Higher order $1/m$ corrections at zero recoil

Thomas Mannel
Theory Division, CERN, CH-1211 Geneva 23, Switzerland
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The general structure of the $1/m$ corrections at zero recoil is studied. The relevant matrix elements are forward matrix elements of local higher dimensional operators and their time ordered products with higher order terms from the Lagrangian. These matrix elements may be classified in a simple way and the analysis at the nonrecoil point for the form factor of heavy quark currents simplifies drastically. The second-order recoil corrections to the form factor h_{A1} of the axial vector current, relevant for the $|V_{cb}|$ determination from $B \to D^*$ decays, are estimated to be $-5\% < h_{A1} - 1 < 0$.

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I. INTRODUCTION

Heavy quark effective theory is by now the standard description for systems with one heavy quark [1–9]. The additional symmetries appearing in the limit of infinite heavy quark mass yield model independent relations between form factors appearing in the description of heavy hadron exclusive weak decays. Aside from that, heavy quark symmetries also yield statements about the normalization of some form factors at zero recoil, i.e., the point where the velocities of the initial and final hadrons are equal. This fact has a very large phenomenological impact; e.g., it allows one to perform a model independent determination of $|V_{cb}|$ by extrapolating to the end point of the lepton spectrum in the decay $B \to D^* \ell \nu$.

QCD radiative corrections as well as recoil corrections have been studied already to next-to-leading order [10]. While QCD radiative corrections may be studied systematically, the recoil corrections in general need new, nonperturbative input, which may be supplied, for instance, by model estimates. For the case of the determination of $|V_{cb}|$ from $B \to D^* \ell \nu$, the leading recoil corrections vanish at the nonrecoil point due to Lukes theorem [11] and the next-to-leading ones have been considered by Falk and Neubert [12], who parametrized the form factors to order $1/m_Q^2$, also off the point of equal velocities.

However, as will be discussed below, the analysis of the $1/m_Q$ corrections at zero recoil simplifies enormously, since then only forward matrix elements (matrix elements of operators between mesons moving with the same velocity) appear. In addition, the algebra of Dirac matrices simplifies and one may obtain a simple expression for the next-to-leading recoil corrections at the point of equal velocity. This expression involves forward matrix elements of operators of higher dimension and also time ordered products with higher order recoil terms from the Lagrangian. The expressions we obtain have a simple interpretation, but their numerical evaluation needs input beyond heavy quark effective theory.

Recently, the methods of the heavy mass expansion have been applied also to inclusive decays by combining the method of operator product expansion with heavy quark effective theory [13–17]. This approach yields the heavy mass expansion for decay rates and also for decay distributions; the leading term in this expansion is the free quark decay rate and the corrections may be studied systematically. Of course, the higher order corrections need nonperturbative input, which is again forward matrix elements of higher dimensional operators and time ordered products of such operators with higher order terms from the Lagrangian. Thus the same matrix elements appear as in the model independent determination of V_{cb}.

Finally, the relation of the heavy hadron mass to the mass of the heavy quark is also given in terms of a $1/m_Q$ expansion. Higher orders are again given by forward matrix elements of higher dimensional operators needed as nonperturbative input to relate the heavy quark mass with the heavy hadron mass.

In the present paper, a systematic study is performed for these forward matrix elements appearing in all higher order calculations at zero recoil, including the relevant time ordered products with higher order terms of the Lagrangian. It turns out that all the forward matrix elements may be classified very simply and the relevant matrix elements for calculations up to order $1/m_Q^3$ are given explicitly.

The classification performed here allows one to simplify the analysis of the recoil corrections to heavy quark weak decay form factors at $v = v'$ enormously, compared to the case where $v \neq v'$. As an application, the analysis for weak decay form factors is performed at the nonrecoil point up to second order in the heavy mass expansion for the case of $b \to c$ transitions and our results are compared with the ones obtained by Falk and Neubert [12].

In the next section, a general discussion of the parametrization of the generic forward matrix element is given. It is split into three subsections. First, we consider local higher dimensional operators in some detail and give numerical estimates for the matrix elements of operators up to dimension seven. In the second subsection, we shall consider time ordered products with higher order terms from the Lagrangian. Finally, in the third subsection we consider the relation between the mass of the heavy meson and the mass of the heavy quark as a toy example, where the forward matrix elements play a role.

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The general discussion of the forward matrix elements is then applied in Sec. III to the $1/m_Q^2$ corrections of the normalization of the weak decay form factors in the decays $B \to D\ell\nu$ and $B \to D^*\ell\nu$. The relevant form factors h_+ for $B \to D\ell\nu$ and h_{A1} for $B \to D^*\ell\nu$ decays are discussed at the nonrecoil point up to second order in the heavy mass expansion. We include also a semiquantitative analysis and estimate the size of the corrections relevant for the V_{cb} determination to order $1/m_Q^2$.

II. HIGHER DIMENSIONAL OPERATORS AND THEIR FORWARD MATRIX ELEMENTS

Higher order terms in the heavy mass expansion of weak transition matrix elements originate in general from two sources. The first source is the heavy mass expansion of the operators for a heavy quark Q appearing in the weak transition Hamiltonian. At the matching scale m_Q, this amounts to the replacement [9]

$$Q(x) = e^{-im_Q(vx)} \left(1 + \left[\sum_{k=0}^{\infty} \left(\frac{-ivD}{2m_Q}\right)^k\right] \frac{i\not{D}^\perp}{2m_Q}\right) Q_v(x) \ , \quad (1)$$

where v is the velocity of the heavy hadron and Q_v is the operator of a static heavy quark moving with velocity v. Furthermore, it is convenient to define

$$D_\mu^\perp = (g_{\mu\nu} - v_\mu v_\nu)D^\nu, \quad vD^\perp = 0. \quad (2)$$

These terms in general lead to contributions, which are matrix elements of local operators.

Second, the Lagrangian of full QCD is also expanded in $1/m_Q$ and the higher orders in $1/m_Q$ are treated as perturbations. This leads to time ordered products involving these higher order terms of the Lagrangian and the weak transition operator. The corrections of higher orders in the $1/m_Q$ expansion to the Lagrangian are given at tree level by [9]

$$\mathcal{L}_I = \sum_{j=1}^{\infty} \mathcal{L}_I^{(j)} = \sum_{j=1}^{\infty} \left(\frac{1}{2m_Q}\right)^j \bar{Q}_v(-i\not{D}^\perp)(ivD)^{j-1} \times (i\not{D}^\perp)Q_v. \quad (3)$$

In every order j it is convenient to split $\mathcal{L}_I^{(j)}$ into a generalized kinetic energy operator $\mathcal{K}^{(j)}$ and chromomagnetic moment operator $\mathcal{G}^{(j)}$ defined as

$$\mathcal{K}^{(j)} = \bar{Q}_v(iD_\alpha^\perp)(-ivD)^{j-1}(iD^{\perp\alpha})Q_v \ , \quad (4)$$

$$\mathcal{G}^{(j)} = -i\bar{Q}_v(iD_\alpha^\perp)(-ivD)^{j-1}(iD_\beta^\perp)\sigma^{\alpha\beta}Q_v. \quad (5)$$

The interpretation of these time ordered product terms is obvious. The heavy hadron states of full QCD still depend on the heavy quark mass, and this dependence is also treated in a $1/m_Q$ expansion. The leading term is the state taken in the infinite mass limit, the "static state," which is the convenient one for a heavy quark effective theory calculation, since it does not depend on the heavy mass anymore. The time ordered products account for correct mass dependence of the full QCD state, and the matrix elements then have to be evaluated using the "static," mass independent state.

A. Local operators of higher dimension

The generic operator of dimension $n+3$ appearing in the contexts mentioned above is of the form

$$\mathcal{O}^{(\Gamma)}_{\mu_1,\mu_2,\ldots,\mu_n} = \bar{Q}_v(iD_{\mu_1})(iD_{\mu_2})\cdots(iD_{\mu_n})\Gamma Q_v \ , \quad (6)$$

where Γ is an arbitrary Dirac matrix.

The Dirac matrix appearing in (6) may be expanded into the 16 basis Dirac matrices 1, γ_5, γ_μ, $\gamma_5\gamma_\mu$, and $\sigma_{\mu\nu}$. However, the matrix Γ is sandwiched between projectors

$$P_+ = \frac{1}{2}(1+\not{v}) \ ,$$

which are contained in the heavy quark fields Q_v. This projection amounts to the replacements

$$1 \longrightarrow P_+ = \frac{1}{2}(1+\not{v}) \ , \quad \gamma_\mu \longrightarrow P_+\gamma_\mu P_+ = v_\mu P_+ \ , \quad (7)$$

$$\gamma_\mu\gamma_5 \longrightarrow P_+\gamma_\mu\gamma_5 P_+ = s_\mu \ , \quad \gamma_5 \longrightarrow P_+\gamma_5 P_+ = 0 \ , \quad (8)$$

$$(-i)\sigma_{\mu\nu} \longrightarrow P_+(-i)\sigma_{\mu\nu}P_+ = iv^\alpha\epsilon_{\alpha\mu\nu\beta}s^\beta \ , \quad (9)$$

where we have defined the spin matrices s_μ, which are the generalizations of the Pauli matrices for the frame moving with velocity v. They satisfy the relations

$$s_\mu s_\nu = (-g_{\mu\nu} + v_\mu v_\nu)P_+ + i\epsilon_{\alpha\mu\nu\beta}v^\alpha s^\beta \ , \quad v\cdot s = 0 \ . \quad (10)$$

