Canonical quantization in $n \cdot A = 0$ gauges

P. V. Landshoff

DAMTP, University of Cambridge, Cambridge, United Kingdom

P. van Nieuwenhuizen

*Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794
and Teyler Foundation, Instituut Lorentz, Leiden, The Netherlands*

We give a unified derivation of the propagator in the gauges $n \cdot A = 0$ for n^2 timelike, spacelike, or lightlike. We discuss the physical states and other physical questions.

PACS number(s): 11.15.Bt, 11.10.Ef

I. INTRODUCTION

Gauges of the type $n \cdot A = 0$ are widely used, with n either timelike, spacelike, or lightlike. They often simplify calculations, and give a more direct physical interpretation to Feynman diagrams. An example is the derivation of the Altarelli-Parisi equation for deep inelastic scattering [1], where the ladder graphs without crossed rungs dominate, or finite-temperature field theory where the heat bath already breaks the Lorentz invariance [2]. It is useful also to use an axial gauge for the renormalization of composite operators, to avoid mixing with nongauge-invariant operators (which in general contain ghosts) [3]. But it has been surprisingly difficult to derive correct perturbation theory for these gauges [4,5]. This is because the Feynman propagator naively is[1]

$$D_{\mu\nu}(k) = \left[-g_{\mu\nu} + \frac{k_\mu n_\nu + n_\mu k_\nu}{n \cdot k} - n^2 \frac{k_\mu k_\nu}{(n \cdot k)^2} \right] \frac{1}{k^2 + i\epsilon} \tag{1.1}$$

and one must decide how to integrate the pole and double pole at $n \cdot k = 0$. In this note we shall show that, in the gauges $A_0 + \lambda A_3 = 0$, for all λ and so regardless of whether n is timelike, spacelike, or lightlike, straightforward canonical quantization leads to

$$\text{``}n \cdot k\text{''}^{-1} = \frac{1}{k_0 + \lambda k_3 \pm i\epsilon / k_3} \tag{1.2}$$

with ϵ an infinitesimally small positive quantity.

We derive this result in Sec. II. In Sec. III we construct the physical states, and in Sec. IV we discuss various physical questions.

II. DERIVATION OF THE PROPAGATOR

To derive the propagator it is sufficient to consider the interaction-free case. That is, we work with the asymptotic *in* or *out* field, described by the Lagrangian $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Rather than adding a gauge-fixing term, we fix the gauge by eliminating A_0 and replacing it with $-\lambda A_3$. The resulting three field equations read

$$\ddot{A}_r + \lambda \partial_r \dot{A}_3 - \partial_j \partial_j A_r + \partial_r (\partial_j A_j) = 0 \quad (r = 1,2; j = 1,2,3) ,$$

$$\ddot{A}_3 + \lambda \partial_3 \dot{A}_3 + (\lambda^2 - 1)\partial_j \partial_j A_3 + \partial_3 (\partial_j A_j) + \lambda \partial_t (\partial_j A_j) = 0 . \tag{2.1}$$

These equations are easy to solve if one assumes that the solution may be written as a four-dimensional Fourier integral, with $e^{-ik \cdot x} = e^{-ik_0 t + i\mathbf{k} \cdot \mathbf{x}}$. Eliminating $\mathbf{k} \cdot \mathbf{A}(\mathbf{k})$ gives

$$(k_0 + \lambda k_3)^2 (k_0^2 - \mathbf{k}^2) A_3(k) = 0 \tag{2.2a}$$

or

$$(\partial_t + \lambda \partial_3)^2 (\partial_t^2 - \partial_j \partial_j) A_3(t, \mathbf{x}) = 0 . \tag{2.2b}$$

(We always use i and j to range over the values 1,2,3, and r and s to range over 1,2). The general real solution of this equation may be written

$$A_3(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} \left\{ \theta(k_3)[itp_3(\mathbf{k}) + q_3(\mathbf{k})]e^{i(\lambda k_3 t + \mathbf{k} \cdot \mathbf{x})} + \frac{1}{2|\mathbf{k}|} a_3(\mathbf{k})e^{-i(|\mathbf{k}|t - \mathbf{k} \cdot \mathbf{x})} \right\} + \text{H.c.} \tag{2.3}$$

The frequencies $-\lambda k_3$ in the first term range from $-\infty$ to $+\infty$, and so when we add on the Hermitian conjugate we have to include the $\theta(k_3)$ to avoid double counting.

