Energy correlation and asymmetry of secondary leptons in $e^+e^- \to t\bar{t}$

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Top quarks produced in the reaction $e^+e^- \to t\bar{t}$ are predicted to have a strong spin-spin correlation. We show that this correlation reflects itself in a strong energy correlation of the charged leptons produced in the decays $t \to bl^+\nu_l$ ($\bar{t} \to \bar{b}l^-\bar{\nu}_l$). Analytical expressions are given for the two-dimensional distribution $d\sigma/dxdx'$ where x and x' are scaled energy variables of l^+ and l^-. In the presence of a CP-violating term in the $e^+e^- \to t\bar{t}$ amplitude, this correlation acquires an antisymmetric component which is also calculated. Our formalism yields compact expressions for the single-particle energy spectra of l^+ and l^- and the asymmetry between them.

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I. INTRODUCTION

Top quarks produced in the reaction $e^+e^- \to t\bar{t}$ are predicted to have strong polarization and spin-spin correlation [1–3]. A question of great interest is to what extent these spin properties will reflect themselves in the spectrum of the secondary leptons l^+ and l^- produced in the decays $t \to bl^+\nu_l(\bar{t} \to \bar{b}l^-\bar{\nu_l})$. In a recent paper, the authors described a technique (based on a proposal of Kawasaki, Shirafuji, and Tsai [4]) for obtaining the angular distribution of the secondary leptons in the e^+e^- c.m. frame and the angular correlation between them. In this paper, we show that this procedure yields analytical expressions for the energy spectrum and the energy-energy correlation of the secondary particles.

The calculation involves two ingredients. (i) The first is the differential production cross section $(d\sigma/d\Omega_t)(s_+, s_-)$ for $e^+e^- \to t\bar{t}$, for arbitrary polarizations s_+, s_- of the $t, \bar{t}$ quark. This cross section was obtained by Kühn, Reiter, and Zerwas [5], assuming γ and Z exchange, and is reproduced in the Appendix. (ii) The second is the differential decay rate for an unpolarized top quark [6]:

$$\frac{1}{\Gamma}\frac{d\Gamma_l}{d^3q/(2q_0)}(t(p_t) \to l^+(q) + \cdots) = \frac{12B_l}{\pi m_t^4 W}(m_t^2 - 2p_t q) \ , \tag{1}$$

where

$$W = \left(1 - \frac{m_W^2}{m_t^2}\right)^2 \left(1 + 2\frac{m_W^2}{m_t^2}\right) \ , \tag{2}$$

B_l is the branching ratio for $t \to l^+ + \cdots$, and we neglect the final fermion masses. [The decay $t \to l^+ + \cdots$ is here treated as a sequence of two-body decays $t \to bW^+$, $W^+ \to l^+\nu_l$, employing the narrow width approximation for the W. This is in accordance with evidence that the mass of the top quark is considerably higher than that of the W [7].]

In the final section of the paper, we consider the effects of CP violation, introduced in the $e^+e^- \to t\bar{t}$ amplitude through electric-dipole-type couplings. This leads to an antisymmetric term in the energy-energy correlation of the secondary leptons and an asymmetry between the l^+ and l^- spectra. These results are compared with those in previous work [8]. Some of the essential steps in the formalism of Kawasaki, Shirafuji, and Tsai [4] are recapitulated in Appendix A.

II. ENERGY SPECTRUM OF A SINGLE LEPTON

As an illustration of our formalism, we begin with the inclusive distribution of a single decay lepton l^+ in the reaction $e^+e^- \to l^+ \cdots$. This is given by

$$\frac{d\sigma}{d^3q/(2q_0)}(e^+e^- \to l^+ + \cdots) = 4\int d\Omega_t \left\{ \frac{d\sigma}{d\Omega_t}(n, 0)\frac{1}{\Gamma}\frac{d\Gamma_l}{d^3q/(2q_0)}(t(p_t) \to l^+(q) + \cdots) \right\} \ , \tag{3}$$

where $(d\sigma/d\Omega_t)(n, 0)$ is obtained from $(d\sigma/d\Omega_t)$ (s_+, s_-) by replacing

$$s_+^\mu \to n^\mu = \left(g^{\mu\nu} - \frac{p_t^\mu p_t^\nu}{m_t^2}\right)\frac{m_t}{p_t q}q_\nu \tag{4}$$

and setting $s_- = 0$. To obtain the energy spectrum of the l^+, we write

$$\frac{d^3q}{2q_0} = \frac{1}{2}E\,dE\,d\Omega_l = \frac{\pi}{4}\frac{1+\beta}{\beta}dx\,d\mu^2 \ , \tag{5}$$

where $\mu^2 = (p_t - q)^2$ is the missing mass squared in the decay $t \to l^+ + \cdots$, $\beta = \sqrt{1 - 4m_t^2/s}$, and x is the reduced energy of the lepton, defined by

$$x = \frac{2E}{m_t}\left(\frac{1-\beta}{1+\beta}\right)^{1/2}, \tag{6}$$

E being the energy of the lepton in the e^+e^- c.m. system. The variable μ^2 is constrained by the inequalities

