

Decays of B mesons to two light pseudoscalars

Michael Gronau

Department of Physics, Technion-Israel Institute of Technology, Haifa 32000, Israel

Oscar F. Hernández* and David London†

Laboratoire de Physique Nucléaire, Université de Montréal, Montréal, PQ, Canada H3C 3J7

Jonathan L. Rosner

Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637

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The decays $B \rightarrow PP$, where P denotes a light pseudoscalar meson, are analyzed. Numerous triangle relations for amplitudes hold within flavor SU(3) symmetry, relating (for example) the decays $B^+ \rightarrow \pi^+\pi^0$, $\pi^0 K^+$, and $\pi^+ K^0$. Such relations can improve the possibilities for learning about phases of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and for early detection of CP -violating asymmetries. Within the context of a graphical analysis of decays, relations are analyzed among SU(3) amplitudes which hold if some graphs are neglected. These relations allow the determination of weak and strong phases from rate measurements alone. Estimates of SU(3) breaking are included.

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I. INTRODUCTION

The present explanation of CP violation in the neutral kaon system makes use of phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix describing the charge-changing weak transitions of quarks. The CP -violating decays of mesons (“ B ” mesons) containing a b or \bar{b} quark provide numerous ways to check this picture. However, it is not necessary to observe a CP -violating quantity (such as a rate asymmetry) in order to measure CKM phases. In the present article we undertake a general flavor SU(3) analysis of the decays $B \rightarrow PP$, where P denotes a light pseudoscalar meson, and find that information on CKM phases can be obtained from the study of rates alone. We also find that the SU(3) relations are of use in interpreting and anticipating CP -violating asymmetries in these decays. We begin by mentioning why $B \rightarrow PP$ decays have been found interesting in previous works.

The study of rate asymmetries in decays of neutral B mesons to CP eigenstates provides crisp information on CKM phases, but at several costs. (1) One must know whether a neutral B meson was a B^0 or \bar{B}^0 at the time of production, and one must usually study time-dependent rate asymmetries. (2) In decays to CP eigenstates, the possibility of two distinct weak subprocesses contributing to each direct decay amplitude requires a somewhat more elaborate analysis. As an example, the dominant subprocess in $B^0 \rightarrow \pi^+\pi^-$, is expected to be one involving $\bar{b} \rightarrow \bar{d}u\bar{u}$. However, if direct $\bar{b} \rightarrow \bar{d}$ (“penguin”)

transitions mediated by virtual u , c , and t quarks and gluons also contribute, the observed CP -violating asymmetries are affected [1]. Here, the study of rates for all possible charge states in $B \rightarrow \pi\pi$ and $\bar{B} \rightarrow \pi\pi$ decays and the time-dependence of $B^0(t) \rightarrow \pi^+\pi^-$ allows one to separate by isospin all the relevant effects and to obtain information on CKM phases [2].

In B decays to “self-tagging” modes such as $B^+ \rightarrow \pi^0 K^+$ and $B^+ \rightarrow \pi^+ K^0$, a difference between the branching ratio for the mode and its charge conjugate immediately signals CP violation. In order that a difference in rates be observable, it is necessary to have a nonzero difference between the phase shifts in two strong eigenchannels ($I = 1/2$ and $I = 3/2$ in the case of πK), as well as two different weak subprocesses contributing to the decay. In $B \rightarrow \pi K$ the two weak subprocesses are $\bar{b} \rightarrow \bar{u}u\bar{s}$ and the CKM-favored $\bar{b} \rightarrow \bar{s}$ penguin amplitude. Using isospin, it is possible to separate these two subprocesses in order to extract the CKM phase from decays of neutral B mesons to the CP eigenstate $\pi^0 K_s$. For this purpose one must measure the time dependence of $B^0(t) \rightarrow \pi^0 K_s$ as well as the rates for decays of all possible charge states in $B \rightarrow \pi K$ and $\bar{B} \rightarrow \pi \bar{K}$ processes [3–5].

Information about final-state phases *per se* has already been obtained in decays of charmed mesons to $\pi \bar{K}$, and phase shift differences have been found significant in some channels [6]. No evidence for strong phase shift differences in B decays has been found yet [7].

Decays such as $B \rightarrow \pi\pi$ and $B \rightarrow \pi K$ are related to one another by SU(3) symmetry [8–13]. It is then natural to ask whether the isospin analyses of Refs. [2–5] can be generalized to SU(3) in order to obtain further information about the prospects for observing CP violation in such decays as $B \rightarrow \pi K$, or for measuring weak and final-state phases. In the present paper we report the

*Electronic address: oscarh@lps.umontreal.ca

†Electronic address: london@lps.umontreal.ca

results of efforts to find such information.

We have undertaken a systematic review of the SU(3) predictions for the decays $B \rightarrow PP$. We have examined a number of linear relations among amplitudes which follow from previous SU(3) analyses, but whose simplicity seems to have gone unnoticed. Among these is a relation between the amplitude for $B^+ \rightarrow \pi^+\pi^0$ and pairs of amplitudes for $B \rightarrow \pi K$. This relation is expressed as a triangle in the complex plane. It is a necessary (but not sufficient) condition for the observability of CP violation in $B^\pm \rightarrow \pi K$ decays that this triangle have nonzero area. The triangle relation simplifies the analysis used to obtain information on CKM angles from the study of time-dependent neutral B decays to the CP eigenstate $\pi^0 K_s$, which uses a somewhat more involved quadrangle relation [3–5].

This triangle is also of great utility in another respect. Using only charged B decays, it allows the clean extraction of a weak CKM phase. Furthermore, using it and other triangle relations involving the decays $B \rightarrow \pi\pi$ and $B \rightarrow \pi K$, along with the rates for $B \rightarrow K\bar{K}$, one can also obtain information about two weak CKM phases, as well as about final-state phases. A possible experimental advantage of these methods for obtaining CKM phase information is that no time-dependent measurements are necessary—only decay rates are needed. We have also found relations between certain final-state phase differences in $B \rightarrow \pi\pi$ and $B \rightarrow \pi K$ decays.

Our analysis is performed using a simple graphical method which has been shown equivalent [8] to a decomposition in terms of SU(3) reduced matrix elements. The graphical description is overcomplete in the sense that we can write six different graphs, but they always appear in the form of five linear combinations, corresponding to the five reduced matrix elements in an SU(3) decomposition.

It is possible that the graphs which we use to construct decay amplitudes take on a more direct meaning. Thus, a process which could take place only as a result of a certain graph (such as quark-antiquark annihilation) might not be fed by rescattering from another graph. We thus make a systematic study of the effects of neglecting certain graphs whose contributions are expected to be small in the limit of infinite mass of the b quark. These correspond to W exchange, W in the direct channel (“annihilation”), and annihilation through a penguin graph (vacuum flavor quantum numbers in the direct channel). We are left with three other types of graph and, correspondingly, three independent combinations of reduced matrix elements.

It is thus our purpose to draw attention to a number of interesting questions about *amplitudes* and their phases which can be addressed purely from the standpoint of *rates* in $B \rightarrow PP$ decays. A full discussion of amplitudes was performed in Ref. [8], but without much emphasis on the simple linear relations among them. The treatment in Ref. [9] dealt primarily with rates, for which few direct relations exist in the $\Delta C = 0$ sector. There does not appear to have been a previous discussion of the effects of neglect of certain contributions in the graphical description of decays.

The effects of SU(3) breaking can be taken into account in factorizable amplitudes in which a weak current

produces a pion or kaon by the use of the appropriate meson decay constant f_π or $f_K \approx 1.2f_\pi$. It may be necessary to test factorization for the full set of amplitudes we consider in order to justify this approach fully.

In Sec. II we recapitulate the full SU(3) analysis of Ref. [8] in terms of graphical contributions. We stress the wide variety of relations that hold among different amplitudes. These include not only separate relations among $\Delta S = 0$ and $|\Delta S| = 1$ transitions, but also relations *between* the two sectors. We give specific examples relating to $B \rightarrow \pi\pi$ and $B \rightarrow \pi K$ decays, and show how to extract useful final-state interaction information from these processes. For completeness, we also quote results involving an octet η , which we denote η_s , and mention their limited usefulness. While we concentrate on $\Delta C = 0$ transitions, we briefly treat decays to final states involving charm, in the context of extraction of final-state phase differences from decay rates.

We next specialize, in Sec. III, to the case in which certain diagrams contributing to decay amplitudes are neglected. We translate this assumption into linear relations among SU(3) reduced matrix elements, and discuss the corresponding relations among amplitudes for decays. The physical consequences for observability of CP violation in various channels are mentioned. The neglect of some diagrams permits one to determine the weak CKM phases, the strong final-state phases, and the sizes of the relevant contributing diagrams from rate measurements alone. For example, the measurement of charged B -meson decays to $\pi\pi$ and πK permits the extraction of the CKM phase γ .

With appropriate warnings, we treat the case of a “physical” η and η' using graphical methods in Sec. IV. Some possible effects of SU(3) breaking are mentioned in Sec. V. We summarize our results and discuss experimental prospects in Sec. VI. An Appendix lists the decomposition of reduced matrix elements labeled by SU(3) representations into graphical contributions, in order to display explicitly the connection between the two languages.

II. SU(3) ANALYSIS

A. Definitions and counting of reduced matrix elements

Adopting the same conventions as Ref. [8], we take the u , d , and s quarks to transform as a triplet of flavor SU(3), and the $-\bar{u}$, \bar{d} , and \bar{s} to transform as an antitriplet. The mesons are defined in such a way as to form isospin multiplets without extra signs. Thus, the pions will belong to an isotriplet if we take

$$\pi^+ \equiv u\bar{d}, \quad \pi^0 \equiv (d\bar{d} - u\bar{u})/\sqrt{2}, \quad \pi^- \equiv -d\bar{u}, \quad (1)$$

while the kaons and antikaons will belong to isodoublets if we take

$$K^+ \equiv u\bar{s}, \quad K^0 \equiv d\bar{s}, \quad (2)$$

$$\bar{K}^0 \equiv s\bar{d}, \quad K^- \equiv -s\bar{u}. \quad (3)$$

We choose $\eta_8 \equiv (2s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{6}$. The true η looks more like an octet-singlet mixture with a mixing angle of around 19° to 20° [14], such that $\eta \approx (s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{3}$. In order to treat such a state correctly, we would have to introduce additional reduced matrix elements in the context of SU(3), additional graphs within the context of a graphical analysis, and decays involving η' . We expect the predictive power of such an approach would be minimal. Thus, we quote results for octet η 's mainly for completeness.

