Model for a light Z' boson

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A model of a light Z' boson is constructed and phenomenological bounds are derived. This Z' boson arises from a very simple extension to the standard model, and it is constrained to be light because the vacuum expectation values which generate its mass also break the electroweak gauge group. It is difficult to detect experimentally because it couples exclusively or primarily (depending on symmetry-breaking details) to second and third generation leptons. However, if the Z' boson is sufficiently light, then there exists the possibility of the two-body decay $\tau \to \mu Z'$ occurring. This will provide a striking signature to test the model.

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The success of the standard model (SM) has led many people to believe that it is the correct low-energy theory for physics below about 100 GeV. Despite this success there are still many ways in which the SM might be incomplete. For example, experiments may reveal neutrino masses. Another possibility is that the gauge sector is incomplete.

The SM uses the gauge group $G_{\rm SM}$=SU$(3)_c$ $\otimes$SU$(2)_L\otimes$U$(1)_Y$ with the fermion transformation laws being

$$\begin{aligned} Q_L &\sim (3,2)(1/3), \quad u_R \sim (3,1)(4/3), \\ d_R &\sim (3,1)(-2/3), \\ \ell_L &\sim (1,2)(-1) \quad \text{and} \\ \ell_R &\sim (1,1)(-2). \end{aligned} \tag{1}$$

Before the discovery of neutral currents the theoretical need for the U(1) factor in $G_{\rm SM}$ was recognized since this was the minimal way to incorporate the U(1) of electromagnetism and the SU(2) which contained the charged current weak interactions [1]. [The U(1) inside the SU(2) could not be used because it gave the wrong electric charges.] While not strictly necessary theoretically, it might be that there are other gauged U(1) symmetries. In other words, the gauge symmetry (below some scale) may effectively be given by $G_{\rm SM}\otimes$U$(1)'$. In this paper, we are interested in examining the possibility that nature is effectively described by a gauge theory with gauge group $G_{\rm SM}\otimes$U$(1)'$ with all gauge boson masses less than about 100 GeV. This is an important question, since it would mean that low-energy physics is not described by $G_{\rm SM}$ but rather by $G_{\rm SM}\otimes$U$(1)'$.

How are we to choose the spectrum of U$(1)'$ charges? We impose three constraints: First, we will assume that the new gauge group $G_{\rm SM}\otimes$U$(1)'$ is anomaly-free under the condition that the standard quarks and leptons are the only fermions in the model. To keep the fermion spectrum minimal we will in particular exclude right-handed neutrinos. Second, the nonzero vacuum expectation values (VEV's) which break U$(1)'$ should also break the electroweak gauge group. This ensures that the symmetry-breaking scale for U$(1)'$ cannot be made arbitrarily high. We will also demand that all Higgs multiplets couple to fermions through Yukawa terms. This serves to connect the U$(1)'$ charges of the fermions with those of the Higgs bosons. Third, we would like the Z' coupling constant to be as large as phenomenology allows. This will maximize the testability of our model.

The condition of anomaly freedom informs us that the U$(1)'$ must couple differently to the different generations. This is because U$(1)_Y$ is the only generation blind symmetry that is anomaly-free with respect to $G_{\rm SM}$. By using this piece of information together with the third criterion stated above we can narrow down the choices considerably. Most experiments are done by using the interactions of the first generation fermions, since these comprise ordinary matter. Any Z' which couples to the first generation will be more heavily constrained than one which couples to second and third generation fermions only. Since we are interested in the possibility of a very light Z', we thus assume that the U$(1)'$ charges of the first generation fermions are all zero. Another stringent constraint on the Z' interactions arises from flavor-changing neutral current (FCNC) processes. If it does not couple universally to quarks, then in general the interactions of the Z' will not conserve flavor. This means there will be no Glashow-Iliopoulos-Maiani (GIM) mechanism, and experimental bounds on processes such as K-$\overline{K}$ mixing will render the Z' coupling constant rather small. We are thus lead to suppose that our Z' couples only to second

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and third generation leptons.

So, we start by assuming the most general U(1)$'$ charge assignments consistent with the above assumptions:

$$\ell_{2L} \sim (1,2)(-1,a_1), \qquad e_{2R} \sim (1,1)(-2,b_1),$$
$$\ell_{3L} \sim (1,2)(-1,a_2), \qquad e_{3R} \sim (1,1)(-2,b_2). \qquad (2)$$

Anomaly cancellation implies that $a_1 = -a_2$, $b_1 = -b_2$, and $a_1 = \pm b_1$. The sign ambiguity in the last equation is of no consequence since one can be obtained from the other by renaming $e_{2,3R}$ as $e_{3,2R}$; we choose the plus sign. Note that the fields in Eq. (2) are the weak eigenstates. In general, the mass eigenstates will be linear combinations of the weak eigenstates.

There are only three choices of weak eigenstates which have a type of GIM mechanism: (1) $\ell_{2L} = \mu_L, \ell_{3L} = \tau_L, e_{2R} = \mu_R, e_{3R} = \tau_R$; (2) $\ell_{2L} = \mu_L, \ell_{3L} = \tau_L, e_{2R} = \tau_R, e_{3R} = \mu_R$; (3) $\ell_{2L} = (\mu+\tau)_L/\sqrt{2}, \ell_{3L} = (\mu-\tau)_L/\sqrt{2}, e_{2R} = (\mu+\tau)_R/\sqrt{2}, e_{3R} = (\mu-\tau)_R/\sqrt{2}$. (In this equation we have denoted mass eigenstates by μ and τ.) The first case corresponds to gauged $L_\mu - L_\tau$ and has been discussed previously [2]. Note that since $L_\mu - L_\tau$ is a symmetry of the standard model, this symmetry is not broken by fermion masses (assuming the minimal fermion content of 15 Weyl fields per generation). The second case [(2) above] corresponds to gauged axial $L_\mu - L_\tau$. This case has not been discussed previously as far as we are aware. In this case, since the μ and τ masses break axial $L_\mu - L_\tau$, the symmetry breaking of the new U(1) is related to electroweak symmetry breaking. While this is an interesting model, we choose not to examine it here. The last case has a type of GIM mechanism because the mixing is maximal. Here decays of the τ such as $\tau \to \mu\mu\mu$ are not induced by tree-level Z' exchange if the states are mass eigenstates also, although other flavor-changing decay modes are possible (as we will discuss). In this paper, it turns out that we will be led to concentrating on the third possibility, but we will also consider a case near the end of the paper where the weak and mass eigenstates are not related in any of the above ways.

We now discuss the model in detail. For second and third generation leptons we have that

$$\ell_{2L} \sim (1,2)(-1,2a), \qquad e_{2R} \sim (1,1)(-2,2a),$$
$$\ell_{3L} \sim (1,2)(-1,-2a), \qquad e_{3R} \sim (1,1)(-2,-2a). \qquad (3)$$

It is interesting to note that a number of benefits can be gained by instituting an exact discrete symmetry under

$$\ell_{2L} \leftrightarrow \ell_{3L}, \quad e_{2R} \leftrightarrow e_{3R}, \quad B^\mu \leftrightarrow B^\mu,$$

and (4)

$$Z'^\mu \leftrightarrow -Z'^\mu,$$

where B^μ and Z'^μ are the gauge fields for U(1)$_Y$ and U(1)$'$, respectively. The benefits are (i) it forces the number of free parameters in the gauge covariant derivative to be reduced by one, as we explain below, (ii) if unbroken, it ensures that the mass eigenstates will be maximally mixed combinations of the weak eigenstates so that a type of GIM mechanism ensues (as discussed above), and (iii) if unbroken it also forbids Z-Z' mixing to all orders, which simplifies the phenomenological analysis since the important experimental constraints on such mixing [3] are automatically satisfied. We will at first be concerned with the version of the model maintaining the discrete symmetry as exact. We will then briefly consider the case where the discrete symmetry is broken by the vacuum.