Consequently, the Dirac matrix Γ sandwiched between the projectors may be expanded into the four matrices 1 and s_μ

$$P_+\Gamma P_+ = \frac{1}{2}P_+\mathrm{Tr}\left\{P_+\Gamma\right\} - \frac{1}{2}s_\mu\mathrm{Tr}\left\{s^\mu\Gamma\right\}, \quad (11)$$

and it is sufficient to consider only the two operators

$$\mathcal{O}^{(1)}_{\mu_1,\mu_2,\ldots,\mu_n} = \bar{Q}_v(iD_{\mu_1})(iD_{\mu_2})\cdots(iD_{\mu_n})Q_v \quad (12)$$

$$\mathcal{O}^{(s)}_{\mu_1,\mu_2,\ldots,\mu_n;\lambda} = \bar{Q}_v(iD_{\mu 1})(iD_{\mu 2})\cdots(iD_{\mu n})s_\lambda Q_v. \quad (13)$$

In the following, we shall consider the matrix elements between the ground state pseudoscalar and vector mesons. There are two different cases to be studied. In the first case the initial and the final state are both either 0^- or 1^-; in the second case the initial state is 0^- and the final state is 1^- or vice versa. All these different cases are related by heavy quark spin symmetry, which implies the relations

$$Q_v|H(v)\rangle = \gamma_5\not{\epsilon}Q_v|H^*(v,\epsilon)\rangle \ ,$$
$$Q_v|H^*(v,\epsilon)\rangle = \gamma_5\not{\epsilon}Q_v|H(v)\rangle \ , \quad (14)$$

where $|H(v)\rangle$ and $|H^*(v,\epsilon)\rangle$ denotes the 0^- and 1^- ground state meson, respectively.

Further restrictions on the structure of the matrix elements may be obtained from the equations of motion for the heavy quark and the gluons

$$ivDQ_v = 0 \ , \quad [(iD^\mu),[(iD_\mu),(iD_\nu)]]_{ab} = 4\pi\alpha_s \sum_q \left[\bar{q}_b\gamma_\mu q_a - \frac{1}{N_c}\delta_{ab}\,\bar{q}_d\gamma_\mu q_d\right] \ , \tag{15}$$

where the sum runs over all quark flavors and a, b, d are color indices.

We shall start the discussion with the case where both initial and final states are either 0^- or 1^-. Spin symmetry relates the matrix elements of 0^- mesons with the ones of the 1^- case in the following way

$$\langle H(v)|\mathcal{O}^{(1)}_{\mu_1,\mu_2,\ldots,\mu_n}|H(v)\rangle = \langle H^*(v,\epsilon)|\mathcal{O}^{(1)}_{\mu_1,\mu_2,\ldots,\mu_n}|H^*(v,\epsilon)\rangle \ , \tag{16}$$

$$\langle H(v)|\mathcal{O}^{(s)}_{\mu_1,\mu_2,\ldots,\mu_n;\lambda}|H(v)\rangle = -\frac{1}{3}\langle H^*(v,\epsilon)|\mathcal{O}^{(s)}_{\mu_1,\mu_2,\ldots,\mu_n;\lambda}|H^*(v,\epsilon)\rangle, \tag{17}$$

and we shall consider in the following only the 0^- case.

The equations of motions for the heavy quark imply that the matrix elements of both $\mathcal{O}^{(1)}$ and $\mathcal{O}^{(s)}$ have to vanish, if the first or the last index, i.e., μ_1 or μ_n, is contracted with the velocity v. Furthermore, the spin vector s is also orthogonal to the velocity and thus the matrix elements have to satisfy

$$\begin{aligned} v^{\mu_1}\langle H(v)|\mathcal{O}^{(1)}_{\mu_1,\mu_2,\ldots,\mu_n}|H(v)\rangle &= v^{\mu_n}\langle H(v)|\mathcal{O}^{(1)}_{\mu_1,\mu_2,\ldots,\mu_n}|H(v)\rangle = 0 \ , \\ v^{\mu_1}\langle H(v)|\mathcal{O}^{(s)}_{\mu_1,\mu_2,\ldots,\mu_n;\lambda}|H(v)\rangle &= v^{\mu_n}\langle H(v)|\mathcal{O}^{(s)}_{\mu_1,\mu_2,\ldots,\mu_n;\lambda}|H(v)\rangle = 0 \ , \\ v^{\lambda}\langle H(v)|\mathcal{O}^{(s)}_{\mu_1,\mu_2,\ldots,\mu_n;\lambda}|H(v)\rangle &= 0. \end{aligned} \tag{18}$$

Note that contractions with any other index may be related to a gluon field strength $[(iD_\mu),(iD_\nu)] = igG_{\mu\nu}$, e.g.,

$$(ivD)(iD_{\mu_n})Q_v|H(v)\rangle = -ig\,v^\alpha G_{\alpha\mu_n}Q_v|H(v)\rangle, \tag{19}$$

and are thus in general nonzero.

Combining the information from the spin structure and the restrictions from the equation of motion of the heavy quark, one obtains for the forward matrix element of $\mathcal{O}^{(1)}$ the general expression

$$\langle H(v)|\bar{Q}_v(iD_\alpha)(iD_{\nu_1})\cdots(iD_{\nu_{n-2}})(iD_\beta)Q_v|H(v)\rangle = 2M_H[g_{\alpha\beta} - v_\alpha v_\beta]A_{\nu_1\cdots\nu_{n-2}}. \tag{20}$$

The tensor A is constructed from $g_{\mu_i\mu_j}$ and v_{μ_i}. It is a simple combinatorical exercise to show that the number N of independent scalar parameters is

$$N(n) = 1 + (n-2)! \sum_{k=1}^{[n]/2-1} \left(\frac{1}{2}\right)^k \frac{1}{[n-2(k+1)]!} \ , \tag{21}$$

where $n > 2$ and $[n] = n$ for n even and $[n] = n-1$ for n odd. The number of independent parameters grows rapidly, the first few are $N(4) = 2$, $N(5) = 4$, $N(6) = 13$, $N(7) = 41$, and $N(8) = 196$.

The matrix elements of $\mathcal{O}^{(s)}$ are parity odd quantities. The general form of these matrix elements, which is compatible with the restrictions (18), is given by

$$\begin{aligned} \langle H(v)|\mathcal{O}_{\alpha\mu_1\cdots\mu_{n-2}\beta;\lambda}|H(v)\rangle = {} & 2M_H d_H i\varepsilon_{\nu\alpha\beta\lambda}v^\nu B_{\nu_1\cdots\nu_{n-2}} + 2M_H d_H[g_{\alpha\beta} - v_\alpha v_\beta]C^{(1)}_{\nu_1\cdots\nu_{n-2};\lambda} \\ & + 2M_H d_H[g_{\alpha\lambda} - v_\alpha v_\lambda]C^{(2)}_{\nu_1\cdots\nu_{n-2};\beta} + 2M_H d_H[g_{\beta\lambda} - v_\beta v_\lambda]C^{(3)}_{\nu_1\cdots\nu_{n-2};\alpha} \ , \end{aligned} \tag{22}$$

where $d_H = 3$ for a pseudoscalar meson and $d_H = -1$ for a vector meson. The tensors $C^{(k)}$ are parity odd and vanish, if the last index is contracted with v.

Up to dimension seven the number of parameters is still manageable, and some of them are more or less well known numerically. The only nonvanishing matrix element between heavy meson states of the dimension three operators is

$$\langle H(v)|\bar{Q}_v Q_v|H(v)\rangle = 2M_H \tag{23}$$

and its value is given by the choice of the normalization. Here M_H is the mass of the heavy meson in the static limit.

All matrix elements of the dimension four operators vanish due to the equations of motion; all matrix elements of the dimension five operators are given in terms of two parameters λ_1 and λ_2

$$\langle H(v)|\bar{Q}_v(iD_\alpha)(iD_\beta)Q_v|H(v)\rangle = 2M_H[g_{\alpha\beta} - v_\alpha v_\beta]\frac{1}{3}\lambda_1 \ , \tag{24}$$

$$\langle H(v)|\bar{Q}_v(iD_\alpha)(iD_\beta)s_\lambda Q_v|H(v)\rangle = 2M_H d_H i\varepsilon_{\nu\alpha\beta\lambda}v^\nu\frac{1}{6}\lambda_2 \ , \tag{25}$$

where the prefactors are chosen to comply with the definition in [12]. The parameter λ_2 corresponds to the leading term in $1/m_Q$ for the mass splitting between the ground state 1^- and 0^- mesons [18], while the kinetic energy parameter λ_1 is not related in an easy way to a measurable quantity. From QCD sum rule analyses, one obtains values of $\lambda_1 = -0.54 \pm 0.12$ GeV^2 [19], but these calculations have been criticized recently and a much lower value of λ_1 has been suggested using an improved sum rule technique [20]. On the other hand, bounds have been derived in a quantum mechanical framework indicating that $\lambda_1 < -0.18$ GeV^2 [21]. In the numerical studies presented below, we shall vary λ_1 in some range and hence we shall use the values

$$-0.3 \text{ GeV}^2 < \lambda_1 < -0.1 \text{ GeV}^2 \ , \quad \lambda_2 = 0.12 \text{ GeV}^2. \tag{26}$$

The parameter λ_2 is scale dependent and we define $\lambda_2 = \lambda_2(m_b)$.

