Generalized gauge-invariant regularization of the Schwinger model

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The Schwinger model is studied with a one-parameter class of gauge-invariant regularizations that may be thought of as generalizing the regularization schemes normally used for this model. The spectrum is found to be qualitatively unchanged, except for a limiting value of the regularizing parameter, where free fermions appear in the spectrum.

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I. INTRODUCTION

The Schwinger model [1], namely, the theory of massless quarks interacting with an Abelian gauge field in two-dimensional spacetime, has been extensively studied over the years and has provided theorists much insight into the phenomena of mass generation and confinement [2,3]. The quark disappears from the physical spectrum in this model, leaving only a free massive particle associated with the gauge field. Exact solutions are available for various operators and Green functions.

The regularization underlying the conventional study of the Schwinger model is such that the physical mass of the particle becomes equal to $\frac{1}{\sqrt{\pi}}$ times the bare gauge coupling constant. This regularization maintains the gauge invariance of the theory although a mass is generated for the gauge field. Other regularizations that give up gauge invariance have recently been studied [4] and lead to different physical results – the quark gets liberated in that situation, much as in the closely related chiral Schwinger model [5]. However, even if gauge invariance is not abandoned, it is possible to make the regularization more flexible, for example in the context of the Fujikawa regularization scheme. The nature of the solution is not qualitatively changed; only the relation between the physical mass and the bare coupling is generalized. In one sense, the theory is not changed at all, for there is only one physical quantity in the picture, the mass of the particle, and it is a dimensional object in two dimensions, so that its *value* is not relevant. In another sense, this regularization gives rise to a new relation between fermionic bilinears and bosons so that one effectively has a different bosonization scheme. This flexibility allows an unusual limit to be taken, whereby the physical mass can be made zero. This opens up a new scenario in this model. It is to the consideration of the new regularized version of the Schwinger model and the special limiting case that the present paper is devoted.

The plan of the paper is as follows. We first explain how the regularization of the Schwinger model allows an extra flexibility in the effective action of the model. This implies a generalized expression for the anomaly in the axial vector current of the theory. It is shown that the usual form of the fermion operator of the Schwinger model allows the current to be constructed in such a way that this generalized expression is obtained for the anomaly. The gauge field equation of motion is then satisfied only if the physical mass is related to the gauge coupling constant in a specific way depending on the regularization. This fixes the effective action of the theory in terms of the gauge coupling constant and the regularization. The quark-antiquark potential following from this effective action is worked out and the behavior of the propagator in the special limiting case investigated. The phenomenon of cluster violation is clarified.

II. OPERATOR SOLUTION OF EQUATIONS OF MOTION

The Schwinger model is described by the Lagrangian density [1]

$$\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ + eA\!\!\!/)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \tag{1}$$

where the indices take the values 0,1 corresponding to a (1+1)-dimensional spacetime and the notation is standard. In two dimensions we can always set

$$A_\mu = -\frac{\sqrt{\pi}}{e}(\widetilde{\partial}_\mu\sigma + \partial_\mu\widetilde{\eta}), \tag{2}$$

where

$$\widetilde{\partial}_\mu = \epsilon_{\mu\nu}\partial^\nu \ , \tag{3}$$

with $\epsilon_{01} = +1$ and $\sigma, \widetilde{\eta}$ are scalar fields.

In this section we shall restrict ourselves to the Lorentz gauge, where from (2) we see that the field $\widetilde{\eta}$ can be taken as a massless field with $\Box\widetilde{\eta} = 0$. We can then introduce its dual through

$$\widetilde{\partial}_\mu\eta(x) = \partial_\mu\widetilde{\eta}(x). \tag{4}$$

These massless fields have to be regularized because in two dimensions the two point function of a massless scalar

*Electronic address: mitra@saha.ernet.in field diverges [2]. We shall not need the explicit form of the regularization here.

