"Theory of theories" approach to string theory

Hiroyuki Hata*
Department of Physics, Kyoto University, Kyoto 606-01, Japan
(Received 18 October 1993)

We propose a new formulation of gauge theories as a quantum theory which has the gauge theory action S as its dynamical variable. This system is described by a simple actional $I(S)$ (that is, an action for the action S) whose equation of motion gives the Batalin-Vilkovisky (BV) master equation for S. Upon quantization we find that our new formulation is reduced to something like a topological field theory having a BRST exact gauge-fixed actional. Therefore the present formulation can reproduce ordinary gauge theories since the path integral over S is dominated by the classical configuration which satisfies the BV master equation. This "theory of theories" formulation is intended to be applied to closed-string field theory.

PACS number(s): 11.25.Sq, 11.15.Tk

I. INTRODUCTION

In this paper we propose a new formulation of gauge field theory. It applies in principle to any kind of gauge theory, but our interest is mainly in the reformulation of string field theory.

A field theory having a dynamical variable φ is described by an action $S(\varphi)$ as

$$\langle A\rangle = \int \mathcal{D}\varphi\, A(\varphi) \exp\left[\frac{1}{\hbar} S(\varphi)\right]. \tag{1.1}$$

In gauge theories S must be modified so that the local gauge invariance is fixed and correspondingly the Faddeev-Popov (FP) ghosts are introduced. This procedure for quantizing gauge theories is most efficiently carried out using the Becchi-Rouet-Stora-Tyutin (BRST) or the Batalin-Vilkovisky (BV) [1] formalism. According to the BV formalism, the quantum action S has to satisfy the master equation

$$\hbar\Delta S + \frac{1}{2}\{S, S\} = 0, \tag{1.2}$$

where $\{*,*\}$ and Δ are operators whose precise definitions will be given in Sec. II. The master equation expresses the quantum BRST invariance of the system: the term proportional to $\hbar$ takes into account the variation of the path-integral measure under the BRST transformation. [Precisely speaking, the action S in Eq. (1.1) for a gauge theory is obtained from S satisfying the master equation (1.2) by the restriction to a Lagrangian submanifold. In this section we do not distinguish these two S's to avoid unnecessary complication.]

In "simple" systems such as the Yang-Mills theory, the measure term $\hbar\Delta S$ can be consistently neglected by using dimensional regularization. However, for closed-string field theory, which is recognizable as a gauge theory having an infinite number of gauge symmetries, the measure term is essential in obtaining a consistent theory. For such a system the quantum action S satisfying Eq. (1.2) is given as an infinite power series in $\hbar$:

$$S = \sum_{n=0}^{\infty} \hbar^n S^{(n)}. \tag{1.3}$$

Construction of the quantum action (1.3) for closed-string field theory has been carried out in Refs. [2–4]. Unfortunately, the resulting S looks too complicated to be used in the investigation of (possible) nonperturbative aspects of string theory. Invention of another, much simpler, reformulation of string (field) theory is greatly desired.

Our attempt in this paper is to present such a reformulation of gauge theories, and, in particular, of closed-string field theory, without referring to the explicit form of the action $S(\varphi)$. Instead we promote the action $S(\varphi)$ from a fixed functional of φ to a *dynamical variable* which should be path-integrated out. Since the kind of field φ which may be used as the argument of S is fixed, the coupling constants in $S(\varphi)$ may be regarded as the dynamical variables. Roughly speaking, we consider a theory described by the path integral

$$\langle\!\langle \mathcal{O}\rangle\!\rangle = \int \mathcal{D}S\, \mathcal{O}(S) \exp\left[\frac{1}{\lambda} I(S)\right], \tag{1.4}$$

where $I(S)$ is the action for the action (hereafter called the *actional*). Since $S(\varphi)$ specifies a theory, the present formulation may be called a "theory of theories" (TT).

