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Mass and width of the ρ^0 from an S-matrix approach to $e^+e^- \to \pi^+\pi^-$

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We determine the mass width of the ρ^0 meson by applying the S-matrix formalism to the reaction $e^+e^- \to \pi^+\pi^-$ in the timelike region. We obtain $M_\rho = (757.5 \pm 1.5)$ MeV and $\Gamma_\rho = (142.5 \pm 3.5)$ MeV, which are significantly smaller than the values quoted by the Particle Data Group. These values are almost independent of the specific form used to model the background.

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I. INTRODUCTION

Mass and width are physical properties of an unstable particle. They should be independent of the theoretical model for the production and decay mechanisms chosen to determine their values from experimental data.

The usual approach to analyze processes involving resonances has been to use a Breit-Wigner formula which includes an energy-dependent width (see p. III.51 of Ref. [1]). This choice relies on dynamical and kinematical assumptions regarding the production and decay mechanisms of the resonance (see, for example, Refs. [2,4,10,11] for hadronic resonances and the on-shell scheme for the Z^0 gauge boson). However, it could happen that this approach is not adequate when we deal with wide hadronic resonances, because of the strong model dependences involved there.

As is well known, the S-matrix approach to scattering and decay amplitudes provides an alternative definition of the mass (M) and width (Γ) of a resonance [3]. In the case of an s-channel resonance, these parameters can be defined from the position of the pole of the S-matrix amplitude in the complex s as

$$s_{\text{pole}} \equiv M^2 - iM\Gamma. \tag{1}$$

This definition of the mass and width of an unstable particle is independent of the process used to determine these parameters, and it is also independent of the theoretical model for the production and decay mechanisms of the particle.

In the present paper we are concerned with the determination of the mass and width (M_ρ and Γ_ρ) of the ρ^0 vector meson from the experimental data on the $\pi^\pm$ electromagnetic form factor $F_\pi(s)$. As can be easily checked, the values of these parameters as quoted by the Particle Data Group [1], $M_\rho = (7.68.1 \pm 0.5)$ MeV and $\Gamma_\rho = (151.5 \pm 1.2)$ MeV, arise mainly from photoproduction and $\pi^+N \to \rho N$ experiments for the ρ^0 mass, whereas the weighted average of the ρ^0 width is strongly influenced by the $e^+e^- \to \pi^+\pi^-$ data. Those determinations could be inconsistent because different Breit-Wigner parametrizations have been used to model the resonant production of the $\pi^+\pi^-$ pair. In particular, it should be noted that the Breit-Wigner formulas used to extract M_ρ and Γ_ρ contain an energy-dependent width.

In this work we apply the S-matrix approach to determine the ρ^0 mass and width from the $e^+e^- \to \pi^+\pi^-$ data [4] in the timelike region $2m_\pi \leq \sqrt{s} \leq 1.1$ GeV. As will be shown below, our results for M_ρ and Γ_ρ are systematically smaller than the ones quoted in Ref. [1]. They are also independent either (i) of the particular form that we choose to parametrize the energy dependence of the background or (ii) of the choice of the normalization condition $F_\pi(0) = 1$. We also find a simple rule to derive M_ρ and Γ_ρ from the corresponding parameters entering the Breit-Wigner formulas that use an energy-dependent width.

Finally, we explore some of the main consequences that could arise from the new values of ρ^0 parameters for some decays of light mesons.

II. $\pi^\pm$ ELECTROMAGNETIC FORM FACTOR

Our purpose in this section is to determine the ρ^0 parameters from the e^+e^- annihilation into a $\pi^+\pi^-$ pair by using data in the timelike region $2m_{\pi^+} \leq \sqrt{s} \leq 1.1$ GeV [4], where $\sqrt{s}$ is the total center-of-mass energy.

The total cross section for this reaction can be written as

$$\sigma(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2}{3s^{5/2}}(s - 4m_\pi^2)^{3/2}|F_\pi(s)|^2. \tag{2}$$

The $\pi^\pm$ electromagnetic form factor $F_\pi(s)$ is an ana-

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lytic function in s, with a cut starting at $s = 4m_\pi^2$, and it is normalized such that $F_\pi(0) = 1$. $F_\pi(s)$ has also a pole corresponding to the resonant production of $\pi^+\pi^-$. Thus it can be represented as

$$F_\pi(s) = \frac{A}{s - s_\rho} + B(s), \tag{3}$$

where s_ρ is the position of the pole, A is the residue at the pole, and $B(s)$ represents the nonresonant background in the region near s_ρ. As has been emphasized also in Ref. [5], the position of the pole is a physical property of the S-matrix amplitude. Thus it provides a good definition of the mass and width for an unstable particle through Eq. (1).

In the model for $F_\pi(s)$, we should take also into account the contribution of the $\omega(782)$, since isospin breaking allows the coupling $\omega\pi\pi$ through the ρ-ω mixing [6]. By using the vector dominance model (VDM) [7], we can represent both contributions to $F_\pi(s)$ as shown in Fig. 1.