The B mesons are taken to be $B^+ \equiv \bar{b}u$, $B^0 \equiv \bar{b}d$, and $B_s \equiv \bar{b}s$. Their charge conjugates are defined as $B^- \equiv -b\bar{u}$, $\bar{B}^0 \equiv b\bar{d}$, and $\bar{B} \equiv b\bar{s}$.

We now count reduced matrix elements for transitions to charmless PP final states. The weak Hamiltonian operators associated with the transitions $\bar{b} \rightarrow \bar{q}u\bar{u}$ and $\bar{b} \rightarrow \bar{q}(q = d \text{ or } s)$ can transform as a $\mathbf{3}^*$, $\mathbf{6}$, or $\mathbf{15}^*$ of SU(3). When combined with the triplet light quark in the B meson, these operators then lead to the following representation in the direct channel:

$$\mathbf{3}^* \times \mathbf{3} = \mathbf{1} + \mathbf{8}_1, \quad (4)$$

$$\mathbf{6} \times \mathbf{3} = \mathbf{8}_2 + \mathbf{10}, \quad (5)$$

$$\mathbf{15}^* \times \mathbf{3} = \mathbf{8}_3 + \mathbf{10}^* + \mathbf{27}. \quad (6)$$

We are concerned with couplings of these representations to the symmetric product of two octets (the pseudoscalar mesons, which are in an S -wave final state). Since $(\mathbf{8} \times \mathbf{8})_s = \mathbf{1} + \mathbf{8} + \mathbf{27}$, the singlet, octet, and 27-plet each have unique couplings to this pair of mesons, while the decimets cannot couple to them. Thus, the decays are characterized by one singlet, three octet, and one 27-plet amplitude. The decomposition in terms of these amplitudes is given in Ref. [8], and is implied by the results of the Appendix. Separate amplitudes apply to the cases of strangeness-preserving and strangeness-changing transitions. As we shall see, there are relations between linear combinations of the strangeness-preserving and strangeness-changing amplitudes.

B. Amplitudes in terms of graphical contributions

The SU(3) analysis of $\Delta C = 0$ $B \rightarrow PP$ decays is equivalent to a decomposition of amplitudes in terms of graphical contributions. We shall adopt a notation in which an unprimed amplitude stands for a strangeness-preserving decay, while a primed contribution stands for a strangeness-changing decay. The relevant graphs are illustrated in Fig. 1. They consist of the following [10]: (1) a (color-favored) “tree” amplitude T or T' , associated with the transition $\bar{b} \rightarrow \bar{q}u\bar{u}$ ($q = d$ or s) in which the $\bar{q}u$ system forms a color-singlet pseudoscalar meson while the \bar{u} combines with the spectator quark to form the other pseudoscalar meson; (2) a “color-suppressed” amplitude C or C' , associated with the transition $\bar{b} \rightarrow \bar{u}u\bar{q}$ in

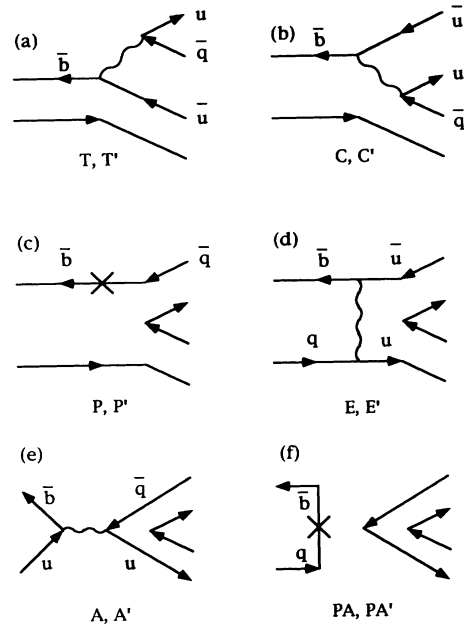


FIG. 1. Diagrams describing decays of B mesons to pairs of light pseudoscalar mesons. Here $\bar{q} = \bar{d}$ for unprimed amplitudes and \bar{s} for primed amplitudes. (a) “Tree” (color-favored) amplitude T or T' ; (b) “color-suppressed” amplitude C or C' ; (c) “penguin” amplitude P or P' (we do not show intermediate quarks and gluons); (d) “exchange” amplitude E or E' ; (e) “annihilation” amplitude A or A' ; (f) “penguin annihilation” amplitude PA or PA' .

which the $\bar{u}u$ system is incorporated into a neutral pseudoscalar meson while the \bar{q} combines with the spectator quark to form the other meson; (3) a “penguin” amplitude P or P' associated with the transition $\bar{b} \rightarrow \bar{q}u$ involving virtual quarks of charge $2/3$ coupling to one or more gluons in a loop; (4) an “exchange” amplitude E or E' in which the \bar{b} quark and an initial q quark in the decaying (neutral) B meson exchange a W and become a $\bar{u}u$ pair; (5) an “annihilation” amplitude A or A' contributing only to charged B decay through the subprocess $\bar{b}u \rightarrow \bar{q}u$ by means of a W in the direct channel; and (6) a “penguin annihilation” amplitude PA or PA' in which an initial $\bar{b}q$ state annihilates into vacuum quantum numbers.

This set of amplitudes is over-complete. The physical processes of interest involve only five distinct linear combinations of the amplitudes, which are given in terms of SU(3) direct-channel representations in the Appendix. Here we shall simply mention acceptable linearly independent sets of amplitudes once we have expanded all processes of interest in terms of the above contributions.

The results for $\Delta S = 0$ transitions are shown in Table I, while those for $|\Delta S| = 1$ transitions are shown in Table II. We can identify the following linearly independent combinations of $\Delta S = 0$ amplitudes: for example, the combination $C + T$ occurs in $B^+ \rightarrow \pi^+\pi^0$; the combination $C - P$ occurs in $B_s \rightarrow \pi^0\bar{K}^0$; the combination

TABLE I. Decomposition of $B \rightarrow PP$ amplitudes for $\Delta C = \Delta S = 0$ transitions in terms of graphical contributions.

Final state	T	C	P	E	A	PA
$B^+ \rightarrow \pi^+\pi^0$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	0	0	0	0
$K^+\bar{K}^0$	0	0	1	0	1	0
$\pi^+\eta_8$	$-1/\sqrt{6}$	$-1/\sqrt{6}$	$-2/\sqrt{6}$	0	$-2/\sqrt{6}$	0
$B^0 \rightarrow \pi^+\pi^-$	-1	0	-1	-1	0	-1
$\pi^0\pi^0$	0	$-1/\sqrt{2}$	$1/\sqrt{2}$	$1/\sqrt{2}$	0	$1/\sqrt{2}$
K^+K^-	0	0	0	-1	0	-1
$K^0\bar{K}^0$	0	0	1	0	0	1
$\pi^0\eta_8$	0	0	$-1/\sqrt{3}$	$1/\sqrt{3}$	0	0
$\eta_8\eta_8$	0	$1/3\sqrt{2}$	$1/3\sqrt{2}$	$1/3\sqrt{2}$	0	$1/\sqrt{2}$
$B_s \rightarrow \pi^+K^-$	-1	0	-1	0	0	0
$\pi^0\bar{K}^0$	0	$-1/\sqrt{2}$	$1/\sqrt{2}$	0	0	0
$\eta_8\bar{K}^0$	0	$-1/\sqrt{6}$	$1/\sqrt{6}$	0	0	0

$P + A$ occurs in $B^+ \rightarrow K^+\bar{K}^0$; the combination $P + PA$ occurs in $B^0 \rightarrow K^0\bar{K}^0$; the combination $E + PA$ occurs in $B^0 \rightarrow K^+K^-$.

Similarly, for the amplitudes in Table II, the combi-

nation $P' + T'$ occurs in $B^0 \rightarrow \pi^-K^+$, the combination $C' - P'$ occurs in $B^0 \rightarrow \pi^0K^0$, the combination $P' + A'$ occurs in $B^+ \rightarrow \pi^+K^0$, the combination $P' + PA'$ occurs in $B_s \rightarrow K^0\bar{K}^0$, and the combination $E' + PA'$ occurs in $B_s \rightarrow \pi^+\pi^-$.

It is not possible to identify linear combinations of decay amplitudes which depend upon the six graphical contributions separately.

C. Linear relations among amplitudes

Since each table contains 12 decay amplitudes while there are only five linearly independent reduced matrix elements, it must be possible to find seven amplitude relations for each. This is indeed the case. We write relations in terms of decay amplitudes, and then translate them into statements about the corresponding combinations of graphical contributions.

1. $\Delta S = 0$ processes

A familiar isospin relation for $B \rightarrow \pi\pi$ decays, expressing the fact that there is just one $I = 0$ and one $I = 2$ amplitude as a result of the form of the interaction giving rise to the decay, is [2]

$$A(B^0 \rightarrow \pi^+\pi^-) + \sqrt{2}A(B^0 \rightarrow \pi^0\pi^0) = \sqrt{2}A(B^+ \rightarrow \pi^+\pi^0), \quad (7)$$

or

$$-(T + P + E + PA) + (-C + P + E + PA) = -(C + T). \quad (8)$$

If the rates are such that the triangle must have nonzero area, we conclude either that there are different interactions in the $I = 0$ and $I = 2$ final states, or that there are important contributions from amplitudes with different CKM phases (such as T and P), or both.

There are two other triangular relations involving only pions and kaons in the final state:

TABLE II. Decomposition of $B \rightarrow PP$ amplitudes for $\Delta C = 0, |\Delta S| = 1$ transitions in terms of graphical contributions.

