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Global analysis of the top quark couplings to gauge bosons

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We propose to probe the electroweak symmetry-breaking sector by measuring the effective couplings of the top quark to gauge bosons. Using precision CERN LEP data, we constrain the nonuniversal couplings of t-t-Z and t-b-W, parametrized by $\kappa_L^{\rm NC}$, $\kappa_R^{\rm NC}$, $\kappa_L^{\rm CC}$ and $\kappa_R^{\rm CC}$, in the electroweak chiral Lagrangian framework. Different scenarios of electroweak symmetry breaking will imply different correlations among these parameters. We find that at the order of $m_t^2 \ln \Lambda^2$, in which $\Lambda \sim 4\pi v$ is the cutoff scale of the effective theory, $\kappa_L^{\rm NC}$ is already constrained by LEP data. In models with an approximate custodial symmetry, a positive $\kappa_L^{\rm CC}$ is preferred. $\kappa_R^{\rm CC}$ can be constrained by studying the direct detection of the top quark at the Fermilab Tevatron and the CERN LHC. At the NLC, $\kappa_L^{\rm NC}$ and $\kappa_R^{\rm NC}$ can be better measured.

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I. INTRODUCTION AND MOTIVATIONS

Despite the success of the standard model (SM) [1,2], there is little faith that the SM is the final theory. The reasons behind this are fundamental and basic [3], e.g., the SM contains many arbitrary parameters with no apparent connections. In addition, the SM provides no satisfactory explanation for the symmetry-breaking mechanism which takes place and gives rise to the observed mass spectrum of the gauge bosons and fermions. In this paper, we study how to use the top quark to probe the origin of the spontaneous symmetry breaking and the generation of fermion masses.

There are strong experimental and theoretical arguments suggesting the top quark must exist [4]; e.g., from the measurement of the weak isospin quantum number of the left-handed b quark we know the top quark has to exist. From the direct search at the Tevatron, assuming SM top quark, m_t has to be larger than 131 GeV [5]. Recently, data were presented by the CDF group at Fermilab to support the existence of a heavy top quark with mass $m_t \sim 174 \pm 20$ GeV [6]. Furthermore, studies on radiative corrections concluded that the mass (m_t) of a standard top quark has to be less than 200 GeV [1]. However, there are no compelling reasons to believe that the top quark couplings to light particles should be of the SM nature. Because the top quark is heavy relative to other observed fundamental particles, one expects that any underlying theory at high energy scale $\Lambda \gg m_t$ will easily reveal itself at low energy through the effective interactions of the top quark to other light particles. Also because the top quark mass is of the order of the Fermi scale $v = (\sqrt{2}G_F)^{-1/2} = 246$ GeV, which characterizes the electroweak symmetry-breaking scale, the top quark may be a useful tool to probe the symmetry-breaking sector. Since the fermion mass generation can be closely related to the electroweak symmetry breaking, one expects some residual effects of this breaking to appear in accordance with the generated mass [7,8]. This means new effects should be more apparent in the top quark sector than any other light sector of the theory. Therefore, it is important to study the top quark system as a direct tool to probe new physics effects [4].

Undoubtedly, any real analysis including the top quark cannot be completed without actually discovering it. In the SM, which is a renormalizable theory, the couplings of the top quark to gauge bosons are fixed by the linear realization of the gauge symmetry $SU(2)_L \times U(1)_Y$. However, the top quark mass remains a free parameter in the theory (SM). If the top quark is not a SM quark, then in addition to the unknown top mass, the couplings of the top quark to gauge bosons are not known. Also, the effective theory describing the top quark interactions at low energy can be non-renormalizable. Therefore, to conclude the properties of the top quark from the radiative corrections is less vital and predictive. Still, precision data at this stage are our best hope to look for any possible deviation in the top quark sector from the SM.

The goal of this paper is to study the couplings of the top quark to gauge bosons from the precision data at the CERN e^+e^- collider LEP and examine how to improve our knowledge about the top quark at the current and future colliders. Also we will discuss how to use this knowledge to probe the symmetry-breaking mechanism.

Generally one studies a specific model (e.g., a grand unified theory) valid up to some high energy scale and evolves that theory down to the electroweak scale to compare its predictions with the precision LEP data [8–10]. Such an approach provides a consistent analysis for low energy data. In addition to such a model by model study, one can incorporate new physics effects in a model-independent way formulated in terms of either a set of variables [11–14] or an effective Lagrangian [15–17]. In this paper, we will adopt the latter approach. We simply address the problem in the following way. Assume there is an underlying theory at some high energy scale. How does this theory *appreciably* manifest itself at low energy? Because we do not know the shape of the underlying theory and because a general treatment is usually very complicated, we cannot provide a satisfying answer. Still, one can get some crude answers to this question based on a few *negotiable* arguments suggested by the status of low energy data with the application of the electroweak chiral Lagrangian.

It is generally believed that new physics is likely to come in via processes involving longitudinal gauge bosons (equivalent to Goldstone bosons) and/or heavy fermions such as the top quark. One commonly discussed method to probe the electroweak symmetry sector is to study the interactions among the longitudinal gauge bosons in the TeV region. Tremendous work has been done in the literature [18]. However, this is not the subject of this paper. As we argued above, the top quark plays an important role in the search for new physics. Because of its heavy mass, new physics will feel its presence easily and eventually may show up in its couplings to the gauge bosons. If the top quark is a participant in a dynamical symmetry-breaking mechanism, e.g., through the $\bar{t}t$ condensate (top mode standard model) [19] which is suggested by the fact that its mass is of the order of the Fermi scale v, then the top quark is one of the best candidates for search of new physics.

An attempt to study the nonuniversal interactions of the top quark has been carried out in Ref. [7] by Peccei *et al.* However, in that study only the vertex t-t-Z was considered based on the assumption that this is the only vertex which gains a significant modification due to a speculated dependence of the coupling strength on the fermion mass: $\kappa_{ij} \le O\left(\frac{\sqrt{m_i m_j}}{v}\right)$, where κ_{ij} parameterizes some new dimensional–four interactions among gauge bosons and fermions i and j. However, this is not the only possible pattern of interactions, e.g., in some extended technicolor models [8] one finds that the nonuniversal residual interactions associated with the vertices b_L-b_L-Z, t_L-t_L-Z, and t_L-b_L-W to be of the same order. In Sec. IV we discuss the case of the SM with a heavy Higgs boson ($m_H > m_t$) in which we find the size of the nonuniversal effective interactions t_L-t_L-Z and t_L-b_L-W to be of the same order but with a negligible b_L-b_L-Z effect.

Here is the outline of our approach. First, we use the chiral Lagrangian approach [20–23] to construct the most general $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ invariant effective Lagrangian including up to dimension-four operators for the top and bottom quarks. Then we deduce the SM (with and without a scalar Higgs boson) from this Lagrangian, and only consider new physics effects which modify the top quark couplings to gauge bosons and possibly the vertex b_L-b_L-Z. With this in hand, we perform a comprehensive analysis using precision data from LEP. We include the contributions from the vertex t-b-W in addition to the vertex t-t-Z, and discuss the special case of having a comparable size in b-b-Z as in t-t-Z. Second, we build an effective model with an approximate custodial symmetry ($\rho \approx 1$) connecting the t-t-Z and t-b-W couplings. This reduces the number of parameters in the effective Lagrangian and strengthens its structure and predictability. After examining what we have learned from the LEP data, we study how to improve our knowledge on these couplings at the SLAC Linear Collider (SLC), the Fermilab Tevatron, the CERN Large Hadron Collider (LHC), and the Next Linear Collider (NLC) [24]. (We use the NLC to represent a generic e^-e^+ supercollider.)

The rest of this work is organized as follows. In Sec. II we provide a brief introduction to the chiral Lagrangian with an emphasis on the top quark sector. In Sec. III we present the complete analysis of the top quark interactions with gauge bosons using LEP data for various scenarios of symmetry-breaking mechanism. In Sec. IV we discuss the heavy Higgs limit ($m_H > m_t$) in the SM model as an example of our proposed effective model at the top quark mass scale. In Sec. V we discuss how the SLC, Tevatron, LHC, and NLC can contribute to the measurement of these couplings. Some discussion and conclusions are given in Sec. VI.

II. INTRODUCTION TO THE CHIRAL LAGRANGIAN

The chiral Lagrangian approach has been used in understanding the low energy strong interactions because it can systematically describe the phenomenon of spontaneous symmetry breaking [20]. Recently, the chiral Lagrangian technique has been widely used in studying the electroweak sector [16,23,25–29], to which this work has been directed.

The chiral Lagrangian can be constructed solely on symmetry with no other assumptions regarding explicit dynamics. Thus, it is the most general effective Lagrangian that can accommodate any truly fundamental theory possessing that symmetry at low energy. Since one is interested in the low energy behavior of such a theory, an expansion in powers of the external momentum is performed in the chiral Lagrangian [21].

In general one starts from a Lie group G which breaks down spontaneously into a subgroup H, hence a Goldstone boson for every broken generator is to be introduced [22]. Consider, for example, $G = \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ and $H = \mathrm{U}(1)_{\mathrm{em}}$. There are three Goldstone bosons generated by this breakdown, ϕ^a, $a = 1, 2, 3$ which are eventually eaten by $W^\pm$ and Z and become the longitudinal degree of freedom of these gauge bosons.

The Goldstone bosons transform nonlinearly under G but linearly under the subgroup H. A convenient way to handle this is to introduce the matrix field

$$\Sigma = \exp\left(i\frac{\phi^a\tau^a}{v_a}\right) , \tag{1}$$

where τ^a, $a = 1, 2, 3$ are the Pauli matrices normalized as $\mathrm{Tr}(\tau^a\tau^b) = 2\delta_{ab}$. Because of $\mathrm{U}(1)_{\mathrm{em}}$ invariance $v_1 = v_2 = v$, but is not necessarily equal to v_3. The matrix field Σ transforms under G as

$$\Sigma \to \Sigma' = \exp\left(i\frac{\alpha^a\tau^a}{2}\right) \Sigma \exp\left(-iy\frac{\tau^3}{2}\right) , \tag{2}$$

where $\alpha^{1,2,3}$ and y are the group parameters of G.

In the SM, being a special case of the chiral Lagrangian, $v = 246$ GeV is the vacuum expectation value of the Higgs boson field. Also $v_3 = v$ arises from the approximate custodial symmetry in the SM. It is this symmetry that is responsible for the tree-level relation

$$\rho = \frac{M_W}{M_Z \cos\theta_W} = 1 \tag{3}$$

in the SM, where θ_W is the electroweak mixing angle. In this paper, we assume the full theory guarantees that $v_1 = v_2 = v_3 = v$.

Out of the Goldstone bosons and the gauge boson fields one can construct the bosonic gauge invariant terms in the chiral Lagrangian

$$\mathcal{L}_B = -\frac{1}{4}W^a_{\mu\nu}W^{\mu\nu a} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{1}{4}v^2\mathrm{Tr}(D_\mu\Sigma^\dagger D^\mu\Sigma)\,, \tag{4}$$

where the covariant derivative

$$D_\mu\Sigma = \partial_\mu\Sigma - igW^a_\mu\frac{\tau^a}{2}\Sigma + ig'\Sigma B_\mu\frac{\tau^3}{2}\,. \tag{5}$$

In the unitary gauge $\Sigma = 1$, one can easily see how the gauge bosons acquire a mass. In Eq. (3), $M_W = gv/2$ is the mass of $W^\pm_\mu = (W^1_\mu \mp iW^2_\mu)/\sqrt{2}$), $M_Z = gv/2/\cos\theta_W$ is the mass of $Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu$. The photon field will be denoted as $A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu$.