The effect of ρ-ω mixing is to multiply the resonant term in Eq. (3) by the factor

$$y \frac{M_\omega^2}{s - s_\omega}, \tag{4}$$

where y is a small dimensionless parameter which quantifies the isospin-breaking contribution and $s_\omega \equiv M_\omega^2 - iM_\omega\Gamma_\omega$. Within the VDM approach we have

$$y = \frac{m_{\rho\text{-}\omega}^2}{M_\rho^2} \frac{f_\rho}{f_\omega}, \tag{5}$$

where eM_V^2/f_V defines the vector-meson–photon coupling and $m_{\rho\text{-}\omega}$ describes the strength of the ρ-ω mixing, $m_{\rho\text{-}\omega}^2 \equiv < \rho|HA^{\Delta I=1}|\omega >$. M_ω and Γ_ω are the mass and width of the ω meson, which we will fix in the following to their values given in Refs. [1,8]:

$$M_\omega = (781.95 \pm 0.14) \text{ MeV},$$

$$\Gamma_\omega = (8.43 \pm 0.10) \text{ MeV}.$$

Thus, taking into account ρ-ω mixing, Eq. (3) becomes either

$$F_\pi(s) = \left(\frac{A}{s - s_\rho} + B(s)\right)\left(1 + y\frac{M_\omega^2}{s - s_\omega}\right) \tag{6}$$

or

$$F_\pi(s) = \frac{A}{s - s_\rho}\left(1 + y\frac{M_\omega^2}{s - s_\omega}\right) + B(s). \tag{7}$$

In the next section we will perform several fits to the $F_\pi(s)$ data by assuming different forms for the background term. Furthermore, we will set A to be a real constant and introduce a dimensionless constant a through the relation $A \equiv -aM_\rho^2$.

III. FITS TO EXPERIMENTAL DATA OF $F_\pi(S)$

Our aim in this section is to convince the reader that the extracted values for M_ρ and Γ_ρ are almost independent of the specific choice we make to parametrize the background term $B(s)$. In Secs. III A—III C we present the fits when we relax the condition of $F_\pi(0)$, and in Sec. III D we discuss the same cases when we fix $F_\pi(0) = 1$.

A. Fit with real constants a and B

In the present fit we choose the simplest case $B(s) = b$, with b a real constant. In this case, Eqs. (6) and (7) take the explicit forms

$$F_\pi^{(1)}(s) = \left(-\frac{aM_\rho^2}{s - M_\rho^2 + iM_\rho\Gamma_\rho} + b\right) \times \left(1 + \frac{yM_\omega^2}{s - M_\omega^2 + iM_\omega\Gamma_\omega}\right) \tag{8}$$

and

$$F_\pi^{(2)}(s) = -\frac{aM_\rho^2}{s - M_\rho^2 + iM_\rho\Gamma_\rho}\left(1 + \frac{yM_\omega^2}{s - M_\omega^2 + iM_\omega\Gamma_\omega}\right) + b. \tag{9}$$

Both Eqs. (8) and (9) contain five free parameters to be determined from experiment: M_ρ, Γ_ρ, a, b, and y.

These formulas provide, respectively, a good fit to experimental data (see Figs. 2 and 3) without the need to fix $F_\pi(0)$. We obtain, respectively, the resonant parameters

$$M_\rho = (756.76 \pm 0.82) \text{ MeV}, \quad \Gamma_\rho = (143.91 \pm 1.15) \text{ MeV} \tag{10}$$

and

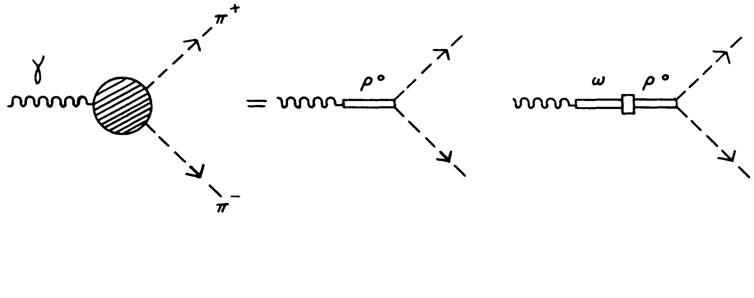

FIG. 1. Vector-meson contributions to the $\pi^\pm$ electromagnetic form factor.