$$0 \le \frac{\mu^2}{m_t^2} \le 1 - \frac{m_W^2}{m_t^2} \tag{7}$$

and

$$1 - x\frac{1+\beta}{1-\beta} \le \frac{\mu^2}{m_t^2} \le 1 - x, \tag{8}$$

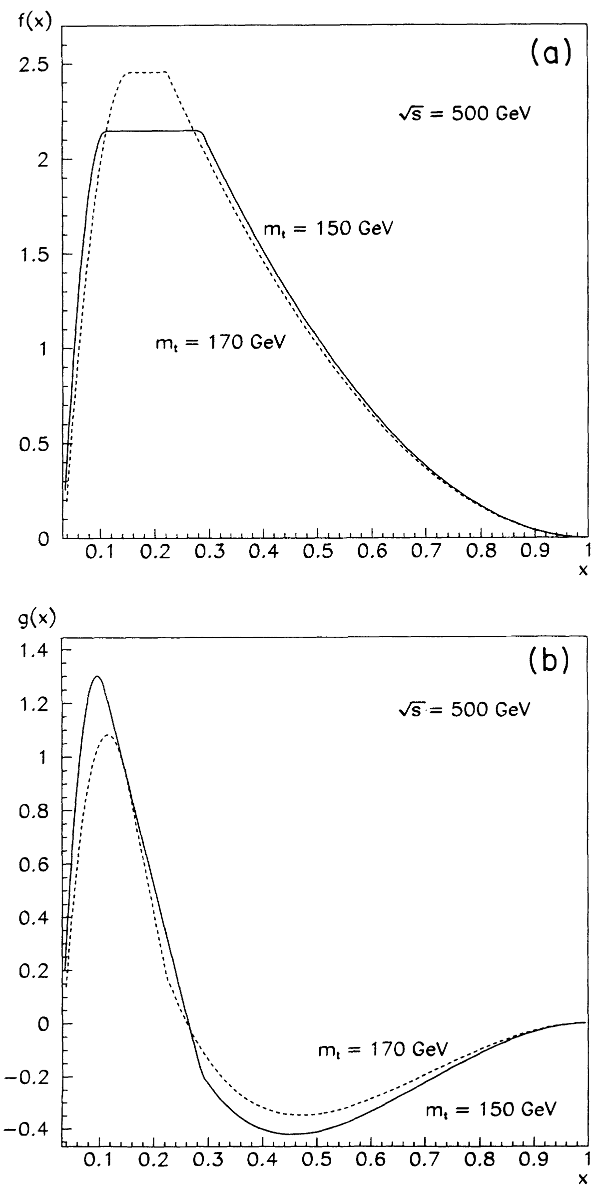

FIG. 1. Functions $f(x)$ and $g(x)$, which appear in the energy distribution of secondary leptons for two different top quark masses and for a center-of-mass energy $\sqrt{s} = 500$ GeV.

while the reduced energy x is bounded by

$$\frac{m_W^2}{m_t^2}\frac{1-\beta}{1+\beta} \le x \le 1. \tag{9}$$

Integration over the variable[1] μ^2 then yields the following normalized energy distribution of the secondary lepton l^+ originating in the reaction $e^+e^- \to t\bar{t}$:

$$\frac{1}{B_l\sigma(e^+e^- \to t\bar{t})}\frac{d\sigma}{dx}(e^+e^- \to l^+ + \cdots)$$

$$= f(x) + \frac{4\,\mathrm{Re}D_{VA}}{(3-\beta^2)D_V + 2\beta^2 D_A}g(x), \tag{10}$$

where the quantities D_V, D_{VA}, and D_A, which depend on electroweak couplings of the top quark and the electron, are given in Appendix B, and the functions $f(x)$ and $g(x)$ are defined as

$$f(x) = \frac{3}{W}\frac{1+\beta}{\beta}\int d\left(\frac{\mu^2}{m_t^2}\right)\frac{\mu^2}{m_t^2},$$

$$g(x) = \frac{3}{W}\frac{1+\beta}{\beta}\int d\left(\frac{\mu^2}{m_t^2}\right)\frac{\mu^2}{m_t^2}\left[1 - \frac{x(1+\beta)}{1-\mu^2/m_t^2}\right]. \tag{11}$$

Carrying out the integration yields

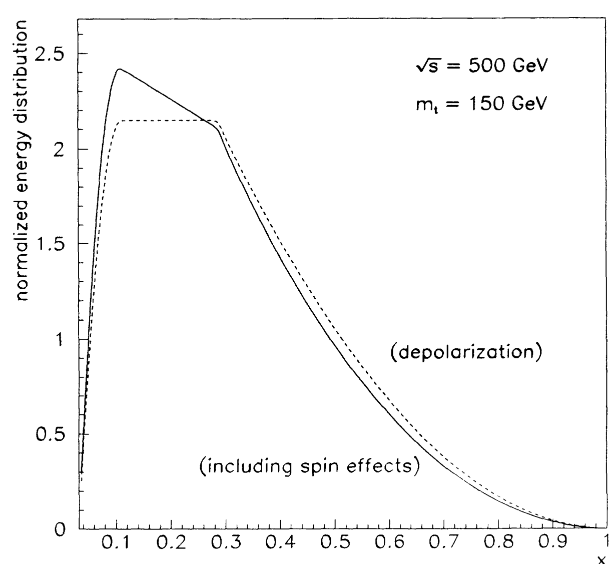

FIG. 2. Normalized energy distribution of charged leptons l^+ from the reaction $e^+e^- \to t\bar{t}$ including spin effects (solid curve) and if the top quarks are depolarized before their decay (dashed curve).

[1]Note that in terms of the variables x and μ^2 the decay spectrum has the form

$$\frac{1}{\Gamma}\frac{d\Gamma_l}{dx\,d\mu^2} = \frac{1+\beta}{\beta}\frac{3B_l}{m_t^4 W}\mu^2.$$

(1) $\frac{m_W^2}{m_t^2} \geq \frac{1-\beta}{1+\beta}$,

$$f(x) = \frac{3}{2W}\frac{1+\beta}{\beta} \times \begin{cases} -2\frac{m_W^2}{m_t^2} + \frac{m_W^4}{m_t^4} + 2x\frac{1+\beta}{1-\beta} - x^2\left(\frac{1+\beta}{1-\beta}\right)^2 : I_1 , \\ 1 - 2\frac{m_W^2}{m_t^2} + \frac{m_W^4}{m_t^4} : I_2 , \\ 1 - 2x + x^2 : I_3 , \end{cases}$$

$$g(x) = \frac{3}{W}\frac{(1+\beta)^2}{\beta} \times \begin{cases} \left\{ -x\frac{m_W^2}{m_t^2} + x^2\frac{1+\beta}{1-\beta} + x\ln\frac{m_W^2}{m_t^2} - x\ln\left(x\frac{1+\beta}{1-\beta}\right) \right. \\ \left. + \frac{1/2}{1+\beta}\left[-2\frac{m_W^2}{m_t^2} + \frac{m_W^4}{m_t^4} + 2x\frac{1+\beta}{1-\beta} - x^2\left(\frac{1+\beta}{1-\beta}\right)^2 \right] \right\} : I_1 , \\ x - x\frac{m_W^2}{m_t^2} + x\ln\frac{m_W^2}{m_t^2} + \frac{1/2}{1+\beta}\left[1 - 2\frac{m_W^2}{m_t^2} + \frac{m_W^4}{m_t^4} \right] : I_2 , \\ x - x^2 + x\ln x + \frac{1/2}{1+\beta}[1 - 2x + x^2] : I_3 , \end{cases}$$

where the intervals I_i are given by

$$I_1 : \frac{m_W^2}{m_t^2}\frac{1-\beta}{1+\beta} \leq x \leq \frac{1-\beta}{1+\beta} ,$$