The Dirac equation in the presence of the gauge field is

$$[i\partial\!\!\!/ + eA\!\!\!/]\,\psi(x) = 0. \tag{5}$$

It is easy to check that this equation is satisfied by

$$\psi(x) =:\ e^{i\sqrt{\pi}\gamma_5[\sigma(x)+\eta(x)]}\ :\psi^{(0)}(x), \tag{6}$$

where $\psi^{(0)}(x)$ is a free fermion field satisfying $i\partial\!\!\!/\psi^{(0)}(x) = 0$.

We can calculate the gauge invariant current using the point-splitting regularization. While constructing a gauge invariant bilinear of fermions which in the limit of zero separation would give the usual fermion current, we generalize slightly the conventional construction [1]. We take

$$J_\mu^{\rm reg}(x) = \lim_{\epsilon\to 0}\left[\overline{\psi}(x+\epsilon)\gamma_\mu\ :e^{ie\int_x^{x+\epsilon}dy^\mu\left\{A_\mu(y)-2a\partial^\nu F_{\mu\nu}(y)\right\}}:\psi(x) - v\right] \tag{7}$$

where a is an arbitrary parameter and v is the vacuum expectation value (VEV). The term in the exponent containing this parameter represents our generalization of Schwinger's regularizing phase factor [1]. A similar generalization was made earlier [6] in a nongauge context. Our generalization, which we use in the Schwinger model, can be obtained by the replacement of the nongauge-invariant vector A_μ in the extra phase factor of [6] by the gauge invariant vector $\partial^\nu F_{\mu\nu}$ and hence preserves gauge invariance. It also maintains Lorentz invariance and the linearity of the theory. Now using (2) and (6) together with

$$F_{\mu\nu} = \frac{\sqrt{\pi}}{e}\epsilon_{\mu\nu}\,\Box\,\sigma, \tag{8}$$

we obtain the current which, up to an overall wave function renormalization, is equal to

$$J_\mu^{\rm reg}(x) \approx\ :\overline{\psi^{(0)}}(x)\gamma_\mu\psi^{(0)}(x): -i\sqrt{\pi}\lim_{\epsilon\to 0}\langle 0\mid \overline{\psi^{(0)}}(x+\epsilon)\gamma_\mu\left[(\gamma_5\epsilon\cdot\partial+\epsilon\cdot\widetilde{\partial})(\sigma+\eta)+2a\epsilon\cdot\widetilde{\partial}\,\Box\,\sigma\right]\psi^{(0)}(x)\mid 0\rangle \tag{9}$$

$$=\ :\overline{\psi^{(0)}}(x)\gamma_\mu\psi^{(0)}(x): -\frac{1}{\sqrt{\pi}}\left[\frac{\epsilon_\mu\epsilon_\nu-\widetilde{\epsilon}_\mu\widetilde{\epsilon}_\nu}{\epsilon^2}\widetilde{\partial}^\nu(\sigma+\eta)+2a\frac{\epsilon_\mu\epsilon_\nu}{\epsilon^2}\widetilde{\partial}^\nu\,\Box\,\sigma\right], \tag{10}$$

where we have used the identity

$$\langle 0\mid \overline{\psi^{(0)}}_\alpha(x+\epsilon)\psi_\beta(x)\mid 0\rangle = -i\frac{\epsilon\!\!/_{\beta\alpha}}{2\pi\epsilon^2}. \tag{11}$$