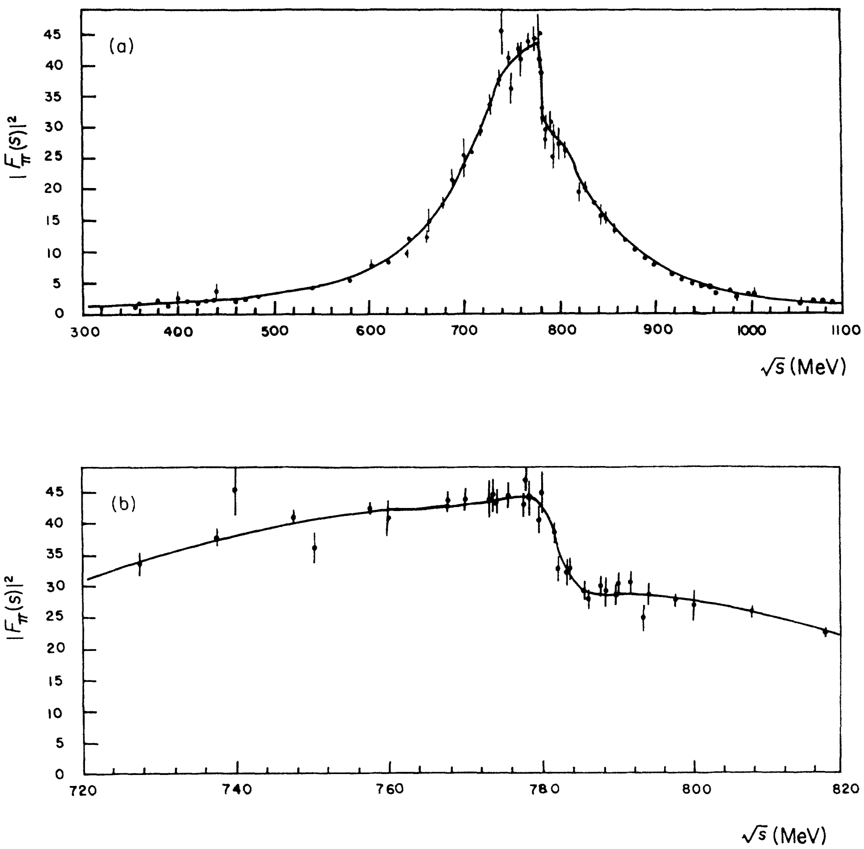

FIG. 2. Fit to the experimental data of $F_\pi(s)$ by using Eq. (8): (a) full range for the center-of-mass energy and (b) kinematical region around ρ and ω.

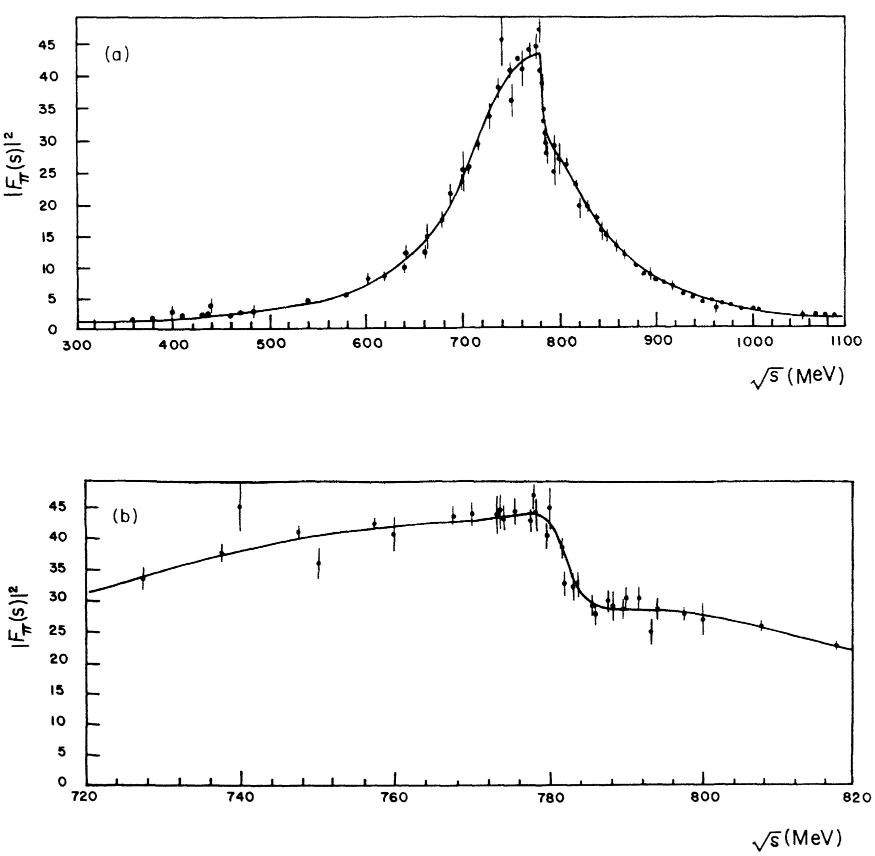

FIG. 3. Same as Fig. 2 by using Eq. (9).

TABLE I. Results of the fits for the parameters a, b, and y appearing in Eqs. (8) and (9) [$F_\pi(0)$ is a free parameter].

Model	y (10^{-3})	a	b	χ^2
$F_\pi^{(1)}(s)$	-1.91 ± 0.15	1.195 ± 0.007	-0.233 ± 0.013	1.00
$F_\pi^{(2)}(s)$	-1.92 ± 0.15	1.194 ± 0.006	-0.234 ± 0.011	1.01

$$M_\rho = (756.63 \pm 0.55)\ \text{MeV}, \qquad \Gamma_\rho = (144.14 \pm 1.13)\ \text{MeV}. \tag{11}$$

The remaining parameters of the fit are shown in Table I.