Fermions can be included in this context by assuming that they transform under $G = \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ as [25]

$$f \to f' = e^{iyQ_f} f\,, \tag{6}$$

where Q_f is the electromagnetic charge of f.

Out of the fermion fields f_1, f_2 and the Goldstone bosons matrix field Σ the usual linearly realized fields Ψ can be constructed. For example, the left-handed fermions [$\mathrm{SU}(2)_L$ doublet] are constructed as

$$\Psi_L = \Sigma F_L = \Sigma\begin{pmatrix} f_1 \\ f_2 \end{pmatrix}_L \tag{7}$$

with $Q_{f_1} - Q_{f_2} = 1$. One can easily show that Ψ_L transforms under G linearly as

$$\Psi_L \to \Psi'_L = g\Psi_L\,, \tag{8}$$

where $g = \exp(i\frac{\alpha^a\tau^a}{2})\exp(i\frac{y}{2}) \in G$. Linearly realized right-handed fermions Ψ_R [$\mathrm{SU}(2)_L$ singlet] simply coincide with F_R: i.e.,

$$\Psi_R = F_R = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}_R\,. \tag{9}$$

Out of those fields with the specified transformations it is straightforward to construct a Lagrangian which is invariant under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$.

Since the interactions among the light fermions and the gauge bosons have been well tested to agree with the SM, we only consider new interactions involving the top and bottom quarks. We ignore all possible mixing of the top quark with light fermions in these new interactions. In case there exists a fourth generation with heavy fermions, there can be a substantial impact on the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{tb}. To be discussed below, this effect is effectively included in the new nonstandard couplings of t-b-W.

Following Ref. [25], define

$$\Sigma^a_\mu = -\frac{i}{2}\mathrm{Tr}(\tau^a\Sigma^\dagger D_\mu\Sigma)\,, \tag{10}$$

which transforms under G as

$$\Sigma^3_\mu \to \Sigma'^3_\mu = \Sigma^3_\mu\,, \tag{11}$$

$$\Sigma^\pm_\mu \to \Sigma'^\pm_\mu = e^{\pm iy}\Sigma^\pm_\mu\,, \tag{12}$$

where

$$\Sigma^\pm_\mu = \frac{1}{\sqrt{2}}(\Sigma^1_\mu \mp i\Sigma^2_\mu)\,. \tag{13}$$

In the unitary gauge $\Sigma = 1$ we have

$$\Sigma^3_\mu = -\frac{1}{2}\frac{gZ_\mu}{\cos\theta_W}\,, \tag{14}$$

$$\Sigma^\pm_\mu = -\frac{1}{2}gW^\pm_\mu\,. \tag{15}$$

Consider the interaction terms up to dimension-four for the t and b quarks. From Eqs. (7) and (9) we denote

$$F = \begin{pmatrix} t \\ b \end{pmatrix} = F_L + F_R\,, \tag{16}$$

with $f_1 = t$ and $f_2 = b$. The SM Lagrangian can be deduced from

$$\begin{aligned}\mathcal{L}_0 = {}& \overline{F}i\gamma^\mu\left[\partial_\mu - ig'\left(\frac{Y}{2} + \frac{\tau^3}{2}\right)B_\mu\right]F - \overline{F}MF \\ & -\overline{F_L}\gamma^\mu\tau^a F_L\Sigma^a_\mu + \mathcal{L}_B\,,\end{aligned} \tag{17}$$

where $Y = 1/3$ and M is a diagonal mass matrix

$$M = \begin{pmatrix} m_t & 0 \\ 0 & m_b \end{pmatrix}\,. \tag{18}$$

$\mathcal{L}_0$ is invariant under G, and the electric charge of fermions is given by $Y/2 + T^3$, where T^3 is the weak isospin quantum number. Taking advantage of the chiral Lagrangian approach, additional nonstandard interaction terms, invariant under G, are allowed [25]

$$\begin{aligned}\mathcal{L} = {}& -\kappa^{\mathrm{NC}}_L\overline{t_L}\gamma^\mu t_L\Sigma^3_\mu - \kappa^{\mathrm{NC}}_R\overline{t_R}\gamma^\mu t_R\Sigma^3_\mu \\ & -\sqrt{2}\kappa^{\mathrm{CC}}_L\overline{t_L}\gamma^\mu b_L\Sigma^+_\mu - \sqrt{2}\kappa^{\mathrm{CC}\dagger}_L\overline{b_L}\gamma^\mu t_L\Sigma^-_\mu \\ & -\sqrt{2}\kappa^{\mathrm{CC}}_R\overline{t_R}\gamma^\mu b_R\Sigma^+_\mu - \sqrt{2}\kappa^{\mathrm{CC}\dagger}_R\overline{b_R}\gamma^\mu t_R\Sigma^-_\mu\,,\end{aligned} \tag{19}$$

where κ^{NC}_L, κ^{NC}_R are two arbitrary real parameters, κ^{CC}_L, κ^{CC}_R are two arbitrary complex parameters, and the superscripts NC and CC denote neutral and charged currents, respectively. In the unitary gauge we derive the following nonstandard terms in the chiral Lagrangian with the symmetry $\frac{\mathrm{SU}(2)_L\times U(1)_Y}{\mathrm{U}(1)_{\mathrm{em}}}$:

$$\mathcal{L} = \frac{g}{4\cos\theta_W}\bar{t}\left[\kappa_L^{\mathrm{NC}}\gamma^\mu(1-\gamma_5)+\kappa_R^{\mathrm{NC}}\gamma^\mu(1+\gamma_5)\right]t\,Z_\mu + \frac{g}{2\sqrt{2}}\bar{t}\left[\kappa_L^{\mathrm{CC}}\gamma^\mu(1-\gamma_5)+\kappa_R^{\mathrm{CC}}\gamma^\mu(1+\gamma_5)\right]b\,W_\mu^+ + \frac{g}{2\sqrt{2}}\bar{b}\left[\kappa_L^{\mathrm{CC}\dagger}\gamma^\mu(1-\gamma_5)+\kappa_R^{\mathrm{CC}\dagger}\gamma^\mu(1+\gamma_5)\right]t\,W_\mu^- . \tag{20}$$

A few remarks are in order regarding the Lagrangian $\mathcal{L}$ in Eqs. (19) and (20).

(1) In principle, $\mathcal{L}$ can include nonstandard neutral currents $\overline{b_L}\gamma_\mu b_L$ and $\overline{b_R}\gamma_\mu b_R$. For the left-handed neutral current $\overline{b_L}\gamma_\mu b_L$ we discuss two cases.

(a) The effective left-handed vertices t_L-t_L-Z, t_L-b_L-W, and b_L-b_L-Z are comparable in size as in some extended technicolor models [8]. In this case, the top quark contribution to low energy observables is of higher order through radiative corrections; therefore, its contribution will be suppressed by $1/16\pi^2$. In this case, as we will discuss in the next section, the constraints derived from low energy data on the nonstandard couplings are so stringent (of the order of a few percent) that it would be a challenge to directly probe the nonstandard top quark couplings at the Tevatron, the LHC, and the NLC.

(b) The effective left-handed vertex b_L-b_L-Z is small as compared to the t-t-Z and t-b-W vertices. We will devote most of this work to the case where the vertex b_L-b_L-Z is not modified by the dynamics of the symmetry breaking. This assumption leads to interesting conclusions to be seen in the next section. In this case one needs to consider the contributions of the top quark to low energy data through loop effects. A specific model with such properties is given in Sec. IV.

(2) We shall assume that b_R-b_R-Z is not modified by the dynamics of the electroweak symmetry breaking. This is the case in the extended technicolor models discussed in Ref. [8]. The model discussed in Sec. IV is another example.

(3) The right-handed charged current contribution κ_R^{CC} in Eqs. (19) and (20) is expected to be suppressed by the bottom quark mass. This can be understood in the following way. If b is massless ($m_b = 0$), then the left- and right-handed b fields can be associated with different global U(1) quantum numbers. [U(1) is a chiral group, not the hypercharge group.] Since the underlying theory has an exact $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ symmetry at high energy, the charged currents are purely left-handed before the symmetry is broken. After the symmetry is spontaneously broken and for a massless b the U(1) symmetry associated with b_R remains exact (chiral invariant) so it is not possible to generate right-handed charged currents. Thus κ_R^{CC} is usually suppressed by the bottom quark mass although it could be enhanced in some models with a larger group G, i.e., in models containing additional right-handed gauge bosons.

We find that in the limit of ignoring the bottom quark mass, κ_R^{CC} does not contribute to low energy data through loop insertion at the order ${m_t}^2\ln\Lambda^2$, therefore we cannot constrain κ_R^{CC} from the LEP data. However, at the Tevatron and the LHC κ_R^{CC} can be measured by studying the direct detection of the top quark and its decays. This will be discussed in Sec. V.

It is worth mentioning that the photon does not participate in the new nonuniversal interactions as described in the chiral Lagrangian $\mathcal{L}$ in Eq. (20) because the $\mathrm{U}(1)_{\mathrm{em}}$ symmetry remains an exact symmetry of the effective theory. Using Ward identities one can show that such nonuniversal terms should not appear. To be precise, any new physics can only contribute to the universal interactions of the photon to charged fields. This effect can simply be absorbed in redefining the electromagnetic fine structure constant α, hence no new t-t-A or b-b-A interaction terms will appear in the effective Lagrangian after a proper renormalization of α.

Here is a final note regarding the physical Higgs boson. It is known that the gauge bosons acquire masses through the spontaneous symmetry-breaking mechanism. In the chiral Lagrangian this can be seen from the last term in $\mathcal{L}_B$ [see Eq. (4)], which only involves the gauge bosons and the unphysical Goldstone bosons. This indicates that the chiral Lagrangian can account for the mass generation of the gauge bosons without the actual details of the symmetry-breaking mechanism. Furthermore, the fermion mass term is also allowed in the chiral Lagrangian,

$$-m_{f_i}\overline{f_i}f_i\,, \tag{21}$$

because it is invariant under G, where the fermion field f_i transforms as in Eq. (6).

From this it is clear the Higgs boson is not necessary in constructing the low energy effective Lagrangian. Indicating that the SM Higgs mechanism is just one example of the possible spontaneous symmetry-breaking scenarios which might take place in nature. Still, a Higgs boson can be inserted in the chiral Lagrangian as an additional field $[\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ singlet$]$ with arbitrary couplings to the rest of the fields. To retrieve the SM Higgs boson contribution at tree level, one can simply substitute the fermion mass m_f by $g_f v$ and v by $v+H$, where g_f is the Yukawa coupling for fermion f and H is the Higgs boson field. Hence, we get the scalar sector Lagrangian

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu H\partial^\mu H - \frac{1}{2}m_H H^2 - V(H) + \frac{1}{2}vH\mathrm{Tr}\left(D_\mu\Sigma^\dagger D^\mu\Sigma\right) + \frac{1}{4}H^2\mathrm{Tr}\left(D_\mu\Sigma^\dagger D^\mu\Sigma\right), \tag{22}$$

where $V(H)$ describes the Higgs boson self-interaction. The coefficients of the last two terms in the above equation can be arbitrary for a chiral Lagrangian with a scalar field other than the SM Higgs boson.

