\frac{\frac{4\pi^2}{9}T^3+\frac{2}{3}\mu^2T}{\frac{2\mu T^2}{3}+\frac{2\mu^3}{3\pi^2}}. \quad (2.5)$$

The bag parameter $B(\mu,T)$ is in general considered to describe the difference between the perturbative and physical vacuum pressure. Thus $B(\mu,T)$ should exhibit similar symmetry properties as the pressure. In particular, it should be an even function of μ and T, that is, $B(\mu,T)=B(-\mu,-T)$. Because of this symmetry, the partial derivatives of $B(\mu,T)$ are of different order in μ. In leading order in μ, $\partial B(\mu,T)/\partial T$ is proportional to μ^2, whereas $\partial B(\mu,T)/\partial\mu$ is linear in μ. Thus, in the limit of $\mu\to 0$, the derivative $\partial B(\mu,T)/\partial T$ in (2.5) gives a subleading contribution and can be neglected as compared with the other terms. For a small μ, the continuity condition (2.5) thus leads to the solution

$$B(\mu,T)_{\mu\to 0}\simeq B_0-\frac{101}{90}\mu^2T^2, \quad (2.6)$$

where B_0 is the integration constant. In the limit of low temperature, the derivative $\partial B(\mu,T)/\partial\mu$ is subleading in T, as it gives contributions of $O(T^2)$ order in (2.5). Thus, keeping only the lowest order in T terms in Eq. (2.5), one finds

$$B(\mu,T)_{T\to 0}\simeq B_0+\frac{8}{27}\mu^2T^2; \quad (2.7)$$

the two limiting solutions of the bag pressure, Eqs. (2.6) and (2.7), can be used then in Eq. (2.3) to calculate the critical curve in the corresponding T and μ regions. By construction, the modified equation of state guarantees the continuity of the entropy per baryon ratio across the critical curve. Thus, the phase transition between quark-gluon plasma and nucleon gas is reversible and appears at the same value of the critical temperature and chemical potential. In particular, in the limit $\mu_c\to 0$, the ratio in Eq. (2.4) becomes unity with the new equation of state.

III. ISENTROPIC EQUILIBRIUM TRANSITION IN A MASSIVE PION-NUCLEON GAS

In the preceding section, we have shown in terms of a simplified model that a modification of the bag pressure in the quark-gluon plasma equation of state is sufficient to obtain an isentropic deconfinement transition at fixed μ and T. In the following, it will be shown that this is also possible when a more realistic equation of state is considered. We assume here the hadronic phase to be composed of massless pions and massive nucleons of mass m. The partition function (2.1) then becomes

$$\frac{T}{V}\ln Z(T,\mu)=\frac{\pi^2T^4}{30}+\frac{4m^2T^2}{\pi^2}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!}\cosh\left(\frac{k\mu}{T}\right)K_2\left(\frac{\mu}{T}\right), \quad (3.1)$$

where the sum takes into account quantum statistics.

In order to implement baryon interactions in the hadronic phase in a semiclassical way, one considers the nucleon as an extended object and includes a hard core nucleon-nucleon repulsion [5]. The resulting excluded volume corrections to the noninteracting gas approximation are effectively taken into account by dividing any thermodynamical quantity for pointlike particles by the factor $[1+v_0n_h(T,\mu)]$; here $v_0=(4\pi R_0^3/3)$, with $R_0\sim 0.8$ fm, is the volume of nucleon, and $n_h(\mu,T)\equiv\partial\ln Z/\partial\mu$ the baryon number density calculated from the partition function (3.1). The finite-volume repulsive corrections to the hadronic equation of state are essential here because in the ideal gas approximation alone there is no unique phase transition for systems with large baryon number [5].

The phase transition between hadron gas and quark-gluon plasma described by the partition function (2.2) and (3.1) is by construction of first order. In this case, the energy, entropy, and baryon number densities are discontinuous across the phase transition curve $\mu_c=\mu_c(T_c)$ defined by the Gibbs criterion (2.3). The actual critical