The matrix elements of the dimension six operators are also given in terms of only two parameters ρ_1 and ρ_2

$$\langle H(v)|\bar{Q}_v(iD_\alpha)(iD_\mu)(iD_\beta)h_v|H(v)\rangle = 2M_H[g_{\alpha\beta} - v_\alpha v_\beta]v_\mu \frac{1}{3}\rho_1 \ , \tag{27}$$

$$\langle H(v)|\bar{Q}_v(iD_\alpha)(iD_\mu)(iD_\beta)s_\lambda Q_v|H(v)\rangle = 2M_H d_H i\varepsilon_{\nu\alpha\beta\lambda}v^\nu v_\mu \frac{1}{6}\rho_2. \tag{28}$$

In order to estimate ρ_1, we may employ the equations of motion for the gluon fields and relate this parameter to a forward matrix element of a four-fermion operator:

$$-4M_H\rho_1 = 4\pi\alpha_s \sum_q \left\langle H(v) \middle| \left[(\bar{Q}_{v,a}Q_{v,b})(\bar{q}_b \not{v} q_a) - \frac{1}{N_c}(\bar{Q}_{v,a}Q_{v,b})(\bar{q}_b \not{v} q_b)\right] \middle| H(v) \right\rangle , \tag{29}$$

where a and b are color indices. Using the Fierz theorem we rearrange the quark fields in order to apply vacuum insertion, after which one is left with matrix elements of heavy light operators between the heavy meson and vacuum. These matrix elements are all related to the heavy meson decay constant f_H due to heavy quark spin symmetry. The estimate for the parameter ρ_1 reads under these assumptions

$$\rho_1 = \frac{1}{2}\pi\alpha_s \frac{N_c^2 - 1}{N_c^2} f_H^2 M_H. \tag{30}$$

A similar estimate has been performed in [21].

However, (30) has the usual problem of a matrix element after factorization. The original matrix element defining ρ_1 is expected to have a different behavior under renormalization group transformations as the result after factorization. In other words, one has to define at which scale factorization is performed. We shall factorize the matrix elements at the scale m_b and thus use the following set of parameters: $\alpha_s = \alpha_s(m_b) = 0.2$ and $M_H = 5.28$ GeV. Varying the heavy meson decay constant between 150 and 200 MeV, we obtain

$$(\rho_1)^{1/3} = (300 - 450) \text{ MeV}. \tag{31}$$

This number is of the same size as $\lambda_2^{1/2} = 350$ MeV.

Finally, the forward matrix elements of the dimension seven operators $\mathcal{O}^{(1)}$ may be written in terms of two parameters η and τ:

$$\langle H(v)|\mathcal{O}^{(1)}_{\alpha\mu_1\mu_2\beta}|H(v)\rangle = 2M_H \frac{1}{3}[g_{\alpha\beta} - v_\alpha v_\beta](g_{\mu_1\mu_2}\eta - v_{\mu_1}v_{\mu_2}\tau) \tag{32}$$

while the general form of the dimension seven operator $\mathcal{O}^{(s)}$ is more complicated:

$$\begin{aligned}\langle H(v)|\mathcal{O}^{(s)}_{\alpha\mu_1\mu_2\beta;\lambda}|H(v)\rangle = & -2M_H d_H i\varepsilon_{\alpha\beta\lambda\nu}v^\nu(g_{\mu_1\mu_2}B_1 - v_{\mu_1}v_{\mu_2}B_2) + 2M_H d_H C^{(1)}[g_{\alpha\beta} - v_\alpha v_\beta]\varepsilon_{\rho\mu_1\mu_2\lambda} \\ & +2M_H d_H C^{(2)}[g_{\alpha\lambda} - v_\alpha v_\lambda]\varepsilon_{\rho\mu_1\mu_2\beta} + 2M_H d_H C^{(3)}[g_{\lambda\beta} - v_\lambda v_\beta]\varepsilon_{\rho\mu_1\mu_2\alpha} \ .\end{aligned} \tag{33}$$

One may apply again the equations of motion for the gluon field to relate this to a matrix element involving the light quark current. In this way one obtains a relation of the form

$$2M_H(4\eta + \tau)(g_{\alpha\beta} - v_\alpha v_\beta) = -4\pi\alpha_s \sum_q \left\langle H(v) \middle| \left[[(iD_\alpha \bar{Q}_{v,a})Q_{v,b}](\bar{q}_b\gamma_\beta q_a) - \frac{1}{N_c}[(iD_\alpha \bar{Q}_{v,a})Q_{v,a}](\bar{q}_b\gamma_\nu q_b)\right] \middle| H(v) \right\rangle . \tag{34}$$

This may again be estimated by using the Fierz theorem and vacuum insertion. After factorization, using

$$\langle H(v)|(iD_\alpha \bar{Q}_v)\gamma_\mu\gamma_5 q|0\rangle = 3\bar{\Lambda} f_H M_H[g_{\alpha\mu} - v_\alpha v_\mu] \ , \tag{35}$$

$$\bar{\Lambda} = M_H - m_Q,$$

one obtains

$$4\eta + \tau = 6\pi\alpha_s \frac{N_c^2 - 1}{N_c^2}\bar{\Lambda} f_H^2 M_H \ . \tag{36}$$

With the same set of parameters and under the same assumptions as above, one obtains the estimate

$$(4\eta + \tau)^{1/4} = (700\text{–}950) \text{ MeV}, \tag{37}$$

where $\bar{\Lambda}$ has been varied between 400 and 600 MeV.

From these estimates it seems that the heavy mass expansion works quite well, at least at the nonrecoil point. All the parameters up to dimension seven behave like the appropriate power of some small scale $\Lambda \sim 200$–500 MeV, which means that the expansion in powers of Λ/m_Q indeed has coefficients of order unity.

The second case to be studied are matrix elements with a 0^- meson in the initial state and a 1^- in the final state, or vice versa. These matrix elements do not introduce any new parameters, since they are related to the ones considered above by heavy quark spin symmetry. However, one should be a little more careful in this case, because one has to rotate the spin of only one of the heavy quarks. The forward matrix elements, which are considered here, involve only one velocity sector of heavy quark effective theory, and spin symmetry is a symmetry holding separately in each velocity sector. In order to rotate only the initial or the final state heavy quark spin, one has to choose in a first step two different velocities for initial and final states, perform the spin rotation of one of the states using (16) in the corresponding velocity sector, and afterwards take the limit $v' \to v$. In this way one obtains the relations

$$\langle H(v)|\bar{Q}_v(iD_{\mu_1})\cdots(iD_{\mu_n})Q_v|H(v)\rangle = -\langle H(v)|\bar{Q}_v(iD_{\mu_1})\cdots(iD_{\mu_n})(s\epsilon)Q_v|H^*(v,\epsilon)\rangle \,, \tag{38}$$

$$\langle H(v)|\bar{Q}_v(iD_{\mu_1})\cdots(iD_{\mu_n})Q_v|H^*(v,\epsilon)\rangle = -\langle H(v)|\bar{Q}_v(iD_{\mu_1})\cdots(iD_{\mu_n})(s\epsilon)Q_v|H(v)\rangle, \tag{39}$$

relating the matrix elements of $\mathcal{O}^{(1)}$ between two 0^- or two 1^- states to the ones of $\mathcal{O}^{(s)}$ between 0^- and a 1^- state, and vice versa.

B. The time ordered products with the Lagrangian

The second type of matrix element appearing in an analysis of higher order $1/m_Q$ corrections at zero recoil are time ordered products of the local operators discussed above and the terms appearing in the heavy mass expansion of the Lagrangian.

We shall consider first the case of two different flavors q_v and Q_v. The simplest terms are the two-point matrix elements

$$(-i)\int d^4x\,\langle H_q(v)|T\left[\bar{q}_v(iD_{\mu_1})\cdots(iD_{\mu_n})Q_v\,\mathcal{K}_Q^{(j)}(x)\right]\times|H_Q(v)\rangle\,, \tag{40}$$

$$(-i)\int d^4x\,\langle H_q(v)|T\left[\bar{q}_v(iD_{\mu_1})\cdots(iD_{\mu_n})Q_v\,\mathcal{G}_Q^{(j)}(x)\right]\times|H_Q(v)\rangle\,, \tag{41}$$

where the operators without argument have to be taken at $x = 0$. $\mathcal{K}_Q$ and $\mathcal{G}_Q$ are the kinetic and chromomagnetic terms for the quark Q as defined above.

The spin structure of the simplest two-point matrix elements may be analyzed in the trace formalism

$$\begin{aligned}(-i)\int d^4x\,\langle H_q(v)|T\left[\bar{q}_v\Gamma Q_v\,\mathcal{K}_Q^{(j)}(x)\right]|H_Q(v)\rangle &= -A\,\mathrm{Tr}\left\{\bar{M}(v)\Gamma M(v)\right\}\,,\\ (-i)\int d^4x\,\langle H_q(v)|T\left[\bar{q}_v\Gamma Q_v\,\mathcal{G}_Q^{(j)}(x)\right]|H_Q(v)\rangle &= \frac{1}{2}B\,\mathrm{Tr}\left\{(-i)\sigma_{\alpha\beta}\bar{M}(v)\Gamma P_+(-i)\sigma^{\alpha\beta}M(v)\right\}\,,\end{aligned} \tag{42}$$

where Γ is a general Dirac matrix, which is a linear combination of 1 and s_μ, and $M(v)$ are the usual representation matrices for the heavy ground state mesons

$$M(v) = \frac{1}{2}\sqrt{M_H}\begin{cases}(\not{v}+1)\gamma_5 & \text{pseudoscalar meson}\,,\\ -(\not{v}+1)\not{\epsilon} & \text{vector meson, polarization } \epsilon,\end{cases} \tag{43}$$

where the normalization is chosen according to (23).

