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Coherent states in null-plane QED

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Light front field theories are known to have the usual infrared divergences of the equal time theories, as well as new "spurious" infrared divergences. The former kind of IR divergences are usually treated by giving a small mass to the gauge particle. An alternative method to deal with these divergences is to calculate the transition matrix elements in a coherent state basis. In this paper we present, as a model calculation, the lowest order correction to the three point vertex in QED using a coherent state basis in the light cone formalism. The relevant transition matrix element is shown to be free of the true IR divergences up to $O(e^2)$.

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I. INTRODUCTION

Light front field theories have been the subject of considerable interest in the past few years as they promise to provide a practical tool for solving the problem of highly relativistic bound states [1,2]. There are two major approaches to obtaining the bound state wave functions in the light-cone framework, the light front Tamm-Dancoff (LFTD) method [1] and the discretized light cone quantization (DLCQ) method [2], both based on diagonalizing the light cone Hamiltonian in the basis of Fock states. Both of these approaches are beset by the usual ultraviolet divergences of field theory coming from large transverse momenta, as well as by infrared (IR) divergences near $k^+ = 0$, where $k^+ = k^0 + k^1$ is the longitudinal momentum. The infrared divergences can be classified into two different categories—"spurious" IR divergences and "true" IR divergences. The spurious IR divergences are just a manifestation of the ultraviolet divergences of equal time theory and can be regularized by an IR cutoff on small values of longitudinal momentum [3,4]. The "true" IR divergences are the bonafide infrared divergences of the equal time theory and are present due to particles being on mass shell. These can be taken care of by giving the photon a small mass [3,4].

In the present work we suggest an alternative treatment of this latter kind of IR divergence. Addressing this excercise may seem unnecessary in the case of QED, since a simple solution (i.e., giving photon a small mass) already exists. However, the suggested formalism may turn out to be useful in future work on non-Abelian gauge theories, where giving mass to the gauge particles violates gauge invariance.

Both the LFTD theory and DLCQ are based on the old-fashioned Hamiltonian perturbation theory, wherein one calculates the matrix elements of the light front Hamiltonian between Fock states. In the present paper, we propose another set of basis states, the coherent states, to calculate these matrix elements. The usefulness of coherent states in the proof of cancellation of IR divergences in equal time theories has been well established [5,6]. Chung has shown that the IR divergences of QED are eliminated to all orders in perturbation theory in the matrix elements by an appropriate choice of initial and final soft photon states [5]. Kulish and Faddeev investigated the asymptotic behavior of the QED Hamiltonian and showed that when a particle mass becomes negligible compared to the energy scale, the asymptotic Hamiltonian need not coincide with the free one. This leads to a redefinition of asymptotic states [6,7]:

$$|n;\pm\rangle = \Omega^A_\pm |n\rangle \ , \qquad (1)$$

where $\Omega^A_\pm$ is the asymptotic evolution operator and $|n\rangle$ is a Fock state. They further showed the cancellation of IR divergences when the matrix elements were calculated between these coherent states. In the light front theories, one can make use of this property of coherent states to separate the two kinds of IR divergences.

As a first step in this direction, we have, in this paper, calculated the lowest order radiative correction to the three point vertex in null plane QED using the coherent state basis. This calculation has been done in Ref. [3], henceforth referred to as I, in the Fock state basis. The main subjects of I are the ultraviolet and spurious IR divergences, and the problem of "true" IR divergences has been treated by giving the photon a small mass. We shall use their expression for the vertex correction *without* giving the photon a mass, and show that the true IR divergences present in this expression are canceled in the coherent state basis by additional contributions coming from the emission and absorption of soft photons.

The present formalism has been developed for continuum light cone QED. However, if applicable to the discretized case, it may provide a natural way of eliminating the troublesome $k^+ = 0, k_\perp = 0$ state from the DLCQ calculations [4,9]. In discretized light-cone QED, one has to eliminate the $k^+ = 0$, $k_\perp = 0$ state by an artificial IR cutoff and then one has to add a Coloumb counter term in order to obtain convergence. The present work was inspired by the hope that the coherent state formalism developed here may provide a natural cutoff on small values of photon momenta, which on discretization will eliminate the $k^+ = 0$, $k_\perp = 0$ state.

The plan of the paper is as follows. In Sec. II, for the sake of completeness we shall set our notation and present the relevant result of I. In Sec. III, we shall define the asymptotic region and coherent states in the light-cone framework. Section IV presents the main result of this paper. We calculate the transition matrix element between coherent states and show the cancellation of true IR divergences. Section V contains some concluding remarks regarding the possible usefulness of coherent state formalism in DLCQ calculations. Appendix A contains some useful properties of coherent states and Appendix B presents the details of the calculation in Sec. IV.

II. PRELIMINARIES

We shall use the notation of I.

The light-cone QED Hamiltonian is given by [3]

$$P^- = H \equiv H_0 + V_1 + V_2 + V_3 \,, \tag{2}$$

where

$$H_0 = \int d^2x_\perp dx^- \left(\frac{i}{2}\bar{\xi}\gamma^- \overleftrightarrow{\partial}_- \xi + \frac{1}{2}(F_{12})^2 - \frac{1}{2}a_+\partial_-\partial_k a_k \right) \tag{3}$$

is the free Hamiltonian as a function of independent degrees of freedom, and

$$V_1 = e\int d^2x_\perp dx^- \bar{\xi}\gamma^\mu \xi a_\mu \tag{4}$$

is the three point vertex interaction. V_2 and V_3 are nonlocal effective four point vertices corresponding to instantaneous fermion and photon exchange, respectively. Detailed expressions for V_2 and V_3 are not needed for our purposes, and can be found in I.