In this analysis we will discuss models with and without a Higgs boson. In the case of a light Higgs boson ($m_H < m_t$) we will include the Higgs boson field in the chiral Lagrangian as a part of the light fields with no new physics being associated with it. In the case of a heavy Higgs boson ($m_H > m_t$) in the full theory, we assume the

Higgs boson field has been integrated out and its effect on low energy physics can be thought of as a new heavy physics effect which is already included in the effective couplings of the top quark at the scale of m_t. Finally, we will consider the possibility of a spontaneous symmetry-breaking scenario without including a SM Higgs boson in the full theory. In this case we consider the effects on low energy data from the new physics parametrized by the nonstandard interaction terms in $\mathcal{L}$ in Eq. (20) and contributions from the SM without a Higgs boson.

III. THE TOP QUARK COUPLINGS TO GAUGE BOSONS

As we discussed in the previous section, one possibility of new physics effects is the modification of the vertices b-b-Z, t-t-Z, and t-b-W in the effective Lagrangian by the same order of magnitude [8]. In this case, only the vertex b-b-Z can have large contributions to low energy data while, based on the dimensional counting method, the contributions from the other two vertices t-t-Z and t-b-W are suppressed by $1/16\pi^2$ due to their insertion in loops.

In this case, one can use Γ_b (the partial decay width of the Z boson to $\bar{b}b$) to constrain the b-b-Z coupling. Denote the nontandard b-b-Z vertex as

$$\frac{g}{4\cos\theta_W}\kappa\gamma_\mu(1-\gamma_5)\,, \tag{23}$$

which is purely left handed. In some extended technicolor models, discussed in Ref. [8], this nonstandard effect arises from the same source as the mass generation of the top quark, therefore κ depends on the top quark mass.

As we will discuss later, the nonuniversal contribution to Γ_b is parametrized by a measurable parameter denoted as ϵ_b [12–14] which is measured to be [12]

$$\epsilon_b\,(10^3) = 4.4 \pm 7.0\,. \tag{24}$$

The SM contribution to ϵ_b is calculated in Refs. [12,13], e.g., for a 150 GeV top quark,

$$\epsilon_b^{\rm SM}\,(10^3) = -4.88\ . \tag{25}$$

The contribution from κ to ϵ_b is

$$\epsilon_b = -\kappa\,. \tag{26}$$

Within a 95% confidence level (C.L.), from ϵ_b we find that

$$-22.9 \le \kappa\,(10^3) \le 4.4\,. \tag{27}$$

As an example, the simple commuting extended technicolor model presented in Ref. [8] predicts that

$$\kappa \approx \frac{1}{2}\xi^2\frac{m_t}{4\pi v}\,, \tag{28}$$

where ξ is of order 1. Also in that model the top quark couplings $\kappa_L^{\rm NC}$, $\kappa_R^{\rm NC}$ and $\kappa_L^{\rm CC}$, as defined in Eqs. (19) and (20), are of the same order as κ. For a 150 GeV top quark, this model predicts

$$\kappa\,(10^3) \approx 24.3\,\xi^2\,. \tag{29}$$

Hence, such a model is likely to be excluded using low energy data.

We will devote the subsequent discussion to models in which the nonstandard b-b-Z coupling can be ignored relative to the t-t-Z and t-b-W couplings. In this case one needs to study their effects at the quantum level, i.e., through loop insertion. We will first discuss the general case where no relations between the couplings are assumed. Later we will impose a relation between $\kappa_L^{\rm NC}$ and $\kappa_L^{\rm CC}$ which are defined in Eqs. (19) and (20) using an effective model with an approximate custodial symmetry.

A. General case

The chiral Lagrangian in general has a complicated structure and many arbitrary coefficients which weaken its predictive power. Still, with a few further assumptions, based on the status of present low energy data, the chiral Lagrangian can provide a useful approach to confine the coefficients parametrizing new physics effects.

In this subsection we provide a general treatment for the case under study with minimal imposed assumptions in the chiral Lagrangian. In this case, we only impose the assumption that the vertex b-b-Z is not modified by the dynamics. In the chiral Lagrangian $\mathcal{L}$, as defined in Eqs. (19) and (20), there are six independent parameters (κ's) which need to be constrained using precision data. Throughout this paper we will only consider the insertion of κ's once in one-loop diagrams by assuming that these nonstandard couplings are small; $\kappa_{L,R}^{\rm NC,CC} \lesssim 1$. At the one-loop level the imaginary parts of the couplings do not contribute to those LEP observables of interest. Thus, hereafter we drop the imaginary pieces from the effective couplings, which reduces the number of relevant parameters to four. Since the bottom quark mass is small relative to the top quark mass, we find that $\kappa_R^{\rm CC}$ does not contribute to low energy data up to the order ${m_t}^2\ln\Lambda^2$ in the $m_b \to 0$ limit. With these observations we conclude that only the three parameters $\kappa_L^{\rm NC}$, $\kappa_R^{\rm NC}$, and $\kappa_L^{\rm CC}$ can be constrained.

A systematic approach can be implemented for such an analysis based on the scheme used in Refs. [12–14], where the radiative corrections can be parametrized by 4 independent parameters, three of those parameters ϵ_1, ϵ_2, and ϵ_3 are proportional to the variables S, U, and T [11], and the fourth one; ϵ_b is due to the Glashow-Iliopoulos-Maiani- (GIM-) violating contribution in $Z \to b\bar{b}$ [12].

These parameters are derived from four basic measured observables, Γ_ℓ (the partial width of Z to a charged lepton pair), A_{FB}^ℓ (the forward-backward asymmetry at the Z peak for the charged lepton ℓ), M_W/M_Z, and Γ_b (the partial width of Z to a $b\bar{b}$ pair). The expressions of these observables in terms of ϵ's are given in Refs. [12,13]. In this paper we only give the relevant terms in ϵ's which might contain the leading effects from new physics.

We denote the vacuum polarization for the W^1, W^2, W^3, and B gauge bosons as

$$\Pi^{ij}{}_{\mu\nu}(q) = -ig_{\mu\nu}\left[A^{ij}(0) + q^2F^{ij}(q^2)\right] + q_\mu q_\nu \,\text{terms}\,, \tag{30}$$

where $i,j = 1,2,3,0$ for W^1, W^2, W^3, and B, respectively. Therefore,

$$\epsilon_1 = e_1 - e_5\,, \tag{31}$$

$$\epsilon_2 = e_2 - c^2 e_5\,, \tag{32}$$

$$\epsilon_3 = e_3 - c^2 e_5\,, \tag{33}$$

$$\epsilon_b = e_b\,, \tag{34}$$

where

$$e_1 = \frac{A^{33}(0) - A^{11}(0)}{M_W^2}\,, \tag{35}$$

$$e_2 = F^{11}(M_W^2) - F^{33}(M_Z^2)\,, \tag{36}$$

$$e_3 = \frac{c}{s}F^{30}(M_Z^2)\,, \tag{37}$$

$$e_5 = M_Z^2\frac{dF^{ZZ}}{dq^2}(M_Z^2)\,, \tag{38}$$

and $c \equiv \cos\theta_W$,

$$c^2 \equiv \frac{1}{2}\left[1 + \left(1 - \frac{4\pi\alpha(M_Z)}{\sqrt{2}G_f M_Z^2}\right)^{1/2}\right]\,, \tag{39}$$

and $s^2 = 1 - c^2$. e_b is defined through the GIM-violating $Z \to b\bar{b}$ vertex

$$V_\mu{}^{\rm GIM}\left(Z \to b\bar{b}\right) = -\frac{g}{2c}e_b\gamma_\mu\frac{1-\gamma_5}{2}\,. \tag{40}$$

ϵ_1 depends quadratically on m_t [12,13] and has been measured to good accuracy, therefore ϵ_1 is sensitive to any new physics coming through the top quark. On the contrary, ϵ_2 and ϵ_3 do not play any significant role in our analysis because their dependence on the top mass is only logarithmic.

Nonrenormalizability of the effective Lagrangian presents a major issue of how to find a scheme to handle both the divergent and the finite pieces in loop calculations [30,31]. Such a problem arises because one does not know the underlying theory; hence, no matching can be performed to extract the correct scheme to be used in the effective Lagrangian [15]. One approach is to associate the divergent piece in loop calculations with a physical cutoff Λ, the upper scale at which the effective Lagrangian is valid [25]. In the chiral Lagrangian approach this cutoff Λ is taken to be $4\pi v \sim 3$ TeV [15]. For the finite piece no completely satisfactory approach is available [30].

To perform calculations using the chiral Lagrangian, one should arrange the contributions in powers of $1/4\pi v$ and then include all diagrams up to the desired power. In the R_ξ gauge ($\Sigma \neq 1$), the couplings of the Goldstone bosons to the fermions should also be included in Feynman diagram calculations. These couplings can be easily found by expanding the terms in $\mathcal{L}$ as given in Eq. (19). We will not give the explicit expressions for those terms here. Some of the relevant Feynman diagrams are shown in Fig. 1. Calculations were done in the 't Hooft–Feynman gauge. We have also checked our calculations in both the Landau gauge and the unitary gauge and found agreement as expected.

We calculate the contribution to ϵ_1 and ϵ_b due to the new interaction terms in the chiral Lagrangian [see Eqs. (19) and (20)] using the dimensional regularization scheme and taking the bottom mass to be zero. At the end of the calculation, we replace the divergent piece $1/\epsilon$ by $\ln(\Lambda^2/m_t^2)$ for $\epsilon = (4-n)/2$ where n is the space-time dimension. Since we are mainly interested in new physics associated with the top quark couplings to gauge bosons, we shall restrict ourselves to the *leading* contribution enhanced by the top quark mass, i.e., of the oder of $m_t^2 \ln \Lambda^2$.

We find

$$\epsilon_1 = \frac{G_F}{2\sqrt{2}\pi^2}3m_t^2(-\kappa_L^{\rm NC} + \kappa_R^{\rm NC} + \kappa_L^{\rm CC})\ln\frac{\Lambda^2}{m_t^2}\,, \tag{41}$$

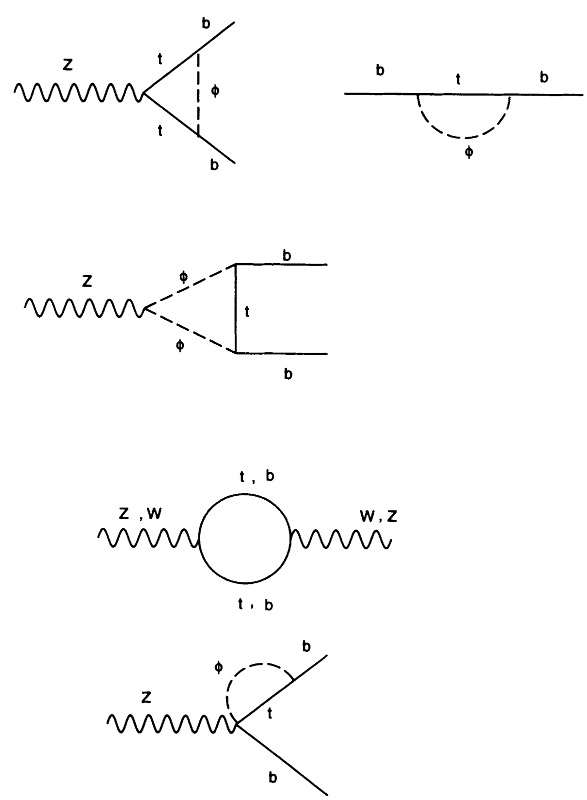

FIG. 1. Some of the relevant Feynman diagrams in the 't Hooft–Feynman gauge, which contribute to the order $O(m_t^2 \ln \Lambda^2)$.

$$\epsilon_b = \frac{G_F}{2\sqrt{2}\pi^2} m_t^2 \left(-\frac{1}{4}\kappa_R^{\rm NC} + \kappa_L^{\rm NC} \right) \ln \frac{\Lambda^2}{m_t^2} \,. \qquad (42)$$

Note that ϵ_2 and ϵ_3 do not contribute at this order. That $\kappa_L^{\rm CC}$ does not contribute to ϵ_b up to this order can be understood from Eq. (20). If $\kappa_L^{\rm CC} = -1$ then there is no net t-b-W coupling in the chiral Lagrangian after including both the standard and nonstandard contributions. Hence, no dependence on the top quark mass can be generated; i.e., the nonstandard $\kappa_L^{\rm CC}$ contribution to ϵ_b must cancel the SM contribution when $\kappa_L^{\rm CC} = -1$, independently of the couplings of the neutral current. From this observation and because the SM contribution to ϵ_b is finite, we conclude that $\kappa_L^{\rm CC}$ cannot contribute to ϵ_b at the order of interest.