$$I_2 : \frac{1-\beta}{1+\beta} \leq x \leq \frac{m_W^2}{m_t^2} ,$$

$$I_3 : \frac{m_W^2}{m_t^2} \leq x \leq 1 .$$

(2) $\frac{m_W^2}{m_t^2} \leq \frac{1-\beta}{1+\beta}$,

$$f(x) = \frac{3}{2W}\frac{1+\beta}{\beta} \times \begin{cases} \times - 2\frac{m_W^2}{m_t^2} + \frac{m_W^4}{m_t^4} + 2x\frac{1+\beta}{1-\beta} - x^2\left(\frac{1+\beta}{1-\beta}\right)^2 : I_4 , \\ -2x + x^2 + 2x\frac{1+\beta}{1-\beta} - x^2\left(\frac{1+\beta}{1-\beta}\right)^2 : I_5 , \\ 1 - 2x + x^2 : I_6 , \end{cases}$$

$$g(x) = \frac{3}{W}\frac{(1+\beta)^2}{\beta} \times \begin{cases} \left\{ -x\frac{m_W^2}{m_t^2} + x^2\frac{1+\beta}{1-\beta} + x\ln\frac{m_W^2}{m_t^2} - x\ln x\left(\frac{x1+\beta}{1-\beta}\right) \right. \\ \left. + \frac{1/2}{1+\beta}\left[-2\frac{m_W^2}{m_t^2} + \frac{m_W^4}{m_t^4} + 2x\frac{1+\beta}{1-\beta} - x^2\left(\frac{1+\beta}{1-\beta}\right)^2 \right] \right\} : I_4 , \\ -x^2 + x^2\frac{1+\beta}{1-\beta} + x\ln\frac{1-\beta}{1+\beta} + \frac{1/2}{1+\beta}\left[-2x + x^2 + 2x\frac{1+\beta}{1-\beta} - x^2\left(\frac{1+\beta}{1-\beta}\right)^2 \right] : I_5 , \\ x - x^2 + x\ln x + \frac{1/2}{1+\beta}[1 - 2x + x^2] : I_6 , \end{cases}$$

with the intervals I_i:

$$I_4 : \frac{m_W^2}{m_t^2}\frac{1-\beta}{1+\beta} \le x \le \frac{m_W^2}{m_t^2} \,,$$

$$I_5 : \frac{m_W^2}{m_t^2} \le x \le \frac{1-\beta}{1+\beta} \,,$$

$$I_6 : \frac{1-\beta}{1+\beta} \le x \le 1 \,.$$

Note that f and g satisfy

$$\int f(x)dx = 1 \,, \qquad \int g(x)dx = 0 \,. \tag{12}$$

These functions are plotted in Fig. 1 for $m_t = 150$ and 170 GeV. The term proportional to $g(x)$ describes explicitly the spin-dependent part of the lepton spectrum and would be absent if, for instance, the t quark were to be depolarized by hadronization effects prior to decay. The predicted energy spectrum is shown in Fig. 2, where the case of no spin correlation $[g(x) = 0]$ is also plotted for contrast. Of particular note is the fact that the spin-independent part of the spectrum is characterised by a plateau in the interval $(1-\beta)/(1+\beta) \le x \le m_W^2/m_t^2$. This plateau changes to an incline when the spin-dependent effects are added.

III. ENERGY CORRELATION OF l^+ AND l^-

We now consider the joint energy distribution of two charged secondary leptons l^+ and l^- originating from $t \to bl^+\nu_l$ and $\bar{t} \to \bar{b}l^-\bar{\nu}_l$. The differential cross section for $e^+e^- \to l^+(q)l^-(q') + \cdots$ is given by

$$\frac{d\sigma}{d^3q/(2q_0)d^3q'/(2q_0')}(e^+e^- \to l^+l^- + \cdots) = 4\int d\Omega_t \left\{ \frac{d\sigma}{d\Omega_t}(n,m)\frac{1}{\Gamma}\frac{d\Gamma_l}{d^3q/(2q_0)}(t(p_t) \to l^+(q) + \cdots) \times \frac{1}{\Gamma}\frac{d\Gamma_l}{d^3q'/(2q_0')}(\bar{t}(p_{\bar{t}}) \to l^-(q') + \cdots) \right\} . \tag{13}$$

Here $(d\sigma/d\Omega_t)(n,m)$ is obtained from the differential cross section $(d\sigma/d\Omega_t)(s_+,s_-)$ by replacing the spin vectors of the top quark (s_+) and the top antiquark (s_-) by

$$s_+^\mu \to n^\mu = \left(g^{\mu\nu} - \frac{p_t^\mu p_t^\nu}{m_t^2}\right)\frac{m_t}{p_t q}q_\nu \,, \qquad s_-^\mu \to m^\mu = -\left(g^{\mu\nu} - \frac{p_{\bar{t}}^\mu p_{\bar{t}}^\nu}{m_t^2}\right)\frac{m_t}{p_{\bar{t}} q'}q'_\nu \,. \tag{14}$$

Carrying out an integration over the angular variables as described above, we obtain the following normalized two-particle spectrum in the energies of l^+ and l^-:

$$\frac{1}{B_l^2\sigma(e^+e^- \to t\bar{t})}\frac{d\sigma}{dx\,dx'}(e^+e^- \to l^+l^- + \cdots) = f(x)f(x') + \frac{1}{\beta^2}\frac{(1+\beta^2)D_V + 2\beta^2 D_A}{(3-\beta^2)D_V + 2\beta^2 D_A}g(x)g(x') + \frac{4\,\mathrm{Re}D_{VA}}{(3-\beta^2)D_V + 2\beta^2 D_A}[f(x)g(x') + f(x')g(x)] \,, \tag{15}$$

where x and x' are the reduced energies,

$$x = \frac{2E}{m_t}\left(\frac{1-\beta}{1+\beta}\right)^{1/2} \,, \tag{16}$$

$$x' = \frac{2E'}{m_t}\left(\frac{1-\beta}{1+\beta}\right)^{1/2} \,, \tag{17}$$

E and E' being the energies of the final leptons l^+ and l^- in the e^+ e^- c.m. system.[2] Equation (15) shows explicitly that the energy spectra of the two leptons are correlated because of the presence of the function $g(x)$ which reflects the spin dependence of the reaction $e^+e^- \to t\bar{t}$.

[2] A form similar to Eq. (15) was obtained in Ref. [9] for $e^+e^- \to f\bar{f} \to l^+l^- + \cdots$, in the case that the fermion f is light compared to m_W. However, the functions f and g, which involve the dynamics of the t decay, are different from those in Ref. [9].

Integrating over x or x' and using the normalization conditions (12), we get back the energy spectrum of a single lepton.