The matrix $\sigma_{\alpha\beta}$ in the expression for the chromomagnetic moment operator appears only between projection operators P_+, and it is convenient to switch to a representation using the Pauli matrices (9). In this representation, one has

$$\mathcal{G}^{(j)} = iv_\mu\epsilon^{\mu\alpha\beta\lambda}\bar{Q}_v(iD_\alpha)(ivD)^j(iD_\beta)s_\lambda Q_v \tag{44}$$

and we write for the second equation of (42)

$$(-i)\int d^4x\,\langle H_q(v)|T\left[\bar{q}_v\Gamma Q_v\,\mathcal{G}_Q^{(j)}(x)\right]|H_Q(v)\rangle = -B\,\mathrm{Tr}\left\{\gamma_\lambda\gamma_5\bar{M}(v)\Gamma s^\lambda M(v)\right\}\,.$$

The representation in terms of the Pauli matrices is very useful, as soon as more than one insertion of a chromomagnetic moment operator appears, since the spin structure of products of chromomagnetic operators correspond to products of the spin matrices s which may be reduced using the relation (10). For example, the product of two chromomagnetic moment operator insertion may be written as

$$\begin{aligned}(-i)^2\int d^4x\,d^4y\langle H_q(v)|T\left[\bar{q}_vQ_v\,\mathcal{G}_Q^{(j)}(x)\mathcal{G}_Q^{(j)}(y)\right]|H_Q(v)\rangle &= -\mathrm{Tr}\left\{\mathcal{T}_{\alpha\mu}\bar{M}(v)s^\alpha s^\mu M(v)\right\}\\ &= -\mathrm{Tr}\left\{\mathcal{T}^{\alpha\mu}(-g_{\alpha\mu}+v_\alpha v_\mu)\bar{M}(v)M(v)\right\}\\ &\quad -\mathrm{Tr}\left\{\mathcal{T}^{\alpha\mu}i\epsilon_{\rho\alpha\mu\nu}v^\rho\bar{M}(v)s^\nu M(v)\right\}\,,\end{aligned} \tag{45}$$

where $\mathcal{T}$ parametrizes the light degrees of freedom

$$\mathcal{T}_{\alpha\beta} = \frac{1}{3}T^{(1)}(v_\alpha v_\beta - g_{\alpha\beta}) + \frac{i}{2}T^{(2)}\epsilon_{\mu\alpha\beta\lambda}v^\mu\gamma^\lambda\gamma_5, \tag{46}$$

and one obtains

$$(-i)^2\int d^4x\,d^4y\,\langle H_q(v)|T\left[\bar{q}_vQ_v\,\mathcal{G}^{(j)}(x)\mathcal{G}_Q^{(j)}(y)\right]|H_Q(v)\rangle = 2M_H(T^{(1)}+d_HT^{(2)}).$$

In this way, one may easily identify the spin symmetry conserving and spin symmetry violating contributions of such products.

The equations of motion also imply restrictions on the matrix elements of time ordered products [16, 22]. In principle, one obtains the same relations as for the local terms, for example

$$\langle H_q(v)|T\left[\bar{q}_v(ivD)Q_v\,\mathcal{K}_Q^{(j)}(x)\right]|H_Q(v)\rangle = 0\ .$$

However, there may be an ambiguity depending whether the derivative acts on the T symbol or not. If one also takes the derivative of the step functions coming from the T symbol, then one obtains a local contribution of the form

$$\langle H_q(v)|T\left[\bar{q}_v(iD_\mu)Q_v\,\mathcal{K}^{(j)}(x)\right]|H_Q(v)\rangle$$

$$\sim i\delta^4(x)v_\mu\langle H_q(v)|\bar{q}_v(iD^\perp_\alpha)(ivD)^j(iD^{\perp\alpha})Q_v|H_Q(v)\rangle.$$

which may in general be reabsorbed into a redefinition of the T product. However, in the applications discussed below only the perpendicular components of the derivatives defined in (2) enter the expressions as

$$\langle H_q(v)|T\left[\bar{q}_v(i\not{D}^\perp)Q_v\,\mathcal{K}_Q^{(j)}(x)\right]|H_Q(v)\rangle = 0,$$

and hence there will be no contribution from such terms.

Finally, the flavor diagonal case may be discussed by inserting first the correction terms for the Lagrangian of the quark q

$$\mathcal{L}^{(j)} = \left(\frac{1}{2m_Q}\right)^j\left(\mathcal{K}_Q^{(j)} + \mathcal{G}_Q^{(j)}\right) + \left(\frac{1}{2m_q}\right)^j\left(\mathcal{K}_q^{(j)} + \mathcal{G}_q^{(j)}\right)$$

and then consider the case $q = Q$. In this case one has insertions in both lines, the one corresponding to q and to Q. When the masses are equal, both insertions are parametrized by the same form factor. However, the spin structure is different; in particular, the insertion of the chromomagnetic moment operator yields a Pauli matrix s to the right of Γ for Q , while s occurs to the left of Γ for q. Thus one obtains for the examples studied above

$$(-i)\int d^4x\,\langle H_Q(v)|T\left[\bar{Q}_v\Gamma Q_v\,\mathcal{K}^{(j)}(x)\right]|H_Q(v)\rangle$$

$$= -2A\,\mathrm{Tr}\left\{\bar{M}(v)\Gamma M(v)\right\}\ , \quad (47)$$

$$(-i)\int d^4x\,\langle H_Q(v)|T\left[\bar{Q}_v\Gamma Q_v\,\mathcal{G}^{(j)}(x)\right]|H_Q(v)\rangle$$

$$= -B\ \mathrm{Tr}\left\{\gamma_\lambda\gamma_5\bar{M}(v)\{\Gamma\,,\,s^\lambda\}M(v)\right\}\ , \quad (48)$$

where $\{,\}$ denotes the anticommutator of the two Dirac matrices.

C. Simple application: The heavy meson mass

The mass of a heavy hadron may be expanded in inverse powers of the heavy quark mass. The lowest order terms of this expansion have been considered and one may extend this analysis to higher orders using the above discussion of the forward matrix elements.

The relation between the heavy meson mass m_H and the mass of the heavy quark is given by

$$m_H = M_H - \frac{1}{2M_H}\left\langle H(v)\left|T\left[\mathcal{L}_I(0)\exp\left(-i\int d^4x\,\mathcal{L}_I(x)\right)\right]\right|H(v)\right\rangle\ , \quad (49)$$

where $M_H = m_Q + \bar{\Lambda}$ is the mass of the heavy hadron in the limit $m_Q \to \infty$.

The $1/m_Q$ expansion of the hadron mass is obtained by inserting expression (3) into the time ordered product. Up to order $1/m_Q^2$ one finds

$$m_H = m_Q + \bar{\Lambda} - \frac{1}{2M_H}\sum_{j=1,2}\left(\frac{1}{2m_Q}\right)^j\left[\langle H(v)|\mathcal{K}^{(j)}(0)|H(v)\rangle + \langle H(v)|\mathcal{G}^{(j)}(0)|H(v)\rangle\right]$$
$$-(-i)\frac{1}{2M_H}\left(\frac{1}{2m_Q}\right)^2\int d^4x\,\langle H(v)|T\left[\left(\mathcal{K}^{(1)}(0)+\mathcal{G}^{(1)}(0)\right)\left(\mathcal{K}^{(1)}(x)+\mathcal{G}^{(1)}(x)\right)\right]|H(v)\rangle + O(1/m_Q^3)\ . \quad (50)$$

The matrix elements appearing here are exactly of the type considered above. To order $1/m_Q$ there are the two parameters λ_1 and λ_2, while to order $1/m_Q^2$ one has not only local operators, but also time ordered products to consider. The two local matrix elements are given in terms of ρ_1 and ρ_2, while the time ordered products are parametrized according to

$$(-i)\int d^4x\,\langle H(v)|T\left[\mathcal{K}^{(1)}(0)\mathcal{K}^{(1)}(x)\right]|H(v)\rangle = -2T_1\mathrm{Tr}\left\{\bar{M}(v)M(v)\right\}\ , \quad (51)$$

$$(-i)\int d^4x\,\langle H(v)|T\left[\mathcal{K}^{(1)}(0)\mathcal{G}^{(1)}(x)\right]|H(v)\rangle = -2T_2\mathrm{Tr}\left\{\gamma_\lambda\gamma_5\bar{M}(v)s^\lambda M(v)\right\}\ , \quad (52)$$

$$(-i)\int d^4x\,\langle H(v)|T\left[\mathcal{G}^{(0)}(0)\mathcal{G}^{(0)}(x)\right]|H(v)\rangle = -\mathrm{Tr}\left\{T^{\alpha\beta}\bar{M}(v)\{s_\alpha\,,\,s_\beta\}M(v)\right\}\ , \quad (53)$$

where

$$T_{\alpha\beta} = \frac{1}{3}T_3^{(1)}(v_\alpha v_\beta - g_{\alpha\beta}) + \frac{i}{2}T_3^{(2)}\epsilon_{\mu\alpha\beta\lambda}v^\mu\gamma^\lambda\gamma_5 \ . \tag{54}$$

Using this parametrization one obtains

$$m_H = M_H - \frac{1}{2m_Q}(\lambda_1 + d_H\lambda_2) - \left(\frac{1}{2m_Q}\right)^2 [\rho_1 + 2T_1 + 2T_3^{(1)} + d_H(\rho_2 + 2T_2)] + O(1/m_Q^3) \ , \tag{55}$$

where the spins symmetry breaking contribution of the double insertion of the chromomagnetic moment operator does not contribute, since we are dealing with the flavor diagonal case.