Note that we set the renormalization scale μ to be m_t, which is the natural scale to be used in our study because the top quark is considered to be the heaviest mass scale in the effective Lagrangian. We have assumed that all other heavy fields have been integrated out to modify the effective couplings of the top quark to gauge bosons at the scale m_t in the chiral Lagrangian. Here we ignore the effect of the running couplings from the top quark mass scale down to the Z boson mass scale which is a reasonable approximation for our study.

To constrain these nonstandard couplings we need to have both the experimental values and the SM predictions of ϵ's. First, we tabulate the numerical inputs, taken from Ref. [12], used in our analysis:

$$\begin{aligned}
\alpha^{-1}(M_Z^2) &= 128.87\,, \\
G_F &= 1.166372 \times 10^{-5}\ \ {\rm GeV}^{-2}\,, \\
M_Z &= 91.187 \pm 0.007\ \ {\rm GeV}\,, \\
M_W/M_Z &= 0.8798 \pm 0.0028\,, \\
\Gamma_\ell &= 83.52 \pm 0.28\ \ {\rm MeV}\,, \\
\Gamma_b &= 383 \pm 6\ \ {\rm MeV}\,, \\
A_{\rm FB}^{\ell} &= 0.0164 \pm 0.0021\,, \\
A_{\rm FB}^{b} &= 0.098 \pm 0.009\,.
\end{aligned}$$

From these values we have [12]

$$\begin{aligned}
\epsilon_1\,(10^3) &= -0.3 \pm 3.4\,, \\
\epsilon_b\,(10^3) &= 4.4 \pm 7.0\,,
\end{aligned}$$

and for completeness

$$\begin{aligned}
\epsilon_2\,(10^3) &= -7.6 \pm 7.6\,, \\
\epsilon_3\,(10^3) &= 0.4 \pm 4.2\,.
\end{aligned}$$

The SM contribution to ϵ's have been calculated in Refs. [12,13]. We will include these contributions in our analysis in accordance with the assumed Higgs boson mass. In the light Higgs boson case ($m_H < m_t$), the calculated values of the ϵ's include both the SM contribution calculated in Refs. [12,13] and the new physics contribution derived from the effective couplings of the top quark to gauge bosons. In the heavy Higgs boson case ($m_H > m_t$) we subtract the Higgs boson contribution from the SM calculations of ϵ's given in Refs. [12,13]. In this case, the Higgs boson contribution is implicitly included in the effective couplings of the top quark to gauge bosons after the heavy Higgs boson field is integrated out. Finally, in a spontaneous symmetry scenario without a Higgs boson the calculations of ϵ's are exactly the same as those done in the heavy Higgs boson case except that the effective couplings of the top quark to gauge bosons are not due to an assumed heavy Higgs boson in the full theory.

Choosing $m_t = 150$ GeV and $m_H = 100$ GeV we span the parameter space defined by $-1 \le \kappa_L^{\rm NC} \le 1$, $-1 \le \kappa_R^{\rm NC} \le 1$, and $-1 \le \kappa_L^{\rm CC} \le 1$. Within 95% C.L. and including both the SM and the new physics contributions, the allowed region of these three parameters is found to form a thin slice in the specified volume. The two–dimensional projections of this slice are shown in Figs. 2, 3, and 4. These nonstandard couplings (κ's) do exhibit some interesting features.

(1) As a function of the top quark mass, the allowed volume for the top quark couplings to gauge bosons shrinks as the top quark becomes more massive.

(2) New physics prefers positive $\kappa_L^{\rm NC}$, see Figs. 2 and 3. $\kappa_L^{\rm NC}$ is constrained within -0.3 to 0.6 (-0.2 to 0.5) for a 150 (175) GeV top quark.

(3) New physics prefers $\kappa_L^{\rm CC} \approx -\kappa_R^{\rm NC}$. This is clearly shown in Fig. 4 which is the projection of the allowed volume in the plane containing $\kappa_R^{\rm NC}$ and $\kappa_L^{\rm CC}$.

In Ref. [7], a similar analysis has been carried out by Peccei *et al.* However, in their analysis they did not include the charged current contribution and assumed only the vertex t-t-Z gives large nonstandard effects. The allowed region they found simply corresponds, in our analysis, to the region defined by the intersection of the allowed volume and the plane $\kappa_L^{\rm CC} = 0$. This gives a small area confined in the vicinity of the line $\kappa_L^{\rm NC} = \kappa_R^{\rm NC}$. This can be understood from the expression of ϵ_1 derived in Eq. (41). After setting $\kappa_L^{\rm CC} = 0$ we find

$$\epsilon_1 \propto \left(\kappa_R^{\rm NC} - \kappa_L^{\rm NC} \right) \,. \qquad (43)$$

In this case we note that the length of the allowed area is merely determined by the contribution from ϵ_b. We

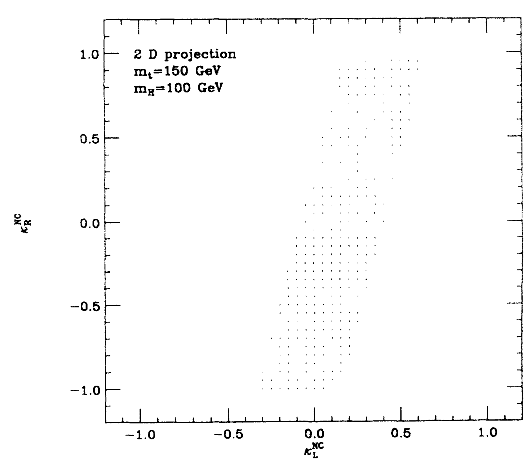

FIG. 2. Two-dimensional projection in the plane of $\kappa_L^{\rm NC}$ and $\kappa_R^{\rm NC}$, for $m_t = 150$ GeV, $m_H = 100$ GeV.

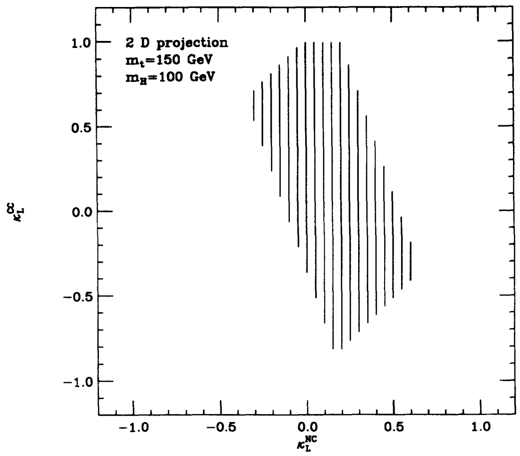

FIG. 3. Two-dimensional projection in the plane of $\kappa_L^{\rm NC}$ and $\kappa_L^{\rm CC}$, for $m_t = 150$ GeV, $m_H = 100$ GeV.

will elaborate on a more quantitative comparison in the second part of this section.

B. Special case

The allowed region in the parameter space obtained in Figs. 2–4 contains all possible new physics (to the order $m_t^2 \ln \Lambda^2$) which can modify the couplings of the top quark to gauge bosons as described by $\kappa_L^{\rm NC}$, $\kappa_R^{\rm NC}$ and $\kappa_L^{\rm CC}$. In this section we would like to examine a special class of models in which an approximate custodial symmetry is assumed as suggested by low energy data.

The SM has an additional (accidental) symmetry called the custodial symmetry which is responsible for the tree-level relation

$$\rho = \frac{M_W}{M_Z \cos\theta_W} = 1 \ . \tag{44}$$

This symmetry is slightly broken at the quantum level by the SU(2) doublet fermion mass splitting and the hypercharge coupling g' [32]. Writing $\rho = 1 + \delta\rho$, $\delta\rho$ would vanish to all orders if this symmetry is exact. Because low energy data indicate that $\delta\rho$ is very close to zero we shall therefore assume an underlying theory with a custodial symmetry. In other words we require the global group $\mathrm{SU}(2)_V$ associated with the custodial symmetry to be a subgroup of the full group characterizing the full theory. We will assume that the custodial symmetry is broken by the same factors which break it in the SM, i.e., by the fermion mass splitting and the hypercharge coupling g'.

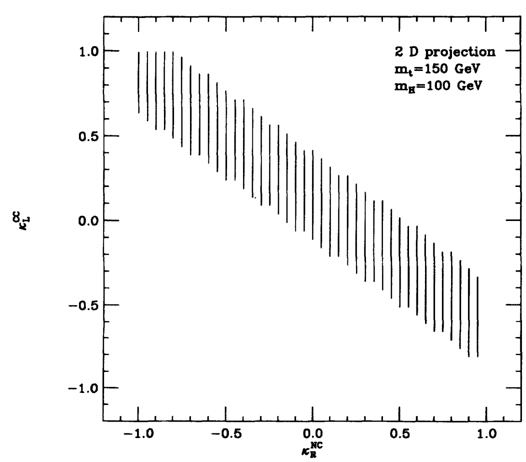

FIG. 4. Two-dimensional projection in the plane of $\kappa_R^{\rm NC}$ and $\kappa_L^{\rm CC}$, for $m_t = 150$ GeV, $m_H = 100$ GeV.

In the chiral Lagrangian this assumption of a custodial symmetry sets $v_3 = v$, and forces the couplings of the top quark to gauge bosons W_μ^a to be equal after turning off the hypercharge and assuming $m_b = m_t$. If the dynamics of the symmetry breaking is such that the masses of the two SU(2) partners t and b remain degenerate then we expect new physics to contribute to the couplings of t-t-Z and t-b-W by the same amount. However, in reality, $m_b \ll m_t$; thus, the custodial symmetry has to be broken. We will discuss how this symmetry is broken shortly. Since we are mainly interested in the *leading* contribution enhanced by the top quark mass at the order ${m_t}^2 \ln \Lambda^2$, turning the hypercharge coupling on and off will not affect the final result up to this order.