---

[1]Our metric is $(+,-,-,-)$, so that $k^2 = k_0^2 - |\mathbf{k}|^2$, $n^2 = 1 - \lambda^2$, $\mathbf{k} \cdot \mathbf{x} = -k_1 x^1 - k_2 x^2 - k_3 x^3$, and $n \cdot k = k_0 + \lambda k_3$. Furthermore, $\mathbf{k} \cdot \mathbf{x} = k^j x^j$ and $\partial_j \partial_j = \partial_1^2 + \partial_2^2 + \partial_3^2$.

By making a similar decomposition of $A_r(t,\mathbf{x})$, but with functions $p_r(\mathbf{k}),q_r(\mathbf{k})$, and $a_r(\mathbf{k})$, and substituting these expressions for A_3 and A_r back into the field equations (2.1), one finds the general solution of the classical field equations:

$$A_r(t,\mathbf{x})=\int \frac{d^3k}{(2\pi)^3}\left\{\theta(k_3)k_r[it(\lambda^2-1)p(\mathbf{k})+q(\mathbf{k})]e^{i(\lambda k_3 t+\mathbf{k}\cdot\mathbf{x})}+\frac{1}{2|\mathbf{k}|}a_r(\mathbf{k})e^{-i(|\mathbf{k}|t-\mathbf{k}\cdot\mathbf{x})}\right\}+\text{H.c.} \qquad (2.4a)$$

$$A_3(t,\mathbf{x})=\int \frac{d^3k}{(2\pi)^3}\left\{\theta(k_3)([it(\lambda^2-1)k_3+\lambda]p(\mathbf{k})+k_3q(\mathbf{k}))e^{i(\lambda k_3 t+\mathbf{k}\cdot\mathbf{x})}-\frac{1}{2|\mathbf{k}|}\frac{k_r a_r(\mathbf{k})}{\lambda|\mathbf{k}|+k_3}e^{-i(|\mathbf{k}|t-\mathbf{k}\cdot\mathbf{x})}\right\}+\text{H.c.} \qquad (2.4b)$$

To quantise, we construct the canonically conjugate momenta $\pi_i=\dot{A}_i-\lambda\partial_i A_3$:

$$\pi_r(t,\mathbf{x})=-i\int \frac{d^3k}{(2\pi)^3}\left\{\theta(k_3)k_r p(\mathbf{k})e^{i(\lambda k_3 t+\mathbf{k}\cdot\mathbf{x})}+\frac{1}{2|\mathbf{k}|}\left[|\mathbf{k}|\delta_{rs}-\frac{\lambda k_r k_s}{\lambda|\mathbf{k}|+k_3}\right]a_s(\mathbf{k})e^{-i(|\mathbf{k}|t-\mathbf{k}\cdot\mathbf{x})}\right\}+\text{H.c.} \qquad (2.5a)$$

$$\pi_3(t,\mathbf{x})=i\int \frac{d^3k}{(2\pi)^3}\left\{\theta(k_3)k_3 p(\mathbf{k})(\lambda^2-1)e^{i(\lambda k_3 t+\mathbf{k}\cdot\mathbf{x})}+\frac{1}{2|\mathbf{k}|}\left[\frac{|\mathbf{k}|+\lambda k_3}{\lambda|\mathbf{k}|+k_3}\right]k_r a_r(\mathbf{k})e^{-i(|\mathbf{k}|t-\mathbf{k}\cdot\mathbf{x})}\right\}+\text{H.c.} \qquad (2.5b)$$