ξ and a_μ can be expanded in terms of creation and annihilation operators as usual:

$$\xi(x) = \int \frac{d^2p_\perp}{(2\pi)^{\frac{3}{2}}} \int \frac{dp^+}{\sqrt{2p^+}} \sum_{s=\pm\frac{1}{2}} [u(p,s)e^{-i(p^+x^- - p_\perp x_\perp)}b(p,s,x^+) + v(p,s)e^{i(p^+x^- - p_\perp x_\perp)}d^\dagger(p,s,x^+)] \,, \tag{5}$$

$$a_\mu(x) = \int \frac{d^2q_\perp}{(2\pi)^{\frac{3}{2}}} \int \frac{dq^+}{\sqrt{2q^+}} \sum_{\lambda=1,2} \epsilon_\mu^\lambda(q)[e^{-i(q^+x^- - q_\perp x_\perp)}a(q,\lambda,x^+) + e^{i(q^+x^- - q_\perp x_\perp)}a^\dagger(q,\lambda,x^+)] \,, \tag{6}$$

where

$$\{b(p,s), b^\dagger(p',s')\} = \delta(p^+ - p'^+)\delta^2(p_\perp - p'_\perp)\delta_{s,s'} = \delta^3(p-p')\delta_{s,s'} \,, \tag{7}$$

$$\{d(p,s), d^\dagger(p',s')\} = \delta(p^+ - p'^+)\delta^2(p_\perp - p'_\perp)\delta_{s,s'} = \delta^3(p-p')\delta_{s,s'} \,, \tag{8}$$

$$[a(q,\lambda), a^\dagger(q',\lambda')] = \delta(q^+ - q'^+)\delta^2(q_\perp - q'_\perp)\delta_{\lambda,\lambda'} = \delta^3(q-q')\delta_{\lambda,\lambda'} \,. \tag{9}$$

The Hamiltonian can be expressed in terms of annihilation and creation operators. For example,

$$V_1 = e\int d^2x_\perp dx^- \int [dp][d\bar{p}][dk] \sum_{s,s',\lambda} [e^{i\bar{p}\cdot x}\bar{u}(\bar{p},s')b^\dagger(\bar{p},s') + e^{-i\bar{p}\cdot x}\bar{v}(\bar{p},s')d(\bar{p},s')] \times\gamma^\mu[e^{-ip\cdot x}u(p,s)b(p,s) + e^{ip\cdot x}v(p,s)d(p,s)^\dagger]\epsilon_\mu^\lambda(k)[e^{-ik\cdot x}a(k,\lambda) + e^{ik\cdot x}a^\dagger(k,\lambda)] \,, \tag{10}$$

where

$$\int [dp] \equiv \int_{-\infty}^{\infty} \frac{d^2p_\perp}{(2\pi)^{\frac{3}{2}}} \int_0^\infty \frac{dp^+}{\sqrt{2p^+}} . \tag{11}$$

Similar expressions for V_2 and V_3 can be obtained and are given in Appendix A of Ref. [3].

In perturbative light cone QED, all graphs are matrix elements of the transition matrix T given by [3]

$$T = V + V\frac{1}{p^- - H_0}V + \cdots \tag{12}$$

between the Fock states. For example, the lowest order correction to the three point vertex $\Lambda^\mu(p,\bar{p})$ is given by the matrix element

$$T_{21} = \epsilon^\lambda_\mu(q)\Lambda^\mu(p,\bar{p})$$
$$= \left\langle \bar{p},\sigma,q,\lambda \middle| V_1 \frac{1}{p^- - H_0} V_1 \frac{1}{p^- - H_0} V_1 \middle| p,s \right\rangle + \left\langle \bar{p},\sigma,q,\lambda \middle| V_2 \frac{1}{p^- - H_0} V_1 \middle| p,s \right\rangle$$
$$+ \left\langle \bar{p},\sigma,q,\lambda \middle| V_3 \frac{1}{p^- - H_0} V_1 \middle| p,s \right\rangle , \quad (13)$$

where $|1\rangle = |p,s\rangle$ is the Fock state containing a single fermion and $|2\rangle = |\bar{p},\sigma,q,\lambda\rangle$ is the Fock state containing one fermion and one photon.

The full set of diagrams corresponding to the above matrix element is given in Ref. [3]. We shall limit ourselves to the calculation of $\Lambda^+(p,\bar{p})$, in which case many of the diagrams do not contribute due to their tensor structure. The only diagrams contributing to $\Lambda^+(p,\bar{p})$ are shown in Fig. 1. Calculation of the diagrams in Fig. 1 has been done in I. Later we shall make one further simplification; i.e., we shall consider only the $q = 0$ case. In this case, only the diagram in Fig. 1(a) contributes to the matrix element in question [3].