We can construct the two Hermitian operators J_L and J_R, which transform under G as

$$J_L^\mu = -i\Sigma D_\mu \Sigma^\dagger \to g_L J_L^\mu g_L^\dagger \ , \tag{45}$$

$$J_R^\mu = i\Sigma^\dagger D_\mu \Sigma \to g_R J_R^\mu g_R^\dagger \ , \tag{46}$$

where $g_L = e^{i\alpha^a \frac{\tau^a}{2}} \in \mathrm{SU}(2)_L$ and $g_R = e^{iy\frac{\tau^3}{2}}$ (note that $v_3 = v$ in Σ). In fact, using either J_L or J_R will lead to the same result. Hence, from now on we will only consider J_R. The SM Lagrangian can be derived from

$$\begin{aligned}\mathcal{L}_0 = {} & \overline{\Psi_L} i\gamma^\mu D_\mu^L \Psi_L \\ & + \overline{\Psi_R} i\gamma^\mu D_\mu^R \Psi_R - (\overline{\Psi_L}\Sigma M \Psi_R + \text{H.c.}) \\ & - \frac{1}{4} W_{\mu\nu}^a W^{\mu\nu a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{v^2}{4} \mathrm{Tr}(J_R^\mu J_{R\mu}) \ ,\end{aligned} \tag{47}$$

where M is a diagonal mass matrix. We have chosen the left-handed fermion fields to be the ones defined in Eq. (7):

$$\Psi_L \equiv \Sigma \begin{pmatrix} t \\ b \end{pmatrix}_L . \tag{48}$$

The right-handed fermion fields t_R and b_R coincide with the original right-handed fields [see Eq. (9)]. Also

$$D_\mu^L = \partial_\mu - igW_\mu^a \frac{\tau^a}{2} - ig' B_\mu \frac{Y}{2} \ , \tag{49}$$

$$D_\mu^R = \partial_\mu - ig' B_\mu \left(\frac{Y}{2} + \frac{\tau^3}{2} \right) \ . \tag{50}$$

Note that in the nonlinear realized effective theories using either set of fields ($\Psi_{L,R}$ or $F_{L,R}$) to construct a chiral Lagrangian will lead to the same S matrix [22].

The Lagrangian $\mathcal{L}_0$ in Eq. (47) is not the most general Lagrangian one can construct based solely on the symmetry of G/H. Taking advantage of the chiral Lagrangian approach we can derive additional interaction terms which deviate from the SM. This is so because in this formalism the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry is nonlinearly realized and only the $\mathrm{U}(1)_{\mathrm{em}}$ is linearly realized.

Because the SM is so successful one can think of the SM (without the top quark) as being the leading term in the expansion of the effective Lagrangian. Any possible deviation associated with the light fields can only come through higher dimensional operators in the Lagrangian. However, this assumption is neither necessary nor preferable when dealing with the top quark because no precise data are available to lead to such a conclusion. In this paper we will include nonstandard dimension-four operators for the couplings of the top quark to gauge bosons. In fact this is all we will deal with and we will not consider operators with dimension higher than four. Note that higher dimensional operators are naturally suppressed by powers of $1/\Lambda$.

One can write J_R as

$$J_R^\mu = J_R^{\mu a}\frac{\tau^a}{2}\,, \tag{51}$$

with

$$J_R^{\mu a} = \mathrm{Tr}\left(\tau^a J_R^\mu\right) = i\mathrm{Tr}\left(\tau^a \Sigma^\dagger D^\mu \Sigma\right)\,. \tag{52}$$

The full operator J_R posses an explicit custodial symmetry when $g' = 0$ as can easily be checked by expanding it in powers of the Goldstone boson fields.

Consider first the left-handed sector. One can add additional interaction terms to the Lagrangian $\mathcal{L}_0$

$$\mathcal{L}_1 = \kappa_1 \overline{\Psi_L}\gamma_\mu \Sigma J_R^\mu \Sigma^\dagger \Psi_L + \kappa_2 \overline{\Psi_L}\gamma_\mu \Sigma \tau^3 J_R^\mu \Sigma^\dagger \Psi_L + \kappa_2^\dagger \overline{\Psi_L}\gamma_\mu \Sigma J_R^\mu \tau^3 \Sigma^\dagger \Psi_L\,, \tag{53}$$

where κ_1 is an arbitrary real parameter and κ_2 is an arbitrary complex parameter. Here we do not include interaction terms such as

$$\kappa_3 \overline{\Psi_L}\gamma_\mu \Sigma \tau^3 J_R^\mu \tau^3 \Sigma^\dagger \Psi_L\,, \tag{54}$$

where κ_3 is real, because it is simply a linear combination of the other two terms in $\mathcal{L}_1$. This can be easily checked by using Eq. (51) and the commutation relations of the Pauli matrices. Note that $\mathcal{L}_1$ still is not the most general Lagrangian one can write for the left-handed sector, as compared to Eq. (19). In fact, it is our insistence on using the fermion doublet form and the full operator J_R that lead us to this form. For shorthand, $\mathcal{L}_1$ can be further rewritten as

$$\mathcal{L}_1 = \overline{\Psi_L}\gamma_\mu \Sigma K_L J_R^\mu \Sigma^\dagger \Psi_L + \overline{\Psi_L}\gamma_\mu \Sigma J_R^\mu K_L^\dagger \Sigma^\dagger \Psi_L\;, \tag{55}$$

where K_L is a complex diagonal matrix.

These new terms can be generated either through some electroweak symmetry-breaking scenario or through some other new heavy physics effects. If $m_b = m_t$ and $g' = 0$, then we require the effective Lagrangian to respect fully the custodial symmetry to all orders. In this limit, $\kappa_2 = 0$ in Eq. (53) and $K_L = \kappa_1 \mathbf{1}$, where $\mathbf{1}$ is the unit matrix and κ_1 is real.

Since $m_b \ll m_t$, we can think of κ_2 as generated through the evolution from $m_b = m_t$ to $m_b = 0$. In the matrix notation this implies K_L is not proportional to the unit matrix and can be parametrized by

$$K_L = \begin{pmatrix} \kappa_L^t & 0 \\ 0 & \kappa_L^b \end{pmatrix}\,, \tag{56}$$

with

$$\kappa_L^t = \frac{\kappa_1}{2} + \kappa_2\,, \tag{57}$$

and

$$\kappa_L^b = \frac{\kappa_1}{2} - \kappa_2\,. \tag{58}$$

In the unitary gauge we get the terms

$$+\frac{g}{2c}2\mathrm{Re}(\kappa_L^t)\overline{t_L}\gamma^\mu t_L Z_\mu + \frac{g}{\sqrt{2}}(\kappa_L^t + {\kappa_L^b}^\dagger)\overline{t_L}\gamma^\mu b_L W_\mu^+ + \frac{g}{\sqrt{2}}(\kappa_L^b + {\kappa_L^t}^\dagger)\overline{b_L}\gamma^\mu t_L W_\mu^- - \frac{g}{2c}2\mathrm{Re}(\kappa_L^b)\overline{b_L}\gamma^\mu b_L Z_\mu\,. \tag{59}$$

As discussed in the previous section, we will assume that new physics effects will not modify the b_L-b_L-Z vertex. This can be achieved by choosing $\kappa_1 = 2\mathrm{Re}(\kappa_2)$ such that $\mathrm{Re}(\kappa_L^b)$ vanishes in Eq. (58). Later, in Sec. IV, we will consider a specific model to support this assumption.

Since the imaginary parts of the couplings do not contribute to LEP physics of interest, we simply drop them hereafter. With this assumption we are left with one real parameter κ_L^t which will be denoted from now on by $\kappa_L/2$. The left-handed top quark couplings to the gauge bosons are

$$t_L - t_L - Z:\ \frac{g}{4c}\kappa_L \gamma_\mu (1-\gamma_5)\,, \tag{60}$$

$$t_L - b_L - W:\ \frac{g}{2\sqrt{2}}\frac{\kappa_L}{2}\gamma_\mu(1-\gamma_5)\,. \tag{61}$$

Notice the connection between the neutral and the charged current, as compared to Eq. (20):

$$\kappa_L^{\mathrm{NC}} = 2\kappa_L^{\mathrm{CC}} = \kappa_L\,. \tag{62}$$

This conclusion holds for any underlying theory with an approximate custodial symmetry such that the vertex b-b-Z is not modified as discussed above.

For the right-handed sector, the situation is different because the right-handed fermion fields are SU(2) singlet, hence the induced interactions do not see the full operator J_R but its components individually. Therefore, we cannot impose the previous connection between the neutral and charged current couplings.

The additional allowed interaction terms in the right-handed sector are given by

$$\mathcal{L}_2 = \frac{g}{2c}\kappa_R^{t^{\rm NC}}\overline{t_R}\gamma^\mu t_R J^3_{R_\mu} + \frac{g}{\sqrt{2}}\kappa_R^{\rm CC}\overline{t_R}\gamma^\mu b_R J_{R_\mu} + \frac{g}{\sqrt{2}}\kappa_R^{\rm CC\dagger}\overline{b_R}\gamma^\mu t_R J^-_{R_\mu} - \frac{g}{2c}\kappa_R^{b^{\rm NC}}\overline{b_R}\gamma^\mu b_R J^3_{R_\mu} \,, \tag{63}$$

where $\kappa_R^{t^{\rm NC}}$ and $\kappa_R^{b^{\rm NC}}$ are two arbitrary real parameters and $\kappa_R^{\rm CC}$ is an arbitrary complex parameter. Note that in $\mathcal{L}_2$ we have one more additional coefficient than we have in $\mathcal{L}_1$ [in Eq. (53)], this is due to our previous assumption of using the full operator J_R in constructing the left-handed interactions. We assume that the b_R-b_R-Z vertex just as the b_L-b_L-Z vertex is not modified, then the coefficient $\kappa_R^{b^{\rm NC}}$ vanishes. Because $\kappa_R^{\rm CC}$ does not contribute to LEP physics in the limit of $m_b = 0$ and at the order $m_t{}^2 \ln\Lambda^2$ we are left with one real parameter $\kappa_R^{t^{\rm NC}}$ which will be denoted hereafter as κ_R. The right-handed top quark coupling to Z boson is

$$t_R - t_R - Z: \ \frac{g}{4c}\kappa_R\gamma_\mu(1+\gamma_5) \,. \tag{64}$$

In the rest of this section we consider models described by $\mathcal{L}_1$ and $\mathcal{L}_2$ with only two relevant parameters κ_L and κ_R. Performing the calculations as we discussed in the previous subsection we find

$$\epsilon_1 = \frac{G_F}{2\sqrt{2}\pi^2}3m_t^2\left(\kappa_R - \frac{\kappa_L}{2}\right)\ln\left(\frac{\Lambda^2}{m_t{}^2}\right) \,, \tag{65}$$

$$\epsilon_b = \frac{G_F}{2\sqrt{2}\pi^2}m_t^2\left(-\frac{1}{4}\kappa_R + \kappa_L\right)\ln\left(\frac{\Lambda^2}{m_t{}^2}\right) \,. \tag{66}$$

These results simply correspond to those in Eqs. (41) and (42) after substituting $\kappa_L^{\rm NC} = 2\kappa_L^{\rm CC} = \kappa_L$ and $\kappa_R^{\rm NC} = \kappa_R$.

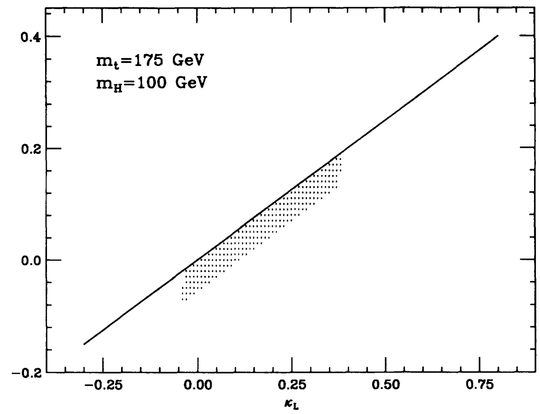

FIG. 6. The allowed region of κ_L and κ_R, for m_t = 175 GeV, m_H = 100 GeV. (Note that $\kappa_L = \kappa_L^{\rm NC} = 2\kappa_L^{\rm CC}$ and $\kappa_R = \kappa_R^{\rm CC}$.)