In Fig. 3(a) we depict the normalized two-particle energy distribution for the process $e^+e^- \to t\bar{t} \to l^+(x)l^-(x') + \cdots$ for a top mass $m_t = 150$ GeV and for an e^+e^- c.m. energy $\sqrt{s} = 500$ GeV. In the case of complete depolarization of top quarks prior to decay, the two-particle distribution is given by the first term $f(x)f(x')$ in Eq. (15) alone. This case, which corresponds to uncorrelated spectra of l^+ and l^-, is exhibited in Fig. 3(b). Estimates of depolarization due to hadronization [10] indicate that such effects will be very small for a top quark as massive as 150 GeV or more. The importance of spin-

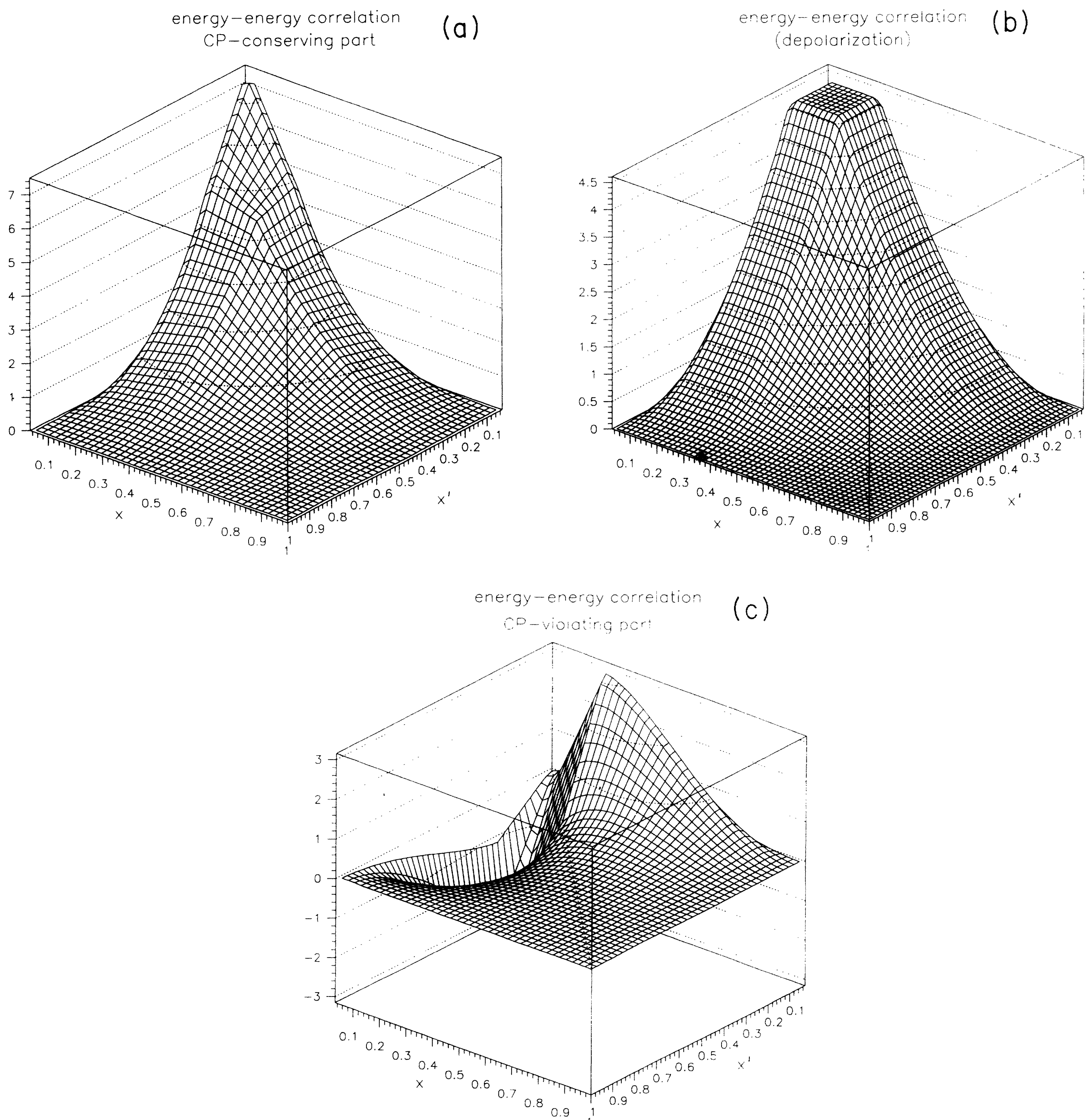

FIG. 3. Normalized energy-energy correlation of two charged secondary leptons for $m_t = 150$ GeV and $\sqrt{s} = 500$ GeV. (a) represents the CP-conserving part of the two-particle spectrum. (b) shows the energy-energy correlation if the top quarks are completely depolarized before their decay. The CP-violating antisymmetric part of the two-dimensional energy distribution $A(x, x')$ is given in (c).

dependent effects may be judged from the fact that in the domain of reduced energies,

$$\frac{m_W^2}{m_t^2}\frac{1-\beta}{1+\beta} \le x, x' \le 0.26 \ , \tag{18}$$

the fraction of events is about 30% higher than in the case of depolarization.

IV. ENERGY CORRELATION AND ENERGY ASYMMETRY IN THE PRESENCE OF CP VIOLATION

We consider in this section the influence of a CP-violating modification in the amplitude of $e^+e^- \to t\bar{t}$ on the energy spectrum and energy correlation of the secondary leptons. Such a modification has the consequence that the $t\bar{t}$ state is no longer an exact CP eigenstate and accordingly can have unequal probabilities for the helicity configurations $t_L\bar{t}_R$ and $t_R\bar{t}_L$. This in turn can lead to an asymmetric term in the two-dimensional distribution $(1/\sigma)\frac{d\sigma}{dx\,dx'}$ and a difference in the energy spectra of l^+ and l^- [8].