Final state	T'	C'	P'	E'	A'	PA'
$B^+ \rightarrow \pi^+K^0$	0	0	1	0	1	0
π^0K^+	$-1/\sqrt{2}$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	0	$-1/\sqrt{2}$	0
η_8K^+	$-1/\sqrt{6}$	$-1/\sqrt{6}$	$1/\sqrt{6}$	0	$1/\sqrt{6}$	0
$B^0 \rightarrow \pi^-K^+$	-1	0	-1	0	0	0
π^0K^0	0	$-1/\sqrt{2}$	$1/\sqrt{2}$	0	0	0
η_8K^0	0	$-1/\sqrt{6}$	$1/\sqrt{6}$	0	0	0
$B_s \rightarrow \pi^+\pi^-$	0	0	0	-1	0	-1
$\pi^0\pi^0$	0	0	0	$1/\sqrt{2}$	0	$1/\sqrt{2}$
K^+K^-	-1	0	-1	-1	0	-1
$K^0\bar{K}^0$	0	0	1	0	0	1
$\pi^0\eta_8$	0	$-1/\sqrt{3}$	0	$1/\sqrt{3}$	0	0
$\eta_8\eta_8$	0	$-\sqrt{2}/3$	$2\sqrt{2}/3$	$1/3\sqrt{2}$	0	$1/\sqrt{2}$

$$\sqrt{2}A(B_s \rightarrow \pi^0 \bar{K}^0) = \sqrt{2}A(B^0 \rightarrow \pi^0 \pi^0) + A(B^0 \rightarrow K^+ K^-), \quad (9)$$

i.e.,

$$(-C + P) = (-C + P + E + PA) - (E + PA), \quad (10)$$

and

$$A(B_s \rightarrow \pi^+ K^-) = A(B^0 \rightarrow \pi^+ \pi^-) - A(B^0 \rightarrow K^+ K^-), \quad (11)$$

i.e.,

$$-(T + P) = -(T + P + E + PA) + (E + PA). \quad (12)$$

We shall anticipate a result of Sec. III in which the $B^0 \rightarrow K^+ K^-$ amplitude may be very small if rescattering effects are negligible. In that case each of the last two relations connects a rate for a charge state of $B_s \rightarrow \pi \bar{K}$ to that for a charge state of $B^0 \rightarrow \pi \pi$. For completeness, we include a fourth triangle relation which can be constructed from the above three, and hence is not independent:

$$A(B_s \rightarrow \pi^+ K^-) + \sqrt{2}A(B_s \rightarrow \pi^0 \bar{K}^0) = \sqrt{2}A(B^+ \rightarrow \pi^+ \pi^0), \quad (13)$$

i.e.,

$$-(T + P) + (-C + P) = -(C + T). \quad (14)$$

In addition there are three equations relating the amplitudes for decays involving a single η_8 to linear combinations of two other amplitudes, and one relation between $A(B^0 \rightarrow \eta_8 \eta_8)$ and a linear combination of three other amplitudes. These last four relations are expected to be of limited usefulness, as we have mentioned, since the physical η is rather far from a pure octet. If the reader wishes to assume that no other graphs contribute to the production of physical η states, the reader is welcome to do so, at the risk of ignoring additional SU(3) octet amplitudes when one η is produced and an additional SU(3) singlet amplitude when two η 's are produced.

2. $|\Delta S| = 1$ processes

The weak Hamiltonian giving rise to $\Delta C = 0$, $|\Delta S| = 1$ decays has pieces which transform as $\Delta I = 0$ and $\Delta I = 1$. In the decays $B \rightarrow \pi K$ there are thus two separate $I = 1/2$ amplitudes and one $I = 3/2$ amplitude. Thus, one can write a relation among the amplitudes for the four different charge states [3-5]:

$$\begin{aligned} & \sqrt{2}A(B^+ \rightarrow \pi^0 K^+) + A(B^+ \rightarrow \pi^+ K^0) \\ &= \sqrt{2}A(B^0 \rightarrow \pi^0 K^0) + A(B^0 \rightarrow \pi^- K^+), \quad (15) \end{aligned}$$

or

$$\begin{aligned} -(T' + C' + P' + A') + (P' + A') &= (-C' + P') \\ &-(T' + P'). \quad (16) \end{aligned}$$

Both sides correspond to the combination $-(C' + T')$ with isospin $3/2$. We shall have a good deal more to say about this relation in what follows. It has been the object of

extensive analyses regarding the possibility of observing direct CP violation in $B \rightarrow \pi K$ decays.

The two other relations involving only pions and kaons are

$$A(B_s \rightarrow \pi^+ \pi^-) = -\sqrt{2}A(B_s \rightarrow \pi^0 \pi^0), \quad (17)$$

i.e.,

$$-(E' + PA') = -(E' + PA') \quad (18)$$

and

$$A(B^0 \rightarrow \pi^- K^+) = A(B_s \rightarrow K^+ K^-) - A(B_s \rightarrow \pi^+ \pi^-), \quad (19)$$

i.e.,

$$-(T' + P') = -(T' + P' + E' + PA') + (E' + PA'). \quad (20)$$

The first of these follows from isospin alone; the $I = 2$ final state is not produced. As we shall see in Sec. III, the $B_s \rightarrow \pi \pi$ branching ratios are expected to be extremely small if rescattering effects do not alter the predictions of diagrams. The second relation implies that if $(B^0 \rightarrow \pi^- K^+)$ is observed with a branching ratio of order 10^{-5} , as appears possible [15], at least one of the processes on the right-hand side must be present with a similar branching ratio. Our conjecture based on the results of Sec. III will be that it is the process $B_s \rightarrow K^+ K^-$.

There are also four linear relations involving decays to η_8 .

3. Relations between $\Delta S = 0$ and $|\Delta S| = 1$ processes

The unprimed and primed amplitudes are related, since they involve different CKM factors but similar hadronic physics. There are two classes of such relations. The primed nonpenguin amplitudes are related to the unprimed ones by the ratio $r_u \equiv V_{us}/V_{ud} \approx 0.23$,

$$T'/T = C'/C = E'/E = A'/A = r_u. \quad (21)$$

The primed penguin amplitudes are related to the unprimed ones by the ratio $r_t \equiv V_{ts}/V_{td}$, since the penguin amplitudes are dominated by the top quark loop [16]. This quantity has a magnitude of about 5 ± 2 . It has a complex phase if CKM phases indeed are the source of the observed CP violation in the kaon system. Thus

$$P'/P = PA'/PA = r_t. \quad (22)$$

It is possible to write three independent amplitude relations which involve only r_u by choosing processes that contain the combinations $C+T$, $T-A$, and $T+E$. The relation involving $C+T$ is particularly useful since it allows one to relate the $B^+ \rightarrow \pi^+\pi^0$ amplitude (with isospin 2) to the linear combination of $B \rightarrow \pi K$ amplitudes already written above with isospin 3/2:

$$\begin{aligned} \sqrt{2}A(B^+ \rightarrow \pi^0 K^+) + A(B^+ \rightarrow \pi^+ K^0) \\ = r_u \sqrt{2}A(B^+ \rightarrow \pi^+ \pi^0), \end{aligned} \quad (23)$$

i.e.,

$$-(T' + C' + P' + A') + (P' + A') = -r_u(C + T). \quad (24)$$

In SU(3) language, these combinations of amplitudes correspond to a pure 27-plet.

A relation involving the combination $T - A$ is

$$\begin{aligned} A(B^+ \rightarrow \pi^+ K^0) + A(B^0 \rightarrow \pi^- K^+) \\ = r_u[A(B^+ \rightarrow K^+ \bar{K}^0) + A(B_s \rightarrow \pi^+ K^-)] \end{aligned} \quad (25)$$

or

$$(P' + A') - (T' + P') = r_u[(P + A) - (T + P)]. \quad (26)$$

A relation involving the combination $T + E$ is

$$\begin{aligned} A(B_s \rightarrow K^+ K^-) + A(B_s \rightarrow K^0 \bar{K}^0) \\ = r_u[A(B^0 \rightarrow \pi^+ \pi^-) + A(B^0 \rightarrow K^0 \bar{K}^0)] \end{aligned} \quad (27)$$

or

$$\begin{aligned} -(T' + P' + E' + PA') + (P' + PA') \\ = r_u[-(T + P + E + PA) + (P + PA)]. \end{aligned} \quad (28)$$

We shall discuss the likely magnitudes of terms in these relations in Sec. III.

The ratio r_t appears in the relation between two amplitudes which involve the combinations $P' + PA'$ or $P + PA$:

$$A(B_s \rightarrow K^0 \bar{K}^0) = r_t A(B^0 \rightarrow K^0 \bar{K}^0). \quad (29)$$

Finally, there should be one relation which involves a mixture of penguin and nonpenguin amplitudes such as $C - P$ or $P + T$. We cannot write such a relation in as simple a form as the others.

SU(3)-breaking effects may be taken into account in any amplitude in which a weak current produces a charged pseudoscalar meson by noting that the amplitude must contain the factor f_π or $f_K \approx 1.2f_\pi$. Thus, a better estimate of the ratio T'/T in Eq. (21) is

$$\tilde{r}_u \equiv r_u(f_K/f_\pi) \approx 1.2r_u. \quad (30)$$

A Fierz rearrangement suggests that this should also be a better estimate of C'/C . Consequently, a more precise version of (23) is

$$\begin{aligned} \sqrt{2}A(B^+ \rightarrow \pi^0 K^+) + A(B^+ \rightarrow \pi^+ K^0) \\ = \tilde{r}_u \sqrt{2}A(B^+ \rightarrow \pi^+ \pi^0). \end{aligned} \quad (31)$$

We shall discuss SU(3) breaking effects more fully in Sec. V.

D. Specific applications to $B \rightarrow \pi\pi$ and $B \rightarrow \pi K$

The amplitude of the decay to a CP eigenstate, $B^0 \rightarrow \pi^+\pi^-$, consists of terms which have two different CKM phases, sometimes denoted by “tree” and “penguin” phases, $\gamma = \arg(V_{ub}^* V_{ud})$ and $-\beta = \arg(V_{tb}^* V_{td})$, respectively [17]. This affects the time-dependent CP asymmetry in this process, which does not measure directly an angle (α) of the CKM unitarity triangle [1].

The effect of the penguin term can be eliminated if one measures, besides the time-dependent rate of $B^0 \rightarrow \pi^+\pi^-$, also the (time-integrated) rates for all possible charge states in $B \rightarrow \pi\pi$ and $\bar{B} \rightarrow \pi\pi$. This method [2] is based on the isospin triangle relation (7) and on its charge conjugate relation.

Similarly, the quadrangle isospin relation (15), for $B \rightarrow \pi K$, can be used to eliminate the effect of the penguin amplitude in order to measure α in the time-dependent rate of $B^0 \rightarrow \pi^0 K_s$ [3-5]. For that purpose one would have to measure all the eight rates of $B \rightarrow \pi K$ and $\bar{B} \rightarrow \pi K$. Since SU(3) relates $B \rightarrow \pi\pi$ to $B \rightarrow \pi K$, as in Eq. (31), one may try to use this symmetry to reduce the number of necessary measurements required to determine a weak phase. Also, final-state interaction phases in these two types of processes may be related. The triangle and quadrangle relations may be used to measure some of these phases, which determine the magnitude of CP asymmetries in charged B decays. Here we will recapitulate this method, in order to demonstrate how this may work within an SU(3) framework.