The constraints on κ_L and κ_R for models with a light Higgs boson or a heavy Higgs boson, or without a physical scalar (such as a Higgs boson) are presented here in order. Let us first consider a standard light Higgs boson with mass m_H = 100 GeV. Including the SM contribution from Ref. [12] we span the plane defined by κ_L and κ_R for top mass 150 and 175 GeV, respectively. Figures 5 and 6 show the allowed range for those parameters within 95% C.L. As a general feature one observes that the allowed range is a narrow area aligned close to the line $\kappa_L = 2\kappa_R$ where for m_t = 150 GeV the maximum range for κ_L is between −0.1 and 0.5. As the top mass increases this range shrinks and moves downward and to the right away form the origin (κ_L,κ_R) = (0,0). The deviation from the relation $\kappa_L = 2\kappa_R$ for various top quark masses is given in Fig. 7 by calculating $\kappa_L - 2\kappa_R$ as a function of m_t. Note that the SM has the solution $\kappa_L = \kappa_R = 0$, i.e., the SM solution lies on the horizontal line shown in Fig. 7. This solution ceases to exist for $m_t \geq 200$ GeV. The special relation $\kappa_L = 2\kappa_R$ is a consequence of the assumption we imposed in connecting

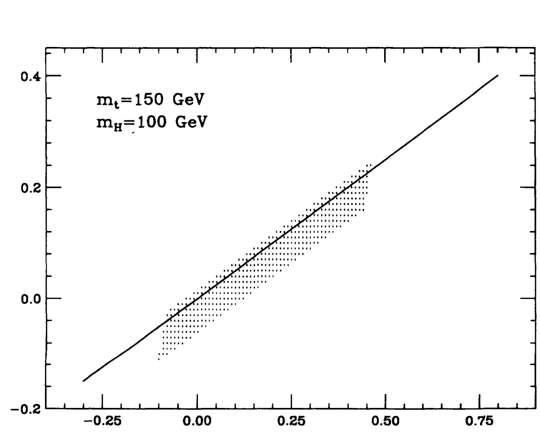

FIG. 5. The allowed region of κ_L and κ_R, for m_t = 150 GeV, m_H = 100 GeV. (Note that $\kappa_L = \kappa_L^{\rm NC} = 2\kappa_L^{\rm CC}$ and $\kappa_R = \kappa_R^{\rm CC}$.)

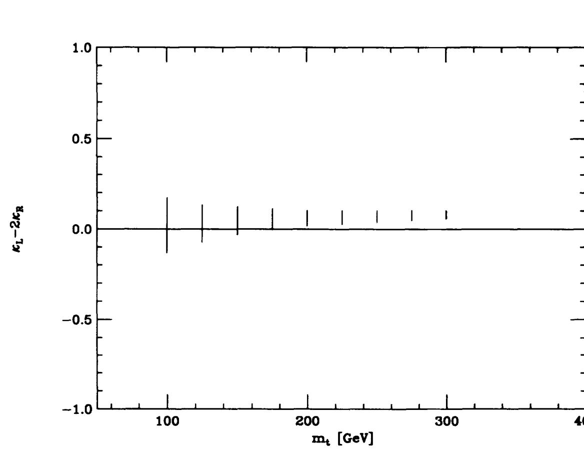

FIG. 7. The allowed range of ($\kappa_L - 2\kappa_R$) as a function of the mass of the top quark. (Note that $\kappa_L = \kappa_L^{\rm NC} = 2\kappa_L^{\rm CC}$ and $\kappa_R = \kappa_R^{\rm CC}$.)

TABLE I. The confined range of the couplings, κ_L and κ_R for various top masses.

m_t (GeV)	κ_L	κ_R
150	−0.10 — 0.50	−0.15 — 0.25
175	−0.05 — 0.40	−0.10 — 0.20
200	0.0 — 0.35	−0.05 — 0.15
300	0.10 — 0.25	0.00 — 0.10

left-handed neutral and charged current. The range of the allowed couplings is summarized in Table I and for different top mass.

It is worth mentioning that the SM contribution to ϵ_b is lower than the experimental central value [12,13]. This is reflected in the behavior of κ_L which prefers being positive to compensate this difference as can be seen from Eq. (66). This means in models of electroweak symmetry breaking with an approximate custodial symmetry, a positive κ_L is preferred. In Fig. 8 we show the allowed $\kappa_L^{CC} = \kappa_L^{NC}/2 = \kappa_L/2$ as a function of m_t. With new physics effects ($\kappa_L \neq 0$) m_t can be as large as 300 GeV, although in the SM ($\kappa_L = 0$) m_t is bounded below 200 GeV.

Now, we would like to discuss the effect of a light SM Higgs boson ($m_H < m_t$) on the allowed range of these parameters. It is easy to anticipate the effect; since ϵ_b is not sensitive to the Higgs boson contribution up to one loop [12], the allowed range is only affected by the Higgs boson contribution to ϵ_1 which affects slightly the width of the allowed area and its location relative to the line $\kappa_L = 2\kappa_R$. One expects that as the Higgs boson mass increases the allowed area moves upward. The reason simply lies in the fact that the standard Higgs boson contribution to ϵ_1 up to one loop becomes more negative for heavier Higgs boson, hence $2\kappa_R$ prefers to be larger than κ_L to compensate this effect. However, this modification is not significant because ϵ_1 depends on the Higgs boson mass only logarithmically [13].

If there is a heavy Higgs boson ($m_H > m_t$), then it should be integrated out from the full theory and its effect in the chiral Lagrangian is manifested through the effective couplings of the top quark to gauge bosons. In this case we simply subtract the Higgs boson contribution from the SM results obtained in Refs. [12,13]. Figure 9 shows the allowed area in the κ_L and κ_R plane for a 175 GeV top quark in such models. Again we find no noticeable difference between the results from these models and those with a light Higgs boson. That is because up to one loop level neither ϵ_1 nor ϵ_b is sensitive to the Higgs boson contribution [12,13].

If we consider a new symmetry-breaking scenario without a fundamental scalar such as a SM Higgs boson, following the previous discussions we again find negligible effects on the allowed range of κ_L and κ_R.

What we learned is that to infer a bound on the Higgs boson mass from the measurement of the effective couplings of the top quark to gauge bosons, we need very precise measurement of the parameters κ_L and κ_R. However, from the correlations between the effective couplings (κ's) of the top quark to gauge bosons, we can infer if the symmetry-breaking sector is due to a Higgs boson or not; i.e., we may be able to probe the symmetry-breaking mechanism in the top quark system. Further discussion will be given in the next section.

Finally, we would like to compare our results with those in Ref. [7]. Figure 10 shows the most general allowed region for the couplings κ_L^{NC} and κ_R^{NC}, i.e., without imposing any relation between κ_L^{NC} and κ_L^{CC}. This region is for top mass 150 GeV and is covering the parameter space $-1.0 \leq \kappa_L^{NC}, \kappa_R^{NC} \leq 1.0$. We find

$$-0.3 \leq \kappa_L^{NC} \leq 0.6\,,$$

$$-1.0 \leq \kappa_R^{NC} \leq 1.0\,.$$

Also shown on Fig. 10 the allowed regions from our model and the model in Ref. [7]. The two regions overlap in the vicinity of the origin (0, 0) which corresponds to the SM case. As $\kappa_L^{NC} \geq 0.1$, these two regions diverge and become separable. One notices that the allowed range predicted in Ref. [7] lies along the line $\kappa_L^{NC} = \kappa_R^{NC}$ whereas in our case the slope is different $\kappa_L^{NC} = 2\kappa_R^{NC}$. This difference comes in because of the assumed dependence of κ_L^{CC} on the other two couplings κ_L^{NC} and κ_R^{NC}. In our case $\kappa_L^{CC} = \kappa_L^{NC}/2$, and in Ref. [7] $\kappa_L^{CC} = 0$.

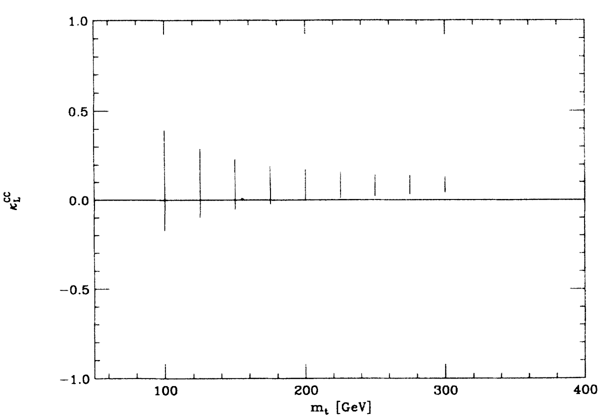

FIG. 8. The allowed range of the coupling $\kappa_L^{CC} = \kappa_L^{NC}/2 = \kappa_L/2$ as a function of the mass of the top quark.

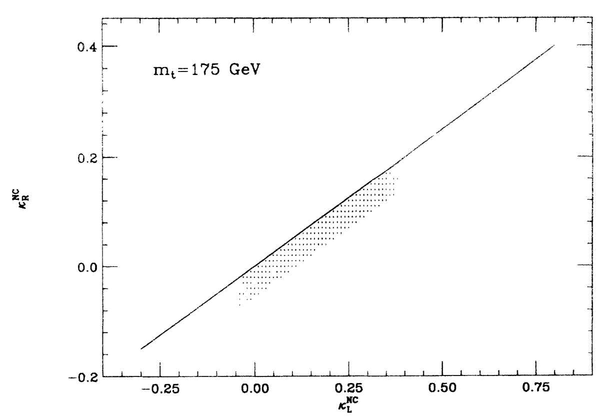

FIG. 9. The allowed region of κ_L^{NC} and κ_R^{NC}, for models without a light Higgs boson. $m_t = 175$ GeV.

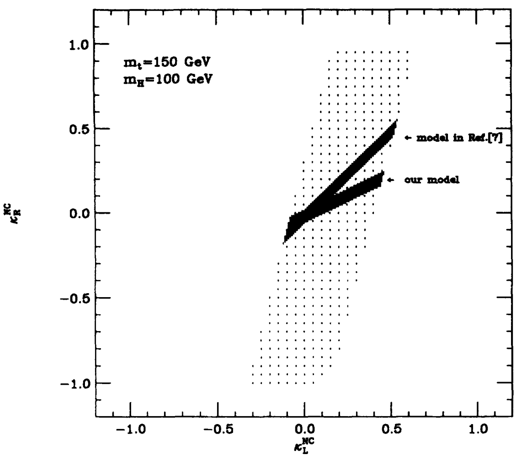

FIG. 10. A comparison between our model and the model in Ref. [7]. The allowed regions in both models are shown on the plane of $\kappa_L^{\rm NC}$ and $\kappa_R^{\rm NC}$, for $m_t = 150$ GeV.

Note that for $m_t \leq 200$ GeV the allowed region of κ's in all models of symmetry breaking should overlap near the origin because the SM is consistent with low energy data at the 95% C.L. If we imagine that any prescribed dependence between the couplings corresponds to a symmetry-breaking scenario, then, given the present status of low energy data, it is possible to distinguish between different scenarios if $\kappa_L^{\rm NC}$, $\kappa_R^{\rm NC}$ and $\kappa_L^{\rm CC}$ are larger than 10%. Better future measurements of ϵ's can further discriminate between different symmetry-breaking scenarios. We will discuss how the SLC, the Tevatron, the LHC, and the NLC can contribute to these measurements in Sec. V. Before that, let us examine a specific model that predicts certain relations among the coefficients $\kappa_L^{\rm CC}$, $\kappa_R^{\rm CC}$, $\kappa_L^{\rm NC}$, and $\kappa_R^{\rm NC}$ of the effective couplings of the top quark to gauge bosons.