The gauge covariant derivative for the electrically neutral gauge bosons is

$$D^\mu = \partial^\mu + ig_2 I_3 W^\mu + i\frac{g_1}{2} Y B^\mu + i\frac{g_1}{2} Y' Z'^\mu, \qquad (5)$$

where $I_3 \equiv \tau_3/2$, W^μ is the neutral weak-isospin gauge boson, and $g_{1,2}$ are the two gauge coupling constants. The coupling constant for U(1)$'$ has been defined to be equal to that for U(1)$_Y$ because the free parameter a in Eq. (3) can be taken to determine the relative strengths of these interactions. The parameter that is eliminated by the discrete symmetry can be identified from an examination of the kinetic energy Lagrangian for the U(1) gauge fields. If the discrete symmetry is ignored then this Lagrangian is given in general by

$$\mathcal{L}_{\rm KE} = k_1 F^{\mu\nu} F_{\mu\nu} + k_2 F'^{\mu\nu} F_{\mu\nu} + k_3 F'^{\mu\nu} F'_{\mu\nu}, \qquad (6)$$

where F and F' are the field strength tensors for B^μ and Z'^μ, respectively. Note that the off-diagonal term is permitted by gauge invariance because the symmetries are Abelian [4]. In order to bring this general kinetic energy Lagrangian into diagonal and canonically normalized form we must rewrite everything in terms of certain linear combinations of B^μ and Z'^μ. If the discrete symmetry is imposed then the mixing term is absent and so $k_2 = 0$. In this case the redefinition required is just a straight rescaling of both B^μ and Z'^μ, and the parameters k_1 and k_3 can be absorbed by g_1 and a. If the discrete symmetry is absent, then the k_2 coefficient is an additional free parameter in the theory (that is, it *cannot* be absorbed into g_1 and a). In the diagonal and conventionally normalized basis for the gauge fields, the freedom represented by k_2 can be incorporated by the substitution $Y' \to Y' + kY$ in the covariant derivative where k is now the arbitrary parameter.

It is convenient to rewrite the gauge covariant derivative in terms of the photon field A^μ and the standard Z^μ field. The rewritten covariant derivative is

$$D^\mu = \partial^\mu + ieQA^\mu + i\frac{e}{s_W c_W}(I_3 - s_W^2 Q)Z^\mu + i\frac{e}{c_W}\frac{Y'}{2}Z'^\mu, \qquad (7)$$

where $s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$, and the weak angle θ_W is defined through $\tan\theta_W \equiv g_1/g_2$. The electromagnetic coupling constant is given as usual by $e \equiv g_2 s_W$ and electric charge Q is $Q \equiv I_3 + Y/2$. Under the discrete symmetry,

$$A^\mu \to A^\mu, \quad Z^\mu \to Z^\mu, \quad \text{and} \quad Z'^\mu \to -Z'^\mu. \qquad (8)$$

This means that if the discrete symmetry remains exact after spontaneous symmetry breaking then there is no

Z-Z' mixing to all orders. This is very important phenomenologically.

The Yukawa coupling Lagrangian is

$$\mathcal{L}_{\rm Yuk} = \lambda(\bar{\ell}_{2L}e_{2R}\phi_1 + \bar{\ell}_{3L}e_{3R}\phi_1) + \lambda'(\bar{\ell}_{2L}e_{3R}\phi_2 + \bar{\ell}_{3L}e_{2R}\phi_3) + \text{H.c.}, \quad (9)$$

where the Higgs doublet transformation laws are

$$\phi_1 \sim (1,2)(1,0),$$
$$\phi_2 \sim (1,2)(1,4a), \quad \text{and} \quad \phi_3 \sim (1,2)(1,-4a). \quad (10)$$

Under the discrete symmetry $\phi_1 \leftrightarrow \phi_1$ and $\phi_2 \leftrightarrow \phi_3$. (If the discrete symmetry is not invoked then one need only introduce the equivalent of one of ϕ_2 and ϕ_3.)

Let us now look at the phenomenology of the model. First note that without any analysis there are two potentially very interesting features of this model due to the hypothesis that the U(1)$'$ breaking is tied to electroweak breaking. The first is that since the mass of Z' is expected to be less than or equal to the Z boson mass, then the Z' boson should have some effect on already measurable low-energy observables provided its coupling to leptons is not too weak. Second, if $m_{Z'} < (m_\tau - m_\mu)$ then the two-body decay $\tau \rightarrow \mu Z'$ can occur. This will provide a striking signature since the final state μ (in the τ rest frame) will have a fixed energy in contrast to the continuum μ energy spectrum from the usual three-body decay mode. It is known that measurements of the μ energy spectrum can provide a sensitive probe of the two-body decay mode. Furthermore, such a signature would easily distinguish this Z' from that of many other models.

Having foreshadowed what to expect we will now proceed to analyze the phenomenological implications of the model. We know that Z-Z' mixing is constrained to be small. Our discrete symmetry affords us the luxury of having this mixing as precisely zero, as discussed above, provided it is not spontaneously broken. We thus adopt, to begin with, that range of parameters in the Higgs potential (which we will display explicitly below) which maintains the discrete symmetry as exact, while breaking both U(1)$'$ and the electroweak gauge group. The VEV pattern required is

$$\langle\phi_1\rangle \equiv u_1 \ (\neq 0 \text{ in general})$$

and (11)

$$|\langle\phi_2\rangle| = |\langle\phi_3\rangle| = u_2 \neq 0.$$

(We will without loss of generality take the phase of $\langle\phi_1\rangle$ to be 1, while the phases of $\langle\phi_2\rangle$ and $\langle\phi_3\rangle$ will be discussed presently.) The Z and Z' masses are then given by

$$m_Z^2 = \frac{1}{2}(g_1^2 + g_2^2)(u_1^2 + 2u_2^2)$$

and (12)

$$m_{Z'}^2 = 16a^2 s_W^2(g_1^2 + g_2^2)u_2^2.$$

As we will soon see, phenomenological bounds force us to consider the Z' to be much lighter than the Z, in contrast to most other Z' models. The reason for this is that in many processes the parameter a cancels out between the Z'-fermion vertices and the Z' propagator when the momentum in the propagator can be neglected relative to the Z' mass. This means that the coupling strength for the Z' in this high mass limit is completely specified by previously measured quantities. It just so happens that this coupling strength is *too* strong. So, we will be led to looking at the $m_{Z'} \ll m_Z$ region of parameter space.

Since the discrete symmetry is exact, all mass eigenstate fields have to be either even or odd under the transformation (this is true for the neutral gauge bosons discussed above for instance). This allows us to write down immediately that the mass eigenstate charged leptons are given by $(e_2 \pm e_3)/\sqrt{2}$. Substituting this into the Z'-lepton interaction Lagrangian we obtain that

$$\mathcal{L}^{\ell}_{\rm int} = \frac{ea}{c_W}(\bar{\mu}\gamma^\mu\tau + \bar{\tau}\gamma^\mu\mu)Z'_\mu. \quad (13)$$

Therefore we see that although the Z' boson mediates flavor-changing neutral currents, these processes are always *purely off diagonal.* (Diagonal terms are forbidden because Z' is odd.) By defining μ and τ neutrinos as those fields that are produced with μ's and τ leptons, respectively, in charged current weak interactions we also see that

$$\mathcal{L}^{\nu}_{\rm int} = \frac{ea}{2c_W}[\bar{\nu}_\mu\gamma^\mu(1-\gamma_5)\nu_\tau + \bar{\nu}_\tau\gamma^\mu(1-\gamma_5)\nu_\mu]Z'_\mu. \quad (14)$$

These Lagrangians will allow us to easily identify the interesting phenomenological constraints on Z'-lepton interactions.