The parameter ρ_1 has been estimated above; this term contributes only about 0.5 MeV to the mass of the B meson. The parameter ρ_2 is more difficult to estimate, but a reasonable guess is certainly $|\rho_2| \sim |\rho_1|$, motivated by the fact that $|\lambda_1| \sim |\lambda_2|$.

The time ordered products are much harder to estimate. They require in general a model describing the dynamics of the light degrees of freedom. We shall not consider this here, but it seems reasonable that the time ordered products are of similar magnitude as the local terms.

III. HIGHER ORDER CORRECTIONS TO THE V_{cb} DETERMINATION

As the main application, we consider the higher order corrections to the semileptonic decay of a B meson into a D or D^* meson. In this case we have to deal with two heavy flavors b and c, and the corresponding static operators are denoted b_v and c_v, respectively. These higher order terms have been considered already in [12] also off the nonrecoil point; however at the point $v = v'$ the analysis simplifies drastically compared to the one off the nonrecoil point.

The form factors to be considered are the ones of the vector and the axial vector current, defined by

$$\langle B(p)|\bar{b}\gamma_\mu c|D(p')\rangle = \sqrt{m_B m_D}\, h_+(vv')2v_\mu + \cdots \ , \tag{56}$$

$$\langle B(p)|\bar{b}\gamma_\mu\gamma_5 c|D^*(p',\epsilon)\rangle = \sqrt{m_B m_{D^*}}\, h_{A1}(vv')(1+vv')\epsilon_\mu + \cdots \ , \tag{57}$$

where the ellipses denote terms which vanish as $v \to v'$ due to their kinematic prefactors. Here b and c are the fields of full QCD and $|B(p)\rangle$ and $|D(p')\rangle$ are the full QCD states. Both form factors h_+ and h_{A1} are normalized at the nonrecoil point $v = v'$ in the heavy quark limit such that $h_+ = h_{A1} = 1$. In addition to these, we also consider the matrix element

$$\langle B^*(p,\epsilon)|\bar{b}\gamma_\mu c|D^*(p',\epsilon')\rangle = \sqrt{m_{B^*}m_{D^*}}\, h_1(vv')(-\epsilon\epsilon')v_\mu + \cdots \ , \tag{58}$$

which we shall need to derive normalization conditions.

Using the $1/m_Q$ expansion (1) for both operators b and c, the contributions to the matrix element at the nonrecoil point may be classified into three species

$$\langle H_b(p)|\bar{b}\Gamma c|H_c(p')\rangle|_{v=v'} = L + T + M + O(1/m_c^3) \ , \tag{59}$$

where $\Gamma = \gamma_\mu, \gamma_\mu\gamma_5$ and H_b and H_c are B, B^* or D, D^*, respectively.

The contribution L are all local terms, i.e., the ones which do not contain any time ordered product. They originate from the expansion of the operators (1) and read

$$\begin{aligned} L = {}& \langle H_b(v)|\bar{b}_v\Gamma c_v|H_c(v)\rangle + \left(\frac{1}{2m_c}\right)\langle H_b(v)|\bar{b}_v\Gamma(i\not{D}^\perp)c_v|H_c(v)\rangle - \left(\frac{1}{2m_b}\right)\langle H_b(v)|\bar{b}_v\,(i\overleftarrow{\not{D}}^\perp)\,\Gamma c_v|H_c(v)\rangle \\ & - \left(\frac{1}{2m_c}\right)^2\langle H_b(v)|\bar{b}_v\Gamma(ivD)(i\not{D}^\perp)c_v|D(v)\rangle - \left(\frac{1}{2m_b}\right)^2\langle H_b(v)|\bar{b}_v\,(i\overleftarrow{\not{D}}^\perp)(iv\overleftarrow{D})\,\Gamma c_v|H_c(v)\rangle \\ & - \left(\frac{1}{4m_b m_c}\right)\langle H_b(v)|\bar{b}_v\,(i\overleftarrow{\not{D}}^\perp)\,\Gamma(i\not{D}^\perp)c_v|H_c(v)\rangle + O(1/m^3) \ , \end{aligned} \tag{60}$$

where now $|H_b(v)\rangle$ and $|H_c(v)\rangle$ are the states in the infinite mass limit.

From the discussion of the forward matrix elements, it follows that only the last term does not vanish. The terms of first order in $1/m_Q$ are forward matrix elements of a dimension four operator and hence zero; the terms of order $1/m_b^2$ and $1/m_c^2$ vanish after a partial integration, which for the forward matrix elements does not yield a surface term. Only the mixed term of order $1/(m_b m_c)$ yields a contribution, which may be related to λ_1 and λ_2.

The second class of terms are the time ordered products of the current to leading order with the terms of order $1/m$ and $1/m^2$ of the Lagrangian. One obtains

$$\begin{aligned} T = &\ (-i)\left(\frac{1}{2m_c}\right)\int d^4x\, \langle H_b(v)|T\left[\bar b_v \Gamma c_v \mathcal{L}_c^{(1)}(x)\right]|H_c(v)\rangle \\ &+(-i)\left(\frac{1}{2m_b}\right)\int d^4x\, \langle H_b(v)|T\left[\mathcal{L}_b^{(1)}(x)\bar b_v \Gamma c_v\right]|H_c(v)\rangle \\ &+(-i)\left(\frac{1}{2m_c}\right)^2\int d^4x\, \langle H_b(v)|T\left[\bar b_v \Gamma c_v \mathcal{L}_c^{(2)}(x)\right]|H_c(v)\rangle \\ &+(-i)\left(\frac{1}{2m_b}\right)^2\int d^4x\, \langle H_b(v)|T\left[\mathcal{L}_b^{(2)}(x)\bar b_v \Gamma c_v\right]|H_c(v)\rangle \\ &+\frac{(-i)^2}{2}\left(\frac{1}{2m_c}\right)^2\int d^4x\, d^4y\, \langle H_b(v)|T\left[\bar b_v \Gamma c_v \mathcal{L}_c^{(1)}(x)\mathcal{L}_c^{(1)}(y)\right]|H_c(v)\rangle \\ &+\frac{(-i)^2}{2}\left(\frac{1}{2m_b}\right)^2\int d^4x\, d^4y\, \langle H_b(v)|T\left[\mathcal{L}_b^{(1)}(x)\mathcal{L}_b^{(1)}(y)\bar b_v \Gamma c_v\right]|H_c(v)\rangle \\ &+(-i)^2\left(\frac{1}{4m_b m_c}\right)\int d^4x\, d^4y\, \langle H_b(v)|T\left[\mathcal{L}_b^{(1)}(x)\bar b_v \Gamma c_v \mathcal{L}_c^{(1)}(y)\right]|H_c(v)\rangle\ , \end{aligned} \tag{61}$$

where here and in the following we suppress the argument of the current $\bar b_v \Gamma c_v$ which is $x = 0$.

Finally, there are the mixed contributions M containing a first-order term of the expansion of the operators (1) and a first-order term of the Lagrangian

$$\begin{aligned} M = &\ (-i)\left(\frac{1}{2m_c}\right)^2\int d^4x\, \langle H_b(v)|T\left[\bar b_v \Gamma (i\not{D}^{\perp}) c_v \mathcal{L}_c^{(1)}(x)\right]|H_c(v)\rangle \\ &-(-i)\left(\frac{1}{2m_b}\right)^2\int d^4x\, \langle H_b(v)|T\left[\mathcal{L}_b^{(1)}(x)\bar b_v\, (i\overleftarrow{\not{D}}^{\perp})\, \Gamma c_v\right]|H_c(v)\rangle \\ &+(-i)\left(\frac{1}{4m_c m_b}\right)\int d^4x\, \langle H_b(v)|T\left[\mathcal{L}_b^{(1)}(x)\bar b_v \Gamma (i\not{D}^{\perp}) c_v\right]|H_c(v)\rangle \\ &-(-i)\left(\frac{1}{4m_b m_c}\right)\int d^4x\, \langle H_b(v)|T\left[\bar b_v\, (i\overleftarrow{\not{D}}^{\perp})\, \Gamma c_v \mathcal{L}_c^{(1)}(x)\right]|H_c(v)\rangle . \end{aligned} \tag{62}$$

As discussed in Sec. II B, these mixed terms all vanish due to the equations of motion.