Now we take the symmetric limit, i.e., average over the point splitting directions ϵ and finally obtain

$$J_\mu^{\rm reg}(x) = -\frac{1}{\sqrt{\pi}}\widetilde{\partial}_\mu(\phi+\sigma+a\,\Box\,\sigma+\eta), \tag{12}$$

where ϕ is a free massless bosonic field satisfying

$$-\frac{1}{\sqrt{\pi}}\widetilde{\partial}_\mu\phi =: \overline{\psi}^{(0)}(x)\gamma_\mu\psi^{(0)}(x): \tag{13}$$

and thus representing the bosonic equivalent of the free fermionic field $\psi^{(0)}$ [7]. This field too has to be understood to be regularized. We find

$$J_{\mu 5}^{\rm reg}(x) = \epsilon_{\mu\nu}J_{\rm reg}^\nu(x) \tag{14}$$

$$= -\frac{1}{\sqrt{\pi}}\partial_\mu(\phi+\eta+\sigma+a\,\Box\,\sigma), \tag{15}$$

so that the anomaly is

$$\partial^\mu J_{\mu 5}^{\rm reg} = -\frac{1}{\sqrt{\pi}}\,\Box\,(\phi+\eta+\sigma+a\,\Box\,\sigma). \tag{16}$$

Note now that Maxwell's equation with sources, viz.,

$$\partial_\nu F^{\nu\mu} + eJ^\mu_{\rm reg} = 0, \tag{17}$$

can be converted to the pair of equations

$$\left[\left(1+\frac{ae^2}{\pi}\right)\Box + \frac{e^2}{\pi}\right]\sigma = 0 \tag{18}$$

and

$$\phi + \eta = 0. \tag{19}$$

The second equation relating two massless free fields will be satisfied in a weak sense by imposing a subsidiary condition

$$(\phi+\eta)^{(+)}\mid \text{phys}\rangle = 0 \tag{20}$$

to select out a physical subspace of states. We shall also ensure that $\phi+\eta$ creates only states with zero norm by taking η to be a negative metric field, i.e., by taking its commutators to have the "wrong" sign. The subsidiary condition then also serves to separate out a subspace with non-negative metric as usual.

We see from (18) that σ is a massive free field, as expected. The only difference from the usual case is the presence of the factor $(1+\frac{ae^2}{\pi})$. Consequently, the spectrum of the theory as regularized here is the same as in the usual case with the mass scaled down by a factor $\sqrt{(1+\frac{ae^2}{\pi})}$. This is reminiscent of the vector meson model of [6], where the mass of the physical particle gets altered because of the change of regularization. Note that in the limit when a tends to infinity, this massive particle becomes massless. Since massless scalars in two dimensions can be regarded as bosonized versions of fermions, it follows that there is a massless fermion in the spectrum in this limit.

III. EFFECTIVE ACTION OF QED_2

In the previous section we regularized the current directly as an operator product of fermion fields. The same regularized current will now be obtained from an effective action which we shall construct through a Fujikawa regularization.

The effective action is defined by the following functional of the Abelian gauge field A_μ:

$$e^{i\Gamma[A]} = \int \mathcal{D}\psi\mathcal{D}\overline{\psi}e^{i\int d^2x\overline{\psi}i\not{D}\psi}, \quad (21)$$

where $D_\mu = \partial_\mu - ieA_\mu$. Notice that by virtue of (2) and the identity

$$\gamma^\mu\epsilon_{\mu\nu} = \gamma^5\gamma_\nu, \quad (22)$$

which holds in two dimensions, we can write

$$\not{D} = \not{\partial} + i\sqrt{\pi}\not{\partial}\widetilde{\eta} + i\sqrt{\pi}\gamma_5\not{\partial}\sigma. \quad (23)$$

It is easy to see that the transformations

$$\psi' = e^{i\sqrt{\pi}(\widetilde{\eta}-\gamma_5\sigma)}\psi, \quad (24)$$

$$\overline{\psi}' = \overline{\psi}e^{-i\sqrt{\pi}(\widetilde{\eta}+\gamma_5\sigma)}, \quad (25)$$

decouple the gauge field from the fermions and the classical action becomes free, i.e.,

$$\overline{\psi}i\not{D}\psi = \overline{\psi}'i\not{\partial}\psi', \quad (26)$$