With these solutions, the equal-time commutators involve terms with t^2,t, and $\theta(k_3)$. In order to obtain the canonical form, the t^2 and t terms should vanish, whereas the $\theta(k_3)$ should combine with $\theta(-k_3)$ to yield unity. The solution to all these conditions is

$$[a_r(\mathbf{k}),a_s^\dagger(\mathbf{k}')]=(2\pi)^3\delta(\mathbf{k}-\mathbf{k}')2|\mathbf{k}|\left\{\delta_{rs}+(\lambda^2-1)\frac{k_r k_s}{(|\mathbf{k}|+\lambda k_3)^2}\right\},$$

$$[p(\mathbf{k}),p^\dagger(\mathbf{k}')]=0,$$

$$[q(\mathbf{k}),q^\dagger(\mathbf{k}')]=(2\pi)^3\delta(\mathbf{k}-\mathbf{k}')\lambda(\lambda^2-1)2k_3\left[\frac{1}{|\mathbf{k}|^2-\lambda^2 k_3^2}\right]^2, \qquad (2.6)$$

$$[q(\mathbf{k}),p^\dagger(\mathbf{k}')]=(2\pi)^3\delta(\mathbf{k}-\mathbf{k}')\frac{1}{|\mathbf{k}|^2-\lambda^2 k_3^2}.$$

Further, the a_r sector commutes with the p,q sector.

In order to derive the Feynman propagator, we must define the vacuum. The issue of what vacua are allowed, and how to define the other physical states in the gauges $n\cdot A=0$, is considered in detail in the next section. Meanwhile, we choose one of the allowed vacua, defined by

$$a_r(\mathbf{k})|0\rangle=0,\quad p(\mathbf{k})|0\rangle=0,\quad q(\mathbf{k})|0\rangle=0. \qquad (2.7)$$

Then, for $x^0>y^0$,

$$\langle 0|TA_r(x^0,\mathbf{x})A_s(y^0,\mathbf{y})|0\rangle=\int \frac{d^3k}{(2\pi)^3}e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}\left[\left\{\delta_{rs}+\frac{(\lambda^2-1)k_r k_s}{(|\mathbf{k}|+\lambda k_3)^2}\right\}\frac{1}{2|\mathbf{k}|}e^{-i|\mathbf{k}|(x^0-y^0)}\right.$$

$$+e^{i\lambda k_3(x^0-y^0)}\theta(k_3)\left\{i(x^0-y^0)(\lambda^2-1)\frac{k_r k_s}{|\mathbf{k}|^2-\lambda^2 k_3^2}\right.$$

$$\left.\left.+\frac{2k_r k_s k_3\lambda(\lambda^2-1)}{(|\mathbf{k}|^2-\lambda^2 k_3^2)^2}\right\}\right], \qquad (2.8a)$$

$$\langle 0|TA_r(x^0,\mathbf{x})A_3(y^0,\mathbf{y})|0\rangle=\int \frac{d^3k}{(2\pi)^3}e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}\left[\left\{\frac{-k_r(\lambda|\mathbf{k}|+k_3)}{(|\mathbf{k}|+\lambda k_3)^2}\right\}\frac{1}{2|\mathbf{k}|}e^{-i|\mathbf{k}|(x^0-y^0)}\right.$$

$$+e^{i\lambda k_3(x^0-y^0)}\theta(k_3)\left\{i(x^0-y^0)(\lambda^2-1)\frac{k_r k_3}{|\mathbf{k}|^2-\lambda^2 k_3^2}+\frac{2k_r k_3^2\lambda(\lambda^2-1)}{(|\mathbf{k}|^2-\lambda^2 k_3^2)^2}\right.$$

$$\left.\left.+\frac{\lambda k_r}{|\mathbf{k}|^2-\lambda^2 k_3^2}\right\}\right], \qquad (2.8b)$$

$$\langle 0 | T A_3(x^0, \mathbf{x}) A_3(y^0, \mathbf{y}) | 0 \rangle = \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})} \left[\left\{ \frac{k_s k_s}{(|\mathbf{k}| + \lambda k_3)^2} \right\} \frac{1}{2|\mathbf{k}|} e^{-i|\mathbf{k}|(x^0-y^0)} \right.$$

$$+ e^{i\lambda k_3(x^0-y^0)} \theta(k_3) \left\{ i(x^0-y^0)(\lambda^2-1) \frac{k_3^2}{|\mathbf{k}|^2 - \lambda^2 k_3^2} + \frac{2k_3^3 \lambda(\lambda^2-1)}{(|\mathbf{k}|^2 - \lambda^2 k_3^2)^2} \right.$$

$$\left. \left. + \frac{2\lambda k_3}{|\mathbf{k}|^2 - \lambda^2 k_3^2} \right\} \right] . \tag{2.8c}$$