We observe that both parametrizations of $F_\pi(s)$ lead to essentially the same set of values for the free parameters. From a simple inspection of Eqs. (8) and (9), this means that the ρ-ω mixing and background terms are very weakly coupled.

B. Fit with an s-dependent background

The next step is to introduce the energy-dependent background in $F_\pi(s)$. In this case the most general form of $F_\pi(s)$ can be found as

$$|F_\pi^{(3)}(s)|^2 = \frac{a^2 M_\rho^4}{(s-M_\rho^2)^2 + M_\rho^2\Gamma_\rho^2} \times \left|1 + y\frac{M_\omega^2}{s - M_\omega^2 + iM_\omega\Gamma_\omega}\right|^2 P\left(\frac{s-M_\rho^2}{M_\rho^2}\right), \tag{12}$$

where P is a polynomial of degree N: namely,

$$P(x) = \sum_{i=0}^{N} c_i x^i, \quad c_0 = 1, \tag{13}$$

which will represent the background.

We will carry out several fits by taking successively $N = 0, \ldots, 4$. Since we are not fixing $F_\pi(0)$, we will have fits with $N + 4$ free parameters corresponding, respectively, to M_ρ, Γ_ρ, a, and y and the N values of the c_i coefficients.

The results of the best fits for each case are given in Table II. We observe that the stability of the fits (namely, the fits for which M_ρ and Γ_ρ remain almost constant when N increases) is attained once we include $c_2 \neq 0$. Observe also that although the fit with $c_i = 0$ for $N \geq 1$ gives a larger value for the ρ mass, it also corresponds to an unacceptably large $\chi^2/N_{\rm DF}$. Finally, note that the effect of introducing higher powers of x in $P(x)$ is compensated by a change in the normalization factor a.

Thus, from the fits shown in Table II and including terms with $c_i \neq 0$ for $N \geq 2$, we extract the values

$$M_\rho = (757.50 \pm 1.50)\ \text{MeV}, \qquad \Gamma_\rho = (141 \pm 2)\ \text{MeV}, \tag{14}$$

which are in good agreement with Eqs. (10) and (11).

C. Fit for timelike and spacelike s

One of the simplest nonlinear backgrounds that can be used for $F_\pi(s)$ in the timelike region is

$$F_\pi^{(4)}(s) = \frac{-aM_\rho^2}{s - M_\rho^2 + iM_\rho\Gamma_\rho}\left(1 + \frac{yM_\omega^2}{s - M_\omega^2 + iM_\omega\Gamma_\omega}\right) \times \left[1 + b\left(\frac{s-M_\rho^2}{M_\rho^2}\right)\right]^{-1}. \tag{15}$$

Again, if we do not use the condition $F_\pi(0) = 1$, we still have a fit with five free parameters. The best fit to the experimental data in the timelike region ($2m_\pi \leq \sqrt{s} \leq 1.1$ GeV) is obtained for the following set of parameters:

$$\begin{aligned} M_\rho &= (757.03 \pm 0.76)\ \text{MeV}, \\ \Gamma_\rho &= (141.22 \pm 1.19)\ \text{MeV}, \\ y &= (-1.87 \pm 0.15) \times 10^{-3}, \\ a &= 1.176 \pm 0.007, \\ b &= -0.192 \pm 0.009, \\ \chi^2 &= 0.90. \end{aligned} \tag{16}$$

As can be easily realized, the ρ mass and width as well as the ρ-ω mixing parameter y remain almost the same as in the previous fits, whereas the effect of the new parametrization, Eq. (15), is to change the size of

TABLE II. Results of the fits for the parameters appearing in Eqs. (12) and (13) [$F_\pi(0)$ is a free parameter].

N	M_ρ (MeV)	Γ_ρ (MeV)	a^2	c_1	c_2	c_3	c_4	y (10^{-3})	χ^2
0	766.90 ± 0.43	143.10 ± 1.09	1.346 ± 0.014					-2.19 ± 0.14	4.555
1	756.90 ± 0.82	145.21 ± 1.16	1.448 ± 0.017	0.378 ± 0.020				-1.94 ± 0.15	1.066
2	756.93 ± 0.59	139.80 ± 1.76	1.359 ± 0.026	0.404 ± 0.020	0.061 ± 0.043			-1.84 ± 0.15	0.922
3	758.60 ± 0.78	141.80 ± 1.93	1.384 ± 0.029	0.278 ± 0.049	0.088 ± 0.051	0.176 ± 0.067		-1.89 ± 0.15	0.875
4	758.74 ± 1.14	141.36 ± 1.14	1.375 ± 0.062	0.267 ± 0.062	0.126 ± 0.035	0.206 ± 0.089	0.044 ± 0.059	-1.89 ± 0.15	0.883

TABLE III. Results of the fits for $F_\pi(s)$ when we impose the constraint $F_\pi(0) = 1$ on Eqs. (8) and (9).