but in the quantum theory this decoupling from the action leads to a nontrivial change in the fermionic measure, which is related to the chiral anomaly. To calculate the Jacobian we must proceed through infinitesimal transformations of the fermionic fields in the path integral. So we define

$$\psi'_\delta = (1 + i\sqrt{\pi}\delta\widetilde{\eta} - i\sqrt{\pi}\gamma_5\delta\sigma)\psi,$$
$$\overline{\psi}'_\delta = \overline{\psi}(1 - i\sqrt{\pi}\delta\widetilde{\eta} - i\sqrt{\pi}\gamma_5\delta\sigma), \quad (27)$$

leading to

$$\overline{\psi}\not{D}\psi = \overline{\psi}'_\delta[\not{\partial} + i\sqrt{\pi}\not{\partial}(\widetilde{\eta} - \delta\widetilde{\eta}) + i\sqrt{\pi}\gamma_5\not{\partial}(\sigma - \delta\sigma)]\psi'_\delta. \quad (28)$$

The Jacobian corresponding to this transformation, defined by

$$\mathcal{D}\psi\mathcal{D}\overline{\psi} = J^\delta\mathcal{D}\psi'_\delta\mathcal{D}\overline{\psi}'_\delta, \quad (29)$$

viz.,

$$J^\delta_{\text{reg}} = e^{2i\sqrt{\pi}\int d^2x \ \text{tr} \ \gamma_5\delta\sigma(x)}, \quad (30)$$

is regularized to

$$J^\delta_{\text{reg}} = e^{2i\sqrt{\pi} \ \text{Tr} \ \gamma_5\delta\sigma(x)e^{t(\not{D}^r)^2}}, \quad (31)$$

where D^r_μ is a regularizing anti-Hermitian differential operator, Tr stands for the full trace, and the limit $t \to 0^+$ is to be taken. Fujikawa chose the operator D^r_μ to be the Euclidean Dirac operator [8]. Other choices, e.g., in [9], correspond to different regularizations. To calculate the trace, it is convenient to take a plane wave basis. Then the exponent in (31) simply gets multiplied by a factor $\frac{a_1}{4\pi}$, which is defined as follows. First the Dirac operator is continued to the Euclidean space; after evaluating the trace it is finally continued back to the Minkowski space. Hence in the following calculation we have to use Euclidean γ matrices (although the same notation is used as for Minkowski gamma matrices). a_1 is given by

$$a_1 = (\not{D}^r)^2 - (D^r)^2. \quad (32)$$

We choose the regularizing Dirac operator to be

$$D^r{}_\mu = \partial_\mu - ieA_\mu + iae\partial_\nu F_{\nu\mu}. \quad (33)$$

We shall see that the anomaly obtained in this regularization matches the one obtained in the point-splitting regularization of Sec. 2. That is the justification for using the same symbol a as before to parametrize the regularizing Dirac operator.

On continuation back to Minkowski spacetime, this gives

$$a_1 = \sqrt{\pi}\gamma_5 \Box (\sigma + a \Box \sigma). \quad (34)$$

The calculation of the effective action goes as follows. By the transformations (27) we can write (21) as

$$e^{i\Gamma[\sigma,\widetilde{\eta}]} = J^\delta_{\text{reg}} \ e^{i\Gamma[\sigma-\delta\sigma,\widetilde{\eta}-\delta\widetilde{\eta}]}. \quad (35)$$

Thus,

$$\delta\Gamma[\sigma] = \frac{1}{i}\ln \ J^\delta_{\text{reg}}[\sigma] = 2\sqrt{\pi}\int d^2x \ \text{tr} \ \gamma_5\delta\sigma(x)\frac{a_1}{4\pi} \quad (36)$$

and

$$\frac{\delta\Gamma}{\delta\widetilde{\eta}} = 0. \quad (37)$$

Using (34) and finally integrating to a finite $\sigma(x)$, we get

$$\Gamma[\sigma] = \frac{1}{2}\int d^2x \ [\sigma \Box \sigma + a \Box \sigma \Box \sigma]. \quad (38)$$