The most convenient way to write the Higgs potential down is

$$\begin{aligned} V = {}& \lambda_1(\phi_1^\dagger\phi_1 - u_1^2)^2 + \lambda_2(\phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3 - 2u_2^2)^2 + \lambda_3(\phi_2^\dagger\phi_2 - \phi_3^\dagger\phi_3)^2 + \lambda_4(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3 - u_1^2 - 2u_2^2)^2 \\ & + \lambda_5[(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) - (\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2)] + \lambda_6[\phi_1^\dagger\phi_1(\phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3) - (\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) - (\phi_1^\dagger\phi_3)(\phi_3^\dagger\phi_1)] \\ & + \lambda_7[\phi_1^\dagger\phi_1(\phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3) - (\phi_1^\dagger\phi_2)(\phi_1^\dagger\phi_3) - (\phi_2^\dagger\phi_1)(\phi_3^\dagger\phi_1)]. \end{aligned} \quad (15)$$

The parameters λ_{1-7} must be real from Hermiticity (we have also redefined to zero a phase that can *a priori* appear in front of the last two terms within the λ_7 term). The symmetry-breaking pattern we require is obtained by choosing $\lambda_{1-7} > 0$. (Other symmetry-breaking patterns can, of course, be induced in other regions of parameter space.) In the $\lambda_{1-7} > 0$ region of parameter space, the Higgs potential is the sum of positive-definite terms. The λ_{1-4} terms are obviously positive definite, while a little algebra shows that

$$(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) - (\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2) = |\phi_3^+\phi_2^0 - \phi_2^+\phi_3^0|^2,$$
$$(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - (\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) = |\phi_2^+\phi_1^0 - \phi_1^+\phi_2^0|^2,$$
$$(\phi_1^\dagger\phi_1)(\phi_3^\dagger\phi_3) - (\phi_1^\dagger\phi_3)(\phi_3^\dagger\phi_1) = |\phi_3^+\phi_1^0 - \phi_1^+\phi_3^0|^2, \quad (16)$$

and

$$\phi_1^\dagger\phi_1(\phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3) - (\phi_1^\dagger\phi_2)(\phi_1^\dagger\phi_3) - (\phi_2^\dagger\phi_1)(\phi_3^\dagger\phi_1)$$
$$= |\phi_1^+\phi_2^- - \phi_1^-\phi_3^+|^2 + |\phi_1^+\phi_2^{0*} - \phi_1^{0*}\phi_3^+|^2 + |\phi_1^+\phi_3^{0*} - \phi_1^{0*}\phi_2^+|^2 + |\phi_1^0\phi_2^{0*} - \phi_1^{0*}\phi_3^0|^2. \quad (17)$$

Since the Higgs potential is written as the sum of positive-definite terms, we know we have a minimum if a VEV pattern renders each term separately zero. The first four terms show that $|\langle\phi_1\rangle| = u_1$, and $|\langle\phi_2\rangle| = |\langle\phi_3\rangle| = u_2$. The λ_3 term is responsible for forcing the last two VEV's to be exactly equal. The $\lambda_{5,6}$ terms force the charged Higgs bosons to have zero VEV's. To see this, first perform an $SU(2)_L$ gauge transformation to define $\langle\phi_1^\pm\rangle = 0$. Then $\langle\phi_1^0\rangle = u_1[\neq 0$ by parameter choice, and it can be made positive and real by the same $SU(2)_L$ transformation]. Minimization of the terms in Eq. (16) forces $\langle\phi_{2,3}^\pm\rangle = 0$. The first three terms on the right-hand side of Eq. (17) are then also zero, while the fourth term tells us that $\langle\phi_1^0\rangle\langle\phi_2^{0*}\rangle = \langle\phi_1^{0*}\rangle\langle\phi_3^0\rangle$. Given that the phase angle for ϕ_1 has been set to 1, this implies that the phase angles for ϕ_2 and ϕ_3 are equal and opposite. However, these phase angles can be removed by a $U(1)'$ transformation and are thus unphysical and will henceforth be set to zero.

Consider the shifted neutral Higgs fields defined through

$$\phi_1^0 \equiv u_1 + \frac{H_1 + i\eta_1}{\sqrt{2}} \quad \text{and} \quad \phi_{2,3}^0 \equiv u_2 + \frac{H_{2,3} + i\eta_{2,3}}{\sqrt{2}}, \quad (18)$$

where the H's are CP-even real Higgs bosons and the η's are CP-odd real Higgs bosons. It is convenient to discuss the $\phi_{2,3}$ fields in the discrete symmetry eigenstate basis given by

$$H_\pm \equiv \frac{H_2 \pm H_3}{\sqrt{2}} \quad \text{and} \quad \eta_\pm \equiv \frac{\eta_2 \pm \eta_3}{\sqrt{2}}, \quad (19)$$

where the subscripts plus and minus denote even and odd fields under the discrete symmetry, respectively.

All three H fields are physical. The odd combination H_- does not mix with the even fields H_1 and H_+. The mass of the former is

$$m_{H_-}^2 = 8\lambda_3 u_2^2 + 2\lambda_7 u_1^2, \quad (20)$$

while the mass matrix for the latter two is

$$m^2(H_1, H_+) = \begin{pmatrix} 4(\lambda_1 + \lambda_4)u_1^2 & 4\sqrt{2}\lambda_4 u_1 u_2 \\ 4\sqrt{2}\lambda_4 u_1 u_2 & 8(\lambda_2 + \lambda_4)u_2^2 \end{pmatrix}. \quad (21)$$

There is one physical η field given by

$$\eta_{\rm phys} = \frac{\sqrt{2}u_2\eta_1 - u_1\eta_+}{\sqrt{u_1^2 + 2u_2^2}} \quad (22)$$

with mass

$$m_{\eta_{\rm phys}}^2 = 2\lambda_7(u_1^2 + 2u_2^2). \quad (23)$$

There are two physical charged Higgs bosons, given by

$$h^+ \equiv \frac{\sqrt{2}u_2\phi_1^+ - u_1 h_+^+}{\sqrt{u_1^2 + 2u_2^2}} \quad \text{and} \quad h_-^+ \equiv \frac{\phi_2^+ - \phi_3^+}{\sqrt{2}}, \quad (24)$$

where $h_+^+ \equiv (\phi_2^+ + \phi_3^+)/\sqrt{2}$. Their masses are

$$m_{h^+}^2 = (\lambda_6 + \lambda_7)(u_1^2 + 2u_2^2)$$
$$\text{and } m_{h_-^+}^2 = (\lambda_6 + \lambda_7)u_1^2 + 2\lambda_5 u_2^2. \quad (25)$$

Note that the odd combination h_-^+ does not mix with the even combination h^+ because of the exact discrete symmetry.