One can calculate the diagram in Fig. 1(a) from

$$T_{21} = \left\langle p',\sigma,q,\lambda \middle| V_1 \frac{1}{p^- - H_0} V_1 \frac{1}{p^- - H_0} V_1 \middle| p,s \right\rangle \quad (14)$$

by substituting V_1 from Eq. (10). Using the relations

$$d_{\mu\nu}(p) = \sum_{\lambda=1,2} \epsilon^\lambda_\mu(p)\epsilon^\lambda_\nu(p) = -g_{\mu\nu} + \frac{\delta_{\mu+}p_\nu + \delta_{\nu+}p_\mu}{p^+} \quad (15)$$

and

$$\sum_{s=\pm 1/2} u(p,s)\bar{u}(p,s) = \not{p} + m , \quad (16)$$

one obtains, after a straightforward calculation [3],

$$\Lambda^\mu_a(p,\bar{p}) = \lambda e^3 \int \frac{d^2k_\perp}{(4\pi)^3} \int \frac{dk^+}{k^+k'^+k''^+} \frac{N^\mu_a + yN^\mu_b}{(p^- - k^- - k'^-)(p^- - k^- - k''^- - q^-)} , \quad (17)$$

where

$$N^\mu_a + yN^\mu_b = \bar{u}(\bar{p},\sigma)\gamma^\alpha(\not{k}' + m) \times \gamma^\beta d_{\alpha\beta}(\not{k}'' + m)\gamma^\mu u(p,s) , \quad (18)$$

and

$$\lambda^{-1} = (2\pi)^{\frac{3}{2}} \sqrt{2p^+}\sqrt{2\bar{p}^+}\sqrt{2q^+} . \quad (19)$$

Reparametrizing the momentum variables as

$$k = \left(xp^+, \frac{(xp_\perp + k_\perp)^2}{2p^+x}, xp_\perp + k_\perp \right) , \quad (20)$$

$$q = y\left(p^+, \frac{p_\perp^2}{2p^+}, p_\perp \right) , \quad (21)$$

$$k' = \left((1-x)p^+, \frac{[(1-x)p_\perp - k_\perp]^2 + m^2}{2(1-x)p^+}, (1-x)p_\perp - k_\perp \right) , \quad (22)$$

$$k'' = \left((1-x-y)p^+, \frac{[(1-x-y)p_\perp - k_\perp]^2 + m^2}{2(1-x-y)p^+}, (1-x-y)p_\perp - k_\perp \right) , \quad (23)$$

$$\bar{p} = \left((1-y)p^+, (1-y)\frac{p_\perp^2}{2p^+} + \frac{m^2}{2p^+(1-y)}, (1-y)p_\perp \right) , \quad (24)$$

and using the properties of γ matrices, one finally obtains, after some algebra,

$$\Lambda_a^+(p,\bar{p}) = \lambda e^3 \int_\alpha^{1-y} dx \int \frac{d^2k_\perp}{(2\pi)^3} \frac{\bar{u}(\bar{p},\sigma)\gamma^+\left(\frac{1-x}{x}\right)\left[\frac{1}{1-x}+1-x\right]u(p,s)}{(k_\perp^2+m^2x^2)}$$
$$+\lambda e^3 \int_\alpha^{1} dx \int \frac{d^2k_\perp}{(2\pi)^3} \frac{\bar{u}(\bar{p},\sigma)2x(1-x)[(1-x)\,\not{p}p^+ -(1-x)m^2\gamma^+ - p^+m]u(p,s)}{[k_\perp^2+m^2x^2][k_\perp^2+g_1m^2)]}\ , \qquad (25)$$

where

$$g_1 = \frac{x(x+y)}{(1-y)}\ . \qquad (26)$$

Putting $q=0$ and ignoring the IR convergent contribution to Λ_a^+, one finally arrives at

$$\Lambda_{\rm IR}^+(p,p) = 4\lambda p^+e^3 \int_\alpha^1 \frac{dx}{x} \int \frac{d^2k_\perp}{(2\pi)^3} \frac{1}{k_\perp^2+m^2x^2} - 4\lambda p^+e^3 \int_\alpha^1 dx \int \frac{d^2k_\perp}{(2\pi)^3} \frac{m^2x}{\left(k_\perp^2+m^2x^2\right)^2}\ , \qquad (27)$$

where α is an infrared regulator, which eliminates the IR divergences near $k^+=0$. The first term in the above expression has ultraviolet divergences also, which are usually regularized by dimensional regularization or by putting a cutoff on large values of $k_\perp$.

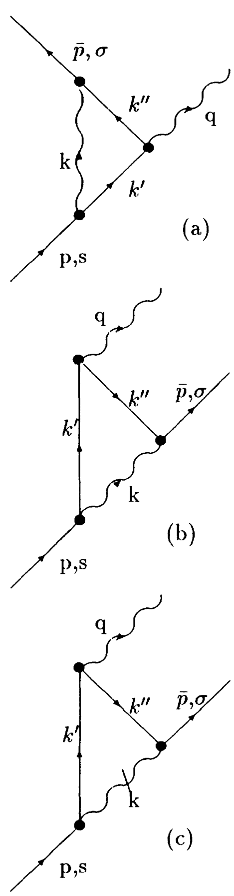

FIG. 1. Vertex correction diagrams that contribute to $\Lambda^+(p,\bar{p})$.