Finally, using the inverse of (2) and (8), we obtain the effective action

$$\Gamma[A] = \int d^2x \left[\frac{e^2}{2\pi}\tilde{\partial}\cdot A\frac{1}{\Box}\tilde{\partial}\cdot A - \frac{ae^2}{4\pi}F_{\mu\nu}F^{\mu\nu}\right]. \quad (39)$$

This effective action can be used to calculate the fermionic currents $eJ_\mu = \frac{\delta}{\delta A^\mu}\,\Gamma[A]$ and

$$\begin{aligned} J_5^\mu &= \epsilon^{\mu\nu}J_\nu \\ &= \frac{e}{\pi}\epsilon^{\mu\nu}\left[A_\nu + a\partial^\rho F_{\rho\nu} - \frac{1}{\Box}\partial_\nu\partial\cdot A\right], \end{aligned} \quad (40)$$

from which we find the anomaly equation to be

$$\partial_\mu J_5^\mu = \frac{e}{2\pi}\epsilon_{\mu\nu}[F^{\mu\nu} + a\,\Box\,F^{\mu\nu}] \quad (41)$$

$$= -\frac{1}{\sqrt{\pi}}\,\Box\,(\sigma + a\,\Box\,\sigma), \quad (42)$$

which, as mentioned above, is consistent with (16) when the subsidiary condition (20) is imposed.

IV. THE BOSONIZATION OF QED$_2$

If we make the above effective action local by introducing an auxiliary field Σ and insert the kinetic energy term for the gauge field, we obtain the bosonized action of two-dimensional QED (QED$_2$) generalized as above:

$$S_B = \int d^2x\left[-\frac{1}{4}\left(1 + \frac{ae^2}{\pi}\right)F^2 + \frac{e^2}{2\pi}A^2 + \frac{1}{2}\partial_\mu\Sigma\partial^\mu\Sigma - \frac{e}{\sqrt{\pi}}A^\mu\partial_\mu\Sigma\right]. \quad (43)$$

The effective action leads to a Hamiltonian through standard constraint analysis as follows. First, the canonical momenta have to be defined. The momenta corresponding to A_0, A_1, and Σ are, respectively,

$$\Pi_0 = 0, \quad (44)$$

$$\Pi^1 = \left(1 + \frac{ae^2}{\pi}\right)(\partial_0 A_1 - \partial_1 A_0), \quad (45)$$

$$\Pi_\Sigma = \dot{\Sigma} - \frac{e}{\sqrt{\pi}}A_0. \quad (46)$$

(44) is recognized to be a constraint. Using all these equations, we obtain the Hamiltonian

$$\mathcal{H} = \frac{(\Pi^1)^2}{2(1+\frac{ae^2}{\pi})} + \frac{1}{2}\Pi_\Sigma^2 + \frac{e}{\sqrt{\pi}}A_0\Pi_\Sigma + \partial_1 A_0\Pi^1 + \frac{e^2}{2\pi}A_1^2 + \frac{1}{2}\Sigma'^2 - \frac{e}{\sqrt{\pi}}\Sigma' A_1. \quad (47)$$

The consistency of (44) under time evolution by this Hamiltonian requires a secondary constraint

$$G \equiv \partial_1\Pi^1 - \frac{e}{\sqrt{\pi}}\Pi_\Sigma = 0. \quad (48)$$

There are no further constraints, and it can be checked that the Poisson brackets of (44) and (48) with one another vanish, so that the constraints are *first class.* This is natural, as we have taken care to maintain gauge invariance in the effective action. As usual, then, we have to fix a gauge to remove gauge degrees of freedom. It is convenient here to consider the physical gauge conditions

$$\Sigma = A_0 = 0. \quad (49)$$

(In the next section we shall use a different kind of gauge fixing.) In the present gauge, the Hamiltonian simplifies to

