Possible reason why leptons are lighter than quarks

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The minimal model of spontaneously broken leptonic color and discrete quark-lepton symmetry predicts that charged leptons have the same masses as their partner charge $+2/3$ quarks up to small radiative corrections. By invoking a different pattern of symmetry breaking, a similar model can be constructed with the structural feature that charged leptons have to be lighter than their partner quarks because of mixing between leptonic colors, provided mixing between generations is not too strong. As well as furnishing a new model-building tool, this is phenomenologically interesting because some of the new physics responsible for the quark-lepton mass hierarchy could exist on scales as low as several hundred GeV.

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The patterns evident in the mass and mixing angle spectrum of quarks and leptons continue to challenge us to provide an explanation. One may broadly categorize these patterns as hierarchies between generations, between weak-doublet partners, and between quarks and leptons. We do not know if these three subproblems can be solved separately, or if an all-encompassing explanation is necessary. In this paper I will introduce a novel suggestion for why the charged lepton is less massive than the charge $+2/3$ quark for a given generation. An analysis of its explanatory success will lead us to discuss how the quark-lepton hierarchy problem might be connected with other hierarchy problems.

The obvious place to look for a reason for quarks to be heavier than leptons is in the dynamics of color. Is there any reason why colored fermions in a generation should be more massive than colorless fermions? There is a well-known answer to this question in the context of ultrahigh-scale unification theories: If quarks and leptons have similar or equal running masses in the 10^{15} GeV to Planck mass regime, then gluonic interactions affect the running to lower energies so as to raise quark masses by roughly the correct amount relative to lepton masses [1]. However, evolution through 13 orders of magnitude or more in energy is required since the masses run only logarithmically. Although this is an interesting observation, it has the observational disadvantage that the new physics of mass generation would be difficult to test properly. Is there a way to understand the quark-lepton mass hierarchy through new color physics at much lower energy scales?

One likely avenue is through a spontaneously broken color group for *leptons* and discrete quark-lepton (q-ℓ) symmetry [2]. These ideas have been pursued for the last few years [3]. The original motivation for them was to connect the quantum numbers of quarks and leptons by new symmetries that could be spontaneously broken at a relatively low scale such as 1 TeV. However, increasing symmetry beyond the SU(3)⊗SU(2)⊗U(1) of the standard model (SM) can also relate parameters such as coupling constants that were previously unrelated. Indeed, it was immediately noticed that in the minimal model discrete q-ℓ symmetry enforced the tree-level mass relations

$$m_{e,\mu,\tau} = m_{u,c,t} \quad \text{and} \quad m^{\text{Dirac}}_{\nu_e,\nu_\mu,\nu_\tau} = m_{d,s,b}. \tag{1}$$

The most constructive way to view the phenomenologically unacceptable charged-lepton up-quark equality is as a springboard for further pondering. Although it is unacceptable *per se*, we after all ultimately do want a theory that will relate quark and lepton masses. I will show how this equality can be transformed into an explanation for why charged leptons are less massive than their up-quark partners. (The $m_\nu^{\text{Dirac}} = m_d$ equality is perfectly acceptable if one uses the seesaw mechanism [4] to explain why the standard neutrinos have such tiny masses.)

We begin by supposing that the SM gauge group G_{SM} is embedded within the larger gauge group $G_{q\ell}$ where

$$G_{q\ell} \equiv \text{SU}(3)_\ell \otimes \text{SU}(3)_q \otimes \text{SU}(2)_L \otimes \text{U}(1)_X, \tag{2}$$

where $\text{SU}(3)_q$ is the usual quark color group, $\text{SU}(3)_\ell$ is leptonic color, and X is an Abelian charge. A fermionic generation is assigned to representations of $G_{q\ell}$ in the following way:

$$\begin{aligned} &Q_L \sim (1,3,2)(1/3),\ u_R \sim (1,3,1)(4/3),\\ &d_R \sim (1,3,1)(-2/3),\ F_L \sim (3,1,2)(-1/3),\\ &E_R \sim (3,1,1)(-4/3),\ N_R \sim (3,1,1)(2/3). \end{aligned} \tag{3}$$

This pattern is anomaly-free, and it enables us to define the discrete symmetry

$$\begin{aligned} &Q_L \leftrightarrow F_L,\ u_R \leftrightarrow E_R,\ d_R \leftrightarrow N_R,\ G_q^\mu \leftrightarrow G_\ell^\mu,\\ &W^\mu \leftrightarrow W^\mu,\ C^\mu \leftrightarrow -C^\mu, \end{aligned} \tag{4}$$

between the quarks and the generalized lepton fields F_L, E_R and N_R, and between the various gauge boson multiplets (G_q^μ are gluons, G_ℓ^μ are leptonic color gluons, W^μ

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are weak gauge bosons, and C^μ is the gauge boson for X).