The specific CP-violating term we introduce is an electric dipole moment coupling of the γ and Z to $t\bar{t}$:

$$\begin{aligned} \gamma t\bar{t} &: -id_\gamma \sigma_{\mu\nu}\gamma_5 (p_t + p_{\bar{t}})^\nu \ , \\ Z t\bar{t} &: -id_Z \sigma_{\mu\nu}\gamma_5 (p_t + p_{\bar{t}})^\nu \ . \end{aligned} \tag{19}$$

This modification produces a new term in the two-dimensional distribution $\frac{d\sigma}{dx\,dx'}$ which now reads

$$\begin{aligned} \frac{1}{B_l^2\sigma(e^+e^-\to t\bar{t})}\frac{d\sigma}{dx\,dx'}(e^+e^-\to l^+l^- + \cdots) = {} & f(x)f(x') + \frac{1}{\beta^2}\frac{(1+\beta^2)D_V + 2\beta^2 D_A}{(3-\beta^2)D_V + 2\beta^2 D_A} g(x)g(x') \\ & + \frac{4\,\mathrm{Re}D_{VA}}{(3-\beta^2)D_V + 2\beta^2 D_A}[f(x)g(x') + f(x')g(x)] \\ & + \frac{E_{\mathrm{dipole}}}{(3-\beta^2)D_V + 2\beta^2 D_A}[f(x)g(x') - f(x')g(x)] \ , \end{aligned} \tag{20}$$

with

$$\begin{aligned} E_{\mathrm{dipole}} = \frac{4m_t}{e}\Bigg[& -e_t \mathrm{Im}d_\gamma - \frac{s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}\frac{(v_e^2 + a_e^2)v_t}{64\sin^3\theta_W\cos^3\theta_W}\mathrm{Im}d_Z \\ & + \frac{s(s-m_Z^2)}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}\left(\frac{v_e v_t}{16\sin^2\theta_W\cos^2\theta_W}\mathrm{Im}d_\gamma + \frac{e_t v_e}{4\sin\theta_W\cos\theta_W}\mathrm{Im}d_Z\right)\Bigg] \ . \end{aligned} \tag{21}$$

The term proportional to E_{dipole} is antisymmetric in x and x', a hallmark of CP violation. Rewriting Eq. (20) as

$$\frac{1}{B_l^2\sigma(e^+e^-\to t\bar{t})}\frac{d\sigma}{dx\,dx'}(e^+e^-\to l^+l^- + \cdots) = S(x,x') + \xi A(x,x') \ , \tag{22}$$

where

$$\begin{aligned} \xi &= \frac{E_{\mathrm{dipole}}}{(3-\beta^2)D_V + 2\beta^2 D_A} \ , \\ A(x,x') &= f(x)g(x') - f(x')g(x) \ , \end{aligned} \tag{23}$$

the functions $S(x,x')$ and $A(x,x')$ represent the symmetric and antisymmetric parts of the two-dimensional distribution. These are plotted in Figs. 3(a) and 3(c). Integration over x or x' yields the single-lepton energy spectra

$$\frac{1}{B_l\sigma(e^+e^-\to t\bar{t})}\frac{d\sigma}{dx}(e^+e^-\to l^\pm + \cdots) = f(x) + \frac{4\,\mathrm{Re}D_{VA}}{(3-\beta^2)D_V + 2\beta^2 D_A}g(x) \mp \xi g(x) \ . \tag{24}$$

Consequently, the asymmetry in the energy spectrum of l^+ and l^-, as a function of the energy x, is

$$\begin{aligned} a(x) &\equiv \frac{\frac{d\sigma}{dx}(e^+e^-\to l^- + \cdots) - \frac{d\sigma}{dx}(e^+e^-\to l^+ + \cdots)}{\frac{d\sigma}{dx}(e^+e^-\to l^- + \cdots) + \frac{d\sigma}{dx}(e^+e^-\to l^+ + \cdots)} \\ &= \xi\frac{g(x)}{f(x) + \frac{4\,\mathrm{Re}D_{VA}}{(3-\beta^2)D_V + 2\beta^2 D_A}g(x)} \ , \end{aligned} \tag{25}$$

which is plotted in Fig. 4. The results contained in Eqs. (21) and (24) agree with those obtained by Chang, Keung, and Phillips [8] using a different method.

Finally, our results for the energy correlation and energy asymmetry of leptons originating from $e^+e^- \to t\bar{t}$

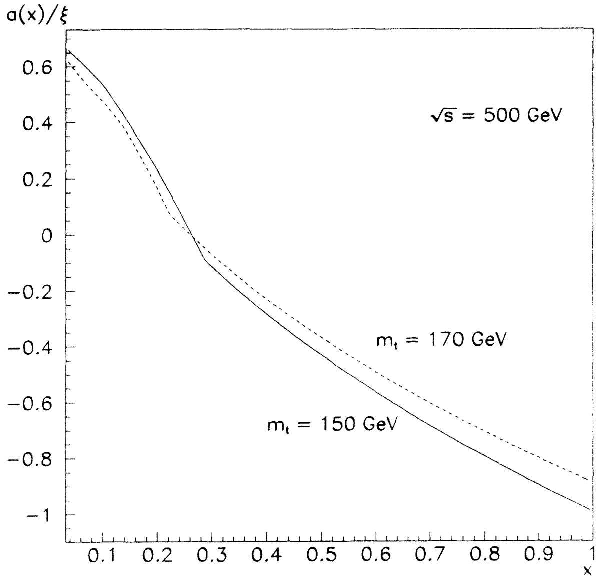

FIG. 4. Asymmetry $a(x)/\xi$ in the energy spectrum of l^+ and l^- for $\sqrt{s} = 500$ GeV.

can be transcribed to the process $q\bar{q} \to t\bar{t}$ by switching off the Z couplings in the production matrix element and replacing the photon by a gluon [11].

As noted in Ref. [8], the energy asymmetry $a(x)$ is a CP-odd, but a T-even observable and requires an imaginary part in the form factors d_γ or d_Z. Explicit calculations of Imd_γ and Imd_Z in a simple Higgs model have been carried out by Bernreuther, Pham, and Schröder [12] and the results confirmed in Ref. [8].

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APPENDIX A

In this appendix we repeat some fundamental steps in the formalism of Kawasaki, Shirafuji, and Tsai [4] in the notation of Bjorken and Drell.