$$\mathcal{H} = \frac{(\Pi^1)^2}{2(1+\frac{ae^2}{\pi})} + \frac{e^2}{2\pi}A_1^2 + \frac{\pi}{2e^2}(\Pi^{1\prime})^2, \quad (50)$$

which may be converted to the familiar form

$$\mathcal{H} = \frac{1}{2}\Pi_\Phi^2 + \frac{1}{2}\Phi'^2 + \frac{1}{2}\frac{e^2}{\pi + ae^2}\Phi^2 \quad (51)$$

by the redefinitions

$$\Phi = \frac{\sqrt{\pi}}{e}\Pi^1, \quad \Pi_\Phi = -\frac{e}{\sqrt{\pi}}A_1. \quad (52)$$

This shows that the physical spectrum of the model contains just a massive boson with mass $\frac{e}{\sqrt{\pi+ae^2}}$. As mentioned before, this mass vanishes in the limit of large a, and a massless fermion emerges. The vanishing of the mass can be seen directly from Eq. (43), where the scaling of A by the factor $\sqrt{1+\frac{ae^2}{\pi}}$ shows that the mass and interaction terms vanish in the limit, leaving the free-field theory.

V. QUARK POTENTIAL, PROPAGATORS, AND CLUSTERING

Let us investigate the nature of the force mediated by the gauge field of this theory between two quarks. First, in the presence of two static external quarks ($q\bar{q}$ pair) of charge Q at $\pm\frac{L}{2}$, the charge density is modified to

$$J_q^0(t,x^1) = \frac{Q}{e}\left[\delta\left(x^1 - \frac{L}{2}\right) - \delta\left(x^1 + \frac{L}{2}\right)\right] + \frac{e}{\pi}A_0 - \frac{e}{\pi}\frac{\partial_0}{\Box}\partial\cdot A + \frac{ae}{\pi}\partial_1 F_{01} = J^0 - \frac{1}{\sqrt{\pi}}\partial_1\chi, \quad (53)$$

where

$$\chi = \frac{\sqrt{\pi}}{e} Q \theta \left(x^1 + \frac{L}{2} \right) \theta \left(\frac{L}{2} - x^1 \right). \tag{54}$$

So the Lagrangian density in the presence of these external quarks can be written as

$$\mathcal{L}_Q = -\frac{1}{4}\Big(1 + \frac{ae^2}{\pi}\Big)F^2 + \frac{e^2}{2\pi}A^2 + \frac{1}{2}\partial_\mu \Sigma \partial^\mu \Sigma - \frac{e}{\sqrt{\pi}}\partial_\mu \Sigma A^\mu - \frac{e}{\sqrt{\pi}}\tilde{\partial}\cdot A\chi. \tag{55}$$

From a constraint analysis similar to the one in Sec. IV, we get the corresponding Hamiltonian density in the physical gauge to be

$$\mathcal{H}_Q = \frac{1}{2}\widetilde{\Pi}_\Phi^2 + \frac{1}{2}\widetilde{\Phi}'^2 + \frac{1}{2}\frac{e^2}{\pi + ae^2}(\widetilde{\Phi} - \chi)^2, \tag{56}$$

where $\widetilde{\Pi}_\Phi = \Pi_\Phi$ and $\widetilde{\Phi} = \Phi + 2\chi$.