For $y^0 > x^0$ we simply interchange the four-vectors x and y.

For Feynman diagrams, we must convert the three-dimensional momentum integral into a four-dimensional integral. Consider first the $(x^0 - y^0)$ term in (2.8a). If we make the replacement

$$e^{i\lambda k_3(x^0-y^0)} \theta(k_3) i(x^0-y^0)(\lambda^2-1) \frac{k_r k_s}{|\mathbf{k}|^2 - \lambda^2 k_3^2} \rightarrow \frac{i}{2\pi} \int dk_0 e^{-ik_0(x^0-y^0)} \frac{(\lambda^2-1)k_r k_s}{(k_0 + \lambda k_3 + i\epsilon/k_3)^2} \frac{1}{(k_0^2 - |\mathbf{k}|^2) + i\epsilon} \tag{2.9}$$

with ϵ an infinitesimal positive quantity, the $i\epsilon/k_3$ correctly reproduces the $\theta(k_3)$ when we close the contour of the k_0 integration. The residue of the double pole reproduces the left-hand side of (2.9), together with another term obtained by differentiating the last factor with respect to k_0. This latter term agrees with the last term in (2.8a). There are also the poles at $k_0 = \pm |\mathbf{k}|$, whose residue reproduces terms involving the other exponential in (2.8a). We could proceed by combining them with a similar term obtained by replacing the factor

$1/2|\mathbf{k}|$ in the first term of (2.8a) by a contour integral over k_0 involving again $(k_0^2 - |\mathbf{k}|^2) + i\epsilon)^{-1}$. However, at this point it is faster to note that the denominator $(k_0 + \lambda k_3 + i\epsilon/k_3)^{-2}$ is just one of the two possible forms of "$(n \cdot k)$"$^{-2}$ as given in (1.2). This suggests that all propagators in (2.8) are given by four-dimensional Fourier transforms of (1.1) with this interpretation of the denominators. This may be verified explicitly. To obtain also the propagators involving A_0 we write it as $-\lambda A_3$. Hence, finally (for the vacuum defined in (2.7))

$$\langle 0 | T A_\mu(x^0, \mathbf{x}) A_\nu(y^0, \mathbf{y}) | 0 \rangle = i \int \frac{d^4 k}{(2\pi)^4} e^{-ik(x-y)} \left[-g_{\mu\nu} + \frac{k_\mu n_\nu + n_\mu k_\nu}{n \cdot k + i\epsilon/k_3} - n^2 \frac{k_\mu k_\nu}{(n \cdot k + i\epsilon/k_3)^2} \right] \frac{1}{k^2 + i\epsilon} . \tag{2.10}$$

For $\lambda < 0$ Wick rotation is possible; for $\lambda > 0$ see Sec. IV.

III. DEFINITION OF PHYSICAL STATES

A general approach to the definition of the physical states is in terms of the Becchi-Rouet-Stora-Tyutin (BRST) operator. When, as we have done, one of the fields has been eliminated, an alternative is to use the lost field equation (here the Gauss law) as a constraint that helps to pick out the physical states. To derive the BRST operator we write the full Lagrangian in terms of the Heisenberg fields, with a gauge-fixing term involving an auxiliary field B and with ghost fields:

$$\mathcal{L} = -\tfrac{1}{4} F_{a\mu\nu} F^{a\mu\nu} + B_a n \cdot A^a + b_a n^\mu (D_\mu c)^a , \tag{3.1}$$