The Yukawa coupling Lagrangian for the H fields is

$$\mathcal{L}_{\rm Yuk}^H = \sum_f \frac{m_f}{\sqrt{2}u_1}\bar{f}fH_1 + \frac{m_\tau + m_\mu}{2\sqrt{2}u_1}(\bar{\tau}\tau + \bar{\mu}\mu)H_1 + \frac{m_\tau - m_\mu}{2\sqrt{2}u_2}\left[(\bar{\tau}\tau - \bar{\mu}\mu)H_+ + (\bar{\mu}\tau - \bar{\tau}\mu)H_-\right], \quad (26)$$

where $f = u, d, c, s, t, b, e$, and m_f is the corresponding mass. The Lagrangian for the physical η field is

$$\mathcal{L}_{\rm Yuk}^\eta = \sum_f \frac{iu_2 m_f}{u_1\sqrt{u_1^2 + 2u_2^2}}\bar{f}\gamma_5 f\eta_{\rm phys} + \frac{i}{2\sqrt{u_1^2 + 2u_2^2}}\left[\frac{u_2}{u_1}(m_\tau + m_\mu) - \frac{u_1}{2u_2}(m_\tau - m_\mu)\right]\bar{\tau}\gamma_5\tau\ \eta_{\rm phys}$$
$$+ \frac{i}{2\sqrt{u_1^2 + 2u_2^2}}\left[\frac{u_2}{u_1}(m_\tau + m_\mu) + \frac{u_1}{2u_2}(m_\tau - m_\mu)\right]\bar{\mu}\gamma_5\mu\ \eta_{\rm phys}. \quad (27)$$

It is interesting to note that the mass eigenstate CP-even Higgs bosons that are superpositions of H_1 and H_+ have flavor-diagonal interactions, as does the physical CP-odd field $\eta_{\rm phys}$. The discrete-symmetry-odd mass eigenstate H_- is flavor-changing in the μ-τ sector, but it is completely off diagonal just like the Z' (and for the same reason of course).

The Lagrangian for fermion coupling to the charged Higgs bosons is

$$\mathcal{L}^{+}_{\text{Yuk}} = \frac{\sqrt{2}u_2}{u_1\sqrt{u_1^2+2u_2^2}}\overline{U}_L M_d D_R h^+ + \frac{\sqrt{2}u_2 m_e}{u_1\sqrt{u_1^2+2u_2^2}}\overline{\nu}_{eL} e_R h^+ + \frac{1}{\sqrt{2}\sqrt{u_1^2+2u_2^2}}\left[\frac{u_2}{u_1}(m_\tau+m_\mu) - \frac{u_1}{2u_2}(m_\tau - m_\mu)\right]\overline{\nu}_{\tau L}\tau_R\, h^+ + \frac{1}{\sqrt{2}\sqrt{u_1^2+2u_2^2}}\left[\frac{u_2}{u_1}(m_\tau+m_\mu) + \frac{u_1}{2u_2}(m_\tau - m_\mu)\right]\overline{\nu}_{\mu L}\mu_R\, h^+ + \frac{m_\tau - m_\mu}{2\sqrt{2}u_2}(\overline{\nu}_{\mu L}\tau_R - \overline{\nu}_{\tau L}\mu_R)h^+_- + \text{H.c.}, \tag{28}$$

where $U \equiv (u,c,t)$, $D^T \equiv (d,s,b)$, and M_d is the undiagonalized down-quark mass matrix.

We now have to identify those processes involving second and third generation leptons which provide significant phenomenological constraints. (Since Z-Z' mixing is absent to all orders, the number of relevant processes is greatly reduced.) There are essentially three important constraints: the anomalous magnetic moment of the muon, a_μ, the gauge boson masses (i.e., we have to ensure that the values of u_1 and u_2 reproduce the measured values for m_W and m_Z, and hence $m_{Z'}$ cannot be arbitrarily large) and the Z' contribution to τ decay [5].

The principal contribution to a_μ is depicted in Fig. 1. Note that the discrete-symmetry-odd field Z' is featured here, because it couples μ to τ leptons. There are similar graphs involving the neutral and charged Higgs bosons which also contribute to a_μ, but they turn out to be much smaller since they are suppressed by the factor $(m_\mu/m_H)^2$ for $m_\mu < m_H$ where m_H is the mass of the generic Higgs field. For the analysis that follows we will assume that all the Higgs bosons in the model are heavier than ~ 40 GeV so that their contributions to a_μ and the decay width of the standard Z boson are negligible.

The Z' contribution to a_μ is given by

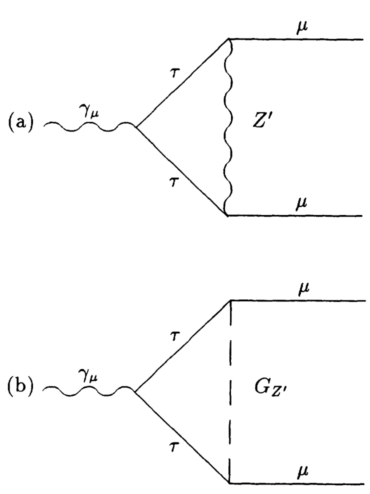

FIG. 1. (a) The one-loop contribution to Δa_μ, and (b) the accompanying diagram with the Goldstone field, $G_{Z'}$, in the R_ζ gauge.

$$\Delta a_\mu^{Z'} = \frac{\alpha_{\text{em}}}{2\pi}\frac{|a|^2}{c_W^2}\left\{\gamma + 2(\beta - \frac{2B}{C}\gamma) + 2M\ln\left(\frac{m_\tau}{m_{Z'}}\right) + \delta\right\}, \tag{29}$$

where

$$\delta = \begin{cases} \frac{NC-MB}{\sqrt{B^2-AC}}\ln\left|\frac{A+B+\sqrt{B^2-AC}}{A+B-\sqrt{B^2-AC}}\right| & \text{if } B^2 > AC, \\ 2\frac{NC-MB}{\sqrt{AC-B^2}}\tan^{-1}\left[\frac{\sqrt{AC-B^2}}{A+B}\right] & \text{if } B^2 < AC. \end{cases} \tag{30}$$

In these equations, α_{em} is the fine-structure constant, and

$$\begin{aligned} &\alpha \equiv 2(m_\tau - m_\mu)/m_\mu, \\ &\beta \equiv 3 - 2(m_\tau/m_\mu) \\ &\quad + \frac{1}{2}\left[(m_\tau/m_\mu)+1\right](m_\tau - m_\mu)^2/m_{Z'}^2, \\ &\gamma \equiv -1 - \frac{1}{2}(m_\tau - m_\mu)^2/m_{Z'}^2, \\ &A \equiv m_{Z'}^2, \quad B \equiv (m_\tau^2 - m_\mu^2 - m_{Z'}^2)/2, \quad C \equiv m_\mu^2, \\ &M \equiv \alpha - \frac{2B}{C}(\beta - \frac{2B}{C}\gamma) \quad \text{and} \quad N \equiv -\frac{A}{C}(\beta - \frac{2B}{C}\gamma). \end{aligned} \tag{31}$$

This expression demonstrates that a large $m_{Z'}$ is phenomenologically disallowed. In the $m_{Z'} \gg m_\tau$ limit, Eq. (29) reduces to the simple result that

$$\Delta a_\mu^{Z'} \simeq \frac{\alpha_{\text{em}}}{2\pi}\frac{|a|^2}{c_W^2}\frac{2m_\mu m_\tau}{m_{Z'}^2} = \frac{m_\mu m_\tau}{64\pi^2 u_2^2} \tag{32}$$

which is independent of $|a|$ and at best about an order of magnitude too large given that u_2 is constrained by the weak scale. So, we will be interested in Z' masses of about a few GeV or less. In any case Eq. (29) can be evaluated numerically. Figures 2(a) and (b) show the allowed region of $(m_{Z'}, |a|)$ parameter space, given the experimental constraint [5] $|\Delta a_\mu| < 10^{-8}$, i.e., given by the region below the dashed curve.

We next consider the constraint coming from the gauge boson masses. By using Eqs. (12) and $m_W^2 = g_2^2(u_1^2 + 2u_2^2)/2$, one finds [6]

$$|a| > \frac{1}{4\tan\theta_W}\frac{m_{Z'}}{m_W} \simeq \left(\frac{m_{Z'}}{175.33\ \text{GeV}}\right). \tag{33}$$

The region allowed by this constraint is the area *above* the solid curve shown in Figs. 2(a) and (b). In other words, for a given value of $m_{Z'}$ there exists a minimum value for $|a|$. When the above two constraints are combined there is a small overlap region remaining. This overlap region where the two constraints are satisfied (roughly $m_{Z'} < 2.5$ GeV) divides into regimes; namely, $m_{Z'} > (m_\tau - m_\mu)$ and $m_{Z'} < (m_\tau - m_\mu)$. In the former, the interesting decay mode $\tau \to \mu Z'$ is not allowed kinematically, whereas in the latter it is. Note that this result makes numerically precise the qualitative observation made earlier that the Z' boson cannot be arbitrarily heavy.