The standard leptons lie within F_L, E_R and N_R. Their precise identification depends on the spontaneous symmetry breakdown pattern of $\mathrm{SU}(3)_\ell \otimes \mathrm{U}(1)_X$. In the models investigated hitherto, the breakdown $\mathrm{SU}(3)_\ell \otimes \mathrm{U}(1)_X \to \mathrm{SU}(2)' \otimes \mathrm{U}(1)_Y$ was employed (see Refs. [2,3] for details). This led to standard leptons being located purely in the $T_8 = -2$ component of the triplets, where $T_8 \equiv \mathrm{diag}(-2,1,1)$ is a generator of $\mathrm{SU}(3)_\ell$. As in the SM, one introduced a single electroweak Higgs doublet $\phi \sim (1,1,2)(1)$. Because of the q-ℓ discrete symmetry of Eq. (4), its Yukawa interactions were constrained to be

$$\mathcal{L}_{\mathrm{Yuk}} = \frac{m_u}{u}(\overline{Q}_L u_R \phi^c + \overline{F}_L E_R \phi) + \frac{m_d}{u}(\overline{Q}_L d_R \phi + \overline{F}_L N_R \phi^c) + \mathrm{H.c.} \tag{5}$$

where $\phi^c \equiv i\tau_2 \phi^*$. Electroweak symmetry breakdown via $\langle\phi\rangle = (0,u)^T$ then produced the quark-lepton mass relations because of the discrete q-ℓ symmetry (under which $\phi \leftrightarrow \phi^c$).

That standard leptons possessed a unique leptonic color was important in this derivation. If standard leptons were a superposition of components of *different* leptonic color, then Eq. (5) would not necessarily produce the mass relations of Eq. (1). The model I construct below is based on this observation.

To proceed, we need to spontaneously break the $\mathrm{SU}(2)'$ subgroup of $\mathrm{SU}(3)_\ell$ that was left exact hitherto. We will use the same gauge group $G_{q\ell}$ and fermion representation content as the usual q-ℓ symmetric model, but a different Higgs sector.

The nonelectroweak Yukawa Lagrangian is given by $\mathcal{L}'_{\mathrm{Yuk}}$ where

$$\begin{aligned}\mathcal{L}'_{\mathrm{Yuk}} = {} & h_1[\overline{(F_L)^c}F_L\chi + \overline{(Q_L)^c}Q_L\chi'] + h_2[\overline{(N_R)^c}E_R\chi + \overline{(d_R)^c}u_R\chi'] + h_3[\overline{(N_R)^c}N_R\xi + \overline{(d_R)^c}d_R\xi'] \\ & + h_4[\overline{(N_R)^c}E_R\Delta + \overline{(d_R)^c}u_R\Delta'] + h_5[\overline{(F_L)^c}F_L\Delta + \overline{(Q_L)^c}Q_L\Delta'] + \mathrm{H.c.}\end{aligned} \tag{6}$$

and the Higgs boson q-ℓ symmetry pairs are

$$\begin{aligned}&\chi \sim (3,1,1)(2/3) \quad \text{and} \quad \chi' \sim (1,3,1)(-2/3),\\ &\xi \sim (3,1,1)(-4/3) \quad \text{and} \quad \xi' \sim (1,3,1)(4/3),\\ &\Delta \sim (\overline{6},1,1)(2/3) \quad \text{and} \quad \Delta' \sim (1,\overline{6},1)(-2/3).\end{aligned} \tag{7}$$

The electroweak Higgs sector again contains one electroweak doublet ϕ, and the Yukawa Lagrangian is the same as Eq. (5).

Spontaneous symmetry breaking proceeds in at least two stages. First, the fields χ, ξ, and Δ gain nonzero vacuum expectation values (VEV's) to break both leptonic color and the discrete q-ℓ symmetry, leaving electroweak $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ unbroken. (The partner Higgs fields χ', ξ' and Δ' must of course have zero VEV's to keep standard color exact.) The nonstandard fermions in the theory gain nonzero masses via $\mathcal{L}'_{\mathrm{Yuk}}$ at this stage. The standard leptons (and quarks) are defined to be those fermions that remain massless. The electroweak gauge symmetry is then broken in the second stage of symmetry breaking via the usual nonzero VEV for ϕ. This also generates nonzero masses for the standard leptons. For phenomenological reasons we will require that $\langle\chi\rangle$, $\langle\xi\rangle$, $\langle\Delta\rangle \gg \langle\phi\rangle$.