Model	M_ρ (MeV)	Γ_ρ (MeV)	a	b	y (10^{-3})	χ^2
$F_\pi^{(1)}(s)$	757.71±0.74	145.42±1.03	1.202±0.006	−0.202±0.006	−1.96±0.15	1.08
$F_\pi^{(2)}(s)$	757.62±0.74	145.66±1.03	1.202±0.006	−0.202±0.006	−1.97±0.15	1.09

the normalization factor a and the background parameter b.

When $s < 4m_\pi^2$, Eq. (15) becomes

$$F_\pi^{(4)}(s) = -\frac{aM_\rho^2}{s - M_\rho^2}\left[1 + b\left(\frac{s - M_\rho^2}{M_\rho^2}\right)\right]^{-1}. \tag{17}$$

With Eqs. (16) and (17) we can also give a satisfactory fit to the experimental data of $F_\pi(s)$ in the spacelike region [9], $-0.253 \leq s \leq -0.015$ GeV^2.

D. Fits to $F_\pi(s)$ when $F_\pi(0) = 1$

In the previous examples we have chosen to fit the experimental data with five free parameters in cases $F_\pi^{(1,2,4)}(s)$ and $N + 4$ parameters ($N \geq 2$) in the case $F_\pi^{(3)}(s)$. In these cases we did not impose the constraint $F_\pi(0) = 1$. Thus, in the first part of this section, we will use the fitted parameters given above to derive the value of $F_\pi(0)$. In the second part we will fit the experimental data by imposing the condition $F_\pi(0) = 1$ to Eqs. (8), (9), and (12) and (15).

When $s < 4m_\pi^2$ the ρ-ω mixing is negligible and the widths are absent. In this case Eqs. (8) and (9) take the form

$$F_\pi^{(1,2)}(s) = -\frac{aM_\rho^2}{s - M_\rho^2} + b, \tag{18}$$

whereas Eqs. (12) and (15) become, respectively,

$$|F_\pi^{(3)}(s)|^2 = \frac{a^2 M_\rho^4}{(s - M_\rho^2)^2} P\left(\frac{s - M_\rho^2}{M_\rho^2}\right) \tag{19}$$

and

$$F_\pi^{(4)}(s) = -\frac{aM_\rho^2}{s - M_\rho^2}\left[1 + b\left(\frac{s - M_\rho^2}{M_\rho^2}\right)\right]^{-1}. \tag{20}$$

By using the values of the parameters given in Eqs. (10), (11), and (16) and Table I into Eqs. (18) and (20) at $s = 0$, we obtain

$$\begin{aligned} F_\pi^{(1)}(0) &= 0.962 \pm 0.020, \\ F_\pi^{(2)}(0) &= 0.960 \pm 0.017, \\ F_\pi^{(4)}(0) &= 0.987 \pm 0.013, \end{aligned} \tag{21}$$

whereas if we use Eq. (19) and Table II, we get

$$F_\pi^{(3)}(0) = \begin{cases} 1.014 \pm 0.052 & \text{if } N = 2, \\ 0.935 \pm 0.131 & \text{if } N = 3, \\ 0.979 \pm 0.176 & \text{if } N = 4. \end{cases} \tag{22}$$

From the values of $F_\pi(0)$ quoted above, we observe that they are in satisfactory agreement with the theoretical expectation $F_\pi(0) = 1$.

Next, we will carry out the fits to experimental data by imposing the condition $F_\pi(0) = 1$ to Eqs. (8), (9), and (18) and (12) and (19). The results of the fits are given in Tables III and IV, which should be compared to the corresponding results given in Eqs. (10) and (11) and Tables I and II.

We can also impose the constraint $F_\pi(0) = 1$ on $F_\pi^{(4)}(s)$ [Eqs. (15) and (20)]. The values of the parameters corresponding to the best fit are

$$\begin{aligned} M_\rho &= (757.30 \pm 0.75) \text{ MeV}, \\ \Gamma_\rho &= (142.29 \pm 0.88) \text{ MeV}, \\ y &= (-1.89 \pm 0.15) \times 10^{-3}, \\ a &= 1.182 \pm 0.005, \\ b &= -0.182 \pm 0.005, \\ \chi^2 &= 0.91, \end{aligned} \tag{23}$$

to be compared to the corresponding values in Eq. (16).

From a comparison of both set of fits, it is very interesting to realize that the constraint $F_\pi(0) = 1$ does not produce a significant shift on the fitted values for M_ρ and Γ_ρ. The effect of imposing this condition is compensated for by changes into the background terms and/or the normalization factor a.

IV. FROZEN ρ-ω MIXING

In the previous fits we have included the isospin-breaking contributions to the electromagnetic form fac-

TABLE IV. Results of the fits for $F_\pi(s)$ when we impose the constraint $F_\pi(0) = 1$ on Eq. (12). The results are given only for $N \geq 2$ in Eq. (13).