where $(D_\mu c)^a = \partial_\mu c^a + g f_{bc}^a A_\mu^b c^c$. We take the auxiliary field B and the ghost c to be Hermitian; then the antighost b must be anti-Hermitian in order to make $\mathcal{L}$ Hermitian. The rigid BRST symmetry of $\mathcal{L}$ under the transformations

$$\delta A_\mu^a = (D_\mu c)^a \Lambda, \quad \delta b^a = \Lambda B^a ,$$

$$\delta c^a = \tfrac{1}{2} g f_{bc}^a c^b c^c \Lambda, \quad \delta B^a = 0 \tag{3.2}$$

leads to the BRST charge as the space integral of the time component of the corresponding Noether current.

Adding $-c^a$ times the field equation of A_μ^a to the Noether current, the BRST charge becomes

$$Q = \int d^3 x [B_a c^a - \tfrac{1}{2} b_a g f_{bc}^a c^b c^c] . \tag{3.3a}$$

There is also a conserved ghost charge

$$Q_{gh} = \int d^3 x [b_a c^a] . \tag{3.4}$$

Redefining the auxiliary field B_a by

$$d_a = B_a + b_b g f_{ac}^b c^c \tag{3.5}$$

the fields d_a, b_a, and c^a all satisfy free-field equations

$$n \cdot \partial d_a = n \cdot \partial b_a = n \cdot \partial c^a = 0 . \tag{3.6}$$

The BRST charge becomes

$$Q = \int d^3 x [d_a c^a + \tfrac{1}{2} b_a g f_{bc}^a c^b c^c] \tag{3.3b}$$

and in this form Q and Q_{gh} are manifestly conserved. The fields d_a, b_a, and c^a can be expanded as

$$d_a(t, \mathbf{x}) = \int \frac{d^3 k}{(2\pi)^3} \theta(k_3) d_a(\mathbf{k}) e^{i(\lambda k_3 t + \mathbf{k} \cdot \mathbf{x})} + \text{H.c.} ,$$

$$b_a(t, \mathbf{x}) = \int \frac{d^3 k}{(2\pi)^3} \theta(k_3) b_a(\mathbf{k}) e^{i(\lambda k_3 t + \mathbf{k} \cdot \mathbf{x})} - \text{H.c.} ,$$

$$c^a(t, \mathbf{x}) = \int \frac{d^3 k}{(2\pi)^3} \theta(k_3) c^a(\mathbf{k}) e^{i(\lambda k_3 t + \mathbf{k} \cdot \mathbf{x})} + \text{H.c.} \tag{3.7}$$

Using Dirac brackets [6], the canonical equal-time commutation relations yield

$$\{c^a(\mathbf{k}), b_b^\dagger(\mathbf{k}')\} = -i\delta_b^a(2\pi)^3\delta(\mathbf{k}-\mathbf{k}') \ ,$$

$$\{c^a(\mathbf{k}), c^{b\dagger}(\mathbf{k}')\} = 0 = \{b_a(\mathbf{k}), b_b^\dagger(\mathbf{k}')\} \ . \tag{3.8}$$

The field equation for d_a is just our gauge condition $A_0 + \lambda A_3 = 0$, and the field equation for A_0^a reads $d_a = (D_j F_{j0})_a$. Further, because the ghosts decouple, we may work in a subspace of the Fock space where the kets contain only the ghost vacuum:

$$| \ \rangle = |\text{nonghost}\rangle \otimes |0 \text{ ghost}\rangle \quad \text{where } b_a(\mathbf{k})|0 \text{ ghost}\rangle = c^a(\mathbf{k})|0 \text{ ghost}\rangle = 0 \ . \tag{3.9}$$

We shall later consider other possible ghost vacua. In this subspace we may omit the last term in Q, because when we express it in terms of creation and annihilation operators each term contains at least one ghost or antighost annihilation operator. Hence

$$Q = \int d^3x \, c^a(\mathbf{x})(D_i F_{i0})_a = -\int d^3x \, c^a(\mathbf{x})(D_i \pi_i)_a \ . \tag{3.3c}$$

Notice that the missing field equation (the Gauss law) is $(D_i F_{i0})^a = 0$, so if this equation were satisfied Q would vanish. In fact, the Gauss law will be satisfied only as a weak condition, as we now discuss.