The difference in ground state energies between $\mathcal{H}_Q$ and $\mathcal{H}$ can be calculated to be

$$E_Q - E = \frac{1}{2}\int dx^1 \left[\frac{e^2}{\pi + ae^2}\chi^2 + \left(\frac{e^2}{\pi + ae^2}\right)^2 \chi \times \left(\partial_1^2 - \frac{e^2}{\pi + ae^2} \right)^{-1} \chi \right]. \tag{57}$$

Hence the potential between the quark-antiquark pair is

$$V(L) = \frac{1}{2}\frac{Q^2}{e^2/\pi}\frac{e}{\sqrt{\pi + ae^2}}\left[1 - e^{-\frac{eL}{\sqrt{\pi + ae^2}}}\right], \tag{58}$$

which is constant for large L, indicating the screening of the charges as in the usual version of the Schwinger model. However, in the limit of massless gauge fields $ae^2 \to \infty$, $V(L) = 0$; i.e., the (external) quarks become free. This is to be contrasted with the limit $e \to 0$ of the usual version of the Schwinger model or simply the free electromagnetic theory, where $V(L) = \frac{1}{2}Q^2 L$, so that there is a linearly rising confining potential. This is consistent with the occurrence of deconfinement in the limit $ae^2 \to \infty$.

The bosonized action can be used to calculate the two point correlation function of the fermions [10] with a gauge-fixing term $-\frac{1}{2\alpha}\int d^2x\,(\partial\cdot A)^2$ added to the action. This yields

$$G_F(x,y) = \exp\left[i\int \frac{d^2p}{(2\pi)^2}\left[-\frac{\pi m^2}{p^2(p^2 - m^2)} + \frac{\alpha e^2}{p^4} \right] \times (1 - e^{-ip\cdot(x-y)}) \right] S_F(x,y), \tag{59}$$

where S_F is the free fermion Green function. It is also possible to calculate the gauge-invariant two-point function [11]. The gauge-invariant two-point function is the vacuum expectation value of the gauge-invariant bilocal operator

$$T(x,y) = \psi(x)\overline{\psi}(y) : e^{ie\int_x^{x+\epsilon} dy^\mu \left\{ A_\mu(y) - a\partial^\nu F_{\mu\nu}(y) \right\}} : . \tag{60}$$

The term $- a\partial_\nu F^{\mu\nu}$ has been included to maintain the identity

$$J^\mu_{\mathrm{reg}}(x) = -\lim_{\epsilon \to 0} \operatorname{tr} \gamma^\mu [T(x, x+\epsilon) - v]. \tag{61}$$

The detailed calculation yields

$$G_F^{\mathrm{GI}}(x,y) = \exp\left[-i\pi \int \frac{d^2p}{(2\pi)^2}\left[-\frac{2}{p^2} + \frac{(x-y)^2}{[p\cdot(x-y)]^2} \right] \times \frac{m^2}{p^2 - m^2}(1 - e^{-ip\cdot(x-y)}) \right] S_F(x,y), \tag{62}$$

which shows that in the limit $ae^2 \to \infty$, $G_F^{\mathrm{GI}} \to S_F$.

There is a deeper way to appreciate why the spectrum changes drastically when a becomes infinite. The confinement of quarks in the usual Schwinger model is understood by imposing the subsidiary condition. Since the operator solutions (6) and (2) for ψ and A do not commute with the operator $\phi + \eta$, they create both physical and unphysical states from the vacuum. It is more convenient to make a gauge transformation and pass to the new set of solutions

$$\psi'(x) =: e^{i\sqrt{\pi}\left(\gamma_5[\sigma(x) + \eta(x)] + \tilde{\eta}(x)\right)} : \psi^{(0)}(x), \tag{63}$$

$$A'_\mu = -\frac{\sqrt{\pi}}{e}\tilde{\partial}_\mu \sigma. \tag{64}$$

Now, according to [12],

$$\psi^{(0)}(x) \propto : e^{i\sqrt{\pi}\left[\gamma_5 \phi(x) + \tilde{\phi}(x)\right]} :, \tag{65}$$

so that, by virtue of the subsidiary condition (20), $\psi'(x)$ is essentially $: e^{i\sqrt{\pi}\gamma_5 \sigma(x)} :$, apart from cluster-violating operators which reduce to c numbers in irreducible sectors [2]. These expressions clarify why there is no fermion in the spectrum for finite a.