The VEVs of the leptonically colored Higgs bosons are induced to take the forms

$$\langle\chi\rangle = \begin{pmatrix} w \\ 0 \\ 0 \end{pmatrix}, \quad \langle\xi\rangle = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} \quad \text{and} \quad \langle\Delta\rangle = \begin{pmatrix} 0 & a & 0 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{8}$$

where Δ is represented by a 3×3 symmetric matrix.[1] This VEV pattern induces the breakdown

$$\begin{aligned}&\mathrm{SU}(3)_\ell \otimes \mathrm{SU}(3)_q \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_X \\ &\quad \to \mathrm{SU}(3)_q \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \equiv G_{\mathrm{SM}},\end{aligned} \tag{9}$$

where weak hypercharge Y is given by

$$Y = X + \frac{T_8}{3} + T_3, \tag{10}$$

with T_3 being the diagonal generator $\mathrm{diag}(0,1,-1)$ of leptonic color. The major difference between this model and the usual q-ℓ symmetric model is the presence of T_3 in the formula for Y.

Using Eq. (10) we see that the leptonic color components of the generalized leptons have weak hypercharges given by

[1] It is important to check that this VEV pattern can be the minimum of the Higgs potential for a range of parameters. The Higgs potential V for this model is quite complicated, and I will not write it down in this paper since I want to focus on the issue of fermion mass. Ideally, one would like to write V in the sum-of-squares form $\Sigma\lambda_i|\text{quadratic form}|^2$, where the λ_i's are chosen to be positive and "quadratic form" is a quadratic function of the Higgs fields. The global minimum of V is then obtained by simply making each quadratic form zero. My analysis shows that most of the terms in V can be written in this manner in such a way that the required alignment of nonzero VEV's ensues. There are a few terms that I have not succeeded in writing thus, so the required region of parameter space may force the coefficients of these recalcitrant terms to be somewhat smaller than the λ_i's. If these coefficients are zero, then it turns out there is an unwanted global U(1) symmetry in V and an unwanted pseudo-Goldstone boson is produced. A rigorous analysis would need to show that this potentially light boson is made sufficiently heavy when the recalcitrant terms are switched on. This issue is beyond the scope of this paper.

$$Y(F_L) = Y\begin{pmatrix}\ell_L\\(f_R)^c\\f_L\end{pmatrix} = \begin{pmatrix}-1\\+1\\-1\end{pmatrix},\quad Y(E_R) = Y\begin{pmatrix}e_{1R}\\\nu_{2R}\\e_{3R}\end{pmatrix} = \begin{pmatrix}-2\\0\\-2\end{pmatrix},\quad Y(N_R) = Y\begin{pmatrix}\nu_{1R}\\(e_{2L})^c\\\nu_{3R}\end{pmatrix} = \begin{pmatrix}0\\+2\\0\end{pmatrix}, \tag{11}$$

where we have used a suggestive notation for the color components of F_L, E_R, and N_R. As a further piece of notation, let the weak-isospin components of the colors of F_L be given by

$$\ell_L = \begin{pmatrix}\nu_L\\e_L\end{pmatrix},\quad (f_R)^c = \begin{pmatrix}(\epsilon_R)^c\\(n_R)^c\end{pmatrix}\quad \text{and}\quad f_L = \begin{pmatrix}n_L\\\epsilon_L\end{pmatrix}. \tag{12}$$

Equation (11) reveals that the generalized leptons contain, per generation, (i) the standard leptons plus a right-handed neutrino, (ii) mirror or vectorlike pairs of ℓ_L-like and e_R-like states, and (iii) two additional ν_R-like particles. After the first stage of symmetry breakdown the vectorlike pairs form massive Dirac fermions, while the ν_R-like states all become massive. The standard ℓ_L and e_R leptons are defined to be the remaining massless states.