In order to proceed further with the time ordered products, one has to split the Lagrangians $\mathcal{L}_{b/c}^{(i)}$ into its kinetic and magnetic terms. In this way one may analyze the spin structure of terms involving products of chromomagnetic moment operators by employing the trace formalism, and by using the fact that all products of Dirac matrices may be reduced using the algebra of the Pauli matrices, Eq. (10). The trace formalism gives for the terms of order $1/m$

$$\begin{aligned} &(-i)\int d^4x\, \langle H_b(v)|T\left[\bar b_v \Gamma c_v \mathcal{K}_c^{(1)}(x)\right]|H_c(v)\rangle = -\chi_1 \mathrm{Tr}\left\{\bar M(v)\Gamma M(v)\right\}\ , \\ &(-i)\int d^4x\, \langle H_b(v)|T\left[\bar b_v \Gamma c_v \mathcal{G}_c^{(1)}(x)\right]|H_c(v)\rangle = -\chi_3 \mathrm{Tr}\left\{\gamma_\lambda\gamma_5 \bar M(v)\Gamma s^\lambda M(v)\right\}\ , \\ &(-i)\int d^4x\, \langle H_b(v)|T\left[\bar b_v \Gamma c_v \mathcal{K}_b^{(1)}(x)\right]|H_c(v)\rangle = -\chi_1 \mathrm{Tr}\left\{\bar M(v)\Gamma M(v)\right\}\ , \\ &(-i)\int d^4x\, \langle H_b(v)|T\left[\bar b_v \Gamma c_v \mathcal{G}_b^{(1)}(x)\right]|H_c(v)\rangle = -\chi_3 \mathrm{Tr}\left\{\gamma_\lambda\gamma_5 \bar M(v) s^\lambda \Gamma M(v)\right\} . \end{aligned} \tag{63}$$

Here only two parameters χ_1 and χ_3 appear, since the matrix element has to be symmetric under the exchange of b and c and the corresponding exchange of initial and final state. The spin structure of the second-order terms of the Lagrangian is the same and one may write in a similar fashion

$$\begin{aligned} &(-i)\int d^4x\, \langle H_b(v)|T\left[\bar b_v \Gamma c_v \mathcal{K}_c^{(2)}(x)\right]|H_c(v)\rangle = -\Xi_1 \mathrm{Tr}\left\{\bar M(v)\Gamma M(v)\right\}\ , \\ &(-i)\int d^4x\, \langle H_b(v)|T\left[\bar b_v \Gamma c_v \mathcal{G}_c^{(2)}(x)\right]|H_c(v)\rangle = -\Xi_3 \mathrm{Tr}\left\{\gamma_\lambda\gamma_5 \bar M(v)\Gamma s^\lambda M(v)\right\}\ , \\ &(-i)\int d^4x\, \langle H_b(v)|T\left[\bar b_v \Gamma c_v \mathcal{K}_b^{(2)}(x)\right]|H_c(v)\rangle = -\Xi_1 \mathrm{Tr}\left\{\bar M(v)\Gamma M(v)\right\}\ , \\ &(-i)\int d^4x\, \langle H_b(v)|T\left[\bar b_v \Gamma c_v \mathcal{G}_b^{(2)}(x)\right]|H_c(v)\rangle = -\Xi_3 \mathrm{Tr}\left\{\gamma_\lambda\gamma_5 \bar M(v) s^\lambda \Gamma M(v)\right\} . \end{aligned} \tag{64}$$

Finally, the double insertions of the first-order terms are parametrized by

$$\begin{aligned}
&\frac{(-i)^2}{2}\int d^4x\,d^4y\,\langle H_b(v)|T\left[\bar{b}_v\Gamma c_v\mathcal{K}_c^{(1)}(x)\mathcal{K}_c^{(1)}(y)\right]|H_c(v)\rangle = -A\,\mathrm{Tr}\left\{\bar{M}(v)\Gamma M(v)\right\},\\
&(-i)^2\int d^4x\,d^4y\,\langle H_b(v)|T\left[\bar{b}_v\Gamma c_v\mathcal{K}_c^{(1)}(x)\mathcal{G}_c^{(1)}(y)\right]|H_c(v)\rangle = -B\,\mathrm{Tr}\left\{\gamma_\lambda\gamma_5\bar{M}(v)\Gamma s^\lambda M(v)\right\},\\
&\frac{(-i)^2}{2}\int d^4x\,d^4y\,\langle H_b(v)|T\left[\bar{b}_v\Gamma c_v\mathcal{G}_c^{(1)}(x)\mathcal{G}_c^{(1)}(y)\right]|H_c(v)\rangle = -\mathrm{Tr}\left\{C^{\alpha\beta}\bar{M}(v)\Gamma s_\alpha s_\beta M(v)\right\},\\
&\qquad = -C_1\mathrm{Tr}\left\{\bar{M}(v)\Gamma M(v)\right\} - C_3\mathrm{Tr}\left\{\gamma_\lambda\gamma_5\bar{M}(v)\Gamma s^\lambda M(v)\right\},
\end{aligned} \tag{65}$$

where we have defined

$$C_{\alpha\beta} = \frac{1}{3}C_1(-g_{\alpha\beta} + v_\alpha v_\beta) + \frac{i}{2}C_3\epsilon_{\mu\alpha\beta\lambda}v^\mu\gamma^\lambda\gamma_5. \tag{66}$$

A similar expression is obtained for the double insertion of the first-order Lagrangian of the b quark, involving the same parameters A, B, C_1, and C_3 due to the exchange symmetry $b \leftrightarrow c$.

Finally, the mixed double insertions need another set of parameters

$$\begin{aligned}
&(-i)\int d^4x\,d^4y\,\langle H_b(v)|T\left[\mathcal{K}_b^{(1)}(x)\bar{b}_v\Gamma c_v\mathcal{K}_c^{(1)}(y)\right]|H_c(v)\rangle = -D\,\mathrm{Tr}\left\{\bar{M}(v)\Gamma M(v)\right\},\\
&(-i)^2\int d^4x\,d^4y\,\langle H_b(v)|T\left[\mathcal{K}_b^{(1)}(x)\bar{b}_v\Gamma c_v\mathcal{G}_c^{(1)}(y)\right]|H_c(v)\rangle = -E\,\mathrm{Tr}\left\{\gamma_\lambda\gamma_5\bar{M}(v)\Gamma s^\lambda M(v)\right\},\\
&(-i)^2\int d^4x\,d^4y\,\langle H_b(v)|T\left[\mathcal{G}_b^{(1)}(x)\bar{b}_v\Gamma c_v\mathcal{K}_c^{(1)}(y)\right]|H_c(v)\rangle = -E\,\mathrm{Tr}\left\{\gamma_\lambda\gamma_5 barM(v)s^\lambda\Gamma M(v)\right\},\\
&(-i)^2\int d^4x\,d^4y\,\langle H_b(v)|T\left[\mathcal{G}_b^{(1)}(x)\bar{b}_v\Gamma c_v\mathcal{G}_c^{(1)}(y)\right]|H_c(v)\rangle = -\mathrm{Tr}\left\{R^{\alpha\beta}\bar{M}(v)s_\alpha\Gamma s_\beta M(v)\right\},
\end{aligned} \tag{67}$$

where R is given in terms of two parameters

$$R_{\alpha\beta} = \frac{1}{3}R_1(-g_{\alpha\beta} + v_\alpha v_\beta) + \frac{i}{2}R_2\epsilon_{\mu\alpha\beta\lambda}v^\mu\gamma^\lambda\gamma_5. \tag{68}$$

A. The $0^- \to 0^-$ and $1^- \to 1^-$ vector current at zero recoil

In order to obtain the vector current, i.e., the two form factors h_+ and h_1, we set $\Gamma = \gamma_\mu$. We shall discuss the case of two 0^- states keeping the parameter $d_H = 3$ explicit. The form factor h_1 may then be obtained by setting $d_H = -1$ and by replacing the masses $m_B \to m_{B^*}$ and $m_D \to m_{D^*}$.

The matrix element of the local term originating from the expansion of the current is

$$\langle B(v)|\bar{b}_v\,(i\overleftarrow{\not{D}}^\perp)\,\gamma_\mu(i\not{D}^\perp)c_v|D(v)\rangle = 2v_\mu\sqrt{M_B M_D}(\lambda_1 + d_H\lambda_2). \tag{69}$$

The traces become trivial and one obtains for (56)

$$\begin{aligned}
\langle B(p)|\bar{b}\gamma_\mu c|D(p')\rangle|_{v=v'} = 2\sqrt{M_B M_D}\;v_\mu\Bigg\{&1 + \left(\frac{1}{2m_c} + \frac{1}{2m_b}\right)(\chi_1 + d_H\chi_3)\\
&+\left(\left(\frac{1}{2m_c}\right)^2 + \left(\frac{1}{2m_b}\right)^2\right)(\Xi_1 + A + C_1 + d_H[\Xi_3 + B + C_3])\\
&+\left(\frac{1}{4m_c m_b}\right)(D + R_1 - \lambda_1 + d_H[2E + R_2 - \lambda_2])\Bigg\}.
\end{aligned} \tag{70}$$

In order to extract the form factor h_+ from this, one has to take into account another trivial source of $1/m$ corrections, which is the normalization of the states. The right-hand side of (56) is expressed in terms of the physical meson masses m_B and m_D. In (70) only the masses of the static limit appear which differ from the physical masses at the order $1/m^2$. To this end, one has to take into account a factor

$$\sqrt{\frac{M_B M_D}{m_B m_D}} = 1 + \left(\frac{1}{2m_c^2} + \frac{1}{2m_b^2}\right)\frac{1}{2}(\lambda_1 + d_H\lambda_2) \tag{71}$$

when extracting h_+ from (70).