III. INFRARED DIVERGENCES AND THE COHERENT STATE BASIS

For massive particles as well as for massless particles with $k_\perp \neq 0$, the condition $k^+ > \alpha$ as in Eq. (27) is equivalent to putting an ultraviolet regulator on large values of k_3 in the usual space-time formulation. However, for a massless particle at $k_\perp = 0$, the divergences near $k^+=0$ are the true IR divergences of equal time theory, and in this region the condition $k^+ > \alpha$ does not follow from the condition of an ultraviolet cutoff on k_3. The IR divergences in equal time QED are canceled in the cross sections when a sum is taken over all possible initial and final states with any number of soft photons having momenta below the threshold of obesrvability. Chung [5] suggested that the origin of IR divergences lies in an inappropriate choice of initial and final states to represent the experimental situation and showed that the matrix elements do not have IR divergences if initial and final states are chosen to be appropriately defined coherent states instead of the usual Fock states. Kulish and Faddeev [6] argued that since the asymptotic Hamiltonian does not coincide with the free one in QED, the matrix elements should be calculated between coherent states instead of the Fock states. They obtained a form for the asymptotic states starting from the asymptotic Hamiltonian. In the following, we shall obtain the form of coherent states in the light-cone formalism following the same procedure. In this way, we will extend the coherent state formalism to the light-cone field theory. The light-cone time dependence of the interaction Hamiltonian is given by

$$H_I(x^+) = e\sum_{i=1}^{3}\int d\nu_i[e^{-\imath\nu_i x^+}\tilde{h}_i(\nu_i) + e^{\imath\nu_i x^+}\tilde{h}_i^\dagger(\nu_i)], \qquad (28)$$

where $\tilde{h}_i(\nu_i)$ are the QED interaction vertices,

$$\tilde{h}_1 = \sum_{s,s',\lambda} b^\dagger(\bar{p},s')b(p,s)a(k,\lambda)\bar{u}(\bar{p},s')\gamma^\mu u(p,s)\epsilon_\mu^\lambda\ , \quad (29)$$

$$\tilde{h}_2 = \sum_{s,s',\lambda} b^\dagger(\bar{p},s')d^\dagger(p,s)a(k,\lambda)\bar{u}(\bar{p},s')\gamma^\mu v(p,s)\epsilon_\mu^\lambda \ , \quad (30)$$

$$\tilde{h}_3 = \sum_{s,s',\lambda} d^\dagger(\bar{p},s')d(p,s)a(k,\lambda)\bar{v}(\bar{p},s')\gamma^\mu v(p,s)\epsilon_\mu^\lambda \ , \quad (31)$$

and ν_i is the light-cone energy transferred at the vertex $\tilde{h}_i$. The integration measure is given by

$$\int d\nu = \frac{1}{(2\pi)^{3/2}} \int \frac{[dp][dk]}{\sqrt{2\bar{p}^+}} \ , \quad (32)$$

$\bar{p}^+$ and $\bar{p}_\perp$ being fixed at each vertex by momentum conservation. For example,

$$\nu_1 = p^- + k^- - \bar{p}^- = \frac{p\cdot k}{p^+ + k^+} \quad (33)$$

is the energy transfer at the $ee\gamma$ vertex.

At asymptotic limits, nonzero contributions to $H_I(x^+)$ come from regions where ν_i goes to zero. It is easy to see that ν_2 is always nonzero, and, hence, $\tilde{h}_2$ does not appear in the asymptotic Hamiltonian. Thus, the asymptotic Hamiltonian is defined by the expression

$$V_{as}(x^+) = e\sum_{i=1,3} \int d\nu_i \Theta_\Delta(k)[e^{-\imath\nu_i x^+}\tilde{h}_i(\nu_i) + e^{\imath\nu_i x^+}\tilde{h}_i^\dagger(\nu_i)] \ , \quad (34)$$

where $\Theta_\Delta(k)$ is a function which takes a value 1 in the asymptotic region and is 0 elsewhere.

One can define the asymptotic region to consist of all points in the phase space for which

$$\frac{p\cdot k}{p^+} < \Delta E \ , \quad (35)$$

where ΔE is an energy cutoff which may be chosen to be the experimental resolution. For simplicity, we shall choose a frame $p_\perp = 0$. In this frame the above condition reduces to

$$\frac{p^+ k_\perp^2}{2k^+} + \frac{m^2 k^+}{2p^+} < \Delta \ , \quad (36)$$

where $\Delta = p^+\Delta E$.

Thus, for all the points satisfying Eq. (36), ν_1 and ν_3 can be approximated by zero. This implies that in this region, the asymptotic Hamiltonian is different from the free Hamiltonian. For the present purposes, i.e., in order to eliminate the true IR divergences, we find it sufficient to choose a subregion of the above-mentioned region as the asymptotic region. We define this subregion to be consisting of all points $(k^+, k_\perp)$ satisfying

$$k_\perp^2 < \frac{k^+\Delta}{p^+} \ , \quad (37)$$

$$k^+ < \frac{p^+\Delta}{m^2} \ . \quad (38)$$

This choice of the asymptotic region leads to the asymptotic interaction Hamiltonian defined by Eq. (34) with

$$\Theta_\Delta(k) = \theta\left(\frac{k^+\Delta}{p^+} - k_\perp^2\right)\theta\left(\frac{p^+\Delta}{m^2} - k^+\right). \quad (39)$$

The asymptotic states can be defined in the usual manner by [7]

$$|n\colon \mathrm{coh}\rangle = \Omega_\pm^A |n\rangle \ , \quad (40)$$

where $|n\rangle$ is a Fock state of charged particles and hard photons and $\Omega_\pm^A$ are the asymptotic Möller operators defined by

$$\Omega_\pm^A = T\left[-i\exp\left(\int_\mp^0 V_{as}(x^+)dx^+\right)\right] \ . \quad (41)$$