IV. HEAVY HIGGS LIMIT IN THE SM

The goal of this study is to probe new physics effects, particularly the effects due to the symmetry-breaking sector, in the top quark system by examining the couplings of top quark to gauge bosons. To illustrate how a specific symmetry-breaking mechanism might affect these couplings, we consider in this section the standard model with a heavy Higgs boson ($m_H > m_t$) as the full theory, and derive the effective couplings $\kappa_L^{\rm NC}$, $\kappa_R^{\rm NC}$, $\kappa_L^{\rm CC}$, and $\kappa_R^{\rm CC}$ at the top quark mass scale in the effective Lagrangian after integrating out the heavy Higgs boson field.

Given the full theory (SM in this case), we can perform matching between the underlying theory and the effective Lagrangian. In this case, the heavy Higgs boson mass acts as a regulator (cutoff) of the effective theory [33].

While setting $m_b = 0$, and only keeping the leading terms of the order $m_t^2 \ln m_H^2$, we find the effective couplings

$$t-t-Z:\ \frac{g}{4c}\frac{G_F}{2\sqrt{2}\pi^2}\left(\frac{-1}{8}m_t^2\gamma_\mu(1-\gamma_5)+\frac{1}{8}m_t^2\gamma_\mu(1+\gamma_5)\right)\ln\left(\frac{m_H^2}{m_t^2}\right)\,, \tag{67}$$

$$t-b-W:\ \frac{g}{2\sqrt{2}}\frac{G_F}{2\sqrt{2}\pi^2}\left(\frac{-1}{16}\right)m_t^2\gamma_\mu(1-\gamma_5)\ln\left(\frac{m_H^2}{m_t^2}\right)\,. \tag{68}$$

From this we conclude

$$\kappa_L^{\rm NC} = 2\kappa_L^{\rm CC} = \frac{G_F}{2\sqrt{2}\pi^2}\left(\frac{-1}{8}\right)m_t^2\ln\left(\frac{m_H^2}{m_t^2}\right)\,, \tag{69}$$

$$\kappa_R^{\rm NC} = \frac{G_F}{2\sqrt{2}\pi^2}\frac{1}{8}m_t^2\ln\left(\frac{m_H^2}{m_t^2}\right)\,, \tag{70}$$

$$\kappa_R^{\rm CC} = 0\,. \tag{71}$$

Note that the relation between the left-handed currents ($\kappa_L^{\rm NC} = 2\kappa_L^{\rm CC}$) agree with our prediction because of the approximate custodial symmetry in the full theory (SM) and the fact that vertex b-b-Z is not modified. The right-handed currents $\kappa_R^{\rm CC}$ and $\kappa_R^{\rm NC}$ are not correlated, and $\kappa_R^{\rm CC}$ vanishes for a massless b. Also note an additional relation in the effective Lagrangian between the left- and right-handed effective couplings of the top quark to Z boson, i.e.,

$$\kappa_L^{\rm NC} = -\kappa_R^{\rm NC}\,. \tag{72}$$

This means only the axial vector current of t-t-Z acquires a nonuniversal contribution but its vector current is not modified.

As discussed in Sec. II, due to the Ward identities associated with the photon field there can be no nonuniversal contribution to either the b-b-A or t-t-A vertex after renormalizing the fine structure constant α. This can be explicitly checked in this model. Furthermore, up to the order of $m_t^2 \ln m_H^2$, the vertex b-b-Z is not modified which agrees with the assumption we made in Sec. II that there exist dynamics of electroweak symmetry breaking so that neither b_R-b_R-Z nor b_L-b_L-Z in the effective Lagrangian is modified at the scale of m_t.

From this example we learn that the effective couplings of the top quark to gauge bosons arising from a heavy Higgs boson are correlated in a specific way: namely,

$$\kappa_L^{\rm NC} = 2\kappa_L^{\rm CC} = -\kappa_R^{\rm NC}\,. \tag{73}$$

[This relation in general also holds for models with a heavy scalar which is not necessarily a SM Higgs boson, i.e., the coefficients of the last two terms in Eq. (22) can be arbitrary, and are not necessarily 1/2 and 1/4, respectively.] In other words, if the couplings of a heavy top quark to the gauge bosons are measured and exhibit large deviations from these relations, then it is likely that the electroweak symmetry breaking is not due to the standard Higgs boson mechanism which contains a heavy SM Higgs boson. This illustrates how the symmetry-breaking sector can be probed by measuring the effective couplings of the top quark to gauge bosons.

V. DIRECT MEASUREMENT OF THE TOP QUARK COUPLINGS

In Sec. III we concluded that the precision LEP data can constrain the couplings $\kappa_L^{\rm NC}$, $\kappa_R^{\rm NC}$, and $\kappa_L^{\rm CC}$, but not $\kappa_R^{\rm CC}$ (the right-handed charged current). In this section we examine how to improve our knowledge on these couplings at the current and future colliders.

A. At the SLC

The measurement of the left-right cross section asymmetry A_{LR} in Z production with a longitudinally polarized electron beam at the SLC provides a stringent test of the SM and is sensitive to new physics.

Additional constraints on the couplings $\kappa_L^{\rm NC}$, $\kappa_R^{\rm NC}$, and $\kappa_L^{\rm CC}$ can be inferred from A_{LR} which can be written as [12]

$$A_{LR} = \frac{2x}{1+x^2}\,, \tag{74}$$

with

$$x = 1 - 4s^2(1+\Delta k')\,, \tag{75}$$

$$\Delta k' = \frac{\epsilon_3 - c^2\epsilon_1}{c^2 - s^2}\,. \tag{76}$$

Up to the order $m_t^2 \ln \Lambda^2$, only ϵ_1 contributes. In our model with the approximate custodial symmetry, i.e., $\kappa_L^{\rm NC} = 2\kappa_L^{\rm CC} = \kappa_L$, the SLC A_{LR} measurement will have a significant influence on the precise measurement of the nonuniversal couplings of the top quark. This influence will be through decreasing the width of the allowed area in the parameter space (κ_L versus κ_R) shown in Figs. 5 and 6. For instance, with an expected uncertainty 0.001 in the 1993 run on the measurement of the effective electroweak mixing angle, $\sin^2\theta_W^{\rm eff} = (1-x)/4$, at the SLC [34], the width of the allowed area shown in Figs. 5 and 6 will shrink by more than a factor of 5. However, there will be no effect on the length of the allowed region which in our approximation is solely determined by ϵ_b. Hence, a more accurate measurement of ϵ_b, i.e., $\Gamma(Z \to b\bar{b})$, is required to further confine the nonuniversal interactions of the top quark to gauge bosons to probe new physics.

B. At the Tevatron and the LHC

In this section we study how to constrain the nonstandard couplings of the top quark to gauge bosons from direct detection of the top quark at hadron colliders.

At the Tevatron and the LHC, heavy top quarks are predominantly produced from the QCD process $gg, q\bar{q} \to t\bar{t}$ and the W-gluon fusion process $qg(Wg) \to t\bar{b}, \bar{t}b$. In the former process, one can probe $\kappa_L^{\rm CC}$ and $\kappa_R^{\rm CC}$ from the decay of the top quark to a bottom quark and a W boson. In the latter process, these nonstandard couplings can be measured by simply counting the production rates of signal events with a single t or $\bar{t}$. More details can be found in Ref. [35].

To probe $\kappa_L^{\rm CC}$ and $\kappa_R^{\rm CC}$ from the decay of the top quark to a bottom quark and a W boson, one needs to measure the polarization of the W boson. For a massless b, the W boson from top quark decay can only be either longitudinally or left-handed polarized for a left-handed charged current ($\kappa_R^{\rm CC} = 0$). For a right-handed charged current ($\kappa_L^{\rm CC} = -1$) the W boson can only be either longitudinally or right-handed polarized. (Note that the handedness of the W boson is reversed for a massless $\bar{b}$ from $\bar{t}$ decays.) In all cases the fraction of longitudinal W from top quark decay is enhanced by $m_t^2/2M_W^2$ as compared to the fraction of transversely polarized W. Therefore, for a more massive top quark, it is more difficult to untangle the $\kappa_L^{\rm CC}$ and $\kappa_R^{\rm CC}$ contributions. The W polarization measurement can be done by measuring the invariant mass ($m_{b\ell}$) of the bottom quark and the charged lepton from the decay of top quark [36]. We note that this method does not require knowing the longitudinal momentum (with twofold ambiguity) of the neutrino from W decay to reconstruct the rest frame of the W boson in the rest frame of the top quark.

Consider the (upgraded) Tevatron as a p̄p collider at $\sqrt{S} = 2$ or 3.5 TeV, with an integrated luminosity of 1 or 10 fb^{-1}. Unless specified otherwise, we will give event numbers for a 175 GeV top quark and an integrated luminosity of 1 fb^{-1}.

The cross section of the QCD process $gg, q\bar{q} \to t\bar{t}$ is about 7 (29) pb at a $\sqrt{S} = 2$ (3.5) TeV collider. In order to measure $\kappa_L^{\rm CC}$ and $\kappa_R^{\rm CC}$ we have to study the decay kinematics of the reconstructed t and/or $\bar{t}$. For simplicity, let us consider the $\ell^\pm + \geq 3\,$jet decay mode, whose branching ratio is $B = 2\frac{2}{9}\frac{6}{9} = \frac{8}{27}$, for $\ell^+ = e^+$ or μ^+. We assume an experimental detection efficiency, which includes both the kinematic acceptance and the efficiency of b tagging, of 15% for the $t\bar{t}$ event. We further assume that there is no ambiguity in picking up the right b ($\bar{b}$) to combine with the charged lepton ℓ^+ (ℓ^-) to reconstruct t ($\bar{t}$). In total, there are $7\,\text{pb} \times 10^3\,\text{pb}^{-1} \times \frac{8}{27} \times 0.15 = 300$ reconstructed $t\bar{t}$ events to be used in measuring $\kappa_L^{\rm CC}$ and $\kappa_R^{\rm CC}$ at $\sqrt{S} = 2$ TeV. The same calculation at $\sqrt{S} = 3.5$ TeV yields 1300 reconstructed $t\bar{t}$ events. Given the number of reconstructed top quark events, one can in principle fit the $m_{b\ell}$ distribution to measure $\kappa_L^{\rm CC}$ and $\kappa_R^{\rm CC}$. We note that the polarization of the W boson can also be studied from the distribution of $\cos\theta_\ell^*$, where θ_ℓ^* is the polar angle of ℓ in the rest frame of the W boson whose z axis is the W bosons moving direction in the rest frame of the top quark [36]. For a massless b, $\cos\theta^*_\ell$ is related to $m^2_{b\ell}$ by

$$\cos\theta^*_\ell \simeq \frac{2m^2_{b\ell}}{m^2_t - M^2_W} - 1\,. \qquad (77)$$

However, in reality, the momenta of the bottom quark and the charged lepton will be smeared by the detector effects and the most serious problem in this analysis is the identification of the right b to reconstruct t. There are two strategies to improve the efficiency of identifying the right b. One is to demand a large invariant mass of the $t\bar{t}$ system so that t is boosted and its decay products are collimated. Namely, the right b will be moving closer to the lepton from t decay. This can be easily enforced by demanding lepton ℓ with large transverse momentum. Another is to identify the nonisolated lepton from $\bar{b}$ decay [with a branching ratio $B(\bar{b} \to \mu^+ X) \sim 10\%$]. Both of these methods will further reduce the reconstructed signal rate by an order of magnitude. How will these affect our conclusion on the determination of the nonuniversal couplings $\kappa^{\rm CC}_L$ and $\kappa^{\rm CC}_R$? This cannot be answered in the absence of detailed Monte Carlo studies.