1. Isospin in $B \rightarrow \pi\pi$

Equation (7) follows from a decomposition in terms of amplitudes into final states with isospin 0 and 2 [2]:

$$\begin{aligned}
A(B^0 \rightarrow \pi^+\pi^-) &= A_2 - A_0, \\
\sqrt{2}A(B^0 \rightarrow \pi^0\pi^0) &= 2A_2 + A_0, \\
\sqrt{2}A(B^+ \rightarrow \pi^+\pi^0) &= 3A_2.
\end{aligned} \tag{32}$$

In terms of graphical contributions,

$$A_2 = -\frac{1}{3}(T + C) \equiv -a_2 e^{i\gamma} e^{i\delta_2},$$

$$\begin{aligned}
A_0 &= \frac{1}{3}[(2T - C + 3E) + (3P + 3PA)] \\
&\equiv a_{0(T)} e^{i\gamma} e^{i\delta_T} + a_{0(P)} e^{-i\beta} e^{i\delta_P}.
\end{aligned} \tag{33}$$

On the right-hand sides the amplitudes are decomposed into terms with different weak phases, $\text{Arg}(V_{ub}^* V_{ud}) = \gamma$, $\text{Arg}(V_{tb}^* V_{td}) = -\beta$, and different final-state interaction phases $\delta_2, \delta_T, \delta_P$. Similar relations hold for the charge-conjugated amplitudes, \bar{A} , in which one only changes the sign of the weak phases. For convenience, let us define $\bar{A} \equiv e^{2i\gamma} \bar{A}$, and let us rotate all amplitudes by a phase factor $e^{-i\gamma} e^{-i\delta_2}$. We then find

$$\begin{aligned}
A_2 &= \bar{A}_2 = -a_2, \\
A_0 &= a_{0(T)} e^{i\Delta_T} - a_{0(P)} e^{i\alpha} e^{i\Delta_P}, \\
\bar{A}_0 &= a_{0(T)} e^{i\Delta_T} - a_{0(P)} e^{-i\alpha} e^{i\Delta_P},
\end{aligned} \tag{34}$$

where $\Delta_i \equiv \delta_i - \delta_2$.

The triangle relations for $B \rightarrow \pi\pi$, Eq. (7), and $\bar{B} \rightarrow \pi\pi$ are shown in Fig. 2. The two triangles are fixed by measurements of all six decay rates, which determine the sides of the triangles. This determines the angle θ . The coefficient of the $\sin(\Delta mt)$ term in the

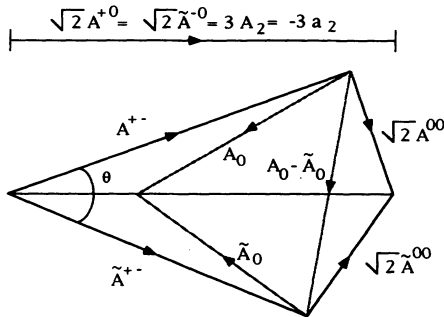


FIG. 2. Isospin triangles for $B \rightarrow \pi\pi$ (upper) and $\bar{B} \rightarrow \pi\pi$ (lower). The lower triangle can also be flipped about the horizontal axis. Here $A^{+-} \equiv A(B^0 \rightarrow \pi^+\pi^-)$, $A^{+0} \equiv A(B^+ \rightarrow \pi^+\pi^0)$, $A_0 \equiv A(B^0 \rightarrow \pi^0\pi^0)$, $\bar{A}^{+-} \equiv \bar{A}(\bar{B}^0 \rightarrow \pi^+\pi^-)$, $\bar{A}^{+0} \equiv \bar{A}(\bar{B}^+ \rightarrow \pi^+\pi^0)$, $\bar{A}_0 \equiv \bar{A}(\bar{B}^0 \rightarrow \pi^0\pi^0)$, and $\bar{A}_0^{-0} \equiv \bar{A}(B^- \rightarrow \pi^-\pi^0)$. The isospin amplitudes A_2 , A_0 , and \bar{A}_0 are defined in the text.

time-dependent decay rate of $B^0 \rightarrow \pi^+\pi^-$ is given by $|A(\bar{B}^0 \rightarrow \pi^+\pi^-)/A(B^0 \rightarrow \pi^+\pi^-)| \sin(2\alpha + \theta)$, which can then be used to determine α .

CP violation in direct decays would be manifested by two triangles with different shapes. This requires final-state interaction phase differences. The final-state interaction phase difference Δ_P can be determined from the phase of

$$A_0 - \bar{A}_0 = -2ia_{0(P)} \sin \alpha e^{i\Delta_P}, \tag{35}$$

as shown in Fig. 2.

2. Isospin in $B \rightarrow \pi K$

In $B \rightarrow \pi K$ the isospin amplitudes consist of two $\Delta I = 1$ “tree” amplitudes, $A_{1/2}$, $A_{3/2}$ into final states with $I = 1/2, 3/2$, and an amplitude $B_{1/2}$ of a $\Delta I = 0$ operator, which is a “tree-penguin” mixture. This leads to the decomposition [3–5]

$$A(B^0 \rightarrow \pi^- K^+) = A_{3/2} + A_{1/2} - B_{1/2},$$

$$\sqrt{2}A(B^0 \rightarrow \pi^0 K^0) = 2A_{3/2} - A_{1/2} + B_{1/2},$$

$$A(B^+ \rightarrow \pi^+ K^0) = A_{3/2} + A_{1/2} + B_{1/2},$$

$$\sqrt{2}A(B^+ \rightarrow \pi^0 K^+) = 2A_{3/2} - A_{1/2} - B_{1/2}. \tag{36}$$

This implies the quadrangle relation (15), where both sides of this equation express one of the diagonals of the quadrangle. A similar quadrangle, with a common diagonal, describes the amplitudes of the charge-conjugated processes $\bar{B} \rightarrow \pi K$. The other two diagonals of the two quadrangles can be shown to have a common midpoint. Except for a class of ambiguities enumerated in Ref. [4], this suffices to specify the shapes of both quadrangles using the eight rate measurements. The relative orientation of the two quadrangles can then be used to eliminate the penguin contribution from the time-dependent rate of the decay to the CP eigenstate $B^0 \rightarrow \pi^0 K_s$, in order to enable a measurement of α .

3. $SU(3)$ relations between amplitudes and strong phases in $B \rightarrow \pi\pi$ and $B \rightarrow \pi K$

Since the previous method involves a large number of rate measurements, with some ambiguity in the shape of the quadrangles, one may want to use $SU(3)$ to relate $B \rightarrow \pi K$ to $B \rightarrow \pi\pi$. Within $SU(3)$ the common diagonal of the two quadrangles is given by

$$3A_{3/2} = -(T' + C') = \tilde{r}_u 3A_2 = \tilde{r}_u \sqrt{2}A(B^+ \rightarrow \pi^+\pi^0) \tag{37}$$

as observed already in (31). The rate of $B^+ \rightarrow \pi^+\pi^0$ determines the magnitude of the common diagonal, and

thus circumvents a good deal of the uncertainty associated with the geometric construction of the two quadrangles. Specifically, to determine this diagonal, one can make use just of rates for charged B decays, as in (31). The quadrangle (15) and the triangle (31) can be combined to yield another triangle

$$\begin{aligned} \sqrt{2}A(B^0 \rightarrow \pi^0 K^0) + A(B^0 \rightarrow \pi^- K^+) \\ = \tilde{r}_u \sqrt{2}A(B^+ \rightarrow \pi^+ \pi^0), \end{aligned} \quad (38)$$

i.e.,

$$-(C' - P') - (T' + P') = -\tilde{r}_u(C + T). \quad (39)$$

An analysis completely analogous to that of $B \rightarrow \pi\pi$ can now be used: the decay rates for the three processes in this triangle and their CP conjugates can be combined with the coefficient of the $\sin(\Delta mt)$ term in the time-dependent rate for $B^0 \rightarrow \pi^0 K_s$ to determine α . This is the first of several examples we present in this paper showing how $SU(3)$ can be used to obtain information about CKM phases.

The triangle (31) relating the three amplitudes of charged B decays to $\pi\pi$ and πK is interesting in its own right, since its construction requires only self-tagging modes. This triangle is very similar to that of $B \rightarrow \pi\pi$. One would like to find a relation between the corresponding measurable final state phases appearing in the two triangles. For that purpose, let us complete the analogy by defining an $I = 1/2$ amplitude (parallel to A_0 in $B \rightarrow \pi\pi$) as

$$\begin{aligned} C_{1/2} &\equiv -(A_{1/2} + B_{1/2}) = -\frac{1}{3}[(T' + C' + 3A') + 3P'] \\ &\equiv a_{1/2(T)} e^{i\gamma} e^{i\delta'_T} - a_{1/2(P)} e^{i\delta'_P}, \end{aligned} \quad (40)$$

where we used Table II for the expressions in terms of graphical contributions, and $\text{Arg}(V_{ub}^* V_{us}) = \gamma$, $\text{Arg}(V_{tb}^* V_{ts}) = \pi$.

Comparing (40) with (33) we note that both A_0 and $C_{1/2}$ consist of two terms with specific weak phases, which, however, involve different graphical contributions, and thus have in general different final-state interaction phases, $\delta'_T \neq \delta_T$, $\delta'_P \neq \delta_P$. Following the arguments which led to (31) one can similarly show that the final-state interaction phase difference $\Delta_{P'} \equiv \delta_{P'} - \delta_{3/2}$ ($\delta_{3/2} = \delta_2$) can be determined from the phase of

$$C_{1/2} - \tilde{C}_{1/2} = 2ia_{1/2(P)} \sin \gamma e^{i\Delta_{P'}}, \quad (41)$$

as shown in Fig. 3.

In the case $E+PA = 0$, to be discussed in Sec. III, only the diagram P in $B \rightarrow \pi\pi$ carries the ‘‘penguin’’ weak phase as in $B \rightarrow \pi K$. Neglecting the phase due to the perturbatively calculated absorptive part of the physical $c\bar{c}$ quark pair in the penguin diagram, which is very small [18], one has $\delta_{P'} = \delta_P$. Thus, in the limit $E+PA = 0$, the two final-state phase differences in $B \rightarrow \pi\pi$ and in $B \rightarrow \pi K$ are equal, $\Delta_{P'} = \Delta_P$.

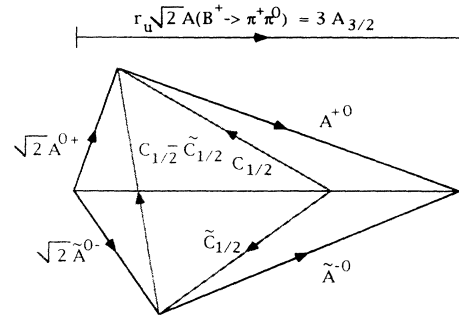


FIG. 3. $SU(3)$ triangles involving decays $B^\pm \rightarrow \pi^\pm \pi^0$ and $B^+ \rightarrow \pi K$ (upper) or $B^- \rightarrow \pi K$ (lower). The lower triangle can also be flipped about the horizontal axis. Here $A^{0+} \equiv A(B^+ \rightarrow \pi^0 K^+)$, $A^{+0} \equiv A(B^+ \rightarrow \pi^+ K^0)$, $\tilde{A}^{0-} \equiv \tilde{A}(B^- \rightarrow \pi^0 K^-)$, $\tilde{A}^{-0} \equiv \tilde{A}(B^- \rightarrow \pi^- K^0)$. The isospin amplitudes $A_{3/2}$, $C_{1/2}$, and $\tilde{C}_{1/2}$ are defined in the text.