As has been first proposed by Kugo and Ojima [7] we require that physical states be annihilated by Q. This generalizes the Gupta-Bleuler condition of QED to general gauges and to non-Abelian fields. As in the previous section, we now pass from the Heisenberg field to the asymptotic *in* or *out* field. We shall omit the color index on the field. In the BRST operator only the terms quadratic in the fields remain, and the terms involving g disappear [7]. So now, using (2.5), up to an overall renormalization [7],

$$Q = \int \frac{d^3k}{(2\pi)^3} \theta(k_3)c^\dagger(\mathbf{k})(|\mathbf{k}|^2 - \lambda^2 k_3^2)p(\mathbf{k}) + \text{H.c.} \tag{3.3d}$$

The terms involving a_r cancel. When we apply Q to a ket in our subspace (3.9), the term with $c(\mathbf{k})$ in (3.3d) vanishes. The kets that are annihilated by Q are those that are annihilated by $p(\mathbf{k})$, for all $\mathbf{k}$ (except when $\mathbf{k}^2 = \lambda^2 k_3^2$). It follows that the Gauss law holds in the weak sense: $\langle A|\partial_i F_{i0}|B\rangle = 0$ whenever the kets A and B are annihilated by Q.

In general it may be shown [7] that the solutions to $Q| \ \rangle = 0$ have either positive norm, or zero norm. The physical states have positive norm, whereas the zero norm states have a vanishing inner product with the physical states and with each other. There may be other conditions required for a state to be physical, for example in QCD it must have zero color. Further, given any ket that represents a physical state, there are an infinite number of other kets, differing from it by a piece that is annihilated by Q and has zero norm, all of which represent the same physical state. In our case, an example of a zero norm state that satisfied $Q| \ \rangle = 0$ is $p^\dagger|0\rangle$, while states created by the $a_r^\dagger$ have positive norm. The standard representative kets for the physical states are obtained by applying a product of $a_r^\dagger$ to the vacuum, and adding further pieces where one or more $p^\dagger$ is applied to such kets represents the same physics. The proof that the norm in the subspace $Q| \ \rangle = 0$ is semipositive-definite relies on the *quartet mechanism* of Kugo and Ojima [7]. A quartet

consists of two BRST doublets with opposite ghost number. In our case, the quartet modes are given by $c(\mathbf{k}), b(\mathbf{k}), p(\mathbf{k})$, and $q(\mathbf{k})$. Indeed, under BRST transformations of the *in* or *out* field $\delta q(\mathbf{k}) \sim c(\mathbf{k})$ and $\delta c(\mathbf{k}) = 0$ since $\delta A_j = \partial_j c$, and $\delta b(\mathbf{k}) \sim B(\mathbf{k}) = d(\mathbf{k}) \sim p(\mathbf{k})$ and $\delta p(\mathbf{k}) = 0$, while $c(\mathbf{k})$ and $b(\mathbf{k})$ have opposite ghost number. All kets in the asymptotic-field Fock space satisfying $Q| \ \rangle = 0$ and $Q_{\text{gh}}| \ \rangle = 0$ consist then of the set with the ghost vacuum (which we have been discussing so far), and further zero norm states with ghost number zero constructed from the quartet modes. An example is the ket

$$[c^\dagger b^\dagger - i(|\mathbf{k}|^2 - \lambda^2 k_3^2)q^\dagger p^\dagger]|\text{nonghost}\rangle \otimes |0 \text{ ghost}\rangle \ .$$

IV. DISCUSSION

(1) We require that the Fock vacuum $|0\rangle$ is a physical state. One allowed vacuum was defined in (2.7) and (3.9). An obvious alternative [8], which is also annihilated by Q, is to replace p, q, b, and c in the definition with their Hermitian conjugates. This has the effect of changing $i\epsilon/k_3$ in the propagator (2.10) to $-i\epsilon/k_3$. If $\lambda < 0$ the first choice is more convenient, and if $\lambda > 0$ the second, because then Wick rotation is possible.