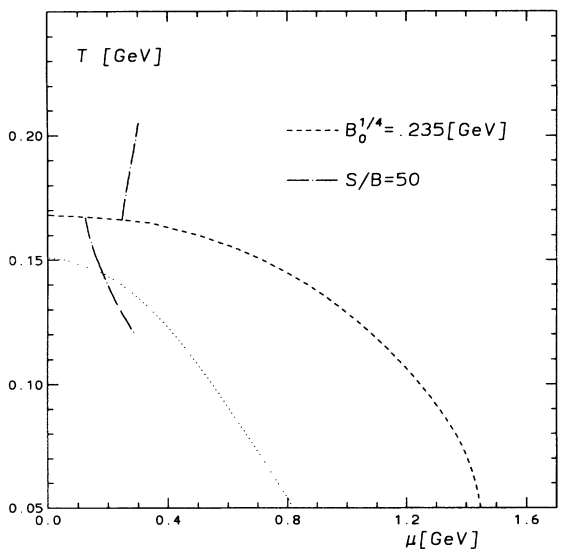

FIG. 1. The critical curve (dashed) and the constant entropy/baryon curve (dotted) obtained in the bag model with constant bag pressure $B_0^{1/4} = 0.235$ GeV. Also shown in the path of constant $(S/B) \simeq 50$ (dashed-dotted).

curve in the (μ, T) plane is determined by the value of the phenomenological bag constant B_0 in Eq. (2.2). In Fig. 1, we show the critical curve for $B_0^{1/4} = 0.235$ GeV.

Comparing the entropy per baryon ratio below and above the phase transition curve one immediately notices its discontinuous behavior. In Fig. 1, the path of constant $(S/B) \simeq 50$ in the (μ, T) plane is shown, indicating the different temperatures and chemical potentials in a quark-gluon plasma and a hadron gas in the vicinity of the deconfinement phase transition. For a given point (μ_c, T_c) on the critical curve, the entropy per baryon of the plasma phase is considerably larger than in the hadronic phase. As also shown in Fig. 1, the condition of equal entropy per baryon in plasma and hadron gas, $(S/B)_q = (S/B)_h$, leads to a (μ, T) curve which is incompatible with the critical curve for a phase transition at fixed μ and T. Thus, the results of Fig. 1 show that it is in general not possible to fulfill the Gibbs criteria of phase equilibrium with further constraint of a continuous entropy per baryon at the same value of μ and T. A possible way out would be to drop the equilibrium condition [12]. However, if we want to retain an equilibrium phase transition, we have to modify the equation of state in order to fulfill the continuity of entropy per baryon across the critical curve. Within a bag model, the conservation of the entropy per baryon across the phase transition can be achieved by modification of the bag pressure,

$$\frac{s_q - \frac{\partial B(\mu,T)}{\partial T}}{n_q - \frac{\partial B(\mu,T)}{\partial \mu}} = \frac{s_h}{n_h}. \tag{3.2}$$

Equation (3.2) corresponds to Eq. (2.5) for the case of massive nucleons; it can be solved analytically in two asymptotic limits of $\mu \to 0$ and $T \to 0$.

A. Matter at low baryon density

In a hot gas at low baryon density we can use classical statistics for nucleons, i.e., we take into account only the first term in the series expansion (3.1). The requirement of a conserved entropy per baryon across the phase boundary leads to the following partial differential equation for the bag pressure $B(\mu, T)$,

$$\frac{\frac{4\cdot 37}{90}\pi^2 T^3 + \frac{2}{9}\mu^2 T - \frac{\partial B}{\partial T}}{\frac{2}{9}\mu T^2 + \frac{2}{81\pi^2}\mu^3 - \frac{\partial B}{\partial \mu}} = \frac{\frac{4\pi^2}{30}T^3 + s_n}{\frac{4m^2}{\pi^2}T K_2\left(\frac{m}{T}\right)\sinh\left(\frac{\mu}{T}\right)}. \tag{3.3}$$

Here s_n refers to the nucleon entropy density and K_2 is the modified Hankel function of the second kind. First we solve this equation in leading order of μ/T. The derivative $\partial B/\partial T$ can be neglected in the first approximation, similar to the case of massless nucleons, and Eq. (3.3) reduces to

$$\frac{\partial B}{\partial \mu} \simeq \frac{2}{9}T^2\mu - \frac{4\cdot 37}{3\pi^2}m^2 K_2\left(\frac{m}{T}\right)\mu, \tag{3.4}$$

with the solution

$$B(\mu, T) \simeq B_0 + \frac{1}{9}T^2\mu^2 - \frac{74}{3\pi^2}m^2 K_2\left(\frac{m}{T}\right)\mu^2. \tag{3.5}$$