Let us first consider the case where $\tau \to \mu Z'$ is not allowed. Although this dramatic two-body decay does not occur, the off-shell Z' contributes to the family lepton-number preserving three-body decay $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ and the family lepton-number-violating decay $\tau^- \to \mu^- \nu_\mu \bar{\nu}_\tau$. We have to check whether or not constraints from the observation of the standard decay mode $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ close the $(m_\tau - m_\mu) < m_{Z'} < 2.5$ GeV window. For this mode, the Z' contribution coherently adds with the standard W-boson contribution yielding

$$R \equiv \frac{\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)}{\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)_{\rm SM}} = 1 - \xi \left[2k(k+1) - \frac{5}{6} - k^2(2k+3) \ln \left| \frac{1+k}{k} \right| \right] + \frac{1}{4}\xi^2 \left[2(2k+1) + k\frac{2k+3}{k+1} - 6k(k+1) \ln \left| \frac{1+k}{k} \right| \right], \quad (34)$$

where

$$k \equiv \frac{m_{Z'}^2}{m_\tau^2} - 1 \quad \text{and} \quad \xi \equiv \frac{\sqrt{2}}{G_F m_\tau^2} \frac{4\pi\alpha_{\rm em}}{c_W^2} |a|^2. \quad (35)$$

Note that the contribution from the finite width of Z' has been neglected in this calculation. This contribution is expected to have its most significant effect near the Z' threshold. However, in this case, the Z' width is given by

$$\Gamma_{Z'} \simeq \Gamma(Z' \to \bar{\nu}_\mu \nu_\tau + \nu_\mu \bar{\nu}_\tau) = \frac{\alpha_{\rm em}}{3c_W^2} |a|^2 m_{Z'}, \quad (36)$$

where it is suppressed by a factor of $|a|^2$ so that the zero width approximation should not be a bad one. The largest contribution comes from the interference term between the W and Z' bosons. (The nonstandard decay $\tau^- \to \mu^- \nu_\mu \bar{\nu}_\tau$ mode will always provide less stringent constraints than the Z' contribution to the standard decay because the decay rate is given by the direct-Z' process only and is thus proportional to $|a|^4$.) The experimental constraint [5]

$$|R - 1| < 0.04 \quad (37)$$

in fact closes the $(m_\tau - m_\mu) < m_{Z'} < 2.5$ GeV window. This is shown in Figs. 2(a) and (b) where the region below the dot-dashed curve is the one allowed by the three-body decay constraint.

So, we are left to consider the kinematic region which permits the two-body decay mode $\tau \to \mu Z'$. First, notice that the three-body decay constraint allows for windows in the $m_{Z'} < 0.2$ GeV and $0.8 < m_{Z'} < 1.0$ GeV regions. There is also a minute region at $m_{Z'} \sim 1.2$ GeV. The second window is caused by the vanishing of the term proportional to ξ in Eq. (34) for values of $m_{Z'}$ in this region, while the third window is due to the cancellation between the ξ and ξ^2 terms in Eq. (34). (This cancellation is possible for large enough values of $|a|$ because the ξ^2 term becomes as important as the ξ term.) We now check to see what effect the two-body decay mode has. The Mark III and ARGUS Collaborations [7] have set limits on two-body decay modes for τ. These experimental groups specifically analyzed the process $\tau \to \mu$ + Goldstone boson and found that the ratio

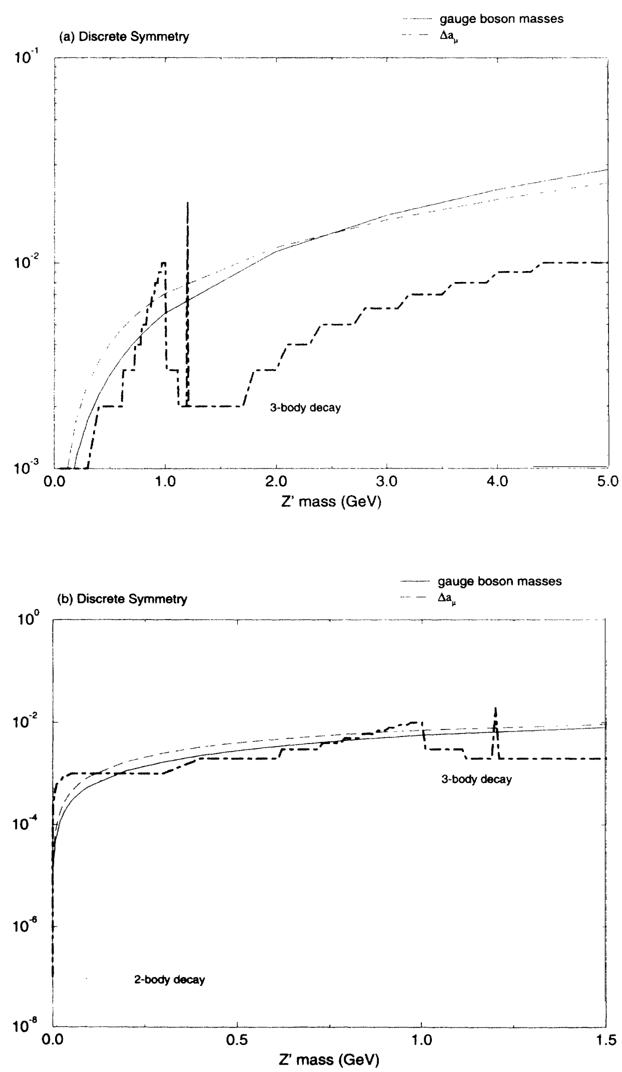

FIG. 2. (a) and (b) Constraints on the $(m_{Z'}, |a|)$ parameter space in the exact discrete symmetry model — (i) gauge boson masses (the allowed region is the area above the solid line), (ii) Δa_μ (area below the dashed line), and (iii) three-body decay (area below the dot-dashed line), (iv) two-body decay (area below the dotted line).

$$\frac{\Gamma(\tau \to \mu Z')}{\Gamma(\tau \to \mu \overline{\nu_\mu} \nu_\tau)} < 0.033 \quad \text{for } m_{Z'} \le 0.1 \text{ GeV}, \tag{38}$$

where the Goldstone boson has been replaced by Z'. (Without going into a detailed reanalysis of the experiment, we expect the above experimental bound to be approximately valid for our case where the final state boson has spin 1.) This bound rises up to 0.071 for $m_{Z'} = 0.5$ GeV. For the exact discrete symmetry case this ratio is given by

$$\frac{\Gamma(\tau \to \mu Z')}{\Gamma(\tau \to \mu \overline{\nu_\mu} \nu_\tau)} = \frac{96}{\sqrt{2}} \pi^2 \tan^2\theta_W \frac{m_W^2}{G_F m_\tau^4} |a|^2 f, \tag{39}$$

where

$$f = \left\{ 1 + \frac{(m_\mu^2 - 2m_{Z'}^2)}{m_\tau^2} - 6\frac{m_\mu}{m_\tau} + \frac{(m_\mu^2 - 2m_{Z'}^2)^2}{m_\tau^2 m_{Z'}^2} \right\} P \tag{40}$$

and

$$P = \sqrt{1 - \frac{(m_\mu + m_{Z'})^2}{m_\tau^2}} \sqrt{1 - \frac{(m_\mu - m_{Z'})^2}{m_\tau^2}}. \tag{41}$$

G_F is the Fermi constant and m_W is the mass of the W boson. By using Eqs. (38) and (39), the region of $(m_{Z'}, |a|)$ parameter space allowed by the two-body decay can be constructed. This is given by the region below the dotted curve in Fig. 2(b). From this one can see that the parameter space for $m_{Z'} < 0.5$ GeV is ruled out (and hence the window of $m_{Z'} < 0.2$ GeV allowed by the three-body constraint).