(2) We require the vacuum also to have unit norm. We have seen in Sec. III that if we define the ghost vacuum to be annihilated by b and c, the nonghost vacuum must be annihilated by p. It is not consistent to require that it be annihilated also by $p^\dagger(\mathbf{k})$, since this would conflict with the vacuum expectation value of $[p, q^\dagger]$. It is not possible to impose the condition that the whole of the Gauss-law operator $D_i F_{i0}$ annihilates the vacuum (or, indeed, other physical states) because its equal-time commutator with A_i is a c number.

(3) Physical states should have positive energy. The free-field Hamiltonian H_0, which governs the time variation of the asymptotic fields according to $\dot{\Phi} = i[H_0, \Phi]$, consists of a nonghost part $H_{0a} + H_{0pq}$ and a ghost part. The former is given by

$$H_{0a} = \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{1}{2} a_r^\dagger(\mathbf{k}) a_r(\mathbf{k}) \right.$$

$$\left. + \frac{(1/2)(1-\lambda^2)k_r k_s}{(\lambda|\mathbf{k}| + k_3)^2} a_r^\dagger(\mathbf{k}) a_s(\mathbf{k}) \right\} \tag{4.1a}$$

and

$$H_{0pq} = \int \frac{d^3k}{(2\pi)^3} \theta(k_3)\{(1-\lambda^2)(|\mathbf{k}|^2 - 3\lambda^2 k_3^2)p^\dagger(\mathbf{k})p(\mathbf{k}) - \lambda k_3(|\mathbf{k}|^2 - \lambda^2 k_3^2)[q^\dagger(\mathbf{k})p(\mathbf{k}) + p^\dagger(\mathbf{k})q(\mathbf{k})]\} \qquad (4.1b)$$

while the ghost part is

$$H_{0bc} = \int \frac{d^3k}{(2\pi)^3} \theta(k_3)i\lambda k_3\{b^\dagger(\mathbf{k})c(\mathbf{k}) - c^\dagger(\mathbf{k})b(\mathbf{k})\} \ . \qquad (4.1c)$$

As a check, one may verify that Q in (3.3d) commutes with H_0. If we define the vacuum $|0\rangle$ to be annihilated by Q, so that $p(\mathbf{k})|0\rangle = 0$, and also to be an eigenket of H_0 with eigenvalue 0, then we need also $q(\mathbf{k})|0\rangle = 0$ as we have required in (2.7). As we have already argued, the ket $|0\rangle p^+(\mathbf{k})|$physical$\rangle$ represents the same physics as $|0\rangle$. However, it is not an eigenstate of H_0, although it gives the same expectation value for H_0 as does $|0\rangle$. The same remarks apply to all physical states. The standard representative kets for the physical states are annihilated by $q(\mathbf{k})$ and are eigenvectors of H_0. One may verify that their eigenvalues are all positive, whatever the value of λ. For example, the state of $a_r^\dagger(\mathbf{k})|0\rangle$ has energy $|\mathbf{k}|$.

(4) If we use the expansion (2.4) of A_i, together with $A_0 = -\lambda A_3$, we see that the terms involving $q(\mathbf{k})$ may be removed by a gauge transformation $A_\mu \to A_\mu + \partial_\mu \Omega$ with $(\partial_t + \lambda\partial_3)\Omega = 0$. Nevertheless we must retain these $q(\mathbf{k})$ modes in the formalism, just as one must keep the longitudinal polarizations of QED in the Feynman gauge.

(5) In our unified treatment, we have seen no basic difference between the cases when n^2 is positive, zero, or negative. For example, in all cases one can perform a Wick rotation. Note, however, that our treatment does not apply to the gauge $A_3 = 0$, though an alternative approach exists for this [9].

(6) It would be interesting to construct the Poincaré generators and investigate how Lorentz boosts relate the results for different λ. At first sight it is not clear how propagators with $i\epsilon/k_3'$ are related to our propagators with $i\epsilon/k_3$. Perhaps such a relation could extend our analysis to the case $A_3 = 0$.