Substituting $\partial B/\partial T$ from Eq. (3.5) into Eq. (3.3) leads to the next order approximation for the bag pressure after integration. This iteration method can then be continued to still higher orders. Noticing that

$$\frac{4m^2}{\pi^2}K_2\left(\frac{m}{T}\right)\left(\cosh\frac{\mu}{T} - 1\right) = P_n - P_n(\mu = 0)$$

one obtains

$$\frac{\partial B}{\partial T} \simeq \frac{2}{9}\mu^2 T - \frac{37}{3}[s_n - s_n(\mu = 0)], \tag{3.6}$$

where P_n is the pressure and s_n the entropy density of nucleons and antinucleons. The main contribution from the nucleon entropy density cancels out in Eq. (3.3), and the remainder is negligible when compared with the other terms. The final expression for the T- and μ-dependent bag pressure from Eqs. (3.3) and (3.6) thus becomes

$$B(\mu, T) \simeq B_0 + \frac{1}{9}T^2\mu^2 + \frac{1}{162\pi^2}\mu^4 - \frac{148}{3\pi^2}m^2 T^2 K_2\left(\frac{m}{T}\right)\left(\cosh\frac{\mu}{T} - 1\right); \tag{3.7}$$

it can be used to construct the phase boundary between quark-gluon plasma and hadron gas at low baryon densities.

B. Matter at high baryon density

Quantum statistics play an important role in cold and baryon-dense matter, leading to a fully degenerate Fermi gas at zero temperature. For temperatures low compared to $(\mu - m)$ one can write the thermodynamical quantities as sums of zero temperature contributions and finite temperature corrections:

$$P_h \simeq P_h^0 + P_h^1 T^2 + P_h^2 T^4,$$
$$n_h \simeq n_h^0 + n_h^1 T^2 + n_h^2 T^4,$$
$$s_h \simeq s_h^1 T + s_h^2 T^3. \tag{3.8}$$

The zero temperature terms are

$$P_h^0 = \frac{1}{6\pi^2}\left[\mu\theta(\theta^2 - 3m^2/2) + \frac{3}{2}m^4 \ln\left(\frac{\mu+\theta}{m}\right)\right], \tag{3.9}$$

$$n_h^0 = \frac{2}{3\pi^2}\theta^3, \tag{3.10}$$

where $\theta \equiv (\mu^2 - m^2)^{1/2}$. The finite temperature corrections are obtained by using a power expansion around $T = 0$ [13]:

$$P_h^1 = \frac{\mu}{3}\theta; \quad P_h^2 = \frac{7\pi^2}{90}\frac{\mu}{\theta}\left(1 - \frac{m^2}{\theta^2}\right) + \frac{\pi^2}{30}, \tag{3.11}$$

$$n_h^1 = \frac{1}{3}\left(\theta + \frac{\mu^2}{\theta}\right); \quad n_h^2 = \frac{7\pi^2}{60}\frac{m^4}{\theta^5}, \tag{3.12}$$

$$s_h^1 = \frac{2}{3}\mu\theta; \quad s_h^2 = \frac{14\pi^2}{45}\frac{\mu}{\theta}\left(1 - \frac{m^2}{\theta^2}\right) + \frac{2\pi^2}{15}. \tag{3.13}$$

In the low temperature limit, $\partial B/\partial T$ dominates over $\partial B/\partial\mu$. Keeping only the leading terms the equality of entropy per baryon across the phase boundary yields

$$\frac{\frac{2}{9}\mu^2 T - \frac{\partial B}{\partial T}}{\frac{2}{81\pi^2}\mu^3} = \frac{s_h^1 T}{n_h^0}, \tag{3.14}$$

resulting in the first approximation of the bag pressure,

$$B \simeq B_0 + \frac{1}{9}\mu^2 T^2 - \frac{1}{81}\frac{\mu^4}{\theta^2}T^2. \tag{3.15}$$

For higher order corrections we insert $\partial B/\partial\mu$ from Eq. (3.15) into Eq. (3.2),

$$\frac{\frac{74}{45}\pi^2 T^3 + \frac{2}{3}\mu^2 T - \frac{\partial B}{\partial T}}{\frac{2}{81}\mu^3 + \frac{4}{81}\frac{\mu^2}{\theta^2}\left(1 + \frac{1}{2}\frac{\mu^2}{\theta^2}\right)T^2} = \frac{s_h^1 T + s_h^2 T^2}{n_h^0 + n_h^1 T^2} \tag{3.16}$$

to obtain by integration

$$\begin{aligned} B(\mu,T) &\simeq B_0 + \frac{37}{90}\pi^2 T^4 + \frac{1}{9}\mu^2 T^2 - \frac{1}{81}\frac{\mu^4}{\theta^2}T^2 - \frac{T^4}{4n_h^0}\left[\frac{2\mu^3}{81\pi^2}\left(s_h^2 - \frac{s_h^1 n_n^1}{n_h^0}\right) + \frac{45 s_h^1 \mu^3}{81\theta^2}\left(1 - \frac{1}{2}\frac{\mu^2}{\theta^2}\right)\right] \\ &= B_0 + \frac{37}{90}\pi^2 T^4 + \frac{1}{9}\mu^2 T^2 - \frac{1}{81}\frac{\mu^4}{\theta^2}T^2 - \frac{\pi^2}{30\cdot 81}\left(\frac{\mu}{\theta}\right)^3\left(\frac{7\mu}{\theta} - \frac{26m^2\mu}{\theta^3} + 3\right)T^4 \end{aligned} \tag{3.17}$$

with the parameters defined as in Eqs. (3.10)–(3.13).