By inputting the VEV's $\langle\chi\rangle$, $\langle\xi\rangle$, $\langle\Delta\rangle$, and $\langle\phi\rangle$ into the Yukawa Lagrangian of Eqs. (6) and (5) we find the charged lepton mass matrix to be given by

$$\mathcal{L}^{\text{ch lept}}_{\text{Yuk}} = \left(\bar{e}_L\,\bar{e}_{2L}\,\bar{\epsilon}_L\right)\begin{pmatrix}m_u & 0 & M_5^\dagger\\ M_4 & M_2 & m_d^T\\ 0 & m_u & M_1\end{pmatrix}\begin{pmatrix}e_{1R}\\e_{3R}\\\epsilon_R\end{pmatrix} + \text{H.c.}, \tag{13}$$

where

$$M_5 \equiv (h_5 - h_5^T)a,\quad M_4 \equiv h_4 a,\quad M_2 \equiv h_2 w,$$

and (14)

$$M_1 \equiv (h_1 + h_1^T)w.$$

The terms m_u, m_d, $M_{1,2,4,5}$ are all 3×3 matrices in generation space. The matrix M_5 is antisymmetric in generation space and thus plays no role in a one-generation toy version of the model. Let us call the full mass matrix in Eq. (13) M_-.

To get a feel for what this mass matrix does, let us turn off the generation structure for the moment (which means that $M_5 = 0$). In the absence of the electroweak contributions m_u and m_d we see that:

(1) The $Q = -1$ field e_L is massless and thus identified as the standard left-handed electron. The other charged members $\epsilon_{L,R}$ of F_L form the left- and right-handed components of a Dirac fermion of mass M_1. (Actually the whole weak-doublet $f = f_L + f_R$ of Dirac fermions has mass M_1.)

(2) The fields $\epsilon'_L \equiv e_{2L}$ and $\epsilon'_R \equiv \sin\phi\; e_{3R} + \cos\phi\; e_{1R}$ form a $Q = -1$ Dirac fermion ϵ' of mass $\sqrt{M_2^2 + M_4^2}$, where $\tan\phi = M_4/M_2$. The right-handed field orthogonal to ϵ'_R is massless and thus identified as the standard right-handed electron: $e_R \equiv \cos\phi\; e_{3R} - \sin\phi\; e_{1R}$.

When the electroweak terms m_u and m_d are switched on, e_L and e_R are connected by a diagonal mass and they also mix with the heavy exotic electronlike states ϵ and ϵ'. The mass of the physical standard electron is the magnitude of the smallest eigenvalue of M_-. Continuing to ignore generation structure, we see that $\det M_- = m_u(M_1M_2 - m_u m_d)$. From $\langle\chi\rangle$, $\langle\xi\rangle$, $\langle\Delta\rangle \gg \langle\phi\rangle$ we expect that $M_{1,2,4} \gg m_{u,d}$ so that $\det M_- \simeq m_u M_1 M_2$. To zeroth order in $m_{u,d}$ the large eigenvalues are still M_1 and $\sqrt{M_2^2 + M_4^2}$, so we see that

$$\text{smallest eigenvalue} \equiv m_e \simeq m_u \cos\phi \le m_u, \tag{15}$$

where

$$\cos\phi \equiv \frac{M_2}{\sqrt{M_2^2 + M_4^2}}. \tag{16}$$

This equation illustrates the central result of this paper: *Mixing between electronlike states of different leptonic color lowers the electron mass from that of its q-ℓ symmetric partner the up quark.*[2]

In the multigeneration real world, each of the ratios m_e/m_u, m_μ/m_c, and m_τ/m_t can be separately adjusted to fit the measurements. This can be trivially seen by supposing we have three generations but no intergenerational mixing. In this case, the charged-lepton of each generation is lighter than the corresponding upquark. Furthermore, each generation can have different values for their corresponding M_2 and M_4 masses, so the corresponding values for $\cos\phi$ can be different. Not unexpectedly, intergenerational mixing complicates this picture. If generations mix strongly, then it is no longer true that all charged lepton masses are *necessarily* lower than their partner upquarks. The precise meaning of "partner" is of course unclear in the presence of intergenerational mixing. I will discuss this issue in more depth after I conclude the technical exposition.