(7) In the case of the temporal gauge $A_0 = 0$ our results agree with previous work [8,10]. But for the light-cone gauge, $\lambda = \pm 1$, we have not retrieved the Leibbrandt-Mandelstam prescription [11] for the propagator: we have $i\epsilon/k_3$ rather than $i\epsilon/(k_3 - k_0)$. Bassetto et al. [12] have given a derivation of the light-cone gauge propaga-

tor which is very similar to ours (when $\lambda^2 = 1$); at a certain point, however, we each have to make an assumption that certain poles whose residues are of order ϵ may be omitted, and we choose different ones. In our analysis, when we pass from (2.8) to (2.10) we have dropped some terms which, superficially, are of order ϵ. Bassetto et al. replace

$$\frac{1}{k_0 - k_3 + i\epsilon\,\mathrm{sgn}k_3} \to \frac{k_0 + k_3}{k_0^2 - k_3^2 + i\epsilon} \ . \qquad (4.2)$$

Such replacements are usually valid when a Wick rotation can be used, but in a delicate calculation such as that of the Wilson loop by Andrasi and Taylor [13] the difference may be important. It is this subtlety [5] which has so far made it impossible to produce a reliable derivation of perturbation theory in $n \cdot A = 0$ gauges.

Note added. It may be useful to repeat that our result for the propagator in $n \cdot A = 0$ gauges (except the case $A_3 = 0$ which is not covered), although derived in a totally straightforward way without any assumptions, differs from that of other authors by terms of order ϵ (the ϵ in $k^2 - i\epsilon$) in the numerator. As usual there is some freedom at the free-field level, and usually such ϵ terms can be omitted, but one has to be careful because the propagator has several poles, and an explicit higher loop calculation in some interacting theory would be welcome. We have identified the subsection of physical states using minimal BRST methods (for example, we do not add a kinetic term for the auxiliary field), but we note that also different vacua can lead to different propagators. Furthermore, we want to make it clear that the canonical formulation of the BRST symmetry has been investigated by many people. The form (3.3c) of the BRST charge is standard (first class constraints times ghosts + more), and in general the BRST charge does not depend on the choice of gauge, as confirmed (3.3c). Also the BRST cohomology for (3.3c) and the crucial role played by the quartet mechanism is well appreciated by workers in the field.

[1] R. G. Robert, *The Structure of the Proton* (Cambridge University Press, Cambridge, England, 1990).
[2] U. Heinz, K. Kajantie, and T. Toimela, Ann. Phys. (N.Y.) **176**, 218 (1987).
[3] W. Furmanski and R. Petronzio, Phys. Lett. B **97**, 437 (1980); W van Neerven, Z. Phys. C **14**, 241 (1982).
[4] S. Carracciolo, G. Curci, and P. Menotti, Phys. Lett. **113B**, 311 (1982).
[5] P. V. Landshoff, review talk in *Proceeding of the Workshop on Nonstandard Gauges*, Lecture Notes in Physics (Springer, Berlin, 1989).
[6] P. A. M. Dirac, *Lectures on Quantum Mechanics* (Yeshiva

University, New York, 1964).
[7] T. Kugo and I. Ojima, Phys. Lett. **73B**, 459 (1978).
[8] P. V. Landshoff, Phys. Lett. B **227**, 427 (1989).
[9] P. V. Landshoff and J. C. Taylor, Phys. Lett. B **231**, 129 (1989).
[10] I. Lazzizzera, Phys. Lett. B **210**, 188 (1988).
[11] S. Mandelstam, Nucl. Phys. **B231**, 149 (1983); G. A. Leibbrandt, Rev. Mod. Phys. **59**, 1067 (1987).
[12] A. Bassetto, M. Dalbosco, I. Lazzizzera, and R. Soldati, Phys. Rev. D **31**, 2012 (1985).
[13] A. Andrasi and J. C. Taylor, Nucl. Phys. **B375**, 341 (1992).