For infinite a, on the other hand, $\sigma(x)$ is a massless field. One can then introduce its dual $\tilde{\sigma}(x)$ through

$$\partial_\mu \tilde{\sigma}(x) = \tilde{\partial}_\mu \sigma(x), \tag{66}$$

and perform a gauge transformation with it to construct new operator solutions of the equations of motion:

$$\psi''(x) =: e^{i\sqrt{\pi}\left[\gamma_5[\sigma(x) + \eta(x)] + \tilde{\sigma}(x) + \tilde{\eta}(x)\right]} : \psi^{(0)}(x), \tag{67}$$

$$A''_\mu = 0. \tag{68}$$

Clearly, after the unphysical fields present in the expression for ψ'' are replaced by c numbers, what is left is a representation of a free massless fermion in terms of σ and its dual; i.e., the analogue of (65) with ϕ replaced by σ. This is how a fermion appears in the limit of infinite a.

VI. CONCLUSION

In this paper we have looked at the Schwinger model with a somewhat generalized regularization. First we point split the current which is formally defined as the product of two fermionic operators. Schwinger has prescribed the insertion of an exponential of a line integral of the gauge field to make the product gauge invariant. However, his choice was only one of many possible choices; see, e.g., [6]. We have inserted an extra factor which involves the field strength of the gauge field and therefore does not interfere with the gauge invariance of the product. It is here that our parameter a enters. Obviously, this is not the most general gauge-invariant regularization possible in this approach, but the introduction of more complicated factors makes the theory difficult to solve. With our regularization, the equations of motion of the Schwinger model can be converted to free field equations exactly as in the usual case, with only the mass of the scalar field altered by a factor involving the new parameter. The conventional indefinite metric treatment has been used and a subsidiary condition imposed to separate out a physical space.

There is one question which may arise in the reader's mind. Have we, in changing the regularization, changed the model? To be more specific, the introduction of $A_\mu - a\partial^\nu F_{\mu\nu}$ instead of just A_μ in the phase factor entering the point-split current may be suspected to amount to the addition of an extra interaction of the form $-aj^\mu\partial^\nu F_{\mu\nu}$. This is not really the case, as the equations of motion of the Schwinger model itself are satisfied. The change is only in the definition of fermion bilinears as composite operators and this is well known to have a lot of flexibility. Formally, in the limit $\epsilon \to 0$, the phase factor does reduce to unity, so that the definition of the bilinears adopted in this paper is by no means unnatural.

After the operator treatment, a Fujikawa regularization is constructed in such a way that it gives the same result as the generalized point splitting procedure. This is used to find the effective action of the theory. The nonlocal terms present here can be recast in a local form as usual by the introduction of a new scalar field, viz., the bosonized equivalent of the fermion field. A Hamiltonian analysis is carried out to establish the physical content of the theory, which is not always clear from the operator solution in an indefinite metric space.

Some properties of the new version of the theory have been studied for the purpose of comparison with the usual version. The potential between external quarks has been calculated. A gauge-invariant propagator has been presented. Last but not least, the operator solution itself has been scrutinized for clustering violation. All these studies point in one direction: there is nothing qualitatively different for finite values of the parameter a, but when this parameter goes to infinity, the fermion reappears.

This paper is limited to the Schwinger model, but it is clear that the ambiguity in regularization that has been exploited here exists in many other models as well. While four-dimensional models may be difficult to handle, we hope to deal with other Abelian and non-Abelian models in two dimensions in a separate publication.

We hope that investigations of this kind will throw more light on the not too well understood phenomenon of quark confinement and its connection with details of regularization. Hand waving arguments about confinement and deconfinement are almost all that there is in four dimensions. The dependence of these phenomena on regularization schemes clearly indicates the need for more quantitative investigations. Much work has of course been done on the lattice, but that is only one regularization. It must be generalized.

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