N	M_ρ (MeV)	Γ_ρ (MeV)	a^2	c_1	c_2	c_3	c_4	y (10^{-3})	χ^2
2	756.78±0.78	140.34±1.42	1.369±0.110	0.409±0.020	0.140± 0.025			−1.85±0.15	0.917
3	757.83±0.98	139.94±1.28	1.357±0.113	0.344±0.041	0.157±0.021	0.077±0.041		−1.85±0.15	0.892
4	757.96±0.93	141.83±1.30	1.387±0.119	0.330±0.036	0.047±0.023	0.078±0.037	0.081±0.019	−1.88±0.15	0.888

tor by a simple product of the two ω and ρ Breit-Wigner shapes. This form is strongly motivated by the VDM. In this section we shall assume that a contact term is allowed for the $\omega\pi\pi$ coupling, i.e., that the factor arising from ρ-ω mixing, $-ayM_\rho^2/(s-M_\rho^2+iM_\rho\Gamma_\rho)$ in Eq. (9), is frozen at a given $s=\bar{s}$ value (see below); as in the previous analysis, we do not observe any shift in the position of the pole of the electromagnetic form factor.

In terms of the quantities ϵ and ϕ which describe, respectively, the isospin-violating contact term and the relative phase between the $\rho\pi\pi$ and $\omega\pi\pi$ couplings, we have

$$F_\pi^{(5)}(s) = -\frac{aM_\rho^2}{s-M_\rho^2+iM_\rho\Gamma_\rho} + \varepsilon e^{i\phi}\frac{M_\omega^2}{s-M_\omega^2+iM_\omega\Gamma_\omega} + b, \qquad (24)$$

and we shall assume ε and ϕ to be real constants.

If the condition $F_\pi(0)=1$ is not fixed in Eq. (24), the fit to experimental data contains six free parameters. The corresponding best parameters of the fit are

$$\begin{aligned} M_\rho &= (757.00 \pm 0.59)\ \text{MeV}, \\ \Gamma_\rho &= (143.41 \pm 1.27)\ \text{MeV}, \\ \varepsilon &= (12.23 \pm 1.20)\times 10^{-3}, \\ \phi &= (116.7 \pm 5.8)^\circ, \\ a &= 1.197 \pm 0.006, \\ b &= -0.231 \pm 0.008, \\ \chi^2 &= 1.00. \end{aligned} \qquad (25)$$

As can be easily checked, the mass and width of the ρ remain the same as the ones found in the previous section.

Now, if we compare Eq. (24) and the model described by Eq. (9), we can establish a correspondence between both models by defining an "average energy" $\bar{s}$ such that

$$\varepsilon(\bar{s})\exp[i\phi(\bar{s})] = -\frac{ayM_\rho^2}{\bar{s}-M_\rho^2+iM_\rho\Gamma_\rho}. \qquad (26)$$

With this relation and the results given in Eq. (25), we obtain

$$\begin{aligned} \sqrt{\bar{s}} &= (M_\rho^2 - M_\rho\Gamma_\rho \cot\phi)^{1/2} \\ &= (792.18 \pm 0.89)\ \text{MeV}, \end{aligned} \qquad (27)$$

which is not too far from the ω mass [$M_\omega = (781.95 \pm 0.14)$ MeV], and

$$\begin{aligned} y &= -\frac{\varepsilon\cos\phi[(\bar{s}-M_\rho^2)^2+M_\rho^2\Gamma_\rho^2]}{aM_\rho^2(\bar{s}-M_\rho^2)} \\ &= (-2.16 \pm 0.35)\times 10^{-3}. \end{aligned} \qquad (28)$$

Thus the effect of introducing a direct isospin-violating coupling in the $I=0$, $G=-1$ channel is to slightly increase the value of an "effective" ρ-ω mixing, but it does not change the resonant properties (the position of the pole) of the ρ^0.

Finally, let us remark that it is very interesting to note that all the above fits, except when $N=0$ in Eq. (13), give very similar curves as those presented in Figs. 2 and 3.

V. COMPARISON WITH RESULTS THAT USE AN ENERGY-DEPENDENT WIDTH

In this section we find a relationship between the ρ mass and width as defined from the pole position and the corresponding parameters which enter in the Breit-Wigner formula with an energy-dependent width. This relation provides values M_ρ and Γ_ρ which are consistent with the ones quoted in previous sections.

Our starting point is to realize that an expression with a single pole can be put into a Breit-Wigner shape with an energy-dependent width by using the transformation

$$[1-ix(s)](s-M_\rho^2+iM_\rho\Gamma_\rho) = s = \bar{M}^2 + i\bar{M}\bar{\Gamma}(s), \qquad (29)$$

where

$$\bar{M}^2 = M_\rho^2 - xM_\rho\Gamma_\rho, \qquad (30a)$$

$$\bar{M}\bar{\Gamma} = M_\rho\Gamma_\rho(1+x^2), \qquad (30b)$$

$$x(s) = \frac{\bar{M}[\bar{\Gamma}-\bar{\Gamma}(s)]}{s-\bar{M}^2}, \qquad (30c)$$

and

$$\bar{\Gamma} \equiv \bar{\Gamma}(s=\bar{M}^2). \qquad (31)$$

Thus $\bar{M}$ and $\bar{\Gamma}$ represent the mass and width parameters that enter the Breit-Wigner shape with an energy-dependent width.