E. Isospin analysis of $B \rightarrow \pi \bar{D}$

Although we have concentrated on decays of B mesons to pairs of light pseudoscalars, some useful evidence regarding final-state interactions can be obtained from the processes $B \rightarrow \pi \bar{D}$. The transition $\bar{b} \rightarrow \bar{d}u\bar{c}$ has $\Delta I = \Delta I_3 = 1$ and hence there are unique amplitudes for $I = 1/2$ and $I = 3/2$ final states. In terms of these, the physical decay amplitudes may be written

$$A(B^+ \rightarrow \pi^+ \bar{D}^0) = A_{3/2}, \quad (42)$$

$$A(B^0 \rightarrow \pi^+ D^-) = \frac{1}{3}A_{3/2} + \frac{2}{3}A_{1/2}, \quad (43)$$

$$A(B^0 \rightarrow \pi^0 \bar{D}^0) = \sqrt{2}(A_{3/2} - A_{1/2})/3. \quad (44)$$

These equations are of the same form as those employed to conclude that significant final-state interactions occur in $D \rightarrow \pi \bar{K}$ decays. They imply the triangle relation

$$\begin{aligned} A(B^0 \rightarrow \pi^+ D^-) + \sqrt{2}A(B^0 \rightarrow \pi^0 \bar{D}^0) \\ = A(B^+ \rightarrow \pi^+ \bar{D}^0). \end{aligned} \quad (45)$$

Experimental data [7] exist for the two processes involving charged pions, but only an upper limit exists at present for $B^0 \rightarrow \pi^0 \bar{D}^0$. These results lead to an upper limit on the phase shift difference [19], $\cos(\delta_{1/2} - \delta_{3/2}) > 0.82$ (90% C.L.), or $|\delta_{1/2} - \delta_{3/2}| < 35^\circ$. With improved data, one may be able to tell whether the triangle has nonzero area. Since these decays are all expected to be governed by the same CKM factor, nonzero area for the triangle would be unambiguous evidence for a difference in final-state phases between the $I = 1/2$ and $I = 3/2$ amplitudes. A similar approach [19] failed to detect any phase shift differences between $I = 1/2$ and $I = 3/2$ amplitudes in the decays $B \rightarrow \pi \bar{D}^*$ and $B \rightarrow \rho \bar{D}$.

III. NEGLECT OF CERTAIN DIAGRAMS

A. Linear relations among reduced amplitudes

The diagrams denoted by E , A , PA involve contributions to amplitudes which should behave as f_B/m_B in comparison with those from the diagrams T , C , and P (and similarly for their primed counterparts). This suppression factor of $f_B/m_B \leq (250 \text{ MeV}/5 \text{ GeV})$ is due to the smallness of the B meson wave function at the origin, and it should remain valid unless rescattering effects are important. Such rescatterings indeed could be responsible for certain decays of charmed particles (such as $D^0 \rightarrow \bar{K}^0 \phi$ [20]), but should be less important for the higher-energy B decays. In addition the diagrams E and A are also helicity suppressed by a factor $m_{u,d,s}/m_B$ since the B mesons are pseudoscalars. We shall investigate in the present section the consequences of assuming that only T , C , P , and the corresponding primed quantities are nonvanishing.

The relations for reduced matrix elements in $SU(3)$ entailed by this assumption are surprisingly simple. The singlet and the octet ($\mathbf{8}_1$) arising from the $\mathbf{3}^*$ operator in the weak Hamiltonian become related to one another, while the 27-plet amplitude and the octet ($\mathbf{8}_3$) which arises from the $\mathbf{15}^*$ operator become related:

$$\{\mathbf{8}_1\} = -\sqrt{5}\{\mathbf{1}\}/4, \quad \{\mathbf{8}_3\} = \{\mathbf{27}\}/4. \quad (46)$$

The amplitude $\{\mathbf{8}_2\}$ which arises from the $\mathbf{6}$ operator remains unconstrained.

B. Relations among decay amplitudes

There are now three independent $SU(3)$ amplitudes for $\Delta S = 0$ transitions expressed in terms of the three independent graphical contributions T , C , and P , and three for $|\Delta S| = 1$ transitions expressed in terms of T' , C' , and P' . The relations (21) and (22) between these two sets noted in Sec. II continue to hold.

When the E , A , PA diagrams and their primed counterparts are neglected, certain decays become forbidden:

$$\Gamma(B^0 \rightarrow K^+ K^-) = 0, \quad (47)$$

$$\Gamma(B_s \rightarrow \pi^+ \pi^-) = \Gamma(B_s \rightarrow \pi^0 \pi^0) = 0. \quad (48)$$

This leaves 13 nonzero amplitudes involving pions and kaons, as shown in Table III. [In this table, the $\sqrt{2}(B^+ \rightarrow \pi^+ \pi^0)$ under the $-(T+C)$ column means that $A(B^+ \rightarrow \pi^+ \pi^0) = -(T+C)/\sqrt{2}$, and similarly for other entries.]

Given the relations (21) and (22), these 13 amplitudes can be expressed in terms of three independent quantities, leading to ten relations among the amplitudes. For $\Delta S = 0$ processes, the triangle relation (7) for $B \rightarrow \pi\pi$ is not simplified by the assumptions of the present section. However, the two triangle relations (9) and (11) of the exact treatment become two rate relations

$$\Gamma(B_s \rightarrow \pi^0 \bar{K}^0) = \Gamma(B^0 \rightarrow \pi^0 \pi^0), \quad (49)$$

$$\Gamma(B_s \rightarrow \pi^+ K^-) = \Gamma(B^0 \rightarrow \pi^+ \pi^-). \quad (50)$$

As for $|\Delta S| = 1$ processes, the quadrangle relation (15) for $B \rightarrow \pi K$ is unchanged. However, as mentioned in Sec. II D, in the context of $SU(3)$ it is more convenient to think of this quadrangle relation as *not* independent, since it can be expressed in terms of two triangle relations. As a result of the vanishing of the $B_s \rightarrow \pi^+ \pi^-$ transition, the triangle relation (19) becomes a rate relation

$$\Gamma(B^0 \rightarrow \pi^- K^+) = \Gamma(B_s \rightarrow K^- K^+). \quad (51)$$

There is one new relation between amplitudes which are pure P ,

$$A(B^+ \rightarrow K^+ \bar{K}^0) = A(B^0 \rightarrow K^0 \bar{K}^0), \quad (52)$$

and one new relation involving pure P' :

$$A(B^+ \rightarrow \pi^+ K^0) = A(B_s \rightarrow K^0 \bar{K}^0). \quad (53)$$

Since above we have listed six relations [counting the triangle relation (7)], there should be four relations between $\Delta S = 0$ and $|\Delta S| = 1$ processes. Three of these, the two triangle relations (23) and (38) and the rate relation (29), are unchanged from the exact treatment. [The πK quadrangle relation (15) is simply related to the two triangle relations (23) and (38).] Finally, give the assumptions of this section, the two quadrangle relations (25) and (27) become equivalent, and can be written in terms of decays of B^0 and B^+ only:

TABLE III. The 13 decay amplitudes in terms of the eight graphical combinations.

$-(T+C)$	$-(C-P)$	$-(T+P)$	(P)
$\sqrt{2}(B^+ \rightarrow \pi^+ \pi^0)$	$\sqrt{2}(B^0 \rightarrow \pi^0 \pi^0)$ $\sqrt{2}(B_s \rightarrow \pi^0 \bar{K}^0)$	$B^0 \rightarrow \pi^+ \pi^-$ $B_s \rightarrow \pi^+ K^-$	$B^+ \rightarrow K^+ \bar{K}^0$ $B^0 \rightarrow K^0 \bar{K}^0$
$-(T'+C'+P')$	$-(C'-P')$	$-(T'+P')$	(P')
$\sqrt{2}(B^+ \rightarrow \pi^0 K^+)$	$\sqrt{2}(B^0 \rightarrow \pi^0 K^0)$	$B^0 \rightarrow \pi^- K^+$ $B_s \rightarrow K^- K^+$	$B^+ \rightarrow \pi^+ K^0$ $B_s \rightarrow K^0 \bar{K}^0$

$$\begin{aligned}
& A(B^+ \rightarrow \pi^+ K^0) + A(B^0 \rightarrow \pi^- K^+) \\
&= \tilde{r}_u [A(B^+ \rightarrow K^+ \bar{K}^0) + A(B^0 \rightarrow \pi^+ \pi^-)], \quad (54)
\end{aligned}$$

i.e.,

$$(P') - (T' + P') = \tilde{r}_u [(P) - (T + P)]. \quad (55)$$

Note that the left-hand side of this relation, and others like it, is likely to involve the cancellation of two nearly equal amplitudes if P' is the dominant effect in $|\Delta S| = 1$ transitions, as we expect to be likely (see below).

To sum up, the assumption of ignoring the diagrams E , A , PA and their corresponding primed quantities leads to ten relations among B decay amplitudes: six rate relations, three triangle relations, and one quadrangle relation. These will be very useful in extracting both weak and strong phase information from B decays.

Finally, there is an additional point: The penguin contributions in $B \rightarrow \pi\pi$ and $B \rightarrow \pi K$ are now related to one another since the amplitude PA is no longer present in $B \rightarrow \pi\pi$. Thus, we expect the strong phase shift difference $\delta_P - \delta_2$ in $B \rightarrow \pi\pi$ to be equal to the difference $\delta_P - \delta_{3/2}$ in $B \rightarrow \pi K$. We showed in Sec. II D how to measure these differences.

C. Measuring the angle γ

Neglecting A' in (24), the triangle relation (31) and its charge conjugate can be used to measure the angle γ [21]. This follows from the fact that the amplitude $-(C + T) = \sqrt{2}A(B^+ \rightarrow \pi^+\pi^0)$ has the tree weak phase γ , whereas the amplitude $P' = A(B^+ \rightarrow \pi^+K^0)$ has the penguin phase π . The two triangles for B^+ and B^- decays are shown in Fig. 4, with the notation defined in the caption. We have drawn the figure such that the strong phase $e^{i\delta_P}$ lies along the horizontal axis. The strong phase δ_2 was discussed in Sec. II D 1.