So, in summary, when all the constraints have been combined, much of the parameter space is ruled out. The remaining allowed regions are for $0.8 < m_{Z'} < 1.0$ GeV ($|a|$ varies between about 0.004 and 0.007) and $m_{Z'}$ around 1.2 GeV. [One might naively think that there ought to be another allowed region for sufficiently small $|a|$, and hence for a sufficiently light Z' boson, since the Z' decouples as $|a| \to 0$. However, as $|a| \to 0$ the local U(1)$'$ gauge symmetry tends toward becoming merely a global symmetry, and the longitudinal component of the Z' turns into its associated Goldstone boson. The two-body decay process considered above then has this Goldstone boson in the final state rather than the Z'. This can be seen explicitly from the fact that the right-hand side Eq. (39) does not go to zero as $|a|$ goes to zero.] It should be noted that we have taken the two-body constraint at face value, i.e., it applies for values of $m_{Z'}$ up to 0.5 GeV. This is the value quoted by the ARGUS Collaboration in Ref. [7]. Actually, the ARGUS experiment is supposed to be able to search for the two-body decay mode for values of $m_{Z'}$ up to about 1.53 GeV, given the experimental cuts and efficiencies. If the current trend of the two-body constraint continues beyond 0.5 GeV (the precise bound will obviously vary with the mass of Z' and becomes several orders of magnitude less severe near threshold) then the remaining allowed windows will be closed and the model will be ruled out.

Finally, we will briefly look at two processes related to the CERN e^+e^- collider LEP that could in principle probe the effects of our low mass Z' boson. First, the Z' will contribute a vertex correction at one-loop level to $Z \to \mu^+\mu^-$ and $Z \to \tau^+\tau^-$. An order of magnitude estimate of this effect yields $\delta\Gamma/\Gamma_{\ell\bar{\ell}} \sim \alpha_{\rm em}|a|^2 f$ where $\delta\Gamma$ is the change in the Z decay rate to $\ell^+\ell^-$ (as given by $\Gamma_{\ell\bar{\ell}}$). The quantity f is a function of mass ratios and is at most of order $\ln(m_Z/m_\mu) \sim 10$. Thus we estimate that $\delta\Gamma/\Gamma_Z \sim 3 \times 10^{-3}|a|^2$ where Γ_Z is the full width. For $|a| < 10^{-2}$ this implies that $\delta\Gamma/\Gamma_Z$ is about three orders of magnitude smaller than the experimental uncertainty of $\sim 4 \times 10^{-4}$ on the full Z width. Second, the Z boson will have the rare three-body decay mode $Z \to \mu^\pm\tau^\mp Z'$. The exact formula for the partial width of this decay yields $\Gamma(Z \to \mu^\pm\tau^\mp Z')/\Gamma_Z \sim 10^{-3}|a|^2$. This should be compared to the experimental bound $\Gamma(Z \to \mu^\pm\tau^\mp)/\Gamma_Z < 4.8 \times 10^{-5}$. For realistic values of $|a|$, the three-body partial width is an order of magnitude or two smaller than this bound.

We now discuss what happens when the discrete symmetry is spontaneously broken by the vacuum. The cal-

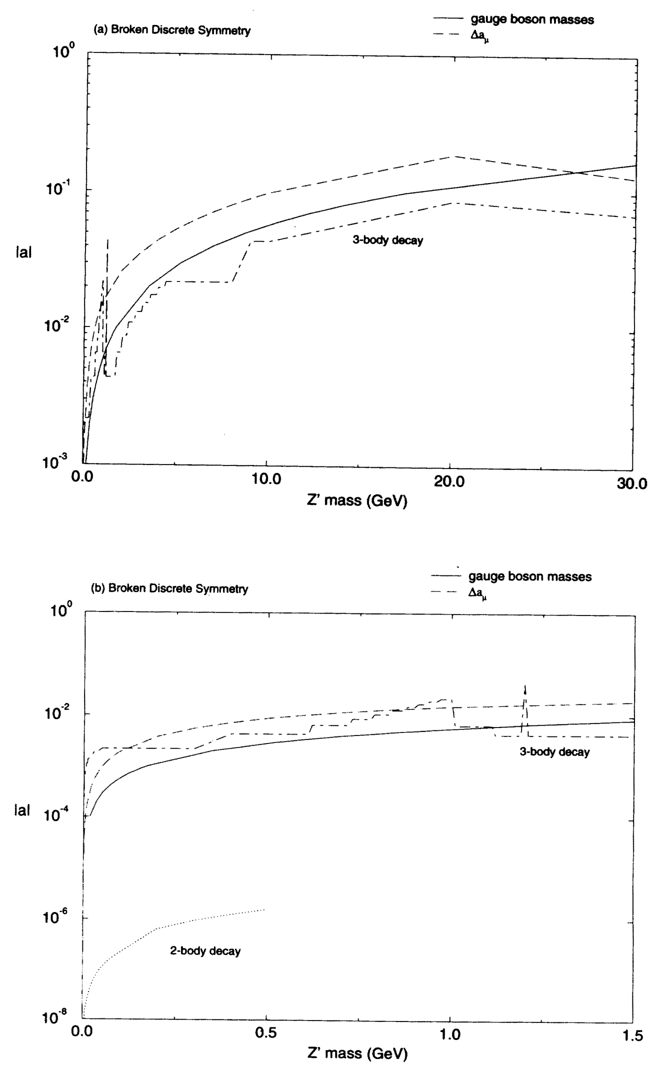

FIG. 3. (a) and (b) The same as Fig. 2 but for the case of spontaneously broken discrete symmetry.

culations of the constraints are similar to those in the unbroken discrete symmetry case (see the appendix for further details). For the gauge boson mass constraint, the calculation uses the mass relations of Eqs. (A1)–(A3) in the Appendix. The calculation of the anomalous magnetic moment [8], two-body decay and three-body decay constraints can be carried over from the exact discrete symmetry case using the substitution $|a| \to |a|\cos\phi\sin 2\theta_L$, where ϕ and θ_L are the gauge boson and μ-τ mixing angles, respectively. (Since the gauge boson mixing is required to be small, we have set $\phi \simeq 0$.) The results are given in Figs. 3(a) and (b) which shows that the broken discrete symmetry case is qualitatively similar to the exact discrete symmetry case. So the conclusions made for the exact discrete symmetry case essentially also hold for this case. The constraints were expected to be less stringent, which they are, but not enough to change anything significantly. For example, there is still no allowed parameter space for $m_{Z'} > (m_\tau - m_\mu)$ since the allowed regions from the gauge boson mass and three-body constraints never overlap for $m_{Z'} > 1.5$ GeV. The reason for this is due to the fact that $\sin 2\theta_L$ cannot be made arbitrarily small [this is a consequence of the discrete symmetry of the Yukawa Lagrangian of Eq. (9)]. It turns out that $\sin 2\theta_L$ cannot be smaller than about 0.46 [see Eq. (A14) in the Appendix]. Therefore, one obvious way to ease the constraints is to abandon the discrete symmetry altogether so that the μ-τ mass mixing remains unconstrained.