Consider a process in which two particles 1 and 2 with four-momenta p_1 and p_2 scatter to give a system of particles a and an unstable spin-$\frac{1}{2}$ particle X. X, which has mass M, four-momentum p, and polarization vector s, then decays into a system of particles b:

$$1 + 2 \to a + X \; , \tag{A1}$$

$$X \to b \; . \tag{A2}$$

The differential cross section for the reaction (A1) and the differential decay rate for the process (A2) are given by

$$d\sigma_X^{(s)} = \frac{1}{4F} |\langle X(s), a|T|1,2\rangle|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p - p_a) \times \frac{1}{(2\pi)^3} \frac{d^3p}{2E} dX_{\mathrm{LIPS}}(a) \; , \tag{A3}$$

$$d\Gamma_b^{(s)} = \frac{1}{2M} |\langle b|T|X(s)\rangle|^2 (2\pi)^4 \delta^4(p - p_b) dX_{\mathrm{LIPS}}(b) \; . \tag{A4}$$

The differential cross section for the reaction $1+2 \to a+b$ reads

$$d\sigma_{X\to b} = \frac{1}{4F} |\langle a, b|T|1,2\rangle|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_a - p_b) \times dX_{\mathrm{LIPS}}(a) dX_{\mathrm{LIPS}}(b). \tag{A5}$$

$F^2 = (p_1 p_2)^2 - p_1^2 p_2^2$ is the invariant flux factor, and $dX_{\mathrm{LIPS}}(a)$ is the Lorentz-invariant phase space element for a system of particles a. The matrix elements $\langle X(s), a|T|1,2\rangle$ and $\langle b|T|X(s)\rangle$ for the processes (A1) and (A2) can be written as

$$\langle X(s), a|T|1,2\rangle = \sqrt{2M} \bar{u}_\alpha(p,s) A_\alpha \; , \tag{A6}$$

$$\langle b|T|X(s)\rangle = \sqrt{2M} \overline{B}_\alpha u_\alpha(p,s) \; . \tag{A7}$$

$u_\alpha(p,s)$ is the Dirac spinor of X normalized as $\bar{u}u = 1$. One finds then

$$|\langle X(s), a|T|1,2\rangle|^2 = 2M \overline{A}_\alpha \left(\frac{\not{p} + M}{2M} \frac{1 + \gamma_5 \not{s}}{2} \right)_{\alpha\beta} A_\beta \; , \tag{A8}$$

$$|\langle b|T|X(s)\rangle|^2 = 2M \overline{B}_\alpha \left(\frac{\not{p} + M}{2M} \frac{1 + \gamma_5 \not{s}}{2} \right)_{\alpha\beta} B_\beta \; . \tag{A9}$$

The matrix element for the combined process is

$$\langle a, b|T|1,2\rangle = \overline{B}_\alpha \left(\frac{\not{p} + M}{p^2 - M^2 + iM\Gamma} \right)_{\alpha\beta} A_\beta \; . \tag{A10}$$

Using the narrow width approximation for the short-lived particle X ($\Gamma \ll M$),

$$\left| \frac{1}{p^2 - M^2 + iM\Gamma} \right|^2 \approx \frac{\pi}{\Gamma M} \delta(p^2 - M^2) \; , \tag{A11}$$

and the identity

$$2[\overline{A}\Lambda_+(p)B][\overline{B}\Lambda_+(p)A] = [\overline{A}\Lambda_+(p)A][\overline{B}\Lambda_+(p)B] + \eta_{\mu\nu}[\overline{A}\Lambda_+(p)\gamma_5\gamma^\mu A] \times [\overline{B}\Lambda_+(p)\gamma_5\gamma^\nu B] \tag{A12}$$

(which can easily be verified in the X rest frame), we find

$$|\langle a,b|T|1,2\rangle|^2 = (2M)^2 \frac{\pi}{M\Gamma}\delta(p^2-M^2)[\overline{B}\Lambda_+(p)B] \times \left(\overline{A}\Lambda_+(p)\frac{1+\gamma_5 \not{n}}{2}A\right) , \qquad (A13)$$

where $\Lambda_+(p) = (\not{p}+M)/2M$ is the projection operator for positive energy states and $\eta_{\mu\nu}$ is defined by

$$\eta_{\mu\nu} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{M^2} . \qquad (A14)$$

The "polarization vector"

$$n_\mu = \eta_{\mu\nu}\frac{\overline{B}\Lambda_+(p)\gamma_5\gamma^\nu B}{\overline{B}\Lambda_+(p)B} \qquad (A15)$$

satisfies $p\cdot n = 0$ and $n^2 = -1$. Finally, one obtains

$$d\sigma_{X\to b} = \frac{1}{4F}\frac{\pi}{M\Gamma}\delta(p^2-M^2)|\langle X(n),a|T|1,2\rangle|^2 \sum_s |\langle b|T|X(s)\rangle|^2(2\pi)^4\delta^4(p_1+p_2-p_a-p_b)dX_{\rm LIPS}(a)dX_{\rm LIPS}(b)$$
$$= 2\times\frac{1}{4F}|\langle X(n),a|T|1,2\rangle|^2(2\pi)^4\delta^4(p_1+p_2-p-p_b)\frac{1}{(2\pi)^3}\frac{d^3p}{2E}$$
$$\times dX_{\rm LIPS}(a)\frac{1}{\Gamma}\frac{1}{2M}\frac{1}{2}\sum_s|\langle b|T|X(s)\rangle|^2(2\pi)^4\delta(p-p_b)dX_{\rm LIPS}(b) \qquad (A16)$$

or symbolically

$$d\sigma_{X\to b} = 2d\sigma_X^{(n)}\frac{d\Gamma_b}{\Gamma} , \qquad (A17)$$

where $d\Gamma_b = \frac{1}{2}\sum_s d\Gamma_b^{(s)}$ is the initial spin-averaged differential decay width for the process $X\to b$.

Using the identity (A12), Eq. (A13) can also be written as

$$|\langle a,b|T|1,2\rangle|^2 = (2M)^2\frac{\pi}{M\Gamma}\delta(p^2-M^2)[\overline{A}\Lambda_+(p)A] \times\left(\overline{B}\Lambda_+(p)\frac{1+\gamma_5\hat{\not{n}}}{2}B\right) , \qquad (A18)$$

with

$$\hat{n}_\mu = \eta_{\mu\nu}\frac{\overline{A}\Lambda_+(p)\gamma_5\gamma^\nu A}{\overline{A}\Lambda_+(p)A} . \qquad (A19)$$

This gives for the combined process the alternative formula

$$d\sigma_{X\to b} = d\sigma_X\frac{d\Gamma_b^{(\hat{n})}}{\Gamma} . \qquad (A20)$$

$d\sigma_X$ is the final spin-summed production cross section for $1+2\to a+X$. Equations (A17) and (A20) are equivalent ways of deriving the distribution of the secondary particle b.