IV. NUMERICAL RESULTS AND DISCUSSION

In the last section we have constructed a phenomenological equation of state which preserves continuity of the entropy per baryon at a fixed (μ_c, T_c) in an isentropic and equilibrium phase transition from a quark-gluon plasma to a hadron gas. In the limit of small chemical potential, the partition function in the deconfined phase is defined by Eq. (2.2) and the effective bag pressure (3.7). In the opposite limit of low temperature and large chemical potential, the bag pressure is replaced by Eq. (3.17). In both we have considered the hadron phase as a gas of pions and nucleons with the partition function (3.1). The contribution of further particle species in a hadron gas would increase the values of all thermodynamical quantities for a given temperature and chemical potential. The entropy per baryon, however, is rather weakly affected by the number of particles included, so that a more complete particle input in Eq. (3.1) will not significantly change our results. The entropy per baryon ratio is also quite insensitive to interactions. If one boosts the entropy density in the mesonic and baryonic sector by mean field terms in order to take into account particle interactions, then the hadronic S/B ratio is left essentially unchanged [14].

The modification of an ideal gas equation of state for the quark-gluon plasma by a bag pressure accounts for its nonperturbative nature close to the deconfinement transition. In the asymptotic limit of high temperature at fixed μ one should, however, recover the Boltzmann limit for noninteracting gas of quarks and gluons. It is straightforward to see that with the modified bag parameter (3.7) and the partition function (2.2) the Boltzmann limit is

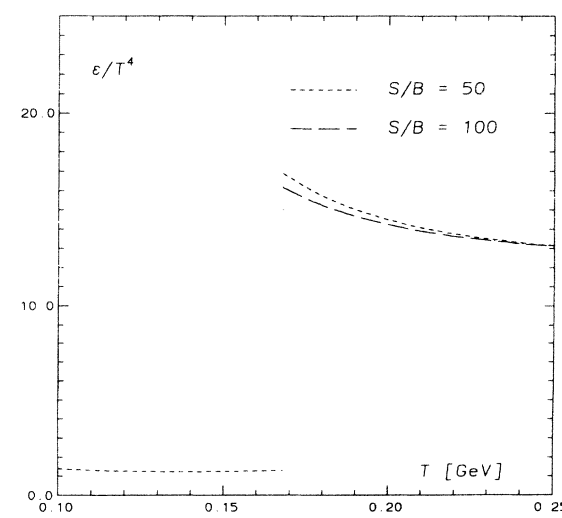

FIG. 2. The energy density as a function of temperature, as given in our model for $(S/B) \simeq 50$ (dashed line). Also shown is the result for the energy density in the plasma phase in the model of constant bag pressure (dashed-dotted line), where the same T_c leads to $(S/B) \simeq 100$.

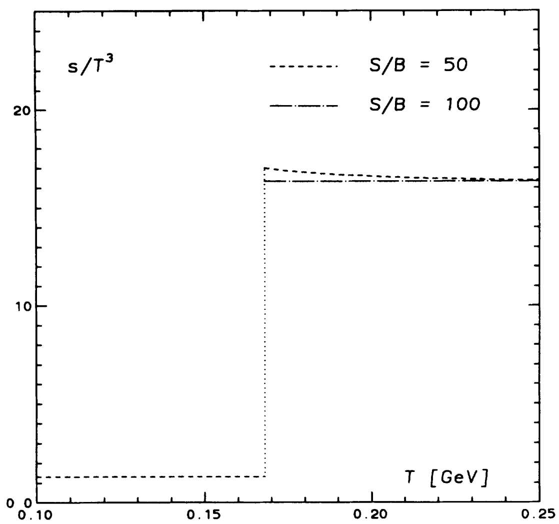

FIG. 3. As in Fig. 2, but for entropy density.

indeed recovered.