Following the standard procedure [6] of substituting $k^+ = 0$, $k_\perp = 0$ in all the slowly varying functions of k, and carrying out the x^+ integration, we arrive at the following expression for the asymptotic states:

$$\Omega_\pm^A|n\colon p_i\rangle = \exp\left[-e\int dp^+ d^2p_\perp \int \sum_{\lambda=1,2} \frac{dk^+}{\sqrt{2k^+}} \int \frac{d^2k_\perp}{(2\pi)^{\frac{3}{2}}} \times [f(k,\lambda\colon p)a^\dagger(k,\lambda) - f^*(k,\lambda\colon p)a(k,\lambda)]\rho(p)\right] \times |n\colon p_i\rangle \ , \quad (42)$$

where

$$f(k,\lambda\colon p) = \frac{p_\mu \epsilon_\lambda^\mu(k)}{p\cdot k}\theta\left(\frac{k^+\Delta}{p^+} - k_\perp^2\right)\theta\left(\frac{p^+\Delta}{m^2} - k^+\right) \ , \quad (43)$$

$$f(k,\lambda\colon p) = f^*(k,\lambda\colon p) \ , \quad (44)$$

if one follows the convention in I for photon polarization and

$$\rho(p) = \sum_n [b_n^\dagger(p)b_n(p) - d_n^\dagger(p)d_n(p)] \ . \quad (45)$$

Applying the operator $\rho(p)$ on the Fock state, we finally obtain

$$\Omega_\pm^A|n\colon p\rangle = \exp\left[-e\int \sum_{\lambda=1,2} \frac{dk^+}{\sqrt{2k^+}} \int \frac{d^2k_\perp}{(2\pi)^{\frac{3}{2}}} [f(k,\lambda\colon p)a^\dagger(k,\lambda) - f^*(k,\lambda\colon p)a(k,\lambda)]\right]|n\colon p\rangle \ . \quad (46)$$

In particular, the coherent state containing one fermion is given by

$$|p,\sigma\colon f(p)\rangle = \exp\left[-e\int \sum_{\lambda=1,2} \frac{dk^+}{\sqrt{2k^+}} \int \frac{d^2k_\perp}{(2\pi)^{\frac{3}{2}}} [f(k,\lambda\colon p)a^\dagger(k,\lambda) - f^*(k,\lambda\colon p)a(k,\lambda)]\right]|p,\sigma\rangle \ , \quad (47)$$

and the coherent state containing one fermion and one hard photon is given by

$$|p,\sigma,q,\lambda\colon f(p)\rangle = \exp\left[-e\int\sum_{\lambda=1,2}\frac{dk^+}{\sqrt{2k^+}}\int\frac{d^2k_\perp}{(2\pi)^{\frac{3}{2}}}[f(k,\lambda\colon p)a^\dagger(k,\lambda)-f^*(k,\lambda\colon p)a(k,\lambda)]\right]|p,\sigma,q,\lambda\rangle\ . \tag{48}$$

Some useful properties of these coherent states are listed in Appendix A.

IV. CALCULATION OF VERTEX CORRECTION IN COHERENT SPACE BASIS

Let us rewrite the IR divergent contribution to the $O(e^2)$ vertex correction as given by Eq. (27) as

$$\Lambda^+_{\rm IR}(p,p) = \int_0^1 dx\int d^2k_\perp I(x,k_\perp)\ . \tag{49}$$

From our discussion in the preceding section, it is natural to split the x and $k_\perp$ integrals in the above equation as

$$\Lambda^+_{\rm IR}(p,p) = \int_0^1 dx\int d^2k_\perp I(x,k_\perp)\theta\left(\frac{k^+\Delta}{p^+}-k_\perp^2\right)\theta\left(\frac{p^+\Delta}{m^2}-k^+\right)$$
$$+\int_{\frac{\Delta}{m^2}}^1 dx\int d^2k_\perp I(x,k_\perp)+\int_\alpha^{\frac{\Delta}{m^2}} dx\int d^2k_\perp I(x,k_\perp)\theta\left(k_\perp^2-\frac{k^+\Delta}{p^+}\right)\ . \tag{50}$$

In what follows, we will show that the vertex correction does not have the true IR divergences such as the first term in the above equation if one calculates the matrix elements of T between coherent states defined by Eqs. (47) and (48).

In the basis of coherent states there are additional $O(e^3)$ contributions to T_{21} coming from

$$T'_{21} = \langle\bar p,\sigma,q,\gamma\colon f(\bar p)|V_1\frac{1}{p^- - H_0}V_1|p,s\colon f(p)\rangle$$
$$= \epsilon^\lambda_\mu(q)\Lambda'^\mu(p,\bar p)\ , \tag{51}$$

corresponding to the emission and absorption of soft photons, as shown in Fig. 2. Details of the calculation of diagrams in Figs. 2(a)–2(c) are given in Appendix B. Here we will present the final expressions for Λ^+_{2a}, Λ^+_{2b}, and Λ^+_{2c}, first:

$$\Lambda^+_{2a} = -\frac{\lambda e^3}{8\pi^3}\int\frac{dk^+}{k^{+2}}\int d^2k_\perp\Theta_\Delta(k)\left(\frac{p^+-k^+}{p\cdot k}\right)$$
$$\times\left(1+\frac{p^+k^- - p^-k^+}{k\cdot p}\right)\ . \tag{52}$$