Since the $\rho\to\pi\pi$ decay proceeds through an $l=1$ wave, $\bar{\Gamma}(s)$ can be parametrized as

$$\bar{\Gamma}(s) = \left(\frac{q_\pi(s)}{q_\pi(\bar{M}^2)}\right)^3 \left(\frac{\bar{M}}{\sqrt{s}}\right)^\lambda \bar{\Gamma}, \qquad (32)$$

where $q_\pi(s) = (s/4 - m_\pi^2)^{1/2}$ is the momentum of the π in the rest frame of the ρ.

As has been assumed explicitly in Eqs. (29) and (30a) and is shown below, $x(s)$ is an almost constant function of s [$x(s) \simeq x(s=\bar{M}^2)$] for two interesting models that use a Breit-Wigner shape with an s-dependent width.

(a) The model used in Ref. [10] corresponds to the choice

$$\begin{aligned} \lambda &= 1, \\ \bar{M} &= 775.4\ \text{MeV}, \\ \bar{\Gamma} &= 149.6\ \text{MeV}, \end{aligned} \qquad (33)$$

which according to Eq. (30c) implies $x(s=\bar{M}^2) \simeq -0.236$. Using Eqs. (30a) and (30b) one obtains

$$\begin{aligned} M_\rho &= 758.5\ \text{MeV}, \\ \Gamma_\rho &= 144.8\ \text{MeV}. \end{aligned} \qquad (34)$$

(b) For the model used in Ref. [11], we can identify

$$\lambda = 1.748,$$
$$\bar{M} = 768.7 \text{ MeV}, \tag{35}$$
$$\bar{\Gamma} = 142.8 \text{ MeV}.$$

According to Eq. (30c), this model will correspond to $x(s = \bar{M}^2) \simeq -0.1589$ and using Eqs. (30a) and (30b) we get

$$M_\rho = 757.55 \text{ MeV},$$
$$\Gamma_\rho = 141.70 \text{ MeV}. \tag{36}$$

The values for M_ρ and Γ_ρ quoted in Eqs. (34) and (36) are in very good agreement with the ones quoted in Sec. III. This becomes obvious if we realize that, under the transformation given in Eq. (29), we can isolate the pole of a Breit-Wigner shape with an energy-dependent width and put into the background its remaining s dependence.

VI. DISCUSSION OF RESULTS AND CONCLUSIONS

In the present paper we have used the S-matrix approach to analyze the experimental data on the electromagnetic form factor in the timelike region. This approach allows a model-independent determination of the mass and width of the ρ^0 vector meson.

In order to study the possible influence of different Breit= Wigner parametrization on the resonant properties of the ρ^0, we have parametrized the background term by assuming different models. From the several fits we have performed, we conclude that the mass and width of the ρ^0 are consistent with the values

$$M_\rho = (757.5 \pm 1.5) \text{ MeV},$$
$$\Gamma_\rho = (142.5 \pm 3.5) \text{ MeV}, \tag{37}$$

which are significantly smaller than the corresponding parameters quoted in Ref. [1]: namely,

$$M_\rho = (768.1 \pm 0.5) \text{ MeV},$$
$$\Gamma_\rho = (151.5 \pm 1.2) \text{ MeV}. \tag{38}$$

As expected in the S-matrix formalism, the extracted values for the mass and width of the ρ^0 have the following properties: (a) They are almost independent of whether or not we impose the normalization condition $F_\pi(0) = 1$ in the different fits; (b) the unique effects of the different parametrizations for $F_\pi(s)$ is to change the size of the residue at the pole and background terms appearing in Eq. (3). We also found [Eqs. (34) and (36)] that M_ρ and Γ_ρ derived from the Breit-Wigner formulas with an energy-dependent width via the transformation given in Eqs. (29) and (30) are also consistent with the results presented in Eqs. (37).

In the following we shall explore some interesting consequences that could have the new values of M_ρ and Γ_ρ.