In conclusion then, the model for both the exact and spontaneously broken discrete symmetry cases is ruled out if $m_{Z'} \leq 0.5$ GeV or $m_{Z'} > (m_\tau - m_\mu)$. For 0.5 GeV $< m_{Z'} \leq (m_\tau - m_\mu)$ there exist windows of allowed parameter space. However, if the trend of the two-body decay constraint continues in this region, then these windows will certainly be closed. *This rather stringent bound from the two-body decay is, nevertheless, very interesting, because it means that the decay $\tau \to \mu Z'$ is by far the best way to test our low-mass Z' model.* One way to view the significance of our model is therefore the following: One should as a matter of phenomenological generality be interested in the possibility that τ might have a rare decay mode into μ plus a spin-1 boson, just as one is in general interested in two-body final states where the boson has spin 0. Our model provides a simple model where this phenomenological possibility is realized. The interesting thing is that the $\tau \to \mu Z'$ decay is essentially the only important piece of new low-energy physics that the model predicts, provided that the Higgs bosons are heavier than a few tens of a GeV.

ACKNOWLEDGMENTS

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APPENDIX: SPONTANEOUSLY BROKEN DISCRETE SYMMETRY

In this appendix some details concerning the model with spontaneously broken discrete symmetry are given. The results given in the following are the ones used to calculate the constraints discussed in the text.

The gauge boson and fermion sector: When the Higgs doublets develop nonzero VEV's, i.e., $|\langle\phi_i\rangle| = u_i$ for $i = 1, 2, 3$, the electroweak and U(1)′ symmetries are broken. This results in a neutral gauge boson mass(-squared) matrix given by

$$\frac{1}{2}\frac{e^2}{c_W^2 s_W^2}\begin{bmatrix} (u_1^2+u_2^2+u_3^2) & -4as_W(u_2^2-u_3^2) \\ -4as_W(u_2^2-u_3^2) & 16a^2s_W^2(u_2^2+u_3^2)\end{bmatrix} \qquad (A1)$$

in the (Z, Z') basis. In terms of mass eigenstates (Z_1, Z_2),

$$Z = \cos\phi Z_1 + \sin\phi Z_2,$$
$$Z' = -\sin\phi Z_1 + \cos\phi Z_2, \qquad (A2)$$

where ϕ is the Z-Z' mixing angle and is given by

$$\tan 2\phi = \frac{8as_W(u_2^2-u_3^2)}{(u_1^2+u_2^2+u_3^2) - 16a^2s_W^2(u_2^2+u_3^2)}. \qquad (A3)$$

In the exact discrete symmetry limit ($u_2 = u_3$), the above reduces to that given in Eq. (12). The mass of the charged $W^\pm$ boson is $m_W^2 = \frac{1}{2}g_2^2(u_1^2+u_2^2+u_3^2)$.

From the Yukawa Lagrangian of Eq. (9), the $\mu-\tau$ mass matrix can be written as

$$\mathcal{L}_{\text{mass}} = \overline{L_L}\mathcal{M}L_R + \text{H.c.}, \qquad (A4)$$

where

$$L_{L,R} = (\mu_{L,R}, \tau_{L,R})^T, \qquad (A5)$$

$$\mathcal{M} = \begin{pmatrix} m_1 & m_2 \\ m_3 & m_1 \end{pmatrix} \qquad (A6)$$

and $m_1 = \lambda u_1$, $m_2 = \lambda' u_2$, and $m_3 = \lambda' u_3$. The matrix $\mathcal{M}$ can be diagonalized by a bi-unitary transformation so that

$$\mathcal{D} = \text{Diag}(m_\mu, m_\tau) = U_L\mathcal{M}U_R^\dagger, \qquad (A7)$$

$$L'_{L,R} = U_{L,R}L_{L,R}, \qquad (A8)$$

where the $L'_{L,R}$ denotes the mass eigenstates. $U_{L,R}$ can be parametrized as

$$U_{L,R} = \begin{pmatrix} \cos\theta_{L,R} & \sin\theta_{L,R} \\ -\sin\theta_{L,R} & \cos\theta_{L,R}\end{pmatrix}\begin{pmatrix} e^{-i\frac{\delta}{2}} & 0 \\ 0 & e^{i\frac{\delta}{2}}\end{pmatrix}, \qquad (A9)$$

where

$$\tan 2\theta_L = \frac{2(m_1m_3^* + m_1^*m_2)}{|m_3|^2 - |m_2|^2} = -\tan 2\theta_R, \qquad (A10)$$

$$\theta_R = (2n+1)\frac{\pi}{2} - \theta_L, \qquad (A11)$$

where n is an integer. In the exact discrete symmetry limit $\theta_L = \theta_R = \frac{\pi}{4}$ and $\delta = 0$. By using the above relations one can rewrite θ_L in terms of the VEV's and the μ and τ masses such that

$$\cos 2\theta_L = \frac{(m_\tau^2 - m_\mu^2)}{(u_3^2 - u_2^2)} \frac{(u_3^2 - u_2^2)^2 - 4u_2^2 u_3^2 \sin^2 2\delta}{[u_2^2 + u_3^2 + 2u_2 u_3 \cos 2\delta]\,[m_\tau^2 + m_\mu^2 + 2m_\tau m_\mu \cos\Delta]}, \tag{A12}$$

where

$$\sin\Delta = \frac{u_2 u_3}{u_2^2 - u_3^2}\frac{m_\tau^2 - m_\mu^2}{m_\tau m_\mu}\sin 2\delta. \tag{A13}$$

Furthermore, one can show that

$$|\cos 2\theta_L| \le \frac{m_\tau - m_\mu}{m_\tau + m_\mu} \quad \Rightarrow \quad |\sin 2\theta_L| \ge 0.46. \tag{A14}$$

Using the foregoing results, the neutral current gauge interactions can be written as

$$\mathcal{L}_{\rm int}^{Z} = -\frac{e}{c_W s_W}\bar{f}\gamma_\mu Z^\mu (I_3 - s_W^2 Q) P_{L,R} f, \tag{A15}$$

$$\mathcal{L}_{\rm int}^{Z'} = -\frac{e}{c_W} a\bar{L}\gamma_\mu Z'^\mu \begin{pmatrix} -\gamma_5\cos 2\theta_L & -\sin 2\theta_L \\ -\sin 2\theta_L & \gamma_5 \cos 2\theta_L \end{pmatrix} L - \frac{e}{c_W} a\bar{N}\gamma_\mu Z'^\mu \begin{pmatrix} \cos 2\theta_L & -\sin 2\theta_L \\ -\sin 2\theta_L & -\cos 2\theta_L \end{pmatrix} P_L N, \tag{A16}$$

where $P_{L,R} = \frac{1}{2}(1 \pm \gamma_5)$, $L = (\mu, \tau)^T$, $N = (\nu_\mu, \nu_\tau)^T$, and $f = L$ or N. The Z and Z' fields can be written in terms of their mass eigenstates by using Eq. (A2). Note that these interactions reduce to the simple form of Eqs. (13) and (14) in the exact discrete symmetry limit.