The quantitative behavior of the model close to the deconfinement phase transition is presented in Figs. 2–5. The energy, entropy, and baryon number density in the region of small μ are shown in Figs. 2–4. All thermodynamical quantities are calculated for $B_0^{1/4} = 0.235$ GeV and given as function of temperature for a fixed value of the entropy per baryon. For each value of the temperature, the chemical potential in the hadron and in the quark-gluon plasma phase was calculated to keep $(S/B) = (S/B)_{(\mu_c,T_c)}$. For the critical temperature $T_c \sim 0.18$ GeV, the entropy per baryon in a hadron gas is $(S/B) \simeq 50$. Also shown in Figs. 2–4 are the corresponding results for the model with constant bag pressure ($B_0^{1/4} = 0.235$ GeV) at the same T_c. In this case, the entropy per baryon in the plasma becomes $(S/B) \simeq 100$, so that S/B increases by a factor 2 between hadronic phase and plasma, while in our model S/B is continuous across the transition. In Figs. 2 and 3 we see that

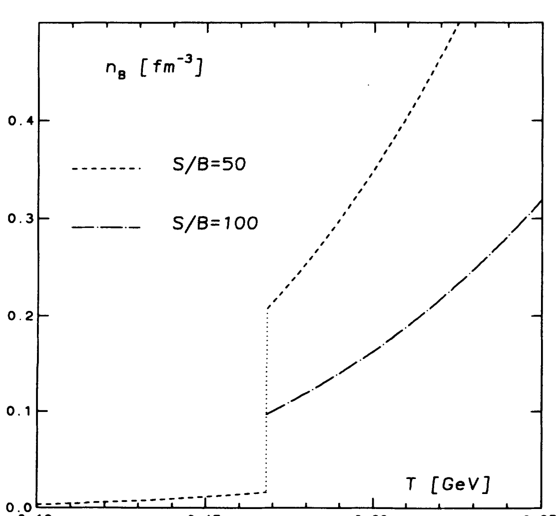

FIG. 4. As in Fig. 2, but for baryon number density.

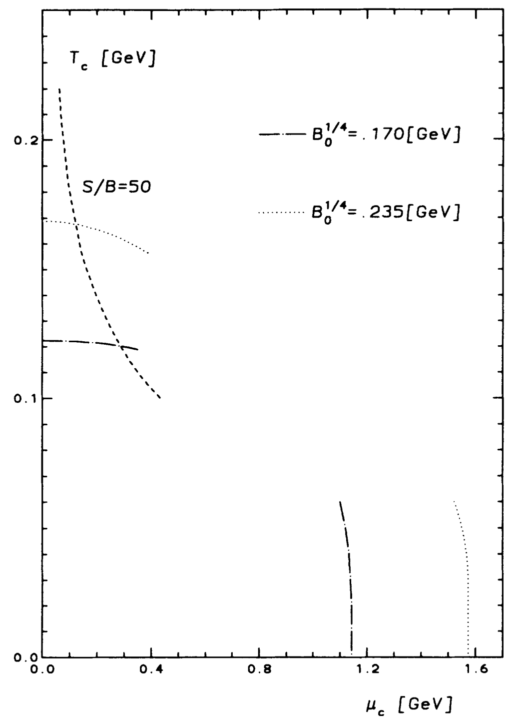

FIG. 5. Critical curve calculated as given by the model defined by Eqs. (3.1), (3.7), and (2.17), for two different values of B_0. Also shown is the curve (dashed line) of constant entropy per baryon, $(S/B) \simeq 50$, in the region of the phase transition.

both energy and entropy density are essentially the same in the two cases. The main effect of making the bag pressure dependent on T and μ is a substantial increase in the baryon density, as shown in Fig. 4. The critical curve in the (μ, T) plane is shown in Fig. 5, in the limit of low temperature and small chemical potential. The actual position of the critical curve in the (μ, T) plane is determined mainly by the value of the bag constant B_0. Therefore, both the standard and modified models lead to a very similar critical curve; of course, the behavior of S/B in these models is very different. In Fig. 5, the path corresponding to $(S/B) \simeq 50$ in the (μ/T) plane is included in order to show explicitly the continuity across the phase transition.

V. CONCLUSIONS

Within a phenomenological model based on an extended bag model, we have analyzed the behavior of the entropy per baryon number in a plasma and in a hadron gas in the vicinity of the phase equilibrium. For a constant bag pressure, the Gibbs construction of the phase transition implies a discontinuous structure in a specific entropy per baryon across the critical curve. We have shown that a temperature and chemical potential dependent bag pressure in the equation of state for the quark-gluon plasma is sufficient to retain the continuity of the entropy per baryon in an isentropic and equilibrium phase transition from a quark-gluon plasma to a hadron gas. In such a model, the plasma can thus hadronize at a common temperature and chemical potential.

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