We now discuss production and subsequent decays of an unstable spin-$\frac{1}{2}$ particle X and the corresponding antiparticle X' with four-momenta p and p' and polarization vectors s and s':

$$1+2\to X+X' , \qquad (A21)$$

$$X\to b,\quad X'\to b' . \qquad (A22)$$

The matrix elements for the decay processes (A22) are

$$\langle b|T|X(s)\rangle = \sqrt{2M}\overline{B}_\alpha u_\alpha(p,s) , \qquad (A23)$$

$$\langle b'|T|X'(s')\rangle = \sqrt{2M}\overline{v}_\alpha(p',s')C_\alpha . \qquad (A24)$$

We denote the spin-dependent differential cross section for the reaction (A21) by $d\sigma_{XX'}^{(s,s')}$ and the differential decay rates for (A22) by $d\Gamma_b^{(s)}$ and $d\Gamma_{b'}^{(s')}$. In analogy to the above disscusion, one finds, for the combined process,

$$d\sigma_{XX'\to bb'} = 4d\sigma_{XX'}^{(n,m)}\frac{d\Gamma_b}{\Gamma}\frac{d\Gamma_{b'}}{\Gamma} , \qquad (A25)$$

with

$$d\Gamma_b = \tfrac{1}{2}\sum_s d\Gamma_b^{(s)} ,\quad d\Gamma_{b'} = \tfrac{1}{2}\sum_{s'} d\Gamma_{b'}^{(s')} . \qquad (A26)$$

$d\sigma_{XX'}^{(n,m)}$ is the production cross section for the process $1+2\to X+X'$ in which the spin vectors s and s' are replaced by

$$s_\mu\to n_\mu = \eta_{\mu\nu}\frac{\overline{B}\Lambda_+(p)\gamma_5\gamma^\nu B}{\overline{B}\Lambda_+(p)B} , \qquad (A27)$$

$$s'_\mu\to m_\mu = \eta'_{\mu\nu}\frac{\overline{C}\Lambda_-(p')\gamma_5\gamma^\nu C}{\overline{C}\Lambda_-(p')C} , \qquad (A28)$$

with $\eta_{\mu\nu} = -g_{\mu\nu}+\frac{p_\mu p_\nu}{M^2}$ and $\eta'_{\mu\nu} = -g_{\mu\nu}+\frac{p'_\mu p'_\nu}{M^2}$. $\Lambda_-(p') = (-\not{p}'+M)/2M$ is the projection operator for negative energy states.

APPENDIX B

In the presence of CP-violating couplings of the top quark, the matrix element for the reaction $e^+e^-\to t\bar{t}$ reads

$$M = \frac{ie^2}{s}\Bigg[(v_e v_t d - e_t)\overline{u}(t)\gamma_\mu v(\overline{t})\overline{v}(e^+)\gamma^\mu u(e^-) + a_e v_t d\overline{u}(t)\gamma_\mu v(\overline{t})\overline{v}(e^+)\gamma_5\gamma^\mu u(e^-) + v_e a_t d\overline{u}(t)\gamma_5\gamma_\mu v(\overline{t})\overline{v}(e^+)\gamma^\mu u(e^-)$$
$$+a_e a_t d\overline{u}(t)\gamma_5\gamma_\mu v(\overline{t})\overline{v}(e^+)\gamma_5\gamma^\mu u(e^-) + \left(4\sin\theta_W\cos\theta_W v_e d\frac{d_Z}{e} - \frac{d_\gamma}{e}\right)\overline{u}(t)\sigma_{\mu\nu}P^\nu\gamma_5 v(\overline{t})\overline{v}(e^+)\gamma^\mu u(e^-)$$
$$+4\sin\theta_W\cos\theta_W a_e d\frac{d_Z}{e}\overline{u}(t)\sigma_{\mu\nu}P^\nu\gamma_5 v(\overline{t})\overline{v}(e^+)\gamma_5\gamma^\mu u(e^-)\Bigg] , \qquad \text{(B1)}$$

with

$$d = \frac{s}{s - m_Z^2 + im_Z\Gamma_Z}\frac{1}{16\sin^2\theta_W\cos^2\theta_W} ,$$
$$v_f = 2I_3^f - 4e_f\sin^2\theta_W , \quad a_f = 2I_3^f . \qquad \text{(B2)}$$

$I_3^f = \pm\frac{1}{2}$ for up or down particles, and e_f is the electric charge in units of the electric charge of the proton.

Neglecting the terms proportional to d_γ and d_Z, the differential cross section for $e^+e^- \to t\overline{t}$ in the Born approximation is given for unpolarized e^+e^- beams as a function of the four-momenta $P \equiv p_{e^-} + p_{e^+}$, $l \equiv p_{e^-} - p_{e^+}$, $Q \equiv p_t - p_{\overline{t}}$ and of the top quark (antiquark) spin vectors $s_+(s_-)$ by [5]

$$\frac{d\sigma}{d\Omega_t}(s_+, s_-) = \frac{3\alpha^2\beta}{8s^3}(D_V\{\tfrac{1}{2}[s^2 + (lQ)^2] + 2m_t^2(ls_+\cdot ls_- - Ps_+\cdot Ps_-)$$
$$+2sm_t^2 - \tfrac{1}{2}[(lQ)^2 - s^2 + 4m_t^2 s]s_+s_- - (s - 2m_t^2)(Ps_+\cdot Ps_- - ls_+\cdot ls_-)$$
$$+lQ(Ps_-\cdot ls_+ - ls_-\cdot Ps_+)\} + D_A\{\tfrac{1}{2}[s^2 + (lQ)^2] + 2m_t^2(ls_+\cdot ls_- - Ps_+\cdot Ps_-)$$
$$-2sm_t^2 + \tfrac{1}{2}[(lQ)^2 - s^2 + 4m_t^2 s]s_+s_- + (s - 2m_t^2)(Ps_+\cdot Ps_- - ls_+\cdot ls_-)$$
$$-lQ(Ps_-\cdot ls_+ - ls_-\cdot Ps_+)\} + 2\,\mathrm{Re}D_{VA}[sm_t(Ps_- - Ps_+) + m_t lQ(-ls_- - ls_+)]$$
$$+2\,\mathrm{Im}D_{VA}[-\tfrac{1}{2}ls_-\varepsilon(l,P,Q,s_+) - \tfrac{1}{2}ls_+\varepsilon(l,P,Q,s_-) + \tfrac{1}{2}lQ\varepsilon(l,Q,s_-,s_+)]$$
$$+E_V 2sm_t(ls_- + ls_+) + E_A 2lQm_t(Ps_+ - Ps_-)$$
$$+2\,\mathrm{Re}E_{VA}[-lQs - 2m_t^2(ls_-\cdot Ps_+ - ls_+\cdot Ps_-)]$$
$$+2\,\mathrm{Im}E_{VA}[-m_t\varepsilon(s_+,l,Q,P) - m_t\varepsilon(s_-,l,Q,P)]) . \qquad \text{(B3)}$$