(i) *The ρ-ω mixing strength.* To calculate the strength of the ρ-ω mixing form Eq. (5), we need the ratio f_ω/f_ρ. In the context of the VDM, the leptonic partial rate of a neutral vector meson is given by

$$\Gamma(V \to e^+e^-) = \frac{4\pi}{3}\left(\frac{\alpha}{f_V}\right)^2 M_V,$$

which gives rise to the ratio

$$\frac{f_\omega}{f_\rho} = \left[\frac{M_\omega \Gamma(\rho \to e^+e^-)}{M_\rho \Gamma(\omega \to e^+e^-)}\right]^{1/2}.$$

Now, by using the experimental partial widths [1] $\Gamma(\rho \to e^+e^-) = (6.77 \pm 0.32)$ keV, $\Gamma(\omega \to e^+e^-) = (0.60 \pm 0.02)$ keV, and the results of Eqs. (37) and Table I, we obtain

$$(m^{(1)}_{\rho\omega})^2 = (-3.735 \pm 0.300) \times 10^{-3} \text{ GeV}^2, \tag{39}$$

whereas if we use the results of Eqs. (25) and (28), we get

$$(m^{(5)}_{\rho\omega})^2 = (-4.225 \pm 0.684) \times 10^{-3} \text{ GeV}^2. \tag{40}$$

Thus the value of $m^2_{\rho\omega}$ obtained from the approximation of Eqs. (24) and (26) is larger, although also with a larger error.

The ρ-ω mixing strength derived from the fit of Sec. III C is

$$(m^{(4)}_{\rho\omega})^2 = (-3.669 \pm 0.300) \times 10^{-3} \text{ GeV}^2, \tag{41}$$

which is consistent with the result given in Eq. (39).

The new values of the ρ^0 parameters, Eqs. (37), and of the ρ-ω mixing, Eq. (41), would slightly modify the rate of the G-parity-violating decay $\omega \to \pi^+\pi^-$; we obtain

$$B(\omega \to \pi^+\pi^-) = \frac{|m^2_{\rho\text{-}\omega}|^2}{(M^2_\omega - M^2_\rho)^2 + M^2_\rho \Gamma^2_\rho}\left(\frac{M^2_\omega - 4m^2_\pi}{M^2_\rho - 4m^2_\pi}\right)^{3/2} \times \frac{M^2_\rho}{M^2_\omega}\frac{\Gamma_\rho}{\Gamma_\omega} \tag{42}$$
$$= (1.85 \pm 0.30)\%, \tag{43}$$

to be compared with $B(\omega \to \pi^+\pi^-) = (2.21 \pm 0.30)\%$ [1].

(ii) *The ratio $\rho^0 \to \pi^0\gamma/\rho^+ \to \pi^+\gamma$.* In the isospin symmetry limit, the ratio $R = \Gamma(\rho^0 \to \pi^0\gamma)/\Gamma(\rho^+ \to \pi^+\gamma)$ is equal to 1. In the context of the vector dominance model, the neutral ρ^0 decay is enhanced by isospin-breaking corrections due to the decay chain $\rho^0 \to \omega \to \pi^0\gamma$. In particular, this correction is very sensitive to the ρ^0 mass because of the narrowness of the $\omega(782)$ [see Eq. (44)].

Including this isospin-breaking correction, we obtain

$$R = \left|1 + \frac{f_\omega}{f_\rho}\frac{m^2_{\rho\text{-}\omega}}{M^2_\rho - M^2_\omega + iM_\omega\Gamma_\omega}\right|^2 \left|\frac{\mathbf{k}_{\pi^0}}{\mathbf{k}_{\pi^+}}\right|^3, \tag{44}$$

where $\mathbf{k}_\pi$ is the corresponding three-momentum of the π in the ρ rest frame.

If we use the results of Eqs. (37) and (41) and if we assume $M_{\rho^+} \simeq M_{\rho^0}$ [1], we obtain, from Eq. (44),

$$R^{\text{theory}} = 1.772(2.395), \tag{45}$$

where the term in parentheses is obtained when $M_{\rho^0} =$ 768.1 MeV [1].

The result given in Eq. (45) should be compared to the experimental value $R^{\text{expt}} = 1.78 \pm 0.49$ (Ref. [1] p. VII.13) and the theoretical prediction $R = 1.11$ given in Ref. [12]

In summary, the new values of the ρ^0 resonance parameters could have interesting consequences for a test of flavor symmetry breaking in the sector of light mesons.

Note added. After we submitted this paper, we became aware of Refs. [13–16] which deal with the determination of the ρ^0 parameters form the pole position of the S matrix.

References [13,14] focus their attention on $\pi\pi$ scattering. In contradistinction to $\pi\pi$ scattering, which involves resonances in different channels ($J = 0, 1$) and where experimental data are not as precise and abundant, the reaction $e^+e^- \to \pi^+\pi^-$ serves as a filter to select precisely the $J = 1$ channel. Thus we might expect to obtain the cleanest values for the ρ parameters in the second case.

On the other hand, Ref. [15] deals with the determination of the pole parameters from the $\pi^\pm$ electromagnetic form factor. Their analysis included available data from the full kinematical (spacelike and timelike) region. Their main result indicated a discrepancy between the spacelike and timelike data for the pion form factor. However, using a different analytical approach, Dubnicka and Martinovic [16] found that no contradiction exists between both sets of data. Their results for the pole parameters, $M_\rho = 761.1 \pm 2.9$ MeV and $\Gamma_\rho = 144.9 \pm 3.7$ MeV, are in very good agreement with ours.

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