Here we propose to probe the couplings $\kappa^{\rm CC}_L$ and $\kappa^{\rm CC}_R$ by measuring the production rate of the single-top quark events. A single-top quark event can be produced from either the W-gluon fusion process $qg\,(W^+g) \to t\bar{b}X$, or the Drell-Yan-type process $q\bar{q} \to W^* \to t\bar{b}$. Including both the single-t and single-$\bar{t}$ events, for a 2 (3.5) TeV collider, the W-gluon fusion rate is 2 (16) pb; the Drell-Yan-type rate is 0.6 (1.5) pb. The Drell-Yan-type event is easily separated from the W-gluon fusion event, therefore will not be considered hereafter [37]. For the decay mode of $t \to bW^+ \to b\ell^+\nu$, with $\ell^+ = e^+ \,\text{or}\, \mu^+$, the branching ratio of interest is $B = \frac{2}{9}$. The kinematic acceptance of this event at $\sqrt{S} = 2$ TeV is found to be 0.55 [37]. If the efficiency of b tagging is 30%, there will be $2\,\text{pb} \times 10^3\,\text{pb}^{-1} \times \frac{2}{9} \times 0.55 \times 0.3 = 75$ single-top quark events reconstructed. At $\sqrt{S} = 3.5$ TeV the kinematic acceptance of this event is 0.50 which, from the above calculation yields about 530 reconstructed events. Based on statistical error alone, this corresponds to a 12% and 4% measurement on the single-top cross section. A factor of 10 increase in the luminosity of the collider can improve the measurement by a factor of 3 statistically.

Taking into account the theoretical uncertainties, we examine two scenarios: 20% and 50% error on the measurement of the single-top cross section, which depends on both $\kappa^{\rm CC}_L$ and $\kappa^{\rm CC}_R$ (see Fig. 11). [Here we assume the experimental data agrees with the SM prediction within 20% (50%).] We found that for a 175 GeV top quark $\kappa^{\rm CC}_L$ and $\kappa^{\rm CC}_R$ are well constrained inside the region bounded by two (approximate) ellipses, as shown in Fig. 11. These results are not sensitive to the energies of the colliders considered here.

The top quark produced from the W-gluon fusion process is almost 100% left-handed (right-handed) polarized for a left-handed (right-handed) t-b-W vertex, therefore the charged lepton ℓ^+ from t decay has a harder momentum in a right-handed t-b-W coupling than in a left-handed coupling. (Note that the couplings of light fermions to W boson have been well tested from the low energy data to be left handed as described in the SM.) This difference becomes smaller when the top quark is more massive because the W boson from the top quark decay tends to be more longitudinally polarized.

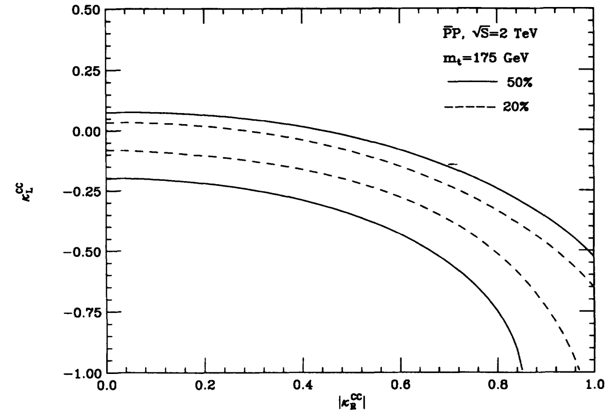

FIG. 11. The allowed $|\kappa^{\rm CC}_R|$ and $\kappa^{\rm CC}_L$ are bounded within the two dashed (solid) lines for a 20% (50%) error on the measurement of the single-top production rate, for a 175 GeV top quark.

A right-handed charged current is absent in a linearly $\text{SU}(2)_L$ invariant gauge theory with massless bottom quark. In this case, $\kappa^{\rm CC}_R = 0$, then $\kappa^{\rm CC}_L$ can be constrained to within about $-0.08 < \kappa^{\rm CC}_L < 0.03$ ($-0.20 < \kappa^{\rm CC}_L < 0.08$) with a 20% (50%) error on the measurement of the the single-top quark production rate at the Tevatron. This means that if we interpret $(1 + \kappa^{\rm CC}_L)$ as the CKM matrix element V_{tb}, then V_{tb} can be bounded as $V_{tb} > 0.9$ (or 0.8) for a 20% (or 50%) error on the measurement of the single-top production rate. Recall that if there are more than three generations, within 90% C.L., V_{tb} can be anywhere between 0 and 0.9995 from low energy data [38]. This measurement can therefore provide useful information on possible additional fermion generations.

We expect the LHC can provide similar or better bounds on these nonstandard couplings when detail analyses are available.

VI. AT THE NLC

The best place to probe $\kappa^{\rm NC}_L$ and $\kappa^{\rm NC}_R$ associated with the t-t-Z coupling is at the NLC through $e^-e^+ \to A, Z \to t\bar{t}$. (We use NLC to represent a generic e^-e^+ supercollider [24].) A detailed Monte Carlo study on the measurement of these couplings at the NLC including detector effects and initial state radiation can be found in Ref. [39]. The bounds were obtained by studying the angular distribution and the polarization of the top quark produced in e^-e^+ collisions. Assuming a 50 fb^{-1} luminosity at $\sqrt{S} = 500$ GeV, we concluded that within a 90% confidence level, it should be possible to measure $\kappa^{\rm NC}_L$ to within about 8%, while $\kappa^{\rm NC}_R$ can be known to within about 18%. A 1 TeV machine can do better than a 500

GeV machine in determining $\kappa_L^{\rm NC}$ and $\kappa_R^{\rm NC}$ because the relative sizes of the $t_R(\bar{t})_R$ and $t_L(\bar{t})_L$ production rates become small and the polarization of the $t\bar{t}$ pair is purer. Namely, it is more likely to produce either a $t_L(\bar{t})_R$ or a $t_R(\bar{t})_L$ pair. A purer polarization of the $t\bar{t}$ pair makes $\kappa_L^{\rm NC}$ and $\kappa_R^{\rm NC}$ better determined. (The purity of the $t\bar{t}$ polarization can be further improved by polarizing the electron beam.) Furthermore, the top quark is boosted more in a 1 TeV machine thereby allowing a better determination of its polar angle in the $t\bar{t}$ system because it is easier to find the right b associated with the lepton to reconstruct the top quark moving direction.

Finally, we remark that at the NLC $\kappa_L^{\rm CC}$ and $\kappa_R^{\rm CC}$ can be studied either from the decay of the top quark pair or from the single–top quark production process, W-photon fusion process $e^-e^+(W\gamma) \to tX$, or $e^-\gamma(W\gamma) \to \bar{t}X$, which is similar to the W-gluon fusion process in hadron collisions.

VII. DISCUSSION AND CONCLUSIONS

In this paper we have applied the electroweak chiral Lagrangian to probe new physics beyond the SM through studying the couplings of the top quark to gauge bosons. We first examined the precision LEP data to extract the information on these couplings, then we discussed how to improve our knowledge at current and future colliders such as at the Tevatron, the LHC, and the NLC.

Because of the non–renormalizability of the electroweak chiral Lagrangian we can only estimate the size of these nonstandard couplings by studying the contributions to LEP observables at the order of $m_t^2 \ln \Lambda^2$, where $\Lambda = 4\pi v \sim 3$ TeV is the cutoff scale of the effective Lagrangian. Already we found interesting constraints on these couplings.

Assuming b-b-Z vertex is not modified, we found that $\kappa_L^{\rm NC}$ is already constrained to be $-0.3 < \kappa_L^{\rm NC} < 0.6$ $(-0.2 < \kappa_L^{\rm NC} < 0.5)$ by LEP data at the 95% C.L. for a 150 (175) GeV top quark. Although $\kappa_R^{\rm NC}$ and $\kappa_L^{\rm CC}$ are allowed to be in the full range of ± 1, the precision LEP data do impose some correlations among $\kappa_L^{\rm NC}$, $\kappa_R^{\rm NC}$, and $\kappa_L^{\rm CC}$. ($\kappa_R^{\rm CC}$ does not contribute to the LEP observables of interest in the limit of $m_b = 0$.) In our calculations, these nonstandard couplings are only inserted once in loop diagrams using dimensional regularization.

Inspired by the experimental fact $\rho \approx 1$, reflecting the existence of an approximate custodial symmetry, we proposed an effective model to relate $\kappa_L^{\rm NC}$ and $\kappa_L^{\rm CC}$. We found that the nonuniversal interactions of the top quark to gauge bosons parametrized by $\kappa_L^{\rm NC}$, $\kappa_R^{\rm NC}$ and $\kappa_L^{\rm CC}$ are well constrained by LEP data, within 95% C.L. The results are summarized in Table I (see also Figs. 5 and 6). Also, the two parameters $\kappa_L = \kappa_L^{\rm NC}$ and $\kappa_R = \kappa_R^{\rm NC}$ are strongly correlated. In our model, $\kappa_L \sim 2\kappa_R$.

We note that the relations among κ's can be used to test different models of electroweak symmetry breaking. For instance, a heavy SM Higgs boson ($m_H > m_t$) will modify the couplings t-t-Z and t-b-W of a heavy top quark at the scale m_t such that $\kappa_L^{\rm NC} = 2\kappa_L^{\rm CC}$, $\kappa_L^{\rm NC} = -\kappa_R^{\rm NC}$, and $\kappa_R^{\rm CC} = 0$.

It is also interesting to note that the upper bound on the top quark mass can be raised from the SM bound $m_t < 200$ GeV to as large as 300 GeV if new physics occurs. That is to say, if there is new physics associated with the top quark, it is possible that the top quark is heavier than what the SM predicts, a similar conclusion was reached in Ref. [7].

With a better measurement of A_{LR} at the SLC, more constraint can be set on the correlation between κ_L and κ_R. To constrain the size of κ_L and κ_R, we need a more precise measurement on the partial decay width $\Gamma(Z \to b\bar{b})$.

Undoubtedly, direct detection of the top quark at the Tevatron, the LHC, and the NLC is crucial to measuring the couplings of t-b-W and t-t-Z. At hadron colliders, $\kappa_L^{\rm CC}$ and $\kappa_R^{\rm CC}$ can be measured by studying the polarization of the W boson from top quark decay in $t\bar{t}$ events. They can also be measured simply from the production rate of the single top quark event. The NLC is the best machine to measure $\kappa_L^{\rm NC}$ and $\kappa_R^{\rm NC}$ which can be measured from studying the angular distribution and the polarization of the top quark produced in e^-e^+ collision. Details about these bounds were given in Sec. V.

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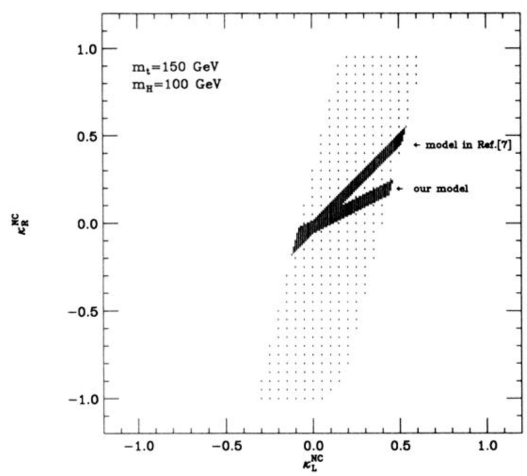

FIG. 10. A comparison between our model and the model in Ref. [7]. The allowed regions in both models are shown on the plane of $\kappa_L^{\rm NC}$ and $\kappa_R^{\rm NC}$, for $m_t = 150$ GeV.