The parameters appearing in (70) are not all independent. The normalization of the flavor diagonal current is known in full QCD for both matrix elements $0^- \to 0^-$ and $1^- \to 1^-$:

$$\langle B(p)|\bar{b}\gamma_\mu b|B(p)\rangle = 2m_B v_\mu = \langle B^*(p,\epsilon)|\bar{b}\gamma_\mu b|B^*(p,\epsilon)\rangle. \tag{72}$$

This may be employed to obtain relations between the parameters. Setting $m_b = m_c$, (72) implies the relations

$$\chi_1 = \chi_3 = 0 \ , \tag{73}$$

$$\begin{aligned} 2(\Xi_1 + A + C_1 + \lambda_1) &= -(D + R_1 - \lambda_1) \ , \\ 2(\Xi_3 + B + C_3 + \lambda_2) &= -(2E + R_2 - \lambda_2). \end{aligned} \tag{74}$$

The first of these equations is Lukes theorem, stating that there are no first-order corrections at the nonrecoil point [11]. Taking the relations between the parameters of the second order into account, one obtains for the form factor h_+ at the non-recoil point

$$h_+(1) = 1 - \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2 \frac{1}{2}(D + R_1 - \lambda_1 + 3[2E + R_2 - \lambda_2]) + O(1/m^3). \tag{75}$$

Similarly, by the appropriate replacements one obtains

$$h_1(1) = 1 - \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2 \frac{1}{2}(D + R_1 - \lambda_1 - [2E + R_2 - \lambda_2]) + O(1/m^3) \ . \tag{76}$$

Looking at the definition of the parameters entering the $1/m^2$ corrections, it turns out that to order $1/m^2$ the only input needed are the two parameters λ_1 and λ_2 from the local dimension five operators and the time ordered product involving insertions of both $\mathcal{L}_b^{(1)}$ and $\mathcal{L}_c^{(1)}$, which is given in terms of four parameters. The results for h_+ and h_1 may also be written as

$$h_+(1) = 1 - \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2 \frac{1}{2}\Bigg(-\lambda_1 - 3\lambda_2 + (-i)^2 \frac{1}{2\sqrt{M_B M_D}} \times \int d^4x\, d^4y\, \langle B(v)|T\left[\mathcal{L}_b^{(1)}(x)\bar{b}_v c_v \mathcal{L}_c^{(1)}(y)\right]|D(v)\rangle\Bigg) + O(1/m^3) \ , \tag{77}$$

$$h_1(1) = 1 - \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2 \frac{1}{2}\Bigg(-\lambda_1 + \lambda_2 + (-i)^2 \frac{1}{2\sqrt{M_B M_D}} \times \int d^4x\, d^4y\, \langle B^*(v,\epsilon)|T\left[\mathcal{L}_b^{(1)}(x)\bar{b}_v c_v \mathcal{L}_c^{(1)}(y)\right]|D^*(v,\epsilon)\rangle\Bigg) + O(1/m^3). \tag{78}$$

These relations have a simple intuitive interpretation. The contributions from the local dimension five operators (λ_1 and λ_2) originate from the matching of the field operators of full QCD to the ones of the effective theory to order $1/m_c$ and $1/m_b$, respectively. However, the states also receive corrections and the matrix element involving the time ordered product corresponds to the corrections to the states to order $1/m_b$ and $1/m_c$, respectively. The local and the nonlocal contributions are of order $1/(4m_c m_b)$, but due to the normalization of the matrix element for the flavor diagonal case, all other terms of order $1/m_b^2$ and $1/m_c^2$ have to be related to these such that the normalization is preserved in the case $m_c = m_b$. This fact leads to the prefactor $(1/m_c - 1/m_b)^2$ in front of the correction terms in (77) and (78).

This result agrees with the one found in [12]. In particular, one may see that at the nonrecoil point the form factors may be expressed in terms of the parameters D_1, D_3, D_4, D_5, and D_6 defined in [12]. However, the representation in terms of the Pauli matrices reveals a relation between the parameters of [12], at least at the nonrecoil point. In total, there are six independent parameters λ_1, λ_2, D, E, R_1, and R_2 at $v = v'$ and one may show that $R_1 = 3(D_4 + D_5)$ and $R_2 = 2(D_5 + D_6)$.

The $1/m_Q^2$ corrections to h_+ and h_1 depend on the spin symmetry conserving contribution $X = D + R_1 - \lambda_1$ and on the spin symmetry breaking combination $Y = 2E + R_2 - \lambda_2$. If one considers in addition the form factor h_{A1}, then a third combination of the six parameters, λ_1, λ_2, D, E, R_1, and R_2, is needed.

In order to perform a model independent extraction of V_{cb} from the decay $B \to D\ell\nu$, one has to take into account also the second form factor h_- of the vector current. However, considering only forward matrix elements, one cannot say anything about this form factor, and one has to consider different velocities along the lines of [12].

B. The $0^- \to 1^-$ axial vector current at zero recoil

For a model independent extraction of V_{cb}, the process $B \to D^* e\nu$ is much more interesting than $B \to D\ell\nu$. The relevant form factor is h_{A1} as defined in (56). To leading order we have at the nonrecoil point due to spin symmetry

$$\langle B(v)|\bar{b}_v c_v|D(v)\rangle = 2\sqrt{M_B M_D} = -\langle B(v)|\bar{b}_v(s\epsilon)c_v|D^*(v,\epsilon)\rangle, \quad (79)$$

and thus h_{A1} is normalized in the same way as h_+ and h_1. To subleading order h_{A1} will differ from both h_+ and h_1 due to spin symmetry breaking. This is, however, calculable in terms of the parameters which have been introduced above.

The local dimension five terms may be expressed in terms of λ_1 and λ_2:

$$\langle B(v)|\bar{b}_v\,(i\overleftarrow{\not{D}}^{\perp})\,\gamma_\mu\gamma_5(i\not{D}^{\perp})c_v|D^*(v,\epsilon)\rangle = 2\sqrt{M_B M_D}\,\epsilon_\mu\left(-\frac{1}{3}\lambda_1-\lambda_2\right) \quad (80)$$

while the contributions from the T products are evaluated by replacing $\Gamma\to\gamma_\mu\gamma_5$ and using the representation matrix for the vector meson in the final state. Taking into account the contributions from the normalization of the states, one obtains for the form factor h_{A1} at zero recoil

$$h_{A1}(1) = 1-\left(\frac{1}{2m_b}\right)^2\frac{1}{2}\left(D+R_1-\lambda_1+3[2E+R_2-\lambda_2]\right)-\left(\frac{1}{2m_c}\right)^2\frac{1}{2}\left(D+R_1-\lambda_1-[2E+R_2-\lambda_2]\right) + \left(\frac{1}{4m_bm_c}\right)\left(D+2E-\frac{1}{3}R_1-R_2+\frac{1}{3}\lambda_1+\lambda_2\right)+O(1/m^3). \quad (81)$$

The structure of this result may be understood from spin symmetry. In the heavy quark limit, spin symmetry relates all the form factors h_+, h_1 and h_{A1}. If one would take into account only the corrections of order $1/m_b^n$, then the spin symmetry of the c quark would still be unbroken and one may rotate the D^* meson into a D meson. Hence the $1/m_b^2$ corrections to h_{A1} have to be the same as the $1/m_b^2$ corrections to h_+. Similarly, and more importantly, the $1/m_c^2$ corrections to h_{A1} have to be the same as the $1/m_c^2$ corrections to h_1 since we may now use the spin symmetry of the b quark to rotate the B meson into a B^* meson. Finally, the mixed insertions break both spin symmetries and thus cannot be expressed in terms of h_1 or h_+.

In total, the three form factors may be reexpressed in terms of three parameters X, Y, and Z:

$$h_+ = 1-\left(\frac{1}{2m_b}-\frac{1}{2m_c}\right)^2\frac{1}{2}(X+3Y)\,, \quad (82)$$

$$h_1 = 1-\left(\frac{1}{2m_b}-\frac{1}{2m_c}\right)^2\frac{1}{2}(X-Y)\,, \quad (83)$$

$$h_{A1} = 1-\left(\frac{1}{2m_b}\right)^2\frac{1}{2}(X+3Y)-\left(\frac{1}{2m_c}\right)^2\frac{1}{2}(X-Y) + \frac{1}{4m_bm_c}\left(-\frac{1}{3}X-Y+Z\right)\,, \quad (84)$$

where

$$X = D+R_1-\lambda_1,\quad Y = 2E+R_3-\lambda_2,\ \text{and}\quad Z = \frac{4}{3}D+4E\,, \quad (85)$$

where X corresponds to the spin symmetry conserving interactions, Y to the spin symmetry breaking ones, while Z is a mixture of spin symmetry conserving and spin symmetry breaking terms.

C. Discussion of the results and quantitative estimates

Finally, we shall discuss the results and try to give a numerical estimate of the corrections. In general, this needs input beyond heavy quark effective theory, e.g., a model. A few things, however, may be said with two plausible assumptions.

The form factors h_+ and h_1 are form factors which are in the heavy quark limit related to matrix elements of conserved currents, which generate heavy flavor symmetry. The operators

$$K_+ = \int d^3\mathbf{x}\;\bar{b}_v c_v\;,\quad K_- = \int d^3\mathbf{x}\;\bar{c}_v b_v\;,\quad K_0 = \int d^3\mathbf{x}\;[\bar{b}_v b_v-\bar{c}_v c_v] \quad (86)$$

are generators of the heavy flavor symmetry satisfying $[K_+\,,\,K_-]=K_0$, and hence one derives in the symmetry limit $\langle B(v)|K_+|D(v)\rangle=\sqrt{M_BM_D}$ and a similar relation for two vector mesons, implying that $h_+(1)=h_1(1)=1$.

In the presence of explicit symmetry breaking, one splits the Hamiltonian in a symmetry conserving piece H_0 and a symmetry breaking term $\lambda H_{\rm SB}$, such that $[K_j\,,\,H_0]=0$. Since now $[K_j\,,\,H]=\lambda[K_j\,,\,H_{\rm SB}]\neq 0$ the generators K_j become time dependent and one has at $x_0=0$

$$1 = \left|\frac{\langle B(v)|K_+|D(v)\rangle}{2\sqrt{M_BM_D}}\right|^2 + \sum_{X\neq D}\left(\frac{\lambda}{E_B-E_X}\right)^2\left|\frac{\langle B(v)|K_+|X(v)\rangle}{2\sqrt{M_BM_X}}\right|^2\,, \quad (87)$$

where $X(v)$ is a hadronic state in which the c quark moves with velocity v. In obtaining (87) we have neglected matrix elements of the form

$$\langle B(v)|\bar{c}_v c_v|B(v)\rangle \ \text{and}$$
$$\langle B(v)|\bar{c}_v(x)b_v(x)\bar{b}_v(0)b_v(0)|B(v)\rangle,$$

which do not contribute at scales, where the c quark is heavy.