For the case $p_\perp = 0$, the above equation reduces to

$$\Lambda^+_{2a} = -\frac{\lambda e^3}{\pi^3}\int\frac{dk^+}{2k^+}(p^+-k^+)\int d^2k_\perp\frac{\Theta_\Delta(k)}{k_\perp^2+\frac{m^2k^{+2}}{p^{+2}}}$$
$$\times\left[1-\frac{\frac{m^2k^{+2}}{p^{+2}}}{k_\perp^2+\frac{m^2k^{+2}}{p^{+2}}}\right]\ . \tag{53}$$

Similarly, we find

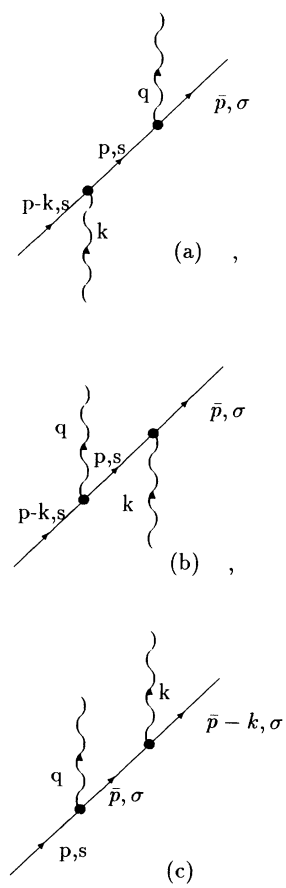

FIG. 2. Contributions to the $O(e^2)$ vertex corrections due to emission and absorption of soft photons.

$$\Lambda^+_{2b} = \Lambda^+_{2c} = -\frac{\lambda e^3}{4\pi^3}\int \frac{dk^+}{k^+}\int d^2k_\perp \Theta_\Delta(k)\frac{1}{p^- - \bar{p}^-} \times\left[1 - \frac{p^- k^+}{k\cdot p}\right] . \quad (54)$$

Both Λ^+_{2b} and Λ^+_{2c} have a vanishing denominator at $q = 0$, and can be evaluated using the Heitler method [3,8], to give

$$\Lambda^+_{2b} + \Lambda^+_{2c} = -\frac{\lambda e^3}{2\pi^2}\int dk^+ \int d^2k_\perp \left(\frac{k_\perp^2}{k_\perp^2 + \frac{m^2 k^{+2}}{p^{+2}}}\right)\Theta_\Delta(k) , \quad (55)$$

which does not have any IR divergence. So, the IR-divergent contribution to $\Lambda'^+(p,\bar{p})$ at $q = 0$, comes from Fig. 2(a) only:

$$\Lambda'^+(p,p) = -\frac{\lambda e^3 p^+}{2\pi^3}\int \frac{dk^+}{k^+}\int d^2k_\perp \frac{\Theta_\Delta(k)}{k_\perp^2 + \frac{m^2(k^+)^2}{(p^+)^2}} \times\left[1 - \frac{\frac{m^2 k^{+2}}{p^{+2}}}{k_\perp^2 + \frac{m^2(k^+)^2}{(p^+)^2}}\right] . \quad (56)$$

Comparing Eq. (27) and Eq. (56) one can easily see that the true IR divergences in Figs. 1 and 2 cancel exactly. This completes the proof of cancellation of "true" IR divergences in the $O(e^2)$ three-point vertex correction in the coherent state basis.

V. CONCLUSION

We have presented a lowest order calculation in continuum light-cone QED to show the cancellation of true IR divergences when a coherent state basis is used to calculate the matrix elements. A similar analysis can be carried out for the lowest order correction to the fermion wave function renormalization constant also (see Eq. (3.27) in Ref. [3]). It can be shown that the use of the coherent state basis eliminates the true IR divergences in Z_2 as well. Since the only effect of using a coherent state basis is to eliminate the true IR divergences from both Z_1 and Z_2, the Ward identity (in the form given in Ref. [3]) is still valid and the physical quantities such as the renormalized charge are in no way affected by our different choice of basis.

The present calculation has been done in the continuum case, but the suggested method of using the coherent states as the asymptotic states in order to calculate the Hamiltonian matrix elements promises to be useful in the discrete case as well. In particular, it may be relevant to the problem arising due to an exchange of a zero mode photon in discretized light-cone QED [4,9]. Tang has shown that in a numerical DLCQ calculation of energy levels of positronium, the lowest energy level diverges with K, the harmonic resolution, if one does not remove the $k^+ = 0$, $k_\perp = 0$ state by an artificial IR cutoff. Kräutgartner *et al.* have analyzed the various approximations to the DLCQ matrix equation for positronium and have discussed the Coulomb singularity occurring due to the exchange of a $k^+ = 0$, $k_\perp = 0$ mode photon. They have claimed that even though one can remove the true IR divergences by eliminating the $k^+ = 0$, $k_\perp = 0$ state by an artificial cutoff or by giving a small mass to the photon, neither of these procedures leads to convergent results. In order to achieve convergence, one has to add and subtract an appropriate term to the light-cone Schrödinger equation. This counterterm removes the discretized IR divergence and replaces the term at small k^+ and $k_\perp$ by the appropriate continuum value. However, if one calculates the Hamiltonian matrix elements between the coherent states *before* discretization is carried out, one may be able to remove the true IR divergences in a natural manner. Work in this direction is in progress.

We would like to emphasize that the coherent state formalism takes care of only the true IR divergences and one would still need an IR cutoff on k^+ for massive fields as well as for nonzero values of $k_\perp$.