$\sqrt{s}$ is the center-of-mass energy, and $\beta = \sqrt{1 - 4m_t^2/s}$ is the velocity of the t quarks in the c.m. system. The symbol $\varepsilon(a,b,c,d)$ means $\varepsilon_{\mu\nu\rho\sigma}a^\mu b^\nu c^\rho d^\sigma$, with $\varepsilon_{0123} = +1$. In the standard model, one finds, for the constants D and E,

$$D_V = |v_e v_t d - e_t|^2 + |a_e v_t d|^2 ,$$
$$D_A = |v_e a_t d|^2 + |a_e a_t d|^2 ,$$
$$D_{VA} = v_e a_t d(v_e v_t d - e_t)^* + a_e a_t d(a_e v_t d)^* ,$$
$$E_V = 2\,\mathrm{Re}[(v_e v_t d - e_t)(a_e v_t d)^*] ,$$
$$E_A = 2\,\mathrm{Re}[v_e a_t d(a_e a_t d)^*] ,$$
$$E_{VA} = v_e a_t d(a_e v_t d)^* + a_e a_t d(v_e v_t d - e_t)^* . \qquad \text{(B4)}$$

The CP-violating part of the differential cross section is

$$\frac{d\sigma}{d\Omega_t}(s_+, s_-)|_{CP} = \frac{3\beta\alpha^2}{8s^3}(\mathrm{Im}F_1\{[(lQ)^2 + 4sm_t^2]P(s_+ + s_-) - lQsl(s_+ - s_-)\}$$
$$+\tfrac{1}{2}\mathrm{Re}F_1[3m_t lQ\varepsilon(s_+,s_-,l,P) + 3sm_t\varepsilon(s_+,s_-,Q,P)$$
$$-m_t ls_-\varepsilon(s_+,l,Q,P) + m_t ls_+\varepsilon(s_-,l,Q,P)]$$
$$+2\,\mathrm{Im}F_2 sm_t(Ps_-\cdot ls_+ + Ps_+\cdot ls_-) + \mathrm{Re}F_2 s[\varepsilon(s_-,l,Q,P) - \varepsilon(s_+,l,Q,P)]$$
$$-2\,\mathrm{Im}F_3 m_t lQ(Ps_-\cdot ls_+ + Ps_+\cdot ls_-) - \mathrm{Re}F_3 lQ[\varepsilon(s_-,l,Q,P) - \varepsilon(s_+,l,Q,P)]$$
$$-\mathrm{Im}F_4[lQsP(s_+ + s_-) - (s^2 - 4sm_t^2)l(s_+ - s_-)]$$
$$-2\,\mathrm{Re}F_4 m_t[Ps_-\varepsilon(s_+,l,Q,P) + Ps_+\varepsilon(s_-,l,Q,P)]) , \qquad \text{(B5)}$$

with

$$
\begin{aligned}
F_1 &= (v_e v_t d - e_t)^* \left(\frac{d_\gamma}{e} - 4 \sin\theta_W \cos\theta_W v_e d \frac{d_Z}{e} \right) - (a_e v_t d)^* 4 \sin\theta_W \cos\theta_W a_e d \frac{d_Z}{e} \ , \\
F_2 &= (a_e v_t d)^* \left(\frac{d_\gamma}{e} - 4 \sin\theta_W \cos\theta_W v_e d \frac{d_Z}{e} \right) - (v_e v_t d - e_t)^* 4 \sin\theta_W \cos\theta_W a_e d \frac{d_Z}{e} \ , \\
F_3 &= (v_e a_t d)^* \left(\frac{d_\gamma}{e} - 4 \sin\theta_W \cos\theta_W v_e d \frac{d_Z}{e} \right) - (a_e a_t d)^* 4 \sin\theta_W \cos\theta_W a_e d \frac{d_Z}{e} \ , \\
F_4 &= (a_e a_t d)^* \left(\frac{d_\gamma}{e} - 4 \sin\theta_W \cos\theta_W v_e d \frac{d_Z}{e} \right) - (v_e a_t d)^* 4 \sin\theta_W \cos\theta_W a_e d \frac{d_Z}{e} \ .
\end{aligned} \tag{B6}
$$

[1] G. L. Kane, G. A. Ladinsky, and C.-P. Yuan, Phys. Rev. D **45**, 124 (1992); C.-P. Yuan, *ibid.* **45**, 782 (1992); C. A. Nelson, *ibid.* **41**, 2805 (1990).

[2] R. H. Dalitz and G. R. Goldstein, Phys. Rev. D **45**, 1531 (1992); Int. J. Mod. Phys. A **9**, 635 (1994), and references therein.

[3] T. Arens and L. M. Sehgal, Nucl. Phys. **B393**, 46 (1993).

[4] S. Kawasaki, T. Shirafuji, and S. Y. Tsai, Prog. Theor. Phys. **49**, 1656 (1973); S. Y. Tsai, Phys. Rev. D **4**, 2821 (1971); see also A. Pais and S. B. Treiman, *ibid.* **14**, 293 (1976).

[5] J. H. Kühn, A. Reiter, and P. M. Zerwas, Nucl. Phys. **B272**, 560 (1986).

[6] M. Ježabek and J. H. Kühn, Nucl. Phys. **B320**, 20 (1989).

[7] Direct evidence for top production, recently reported, gives $m_t = 174 \pm 10^{+13}_{-12}$ GeV [CDF Collaboration, F. Abe *et al.*, Phys. Rev. D **50**, 2966 (1994)]; indirect limits from the CERN e^+e^- collider LEP analysis of data yield $m_t = 162^{+16+18}_{-17-21}$ GeV [J. Lefrançqis, in *Proceedings of the International Europhysics Conference on High Energy Physics*, Marseille, France, 1993, edited by J. Carr and M. Perottet (Editions Frontieres, Gif-sur-Yvette, 1993)].

[8] D. Chang, W.-Y. Keung, and I. Phillips, Nucl. Phys. **B408**, 286 (1993).

[9] S. Matsumoto, K. Tominaga, O. Terazawa, and M. Biyajima, Prog. Theor. Phys. **85**, 631 (1991).

[10] L. H. Orr and J. L. Rosner, Phys. Lett. B **246**, 221 (1990); **248**, 474 (1990); I. Bigi, Y. Dokshitzer, V. Khoze, J. Kühn, and P. Zerwas, *ibid.* **181**, 157 (1986).

[11] C. R. Schmidt and M. E. Peskin, Phys. Rev. Lett. **69**, 410 (1992).

[12] W. Bernreuther, T. N. Pham, and T. Schröder, Phys. Lett. B **279**, 389 (1992).