The neutrino sector before electroweak symmetry breakdown splits into massless left-handed neutrinos ν_L, massive fermions $n \sim n_L + n_R$ which are degenerate with the ϵ's, plus a right-sector mass matrix given by

[2] This type of result was first explicitly calculated by de Jonge [5] in the context of a quark-lepton symmetric model with a Higgs sector different from the one I use here. The primary characteristic of the Higgs sector used in Ref. [5] was the nonminimal combination of two χ-type leptonic triplets.

$$\mathcal{L}_{\text{mass}}^{\text{neut}} = \left(\overline{(\nu_{1R})^c}\,\overline{(\nu_{2R})^c}\,\overline{(\nu_{3R})^c}\right) \frac{1}{2} \begin{pmatrix} 0 & M_4 & M_3 \\ M_4^T & 0 & -M_2^T \\ -M_3 & -M_2 & 0 \end{pmatrix} \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix} + \text{H.c.}, \tag{17}$$

where $M_3 \equiv (h_3^T - h_3)v$ and $M_{2,3,4}$ are 3×3 matrices in generation space. Diagonalization of the whole right-sector neutrino mass matrix yields all eigenvalues as nonzero and of order M_i, so the usual seesaw mechanism [4] for neutrinos will ensue when electroweak symmetry breakdown occurs (note that $M_3 \neq 0$ is essential).

The eight gauge bosons of leptonic color acquire masses of the order of $g_s(\Lambda)\Lambda$ where g_s is the strong coupling constant and $\Lambda \sim \langle\chi\rangle$, $\langle\xi\rangle$, and $\langle\Delta\rangle$. Neutral current and other phenomenology will typically require that these gauge bosons are heavier than about 1 TeV, so that $\Lambda \sim \langle\chi\rangle$, $\langle\xi\rangle$, $\langle\Delta\rangle > 1$ TeV. Note that the $\cos\phi$ suppression factor can be large even if the leptonic color breaking scale is much higher than a few TeV. This is because the mixing between electronlike states of different leptonic color is controlled by M_4 and thus it increases with Λ.

Let us now evaluate the successes and failures of the above scenario: (i) We have succeeded in constructing a quark-lepton symmetric model that has both a minimal electroweak Higgs sector and acceptable quark-lepton mass relations. By contrast, in the usual q-ℓ symmetric model one evades the relations in Eq. (1) by postulating two electroweak Higgs doublets rather than one [6]. (ii) But the most important achievement is the fact that charged leptons are forced to be less massive than upquarks by a *structural ingredient of the model.* This provides us with an interesting new technique in model building, and is the main point of this paper. My mechanism is closely related in spirit to the seesaw mechanism [4] and the universal seesaw mechanism [7]. The former is a way of using fermion mixing to understand why neutrinos are much lighter than any other fermion, while the latter is a way of employing fermion mixing to understand why fermions are generally much lighter than the electroweak scale. My mechanism, on the other hand, is a way to employ fermion mixing to understand why charged leptons are lighter than up quarks. Furthermore, it is a low-energy (or potentially low-energy) alternative to the running mass idea alluded to in the introductory paragraphs. I stress also that the mechanism itself is almost certainly of more interest than the specific model I have chosen by way of illustration here. (This is after all also true of the seesaw and universal seesaw mechanisms.) For instance, there are nonminimal q-ℓ symmetric models [8] that have $m_e = m_d$ rather than $m_e = m_u$ which can also probably be modified to incorporate my mechanism. This could be of great interest since the charged-lepton down-quark hierarchy is less severe than the charged-lepton up-quark hierarchy for the second and third generations, and thus it might be easier to explain. (iii) The specific model examined here easily incorporates both the seesaw mechanism for neutrinos and my new mechanism in a reasonably coherent theoretical structure. (iv) The model, however, fails to account for the quark-lepton mass hierarchy in quantitative detail. In the one-generation case of Eqs. (15) and (16) we require M_4 to be significantly larger than M_2 in order to reproduce any of m_e/m_u, m_μ/m_c, or m_τ/m_t. It is interesting that M_4 is proportional to the sextet Higgs VEV while M_2 is proportional to a triplet Higgs VEV. This indicates that the quark-lepton mass hierarchy might be related to a VEV hierarchy and we would have to search for a fundamental reason for the sextet Higgs to have a larger VEV than the triplet Higgs. But we also note that such a VEV hierarchy is not enough since the additional hierarchy $m_e/m_u > m_\mu/m_c > m_\tau/m_t$ can only be incorporated by adjusting Yukawa coupling constants.

However, I argue that it is inappropriate to demand of the present model that it explain the quark-lepton hierarchy in this much detail, since it does not seek to address any of the other hierarchy subproblems. This echoes the point I made in the opening paragraph that the various subproblems within the global conundrum of fermion mass may well interconnect in nontrivial ways. It would be very surprising in my view if a theory *perfectly* explained one type of hierarchy in the quark-lepton sector but left the others accommodated but unexplained. I have deliberately made no attempt to address the generation, mixing angle, and down-up quark hierarchies in the present model, because I wanted to discuss the quark-lepton mass hierarchy subproblem in an unencumbered context. However, ultimately it is probably true that only those models that seek to explain the whole lot can adequately explain any one specific hierarchy.