The measurements of the four independent rates for $B^+ \rightarrow \pi^0 K^+$, $B^- \rightarrow \pi^0 K^-$, $B^+ \rightarrow \pi^+ K^0$, and $B^+ \rightarrow \pi^+ \pi^0$ can determine γ . If A' can be neglected, the rates for $B^+ \rightarrow \pi^+ K^0$ and $B^- \rightarrow \pi^- \bar{K}^0$ should be equal. We expect $\Gamma(B^+ \rightarrow \pi^+ \pi^0) = \Gamma(B^- \rightarrow \pi^- \pi^0)$ in any case, as noted earlier. The triangle for B^- decays can also be flipped about its horizontal axis, leading to a twofold ambiguity.

If $\delta_P - \delta_2 = 0$, we will not observe a CP -violating difference between the rates for $B^+ \rightarrow \pi^0 K^+$ and $B^- \rightarrow \pi^0 K^-$. Nevertheless, accepting the standard model CKM mechanism for CP violation in the kaon system, we know that $\gamma \neq 0$, which selects the “flipped” solution as shown in the lower figure.

The present data on B^0 decays to pairs of pseudoscalars [15] do not allow one to distinguish between $\pi^- K^+$ and $\pi^+ \pi^-$ final states. The combined branching ratio is about 2×10^{-5} , with equal rates for $\pi^- K^+$ and $\pi^+ \pi^-$ being most likely. If this is true, the amplitudes T and P' have about the same magnitude, so that the short sides of the triangles in Fig. 4 are probably about 1/4 to

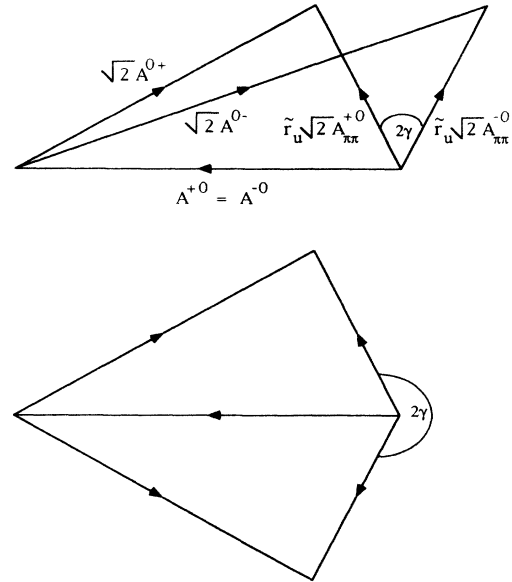


FIG. 4. SU(3) triangles involving decays of charged B 's which may be used to measure the angle γ . Here $A^{0+} \equiv A(B^+ \rightarrow \pi^0 K^+)$, $A^{+0} \equiv A(B^+ \rightarrow \pi^+ K^0)$, $A^{0-} \equiv A(B^- \rightarrow \pi^0 K^-)$, $A^{-0} \equiv A(B^- \rightarrow \pi^- \bar{K}^0)$, $A_{\pi\pi}^{+0} \equiv A(B^+ \rightarrow \pi^+ \pi^0) = -(C + T)/\sqrt{2} = 3a_2 e^{i\gamma} e^{i\delta_2}/\sqrt{2}$, $A_{\pi\pi}^{-0} \equiv A(B^- \rightarrow \pi^- \pi^0) = 3a_2 e^{-i\gamma} e^{i\delta_2}/\sqrt{2}$, and $\tilde{r}_u \equiv (f_K/f_\pi)|V_{us}/V_{ud}|$.

1/3 [$\approx (f_K/f_\pi)r_u$] the lengths of the other two sides. Then the “long” sides of the triangle must be measured with fractional accuracies of about $(f_K/f_\pi)r_u \delta_\gamma$ in order to achieve an accuracy of δ_γ in the angle γ . For example, to measure γ to a statistical accuracy of about 10° , one probably needs fractional errors of about 1/20 in amplitudes, or 10% in rates. This would require at least 100 decays in each channel of interest.

D. Measuring the CKM phases and the strong phase shifts

The relations of Sec. III B can be further used to measure all weak phases, as well as the strong phase shifts and the sizes of the relevant contributing diagrams T , C , P , and P' [22]. Having neglected E , A , and PA , the three triangle relations can now be written schematically [including SU(3) breaking] as

$$(T + C) = (C - P) + (T + P), \quad (56)$$

$$(T + C) = (C' - P')/\tilde{r}_u + (T' + P')/\tilde{r}_u, \quad (57)$$

$$(T + C) = (T' + C' + P')/\tilde{r}_u - (P')/\tilde{r}_u. \quad (58)$$

By SU(3) symmetry the strong phases for the primed graphs are the same as the unprimed ones. We thus have, for the three relations,

$$3a_2 e^{i\gamma} e^{i\delta_2} = (A_C e^{i\gamma} e^{i\delta_C} - A_P e^{-i\beta} e^{i\delta_P}) + (A_T e^{i\gamma} e^{i\delta_T} + A_P e^{-i\beta} e^{i\delta_P}), \quad (59)$$

$$3a_2 e^{i\gamma} e^{i\delta_2} = (A_C e^{i\gamma} e^{i\delta_C} + A_{P'} e^{i\delta_{P'}}/\bar{r}_u) + (A_T e^{i\gamma} e^{i\delta_T} - A_{P'} e^{i\delta_{P'}}/\bar{r}_u), \quad (60)$$

$$3a_2 e^{i\gamma} e^{i\delta_2} = (A_T e^{i\gamma} e^{i\delta_T} + A_C e^{i\gamma} e^{i\delta_C} - A_{P'} e^{i\delta_{P'}}/\bar{r}_u) + A_{P'} e^{i\delta_{P'}}/\bar{r}_u, \quad (61)$$

where a_2 [introduced in Eq. (33)] and the quantities A_T , A_C , A_P , and $A_{P'}$ are real and positive. Multiplying through on both sides by $\exp(-i\gamma - i\delta_2)$ gives

$$3a_2 = (A_C e^{i\Delta_C} + A_P e^{i\alpha} e^{i\Delta_P}) + (A_T e^{i\Delta_T} - A_P e^{i\alpha} e^{i\Delta_P}), \quad (62)$$

$$3a_2 = (A_C e^{i\Delta_C} + A_{P'} e^{-i\gamma} e^{i\Delta_{P'}}/\bar{r}_u) + (A_T e^{i\Delta_T} - A_{P'} e^{-i\gamma} e^{i\Delta_{P'}}/\bar{r}_u), \quad (63)$$

$$3a_2 = (A_T e^{i\Delta_T} + A_C e^{i\Delta_C} - A_{P'} e^{-i\gamma} e^{i\Delta_{P'}}/\bar{r}_u) + A_{P'} e^{-i\gamma} e^{i\Delta_{P'}}/\bar{r}_u, \quad (64)$$

where we again define $\Delta_i \equiv \delta_i - \delta_2$, and note that $-(\beta + \gamma) = \alpha - \pi$.

Consider the two triangle relations in Eqs. (63) and (64). Implicit there is the relation

$$3a_2 = T + C = A_T e^{i\Delta_T} + A_C e^{i\Delta_C}. \quad (65)$$

Thus not only do both these triangles share a common base but they also share a common subtriangle with sides $T + C$, C , T as shown in Fig. 5(a). Furthermore, this subtriangle is completely determined, up to a fourfold

ambiguity, by the two triangles in Eqs. (63) and (64). In other words, if we measure the five rates for $B^0 \rightarrow \pi^- K^+$ (giving $|T + P'/\bar{r}_u|$), $B^0 \rightarrow \pi^0 K^0$ (giving $|C - P'/\bar{r}_u|$), $B^+ \rightarrow \pi^0 K^+$ (giving $|T + C + P'/\bar{r}_u|$), $B^+ \rightarrow \pi^+ K^0$ (giving $|P'/\bar{r}_u|$), and $B^+ \rightarrow \pi^+ \pi^0$ (giving $|T + C|$), we can determine $|T|$, $|C|$, Δ_C , Δ_T , and $\Delta_P - \gamma$. If we also measure the CP conjugate processes then we can also separately determine Δ_P and γ .

Now consider the triangle relations in Eqs. (62) and (63). Since the magnitudes of the penguin diagrams P and P' can be measured by measuring rates, the subtriangle in Eq. (65) (which still holds) is completely determined up to an eightfold ambiguity. This eightfold ambiguity corresponds to the two intersections of the circles drawn from the vertices of the triangles as seen in Fig. 5(b), in addition to the fourfold ambiguity caused by the possibility of reflecting the triangles about their bases. Thus by measuring seven rates we can extract, in addition to the parameters mentioned above, the angle $\Delta_P - \alpha$. Thus we can determine $\beta = \pi - \alpha - \gamma$. By considering the CP conjugate processes we can determine Δ_P and α separately.

An identical construction to that in the previous paragraph holds for relations (63) and (64). This provides an independent way to measure the same quantities and is likely to be of great help in evaluating the size of $SU(3)$ -breaking effects [22].

In Figs. 5(a) and 5(b) more realistic proportions would be $|P|, |C| < |T| < |P'/\bar{r}_u|$. This hierarchy can reduce the possibility of discrete ambiguities. Thus, for example, in Fig. 5(a) two sides of each triangle with base $C + T$ will be of order $|P'/\bar{r}_u|$. One of the two choices of relative orientation of the two triangles will imply that $|C|$ and $|T|$ are each of order $|P'/\bar{r}_u|$, violating this hierarchy. Thus only a *twofold* ambiguity will remain, corresponding to reflection of *each* triangle about the base $C + T$.

E. Results of further specialization

The relative magnitude of penguin effects in the decays $B \rightarrow \pi\pi$ can be estimated either by direct reference to various charge states in $B \rightarrow \pi\pi$ [2], or with the help of $SU(3)$ and some auxiliary assumptions by reference to the process $B^0 \rightarrow \pi^- K^+$ [11]. It appears that some combination of the decays $B^0 \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow \pi^- K^+$ has been observed, with the most likely mixture being approximately equal amounts of each [15]. As a result of the relations (21) and (22) one then concludes that T and C are likely to dominate the $\Delta S = 0$ transitions, while P' is likely to dominate the $|\Delta S| = 1$ transitions. For reference, we quote some results of assuming this to be so.