The Higgs boson sector: The Higgs potential is given by

$$\begin{aligned} V(\phi_1,\phi_2,\phi_3) = & -\mu_1^2(\phi_1^\dagger\phi_1) - \mu_2^2(\phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3) + k_1(\phi_1^\dagger\phi_1)^2 + k_2\left[(\phi_2^\dagger\phi_2)^2 + (\phi_3^\dagger\phi_3)^2\right] \\ & + k_{12}(\phi_1^\dagger\phi_1)\left[(\phi_2^\dagger\phi_2) + (\phi_3^\dagger\phi_3)\right] + k_{12}'\left[(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + (\phi_1^\dagger\phi_3)(\phi_3^\dagger\phi_1)\right] \\ & + k_{23}(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) + k_{23}'(\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2) + k_4 Re(\phi_1^\dagger\phi_2)(\phi_1^\dagger\phi_3) + k_4' Im(\phi_1^\dagger\phi_2)(\phi_1^\dagger\phi_3). \end{aligned} \tag{A17}$$

The minimization conditions are given by

$$\begin{aligned} 0 &= -\mu_1^2 + 2k_1 u_1^2 + 2\tilde{k}_{12}\left(u_2^2 + u_3^2\right) + k_4 u_2 u_3, \\ 0 &= 2\left(-\mu_2^2 + 2k_2 u_2^2 + 2\tilde{k}_{12}u_1^2 + 2\tilde{k}_{23}u_3^2\right) u_2 + k_4 u_1^2 u_3, \\ 0 &= 2\left(-\mu_2^2 + 2k_2 u_3^2 + 2\tilde{k}_{12}u_1^2 + 2\tilde{k}_{23}u_2^2\right) u_3 + k_4 u_1^2 u_2, \end{aligned} \tag{A18}$$

together with $k_4' = 0$, where

$$\tilde{k}_{12} \equiv \frac{1}{2}(k_{12} + k_{12}') \quad \text{and} \quad \tilde{k}_{23} \equiv \frac{1}{2}(k_{23} + k_{23}'). \tag{A19}$$

Equation (A17) reduces to that of Eq. (15) with

$$\begin{aligned} k_1 &= \lambda_1 + \lambda_4, & k_2 &= \lambda_2 + \lambda_3 + \lambda_4 \\ k_{12} &= 2\lambda_4 + \lambda_6 + \lambda_7, & k_{12}' &= -\lambda_6 \\ k_{23} &= 2\lambda_2 - 2\lambda_3 + 2\lambda_4 + \lambda_5, & k_{23}' &= -\lambda_5 \\ k_4 &= -2\lambda_7, & k_4' &= 0. \end{aligned} \tag{A20}$$

(a) The CP-even mass(-squared) matrix, $A_{ij} = A_{ji}$ in the basis (H_1, H_2, H_3) is given by

$$\begin{aligned} A_{11} &= 4k_1 u_1^2, & A_{12} &= 4\tilde{k}_{12}u_1 u_2 + k_4 u_1 u_3, \\ A_{13} &= 4\tilde{k}_{12}u_1 u_3 + k_4 u_1 u_2, & A_{22} &= 4k_2 u_2^2 - \tfrac{1}{2}k_4 u_1^2 (u_3/u_2), \\ A_{23} &= 4\tilde{k}_{23}u_2 u_3 + \tfrac{1}{2}k_4 u_1^2, & A_{33} &= 4k_2 u_3^2 - \tfrac{1}{2}k_4 u_1^2 (u_2/u_3). \end{aligned} \tag{A21}$$

In general, A_{ij} has no zero eigenvalues. So there are three real massive physical scalars.

(b) The CP-odd mass(-squared) matrix, $B_{ij} = B_{ji}$ in the basis (η_1, η_2, η_3) is given by

$$\begin{aligned} B_{11} &= -2k_4 u_2 u_3, & B_{12} &= k_4 u_1 u_3, \\ B_{13} &= k_4 u_1 u_2, & B_{22} &= -\tfrac{1}{2}k_4 u_1^2 (u_3/u_2), \\ B_{23} &= -\tfrac{1}{2}k_4 u_1^2, & B_{33} &= -\tfrac{1}{2}k_4 u_1^2 (u_2/u_3). \end{aligned} \tag{A22}$$

B_{ij} has two zero eigenvalues, and hence there are two Goldstone bosons and one CP-odd real scalar, $\eta_{\rm phys}$. The Goldstone fields corresponding to Z_1 and Z_2 are given respectively by

$$G_{Z_1} = \frac{g_2}{m_{Z_1}\sqrt{2}c_W}\{u_1\cos\phi\eta_1 + u_2(\cos\phi + 4as_W\sin\phi)\eta_2 + u_3(\cos\phi - 4as_W\sin\phi)\eta_3\},$$

$$G_{Z_2} = \frac{g_2}{m_{Z_2}\sqrt{2}c_W}\{u_1\sin\phi\eta_1 + u_2(\sin\phi - 4as_W\cos\phi)\eta_2 + u_3(\sin\phi + 4as_W\cos\phi)\eta_3\}. \quad \text{(A23)}$$

The physical CP-odd field is given by

$$\eta_{\rm phys} = \frac{2u_2u_3\eta_1 - u_1u_3\eta_2 - u_1u_2\eta_3}{\sqrt{u_1^2(u_2^2+u_3^2) + 4u_2^2u_3^2}} \quad \text{(A24)}$$

with its mass given by $-\frac{1}{2}k_4[u_1^2\,(u_2/u_3 + u_3/u_2) + 4u_2u_3]$.

(c) The charged scalar mass(-squared) matrix, $C_{ij} = C_{ji}$ in the basis $(\phi_1^\pm, \phi_2^\pm, \phi_3^\pm)$ is given by

$$C_{11} = -k'_{12}(u_2^2 + u_3^2) - k_4u_2u_3,$$
$$C_{12} = k'_{12}u_1u_2 + \frac{1}{2}k_4u_1u_3,$$
$$C_{13} = k'_{12}u_1u_3 + \frac{1}{2}k_4u_1u_2,$$
$$C_{22} = -k'_{23}u_3^2 - \left(k'_{12} + \frac{1}{2}k_4\frac{u_3}{u_2}\right)u_1^2,$$
$$C_{23} = k'_{23}u_2u_3,$$
$$C_{33} = -k'_{23}u_2^2 - \left(k'_{12} + \frac{1}{2}k_4\frac{u_2}{u_3}\right)u_1^2. \quad \text{(A25)}$$

There are two massive charged scalars and one Goldstone boson associated with the $W^\pm$ boson:

$$G^\pm = \frac{g_2}{m_W\sqrt{2}}\left(u_1\phi_1^\pm + u_2\phi_2^\pm + u_3\phi_3^\pm\right). \quad \text{(A26)}$$

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[6] Note that these equations also give

$$u_1^2 = \frac{s_W^2m_W^2}{2\pi\alpha_{\rm em}}\left\{1 - \frac{1}{16a^2\tan^2\theta_W}\left(\frac{m_{Z'}}{m_W}\right)^2\right\}.$$

For values of a and $m_{Z'}$ close to saturating the bound of Eq. (33), u_1 becomes small so that it is possible for certain quark Yukawa couplings, $\lambda_q = m_q/u_1$ (where m_q is the generic quark mass), to become nonperturbative.

[7] ARGUS Collaboration, H. Albrecht *et al.*, Phys. Lett. B **246**, 278 (1990); Mark III Collaboration, R. M. Baltrusaitis *et al.*, Phys. Rev. Lett. **55**, 1842 (1985).

[8] In the broken discrete symmetry case, there is an additional contribution to $\Delta a_\mu^{Z'}$ which could be potentially large. This comes from a graph similar to that in Fig. 1 where the internal τ lines are replaced by μ ones. This contribution is given by

$$\Delta a_\mu^{Z'} \simeq -\frac{5\alpha_{\rm em}}{3\pi}\left(\frac{m_\mu}{m_{Z'}}\right)^2\frac{|a|^2}{c_W}\cos^2 2\theta_L,$$

which is suppressed by the factor $\left(\frac{m_\mu}{m_{Z'}}\right)^2$ for $m_{Z'} > m_\mu$. Ignoring this contribution will not affect any of our conclusions since the two-body decay constraint is so stringent for $m_{Z'} \le 0.5$ GeV. In this case also, the previously forbidden decay $\tau \to 3\mu$ is now allowed. However, the partial decay rate is proportional to $|a|^4$ and is thus very small.