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APPENDIX A: PROPERTIES OF COHERENT STATES

We denote with $|1{:}p_i\rangle$ the coherent state containing a fermion and a superposition of an infinite number of soft photons as defined by Eq. (47) and with $|2{:}p_i,k_i\rangle$ the coherent state containing a fermion and a hard photon as defined by Eq. (48).

It can be shown easily that the coherent states $|1{:}p_i\rangle$ are the eigenstates of $a(k,\lambda)$:

$$a(k,\rho)|1{:}p_i\rangle = -\frac{e}{(2\pi)^{3/2}}\frac{1}{\sqrt{2k^+}}f(k,\rho{:}p_i)|1{:}p_i\rangle . \quad (A1)$$

Also,

$$a(k,\rho)|2{:}p_i,k_i\rangle = -\frac{e}{(2\pi)^{3/2}}\frac{1}{\sqrt{2k^+}}f(k,\rho{:}p_i)|2{:}p_i,k_i\rangle + \delta^3(k-k_i)\delta_{\rho\lambda_i}|1{:}p_i\rangle , \quad (A2)$$

and

$$a^\dagger(k,\rho)|1{:}p_i\rangle = -\frac{e}{(2\pi)^{3/2}}\frac{1}{\sqrt{2k^+}}f^*(k,\rho : p_i)|1{:}p_i\rangle + |2{:}p_i,k_i\rangle . \quad (A3)$$

In the lowest order, Eq. (A3) reduces to

$$a^\dagger(k,\rho)|1{:}p_i\rangle = |2{:}p_i,k_i\rangle . \quad (A4)$$

Coherent states satisfy the orthonormalization properties

$$\langle 1{:}\,p_f,\sigma_f|1{:}\,p_i,\sigma_i\rangle = \delta^{(3)}(p_i-p_f)\delta_{\sigma_i\sigma_f},$$

$$\langle 2{:}\,p_f,\sigma_f,k_f,\lambda_f|1{:}\,p_i,\sigma_i\rangle = \delta^{(3)}(p_i-p_f)\delta_{\sigma_i\sigma_f} \times f(k_f,\lambda_f{:}\,p_f)\frac{e}{(2\pi)^{3/2}}. \quad \text{(A5)}$$

APPENDIX B: MATRIX ELEMENTS BETWEEN COHERENT STATES

In this appendix, we shall calculate the various contributions to the matrix element considered in Sec. IV:

$$T'_{21} = \left\langle \bar{p},\sigma,q,\lambda{:}\,f(\bar{p})\left|V_1\frac{1}{p^- - H_0}V_1\right|p,s{:}\,f(p)\right\rangle.$$

Substituting for V_1 from Eq. (10) and using Eqs. (A1) and (A2) one obtains

$$T'_{21} = T_{2a}+T_{2b}+T_{2c}\,, \quad \text{(B1)}$$

where T_{2a} and T_{2b} correspond to Feynman diagrams in Figs. 2(a) and 2(b) representing the absorption of soft photons in the initial and final states; T_{2c} corresponds to the emission of soft photons from the final state. There is no diagram corresponding to the emission of soft photons in the initial state to this order as evident from Eq. (A4). T_{2a}, T_{2b}, and T_{2c} are defined by

$$T_{2a} = \left\langle \bar{p},\sigma{:}\,f(\bar{p})\left|V_a^\dagger\frac{1}{p^- - H_0}V_a\right|p,s{:}\,f(p)\right\rangle,$$

$$T_{2b} = \left\langle \bar{p},\sigma{:}\,f(\bar{p})\left|V_a^\dagger\frac{1}{p^- - H_0}V_a^\dagger\right|p,s{:}\,f(p)\right\rangle, \quad \text{(B2)}$$

and

$$T_{2c} = \left\langle \bar{p},\sigma{:}\,f(\bar{p})\left|V_a\frac{1}{p^- - H_0}V_a^\dagger\right|p,s{:}\,f(p)\right\rangle, \quad \text{(B3)}$$

where

$$V_a = e\int d^2x_\perp dx^- \int [dp][d\bar{p}][dk]\sum_{s,s',\lambda}[e^{i(\bar{p}-p-k)\cdot x}\bar{u}(\bar{p},s')\gamma^\mu \times u(p,s)b^\dagger(\bar{p},s')b(p,s)a(k,\lambda)\epsilon_\mu^\lambda(k)]. \quad \text{(B4)}$$

Using Eqs. (16), (A1), and (B4), one obtains, in a straightforward manner,

$$T_{2a} = \epsilon_\mu^\lambda(q)\Lambda_{2a}^\mu(p,\bar{p}) = -\frac{\lambda e^3\epsilon_\mu^\lambda(q)}{2p^+}\int\frac{dk^+d^2k_\perp}{(2\pi)^3 2k^+}\frac{1}{k^- + (p-k)^- - p^-}\sum_{\lambda_1}\epsilon_\nu^{\lambda_1}(k)f(k,\lambda_1{:}\,p) \times\left[\bar{u}(\bar{p},\sigma)\gamma^\mu(\not{p}+m)\gamma^\nu u(p-k,s)\right]. \quad \text{(B5)}$$