1. $B \rightarrow \pi\pi$ without penguins

If the triangle formed by the complex amplitudes in (7) has nonzero area and penguin contributions are known to be small, there must be final-state phase differences between $I = 0$ and $I = 2$ amplitudes. Direct CP viola-

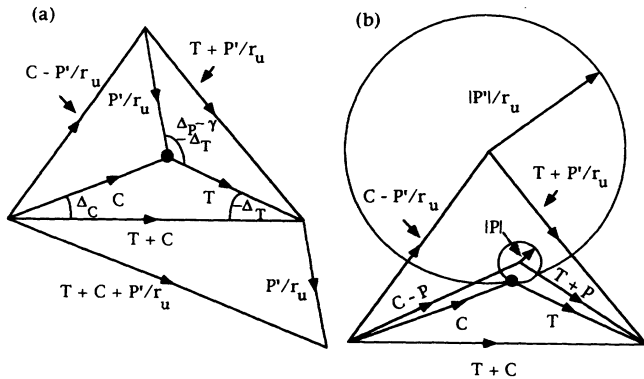


FIG. 5. Triangle relations from which weak phases and strong phase shift differences can be obtained in the limit of neglect of certain diagrams. The black dot corresponds to the solution for the vertex of the triangle in the relation (65). (a) Relation based on (63) (upper triangle) and (64) (lower triangle). (b) Relation based on (62) (lower triangle with small circle about its vertex) and (63) (upper triangle with large circle about its vertex).

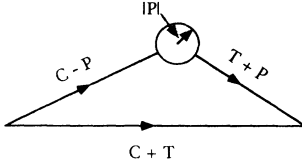


FIG. 6. Isospin triangle for $B \rightarrow \pi\pi$ decays with a small circle of uncertainty associated with unknown final-state phase of the penguin amplitude P . Here $C + t = -\sqrt{2}A(B^+ \rightarrow \pi^+\pi^0)$, $T + P = -A(B^0 \rightarrow \pi^+\pi^-)$, and $C - P = \sqrt{2}A(B^0 \rightarrow \pi^0\pi^0)$.

tion in rates is not observable but the usual measurement of a CP -violating difference between the time-integrated rates for $B^0|_{t=0} \rightarrow \pi^+\pi^-$ and $\bar{B}^0|_{t=0} \rightarrow \pi^+\pi^-$ measures the angle α in the unitarity triangle. Here $\alpha + \beta + \gamma = \pi$, with $\beta = \text{Arg}V_{td}^*V_{tb}$ and $\gamma = \text{Arg}V_{ub}^*V_{ud}$.

If the magnitude $|P|$ is *measured* to be small in comparison with T and C (by the study of $B^+ \rightarrow K^+\bar{K}^0$ and $B^0 \rightarrow K^0\bar{K}^0$ as mentioned below), and all the other graphs give negligible contributions, one can draw the triangle of Eq. (7) as shown in Fig. 6, with a circle of error around one vertex corresponding to the uncertainty in the phase of P . (This assumes we have not yet measured that phase using methods mentioned earlier). If this circle is small enough, we can obtain approximate information on the relative strong phases of the amplitudes C and T , and hence on the phase difference $\delta_2 - \delta_0$ in $B \rightarrow \pi\pi$.

2. $B \rightarrow \pi K$ with penguins alone

All rates are related [9]; those with charged pions are twice those with neutral ones. The quadrangles in Fig. 3 have zero area because their common diagonal receives no contributions from penguin amplitudes and hence vanishes. As mentioned in Sec. II, one expects to be able to tell directly from the $B^+ \rightarrow \pi^+\pi^0$ rate how large this diagonal actually is. If the quadrangles have zero area, the CP -violating difference between the time-integrated rates for $B^0|_{t=0} \rightarrow \pi^0 K_S$ and $\bar{B}^0|_{t=0} \rightarrow \pi^0 K_S$ measures the angle β in the unitarity triangle.

3. Other $B \rightarrow PP$ rate predictions

If P is negligible in comparison with C and T , the processes $B^+ \rightarrow K^+\bar{K}^0$ and $B^0 \rightarrow K^0\bar{K}^0$ have negligible rates in comparison with $B \rightarrow \pi\pi$. (We have already argued that $B^0 \rightarrow K^+K^-$ is likely to be small.) For $|\Delta S| = 1$ transitions, there appear to be no special rate predictions beyond those for $B \rightarrow \pi K$ which follow from the assumption of P' dominance. (We exclude processes involving η_8 's from this discussion, as usual.)

4. Neglect of color-suppressed diagrams

If both P and C are negligible in comparison with T in $\Delta S = 0$ $B \rightarrow PP$ decays, the $B^0 \rightarrow \pi^0\pi^0$ decay does

not occur, and the triangle relation (7) becomes a relation between two amplitudes, entailing a rate prediction $\Gamma(B^+ \rightarrow \pi^+\pi^0) = \Gamma(B^0 \rightarrow \pi^+\pi^-)/2$.

If the color-suppressed amplitudes C' can be neglected in $B \rightarrow \pi K$ in comparison to T' and P' , one obtains the rate relations $2\Gamma(B^0 \rightarrow \pi^0 K^0) = \Gamma(B^+ \rightarrow \pi^+ K^0)$ and $2\Gamma(B^+ \rightarrow \pi^0 K^+) = \Gamma(B^0 \rightarrow \pi^- K^+)$. This may help in extracting the weak phase from the time dependence of $B^0 \rightarrow \pi^0 K_S$.

In decays to charmed final states, the absence of a color-suppressed contributions would lead to equal rates for $B^+ \rightarrow \pi^+\bar{D}^0$ and $B^0 \rightarrow \pi^+D^-$ and a suppression of $B^0 \rightarrow \pi^0\bar{D}^0$. It will be interesting to watch as the data on these processes improve, to learn the actual suppression factor. A similar suppression is expected in $B \rightarrow KD$. The magnitude of color suppression in this class of processes would become crucial for a measurement of the unitarity triangle angle γ from the rates of the self-tagged modes $B^+ \rightarrow K^+D^0$, $B^+ \rightarrow K^+\bar{D}^0$, and $B^+ \rightarrow K^+D_{CP}$, where D_{CP} denotes a CP eigenstate [23]. Too strong a suppression of $B^+ \rightarrow K^+D^0$, which only involves a color-suppressed and an annihilation diagram, would presumably make this method unfeasible.

IV. RESULTS FOR PHYSICAL η AND η'

The physical η and η' appear to be octet-singlet mixtures:

$$\eta = \eta_8 \cos \phi - \eta_1 \sin \phi, \quad \eta' = \eta_8 \sin \phi + \eta_1 \cos \phi, \quad (66)$$

where $\eta_8 \equiv (2s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{6}$ and $\eta_1 \equiv (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$.

For a mixing angle of $\phi = 19.5^\circ = \arcsin^{-1}(1/3)$, close to one obtained in a recent analysis [14], the physical η and η' can be represented approximately as

$$\eta = (s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{3}, \quad \eta' = (u\bar{u} + d\bar{d} + 2s\bar{s})/\sqrt{6}. \quad (67)$$

We shall calculate amplitudes for production of these states in terms purely of the graphical contributions of Figs. 1(a)–1(c), neglecting the small terms associated with exchange, annihilation, or penguin annihilation. We also neglect graphs in which one or two final-state particles are connected to the rest of the diagram by gluons (or vacuum quantum numbers) alone. It would not make sense to neglect such graphs while still continuing, for example, to take account of the disconnected penguin annihilation graph of Fig. 1(f). As mentioned in Sec. II, a full $SU(3)$ analysis would have to take account of all types of disconnected graphs as a result of the singlet components of η and η' . The validity of the neglect of disconnected graphs in charmed-particle decays involving η and η' has been discussed by Lipkin [24]. The results are shown in Tables IV and V.

An interesting feature of these results is the absence of the P contribution to $B_s \rightarrow \eta\bar{K}^0$ and the P' contribution to $B^0 \rightarrow K^0\eta$. The first result says that $B_s \rightarrow \eta\bar{K}^0$ may

TABLE IV. Decomposition of $B \rightarrow PP$ amplitudes involving η and η' for $\Delta C = \Delta S = 0$ transitions in terms of graphical contributions. Here η and η' are defined as in Eq. (67).

	Final state	T	C	P
$B^+ \rightarrow$	$\pi^+\eta$	$-1/\sqrt{3}$	$-1/\sqrt{3}$	$-2/\sqrt{3}$
	$\pi^+\eta'$	$1/\sqrt{6}$	$1/\sqrt{6}$	$2/\sqrt{6}$
$B^0 \rightarrow$	$\pi^0\eta$	0	0	$-2/\sqrt{6}$
	$\pi^0\eta'$	0	0	$1/\sqrt{3}$
	$\eta\eta$	0	$\sqrt{2}/3$	$\sqrt{2}/3$
	$\eta\eta'$	0	$-\sqrt{2}/3$	$-\sqrt{2}/3$
	$\eta'\eta'$	0	$\sqrt{2}/6$	$\sqrt{2}/6$
$B_s \rightarrow$	$\eta\bar{K}^0$	0	$-1/\sqrt{3}$	0
	$\eta'\bar{K}^0$	0	$1/\sqrt{6}$	$3/\sqrt{6}$

be a good probe of the color-suppressed contribution C . This can be tested by comparison with the value of C as extracted using $B \rightarrow \pi\pi$, πK , and KK (Sec. III D). The second result says that the decay $B^0 \rightarrow K^0\eta$ may be considerably suppressed in comparison with $B^0 \rightarrow K^0\pi^0$. The suppression of the P' contribution is complete for the particular mixing scheme (67), but is also considerable [25] for a mixing angle of $\phi \approx 10^\circ$ such that $\eta \approx (\sqrt{2}s\bar{s} - u\bar{u} - d\bar{d})/2$.

V. SU(3)-BREAKING EFFECTS

As pointed out in Ref. [9], there appear to be important SU(3)-breaking effects in charmed meson decays. One expects [26] $\Gamma(D^0 \rightarrow K^+K^-)/\Gamma(D^0 \rightarrow \pi^+\pi^-) = 1$, but this ratio appears to be [6,27] 2.6 ± 0.4 . It is possible to take at least partial account of such effects in the case of B decays. Some of them are expected to be independent of the mass of the decaying quark and some are expected to decrease with increasing quark mass. The independent ways we have described of extracting the same strong and weak phases with SU(3) relations provide a way to measure the size of SU(3) breaking effects.

TABLE V. Decomposition of $B \rightarrow PP$ amplitudes for $\Delta C = 0, |\Delta S| = 1$ transitions involving η and η' as defined in (67) in terms of graphical contributions.