This of course relates to what becomes of my mechanism when mixing between generations is switched on. As stated previously, if generations mix together strongly then it is no longer necessary for all charged-leptons to be lighter than their "partner" up quarks. However, the only known example of fermion mixing in the real world is parametrized by the approximately diagonal Kobayashi-Maskawa matrix. This suggests that nature may suppress mixing between generations in general. On the other hand, this is by no means inevitable or even desirable. The way ahead is unclear because I can make arguments both for and against a strong correlation between the quark-lepton and intergenerational hierarchy subproblems

The case for. One can speculate that the M_2 and M_4 parameters in $\cos\phi$ perhaps should come from different generations so that $M_4 > M_2$ for the same general reason that, say, $m_c > m_u$. This would require a special pattern of generation mixing that might be due, for example, to a very particular horizontal symmetry. It would also remove the motivation for the proposed hierarchy between the sextet and triplet Higgs boson VEV's.

The case against. We know from the Kobayashi-Maskawa matrix that the only observed mixing between generations is suppressed. We also believe the structural feature that charged-leptons are necessarily lighter than up quarks to be an elegant feature of a one-generation toy model version of my theory. It is therefore tempting to combine these observations and so require an extension of the present theory (that nontrivially incorporates generations) that it necessarily prohibit strong mixing between generations. The role of the matrix M_5 is of particular interest in this regard because it is purely nondiagonal, and thus it may be forbidden by this hypothetical extension.

These are difficult issues, and they perhaps go to the core of why the fermion mass problem has proven so intractable. One can either attempt a grand explanation of all features of the problem at once, or one can attack subproblems individually and then try to synthesize the disparate elements later. The grand approach is probably too ambitious, while in the piecemeal approach one is bound to be dissatisfied with the explanatory powers of the individual pieces.

My model follows the piecemeal philosophy, and so one has to be perfectly clear about the scope of the exercise in order to not make inappropriate demands. Our purpose here was to address some "fine structure" within a generation and the model I propose broadly achieves this aim. How this picture should be extended to incorporate (a) multiple generations in a profound way (horizontal symmetry? compositeness?), and (b) the up-quark to down-quark hierarchy, is a task for the future. (Of course, these types of observations can also be made about the seesaw and universal seesaw mechanisms. Neither has anything to say about generations, mixing angles, or the weak-isospin hierarchy.)

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[1] For a recent study, see, H. Arason *et al.*, Phys. Rev. D **46**, 3945 (1992).

[2] R. Foot and H. Lew, Phys. Rev. D **41**, 3502 (1990).

[3] R. Foot, H. Lew, and R. R. Volkas, Phys. Rev. D **44**, 1531 (1991); *ibid.* **44**, 859 (1991); Int. J. Mod. Phys. A **8**, 983 (1993); Mod. Phys. Lett. A **8**, 1859 (1993); R. Foot and H. Lew, Nuovo Cimento A **104**, 167 (1991); Phys. Rev. D **42**, 945 (1990); Mod. Phys. Lett. A **5**, 1345 (1990); *ibid.* **7**, 301 (1992); G. C. Joshi and R. R. Volkas, Phys. Rev. D **45**, 1711 (1992); H. Lew and R. R. Volkas, *ibid.* **47**, 1356 (1993); Y. Levin and R. R. Volkas, *ibid.* **48**, 5342 (1993); C. C. Lassig and R. R. Volkas, University of Melbourne Report No. UM-P-94/21 (unpublished).

[4] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, Proceedings of the Workshop, Stony Brook, New York, 1979, edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, in Proceedings of the Workshop on Unified Theory and Baryon Number of the Universe, Tsukuba, Japan, 1979, edited by A. Sawada and A. Sugamoto [KEK Report No. 79-18, 1979 (unpublished)]; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[5] M. de Jonge, 4th year Honors thesis, University of Melbourne, 1993 (unpublished).

[6] For a detailed study of this, see the paper by Levin and Volkas in [3].

[7] A. Davidson and K. C. Wali, Phys. Rev. Lett. **59**, 393 (1987); S. Rajpoot, Phys. Lett. B **191**, 122 (1987).

[8] See R. Foot, H. Lew, and R. R. Volkas, Phys. Rev. D **44**, 1531 (1991); R. Foot and H. Lew, Mod. Phys. Lett. A **7**, 301 (1992).