Relation (87) is the standard derivation of the Ademollo-Gatto theorem [23, 24], and it allows two observations. The state X is not in the lowest spin symmetry doublet and hence the energy difference $E_B - E_X$ is not of the order λ, but of the order 1. Hence the corrections due to symmetry breaking are of second order in the symmetry breaking interaction, which is the well-known Ademollo-Gatto theorem [23].

Secondly, and more importantly for the present discussion, it shows that one expects that

$$\left|\frac{\langle B(v)|K_+|D(v)\rangle}{2M_BM_D}\right|^2 \leq 1,$$

since the sum on the right-hand side of (87) is positive. This means that $h_+ - 1 \leq 0$ and $h_1 - 1 \leq 0$.

However, it is known that short distance contributions may change this conclusion. For instance, the full one-loop QCD calculation yields for $h_+(1)$ [3]

$$h_+(1) = 1 + \frac{\alpha}{\pi}\left[\frac{m_b + m_c}{m_b - m_c}\ln\left(\frac{m_b}{m_c}\right) - 2\right] , \quad (88)$$

which yields a positive contribution to the normalization. This may be traced back to the matrix elements, which we have neglected when obtaining (87). We shall assume in the following that the positive short distance contribution (88) is compensated by the long distance one, for which (87) holds.

From this requirement one obtains two constraints for the parameters

$$(D + R_1) - \lambda_1 + 3(2E + R_2) - 3\lambda_2 > 0 , \quad (89)$$

$$(D + R_1) - \lambda_1 - (2E + R_2) + \lambda_2 > 0, \quad (90)$$

which are equivalent to $(D + R_1) \geq \lambda_1$ and $-(D + R_1 - \lambda_1)/3 \leq (2E + R_2\lambda_2) \leq (D + R_1 - \lambda_1)$.

In Fig. 1, we plot the spin symmetry conserving contribution of the time ordered products $(D + R_1)$ versus the spin symmetry breaking part $(2E + R_2)$. The allowed region is the one below the dashed and above the solid line, where $h_1 - 1 < 0$ and $h_+ - 1 < 0$.

In order to obtain some numerical estimate, we shall assume that $(D + R_1) \leq -\lambda_1$ which should be a reasonable order of magnitude for the spin symmetry conserving terms of the time ordered products. Thus the parameters for the time ordered product terms should lie within the shaded triangular region in Fig. 1.

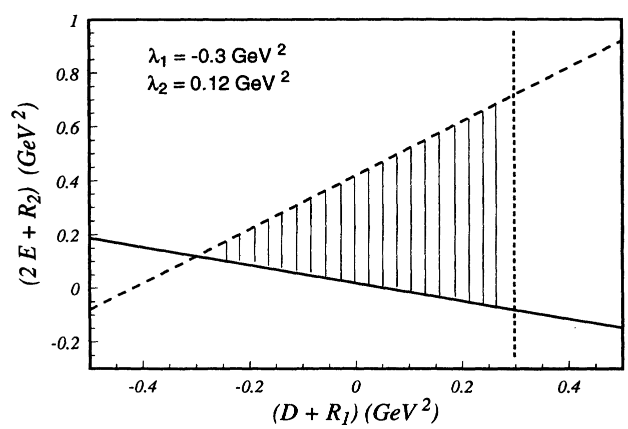

FIG. 1. Allowed region for the spin symmetry conserving $(D + R_1)$ and spin symmetry breaking terms $(2E + R_2)$ of the time ordered product. The solid (dashed) line is from the constraint $h_+(1) < 1$ $(h_1(1) < 1)$, while the vertical dotted line is the assumed upper limit for $(D + R_1)$.

We shall estimate the contributions to $h_{A1}(1)$ by observing that, numerically, the contributions of the order $1/m_c^2$ are by far the largest. As argued above, spin symmetry enforces that these contributions are the same as the ones to the form factor h_1. Maximizing the form factor h_1 in the shaded triangle of Fig. 1, we have for $h_{A1}(1)$

$$-\left(\frac{1}{2m_c}\right)^2\left(-\frac{4}{3}\lambda_1\right) \leq h_{A1}(1) - 1 \leq 0 . \quad (91)$$

Using $\lambda_1 \sim -0.3$, corresponding to the minimal value assumed here, one obtains corrections to $h_{A1} - 1$ ranging between 0 and -5 % . This is consistent with the estimate performed in [12] based on a simple wave function overlap model, once updated values for the parameters are used [25].

A different estimate, based on chiral perturbation theory, has been performed by Randall and Wise [26]. The result obtained in this way exhibits a nonanalytic dependence of the $1/m^2$ corrections on the pion mass. Numerically Randall and Wise obtain for the $1/m^2$ corrections the results

$$h_+(1) - 1 = -1.2\% , \quad h_{A1}(1) - 1 = -2.7\%, \quad (92)$$

which agrees with the present estimate. From (92) we may also extract values for the spin symmetry conserving and spin symmetry breaking terms of the time ordered products

$$D + R_1 \sim 0.51 \text{ GeV}^2 , \quad 2E + R_2 \sim 0.09 \text{ GeV}^2. \quad (93)$$

This yields a large value for the spin symmetry conserving term, which lies slightly outside the shaded region in Fig. 1. This indicates that $(D + R_1) \leq -\lambda_1$ is only a rough estimate. Varying the upper bound, such that the time ordered products satisfy $\lambda_1 \leq (D + R_1) \leq -\alpha\lambda_1$ with $\alpha \sim 1$ shows that the dependence on α is not very strong. For instance, if $\alpha = 2$, one obtains $h_{A1} - 1 > -6\%$ and $1/m_Q^2$ corrections to h_{A1} exceeding 8% in magnitude are very unlikely.

IV. CONCLUSIONS

In this paper we have considered forward matrix elements of local operators of higher dimension and their time ordered products with terms originating from the heavy mass expansion of the Lagrangian. Due to the projection $P_+ = (\not v + 1)/2$ appearing in heavy quark effective theory, the Dirac algebra simplifies and only two types of matrix elements of local operators appear. In addition, the spin structure of the time ordered products of these operators with higher order terms from the Lagrangian may be analyzed in a simple way.

Matrix elements of this type appear in two important applications. Performing a heavy mass expansion for inclusive decays along the lines of Bigi *et al.* [14], these matrix elements parametrize the nonperturbative input required beyond the leading order in the $1/m_Q$ expansion of total rates as well as for inclusive decay spectra.

The second application are the form factors for weak transitions at the nonrecoil point. The symmetries of the heavy quark limit yield the normalization of the weak transitions between heavy quarks; this fact may be employed to perform a model independent determination of $|V_{cb}|$. The recoil corrections to the normalization are given in terms of the forward matrix elements considered here. At the nonrecoil point the analysis simplifies drastically, mainly due to the simplification of the Dirac algebra, as compared to the general analysis.

As an example, we have reconsidered the second order corrections to the semileptonic transition $B \to D^{(*)}$. These corrections have been studied already in [12] for the general case. At the nonrecoil point, the present analysis agrees with the one performed in [12]. However, it turns out that at $v = v'$ some of the parameters given in [12] are in fact not independent.

The second-order corrections of the weak decay form factors are all parametrized in terms of five matrix elements:

$$\lambda_1 = \frac{1}{2M_Q}\langle H(v)|\bar{Q}_v (iD)^2 Q_v|H(v)\rangle \ ,$$

$$\lambda_2 = \frac{1}{2M_Q}\langle H(v)|\bar{Q}_v (iD_\alpha)(iD_\beta)(-i\sigma^{\alpha\beta})Q_v|H(v)\rangle$$

and the matrix elements of double insertions of the first-order correction to Lagrangian:

$$(-i)\frac{1}{2\sqrt{M_B M_D}}\int d^4x\, d^4y\,\langle B(v)|T\left[\mathcal{K}_b^{(1)}(x)\bar{b}_v c_v \mathcal{K}_c^{(1)}(y)\right]|D(v)\rangle \ ,$$

$$(-i)\frac{1}{2\sqrt{M_B M_D}}\int d^4x\, d^4y\,\langle B(v)|T\left[\mathcal{G}_b^{(1)}(x)\bar{b}_v c_v \mathcal{K}_c^{(1)}(y)\right]|D(v)\rangle \ ,$$

$$(-i)\frac{1}{2\sqrt{M_B M_D}}\int d^4x\, d^4y\,\langle B(v)|T\left[\mathcal{G}_b^{(1)}(x)\bar{b}_v c_v \mathcal{G}_c^{(1)}(y)\right]|D(v)\rangle .$$

All other matrix elements of time ordered products are related to these by the normalization condition for the vector current in the full theory.

All these matrix elements are nonperturbative. In principle they may be measured on the lattice and first results have been reported [27]. However, in the meantime one has to rely on some model to estimate their size. In the present paper, we have used a reasonable guess for the spin symmetry conserving contributions of the time ordered products to get some upper limit for the $1/m_Q$ corrections to h_{A1} at the nonrecoil point. The main result of this analysis is that the corrections to $h_{A1}(1)$ are small, ranging between -5% and 0.

Including also the leading and subleading QCD radiative corrections [10] to the normalization of h_{A1}, one concludes that

$$h_{A1}(1) = 0.96 \pm 0.03, \tag{94}$$

and thus the corrections at zero recoil are small.

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