Alternatively,

$$\Lambda_{2a}^\mu(p,\bar{p}) = \frac{\lambda e^3}{2p^+}\int\frac{dk^+}{2k^+}\int\frac{d^2k_\perp}{(2\pi)^3}\frac{1}{p^- - k^- - (p-k)^-}\left[\bar{u}(\bar{p},\sigma)\gamma^\mu(\not{p}+m)\gamma^\nu u(p,s)\right]\sum_\lambda \epsilon_\nu^\lambda(k)\epsilon_{\rho\lambda}(k)f^\rho(k,p)\,, \quad \text{(B6)}$$

where we have used Eqs. (39) and (43) in the above expression, and have approximated $u(p-k,s)$ by $u(p,s)$. We have defined $f^\rho(k,p)$ by

$$f(k,\lambda{:}\,p) = f^\rho(k,p)\epsilon_\rho^\lambda(p)\,. \quad \text{(B7)}$$

For $\mu=+$, Eq. (B6) reduces to

$$\Lambda_{2a}^+(p,\bar{p}) = \frac{\lambda e^3}{8\pi^3}\frac{1}{2p^+}\int\frac{dk^+}{k^{+2}}\int d^2k_\perp\left[\frac{\left(\bar{u}(\bar{p},\sigma)\gamma^+(\not{p}+m)\gamma^\nu u(p,s)\right)\left(-f_\nu(k,p)+\frac{k_\nu}{k^+}f^+(k,p)+\frac{\delta_{\nu+}}{k^+}k\cdot f\right)}{p^- - k^- - (p-k)^-}\right]. \quad \text{(B8)}$$

In a frame where $p_\perp = 0$, one can use Eqs. (43) and the relations [3]

$$p^- - k^- - (p-k)^- = -\frac{p\cdot k}{p^+ - k^+}, \quad \text{(B9)}$$

$$p\cdot k = \frac{p^+}{2k^+}\left(k_\perp^2 + \frac{m^2k^2}{p^{+2}}\right), \quad \text{(B10)}$$

to reduce Eq. (B8) at $q=0$ to

$$\Lambda^{+}_{2a}(p,\bar{p}) = -\frac{\lambda e^3}{4\pi^3}\int\frac{dk^+}{k^{+2}}\int d^2k_\perp\theta\left(\frac{\Delta p^+}{m^2}-k^+\right)\theta\left(\frac{k^+\Delta}{p^+}-k_\perp^2\right)\frac{p^+-k^+}{p^+}\left(1+\frac{p^+k^- - p^-k^+}{k\cdot p}\right), \tag{B11}$$

which has an IR-divergent part given by

$$\Lambda^{+}_{2a\mathrm{IR}}(p,p) = -\frac{\lambda e^3}{2\pi^3}\int\frac{dk^+}{k^+}\int d^2k_\perp\theta\left(\frac{\Delta p^+}{m^2}-k^+\right)\theta\left(\frac{k^+\Delta}{p^+}-k_\perp^2\right)\left(\frac{1}{k_\perp^2+\frac{m^2k^{+2}}{p^{+2}}}\right)\left[1-\frac{\frac{m^2k^{+2}}{p^{+2}}}{k_\perp^2+\frac{m^2k^{+2}}{p^{+2}}}\right]. \tag{B12}$$

Similarly, one can show that

$$\Lambda^{+}_{2b\mathrm{IR}}(p,\bar{p}) = \Lambda^{+}_{2c\mathrm{IR}}(p,\bar{p}) = -\frac{\lambda e^3}{4\pi^3}\int\frac{dk^+}{k^+}\int\left(\frac{p^+}{\bar{p}^+}\right)\frac{1}{p^- - \bar{p}^-}\left[1-\frac{\frac{m^2k^{+2}}{p^{+2}}}{k_\perp^2+\frac{m^2k^{+2}}{p^{+2}}}\right]\Theta_\Delta(k)\,,$$

which has a vanishing denominator at $q=0$ and therefore must be calculated using the Heitler method [3,8]. This is easily seen to be free of IR divergences.

Thus, one finally obtains

$$\Lambda'^{+}_{\mathrm{IR}}(p,p) = -\frac{\lambda e^3}{2\pi^3}\int\frac{dk^+}{k^+}\int d^2k_\perp\frac{\Theta_\Delta(k)}{k_\perp^2+\frac{m^2k^{+2}}{p^{+2}}}\left[1-\frac{\frac{m^2k^{+2}}{2p^+}}{k_\perp^2+\frac{m^2k^{+2}}{p^{+2}}}\right]. \tag{B13}$$

[1] R.J. Perry, A. Harindranath, and K.G. Wilson, Phys. Rev. Lett. **65**, 2050 (1990).
[2] H.C. Pauli and S.J. Brodsky, Phys. Rev. D **32**, 1993 (1985).
[3] D. Mustaki, S. Pinsky, J. Shigemitsu, and K.G. Wilson, Phys. Rev. D **43**, 3411 (1991).
[4] A.C. Tang and S.J. Brodsky, Phys. Rev. D **44**, 1842 (1991); A.C. Tang, Stanford University, Ph.D. thesis, 1990.
[5] V. Chung, Phys. Rev. **140**, B1110 (1965).
[6] P.P. Kulish and L.D. Faddeev, Theor. Math. Phys. **4**, 745 (1970).
[7] V.D. Duca, L. Magnea, and G. Sterman, Nucl. Phys. **B324**, 391 (1989).
[8] W. Heitler, *The Quantum Theory Of Radiation* (Oxford University Press, Oxford, England, 1954).
[9] M. Kräutgartner, H. C. Pauli, and F. Wölz, Phys. Rev. D **45**, 3755(1992).