	Final state	T'	C'	P'
$B^+ \rightarrow$	$\eta\bar{K}^+$	$-1/\sqrt{3}$	$-1/\sqrt{3}$	0
	$\eta'K^+$	$1/\sqrt{6}$	$1/\sqrt{6}$	$3/\sqrt{6}$
$B^0 \rightarrow$	ηK^0	0	$-1/\sqrt{3}$	0
	$\eta'K^0$	0	$1/\sqrt{6}$	$3/\sqrt{6}$
$B_s \rightarrow$	$\pi^0\eta$	0	$-1/\sqrt{6}$	0
	$\pi^0\eta'$	0	$1/\sqrt{3}$	0
	$\eta\eta$	0	$-\sqrt{2}/3$	$\sqrt{2}/3$
	$\eta\eta'$	0	$-1/3\sqrt{2}$	$\sqrt{2}/3$
	$\eta'\eta'$	0	$-\sqrt{2}/3$	$2\sqrt{2}/3$

A. Meson decay constants

In the description of decays via factorization, a charged weak current can materialize either into a pion, with decay constant $f_\pi = 132$ MeV, or into a kaon, with decay constant $f_K = 160$ MeV. Through Fierz identities it is sometimes assumed that neutral quark-antiquark combinations emerging from a weak vertex materialize into neutral pseudoscalar mesons with corresponding decay constants, though this assumption is on shakier ground.

Since $f_K/f_\pi \approx 1.2$, one can expect this effect alone to contribute to deviations in decay rates by more than 40% from the naive SU(3) expectation, independently of the mass of the decaying particle. Such effects have been taken into account in Ref. [11] in relating penguin effects in $B \rightarrow \pi\pi$ to the corresponding ones in $B \rightarrow \pi K$.

B. Form factors and hadronization

The rates for processes like $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$, if calculated using factorization, depend not only on meson decay constants but also on form factors for $D \rightarrow K$ and $D \rightarrow \pi$. If $f_{D \rightarrow K}^+(0)/f_{D \rightarrow \pi}^+(0) > 1.2$, one can understand why the rates for $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ are so different. The cooperation of two different SU(3)-breaking effects (decay constants and form factors) in this case originates in the presence of two different tree subprocesses ($c \rightarrow d\bar{u}\bar{d}$ and $c \rightarrow s\bar{u}\bar{s}$). No similar case arises in B decays, and therefore SU(3) breaking is generally expected to be smaller.

Some evidence that the corresponding ratio $f_{B \rightarrow K}^+(0)/f_{B \rightarrow \pi}^+(0)$ exceeds 1, and could be of the order of 1.1 ± 0.1 , comes from a recent QCD sum rule calculation [28]. In Ref. [11] this ratio of form factors was assumed equal to 1. This ratio may be relevant to the ratio of $b \rightarrow s$ and $b \rightarrow d$ penguin contributions, at least for the decays of nonstrange B 's.

In SU(3) we assume that the probability of producing an extra $s\bar{s}$ pair from the vacuum equals the probability for $u\bar{u}$ and $d\bar{d}$ production. For the heavy B meson this may be a good approximation.

C. Specific applications

The SU(3) relations between $\Delta S = 0$ and $|\Delta S| = 1$ transitions are of two types, Eqs. (21) and (22). For $\Delta S = 0$ processes, the dominant effects are expected to be T and C , while for $|\Delta S| = 1$ we expect P' to dominate. The small admixture of P in $\Delta S = 0$ transitions is estimated from P' using (22), while the small contributions of T' and C' to $|\Delta S| = 1$ transitions are estimated using (21). In both of these cases, since SU(3) is only used in order to estimate the magnitude of the smaller amplitude, the effects of SU(3) breaking will be suppressed by a factor of 4 to 5 in any given decay.

When we come to relations such as (23) in which two large amplitudes nearly cancel, SU(3) breaking is more important. Here, by referring to the graphs which give

rise to the amplitudes C and T , and assuming factorization to govern their behavior, we can expect the main effect of SU(3) breaking to involve the ratio f_K/f_π . We then expect (23) to be replaced by (31), as mentioned above and as used in the subsequent discussions.

The factor f_K/f_π is well motivated in the case of the ratio T'/T , where factorization has been tested for some processes like $B \rightarrow D\pi$ (see, e.g., Ref. [29]). As mentioned above, one relies on a Fierz rearrangement to justify it for C'/C . It may be possible to check this assumption more explicitly by comparing various $B \rightarrow D\pi$ and $B \rightarrow DK$ decays. In addition, the redundancy of the measurements of the strong and weak phases described in Sec. III D could give us an additional handle on the size of SU(3)-breaking effects (beyond f_K/f_π).

It is more difficult to estimate the effects of SU(3) breaking on an equation such as (29) which involves the ratio r_t in (22). Both form factor and hadronization effects enter into corrections to this relation.

VI. SUMMARY AND EXPERIMENTAL PROSPECTS

We have examined the decays of B mesons to two light pseudoscalars within the context of SU(3), looking for amplitude relations, simplifications, and help in sorting out the physics of the $B \rightarrow \pi K$ system. While the SU(3) decompositions we obtain are not new, we have found a number of simple linear relations among amplitudes whose validity tests assumptions at various levels of generality.

An SU(3) analysis without further simplifying assumptions leads to several rate predictions, a number of triangle relations among amplitudes, and one very useful relation (23) [or (31) if SU(3) breaking is taken into account] between the amplitude for $B^+ \rightarrow \pi^+\pi^0$ and the $I = 3/2$ amplitude in $B \rightarrow \pi K$. This last relation can be used in several different ways, including the specification of relative phases of various $B \rightarrow \pi K$ amplitudes and the substitution of a measurement of the $B^+ \rightarrow \pi^+\pi^0$ rate for a measurement of the rates for $B^0 \rightarrow \pi^-K^+$ and the charge-conjugate process in sorting out CKM phases. We have also shown how to extract strong final-state phase differences $\delta_P - \delta_2$ or $\delta_{P'} - \delta_{3/2}$ between the penguin amplitude and the $I = 2$ amplitude in $B \rightarrow \pi\pi$ or the $I = 3/2$ amplitude in $B \rightarrow \pi K$.

Additional predictions are obtained if one is prepared to neglect certain contributions in a manner motivated by a graphical SU(3) language. The neglect of these contributions is predicated on the relative unimportance of strong rescattering effects. These predictions frequently convert triangle relations to relations regarding rates, since in a number of cases they imply that one side of a triangle has vanishing length. One application of these relations is that measurements of the rates for B^+ to $\pi^+\pi^0$, π^+K^0 , and π^0K^+ and the charge-conjugate processes can be used to determine the weak CKM phase γ . With measurements of the remaining rates for B decays to $\pi\pi$, πK , and $K\bar{K}$, one can obtain the CKM phases γ and α and all the relevant differences of strong phase

shifts.

In the more general case, when all amplitudes are considered, we can learn about final-state phase shift differences from the decays $B \rightarrow \pi\bar{D}$, which involve a single CKM factor. If such phase shift differences were small, we would expect them to be even smaller in the decays to pairs of light pseudoscalars in which more energy is released. One relies on the presence of such phase shift differences in order to be able to detect direct CP violation in such processes as self-tagging $B \rightarrow \pi K$ decays.

As we have mentioned earlier, some combination of the decays $B^0 \rightarrow \pi^+\pi^-$ and $B^0 \rightarrow \pi^-K^+$ has been observed [15], with a total branching ratio of about 2×10^{-5} . If this consists of equal amounts of $\pi\pi$ and πK , and if the πK decays are indeed dominated by a $\Delta I = 0$ transition as occurs in penguin graphs, all the charge states of $B \rightarrow \pi K$ should be observable at levels of 10^{-5} for charged pions or half that for neutral pions. Similarly, if color-unsuppressed tree diagrams dominate the $B \rightarrow \pi^+\pi^-$ process, and if it occurs at a level of 10^{-5} , one should see $B^+ \rightarrow \pi^+\pi^0$ at a level of half that. Once these signals are observed, refinements of rate information will be able to test for the presence of subdominant contributions. The next step would be to look for processes which we predict to be suppressed; searches at branching ratios down to 10^{-7} would be able to provide information on amplitudes at the 10% level and could be of great help in sorting out prospects for observing signals of CP violation.

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APPENDIX: SU(3) REDUCED MATRIX ELEMENTS

We relate the SU(3) reduced matrix elements as introduced by Zeppenfeld [8] to the diagrammatic contributions described in Sec. II.

We introduce a shorthand based on the decomposition (4)–(6) in Sec. II. There is a unique singlet amplitude; we denote it by $\{1\}$. The three octet amplitudes arising from the $\mathbf{3}^*$, $\mathbf{6}$, and $\mathbf{15}^*$ operators in (4)–(6) are denoted by $\{8_1\}$, $\{8_2\}$, and $\{8_3\}$, respectively. There is a unique amplitude $\{27\}$. The singlet and first octet receive contributions involving both $V_{uq}V_{ub}^*$ and $V_{iq}V_{ib}^*$ (we can eliminate $V_{cq}V_{cb}^*$ using unitarity), while the second and third

octets and the 27-plet receive only contributions proportional to $V_{uq}V_{ub}^*$. Here q stands for d or s .

It is sufficient to discuss the case of $\Delta C = 0$, $\Delta S = 0$ decays; a corresponding set of relations exists for the strangeness-changing $\Delta C = 0$ amplitudes. Absorbing CKM factors into the definitions of reduced amplitudes, we then have the relations

$$\{1\} = 2\sqrt{3} \left[PA + \frac{2}{3}E + \frac{2}{3}P - \frac{1}{12}C + \frac{1}{4}T \right], \quad (\text{A1})$$

$$\{8_1\} = -\sqrt{\frac{5}{3}} \left[P + \frac{3}{8}(T + A) - \frac{1}{8}(C + E) \right], \quad (\text{A2})$$

$$\{8_2\} = \frac{\sqrt{5}}{4}(C + A - T - E), \quad (\text{A3})$$

$$\{8_3\} = -\frac{1}{8\sqrt{3}}(T + C) - \frac{5}{8\sqrt{3}}(A + E), \quad (\text{A4})$$

$$\{27\} = -\frac{T + C}{2\sqrt{3}}. \quad (\text{A5})$$

All amplitudes are linear combinations of these contributions. A corresponding set of relations exists for the primed quantities, with primed contributions related to unprimed ones by Eqs. (21) and (22).

The singlet amplitude is the only one which contains the penguin annihilation (PA) contribution. It does not receive any contribution from the annihilation graph, which contributes only to direct-channel octet amplitudes. The penguin contribution P appears only in the singlet and first octet amplitudes.

If one neglects E , A , and PA one obtains the relations (46), reducing the number of independent SU(3) amplitudes from five to three. However, the same relations also follow from the less restrictive assumptions $E + A = E + PA = 0$. As one sees from the discussion of Tables I and II, these are the only two independent combinations which contain exclusively contributions of graphs we wish to neglect.

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