The principles we use in constructing the actional $I(S)$ for a TT are as follows. First, we require that the equation of motion of the TT, $\delta I(S)/\delta S = 0$, gives the master equation (1.2). Second, $I(S)$ should be invariant under the "local" gauge transformation

$$\delta_\epsilon S = \hbar\Delta\epsilon + \{S, \epsilon\}, \tag{1.5}$$

*Electronic address: hata@gauge.scphys.kyoto-u.ac.jp, hata@jpnyitp.bitnet

where ϵ is an arbitrary functional of φ. The transformation (1.5) is known to be a symmetry of the master equation (1.2) [5,6]: if S is a solution to the BV equation, so is $S+\delta\epsilon S$ (this can be naively understood if the left-hand side (LHS) of the BV equation (1.2) is regarded as an analogue of the usual field strength $F = dA + A^2$).

An actional $I(S)$ satisfying the above two requirements is easily found (and has already been proposed in Ref. [7]). We want the TT to reproduce the original gauge theory (1.1), since our aim is to present a reformulation of string field theory. This implies that our TT should be a kind of "topological" theory [8,9] which has almost no physical degrees of freedom as a system of the dynamical variable $S(\varphi)$ [recall that an ordinary gauge theory (1.1) is described by a *fixed* action $S(\varphi)$]. Remarkably, we find that this expectation is in fact true. Since the TT is also a gauge theory, we quantize it by again employing the BV formalism. After introducing an auxiliary field, the resulting quantized TT turns out to be a topological theory described by a BRST exact actional. The connection between the TT and the conventional formulation of gauge theories is made through the partition function: in the TT the partition function operator $V_L(S)$ is an observable, whose expectation value is shown to be equal to the partition function of a gauge theory in the conventional formulation.

The organization of the rest of this paper is as follows. In Sec. II we give a brief summary of the BV formalism necessary in the construction of the TT. In Sec. III, which is the main part of this paper, we first introduce the actional (Sec. III A), carry out the BV quantization of the TT (Secs. III B, C, and D), and discuss the relationship to the ordinary formulation of gauge theories (Sec. III E). The final section (Sec. IV) is devoted to a summary and discussion.

II. BV FORMALISM

In this section we shall recapitulate the elements of the BV formalism used in this paper. We follow our previous convention [7]. A more detailed explanation of the BV formalism may be found in Ref. [6].

We consider an (n,n)-dimensional supermanifold $\mathcal{M}$. The coordinates of $\mathcal{M}$ are the field variables. In real gauge theories, a field $\varphi(x)$ has continuous space-time parameter x. Here the index I $(=1,\dots,2n)$ specifying the coordinates of $\mathcal{M}$ should be understood to represent all the (continuous as well as discrete) parameters characterizing the fields.

The supermanifold $\mathcal{M}$ is endowed with an odd symplectic structure defined by a fermionic two-form ω which is nondegenerate and closed, $d\omega=0$, and carries the ghost number $N_{\rm gh}[\omega]=-1$. In a local coordinate system $(z^I)=(z^1,z^2,\dots,z^{2n})$ of $\mathcal{M}$, ω is expressed as

$$\omega = -dz^I\omega_{IJ}(z)\,dz^J = \omega_{JI}(z)\,dz^I\wedge dz^J. \tag{2.1}$$

On $\mathcal{M}$ we also introduce the volume element

$$d\mu(z) = \rho(z)\prod_{I=1}^{2n} dz^I, \tag{2.2}$$

where $\rho(z)$ is the density. Then we can define two basic operators, the antibrackets $\{*,*\}$ and the Δ operator Δ_ρ, by

$$\{A,B\} = A\overleftarrow{\partial}_I\,\omega^{IJ}(z)\,\overrightarrow{\partial}_J B, \tag{2.3}$$

$$\Delta_\rho A = \frac{1}{2\rho}(-)^{z^I}\partial_I\left(\rho\,\omega^{IJ}\partial_J A\right), \tag{2.4}$$

where $\omega^{IJ}(z)$ is the inverse matrix to $\omega_{IJ}(z)$, and $\overrightarrow{\partial}_I = \partial_I = \partial_l/\partial z^I$ and $\overleftarrow{\partial}_I = \partial_r/\partial z^I$ denote the left and right derivatives, respectively.[1] Note that both the antibrackets and Δ raise the ghost number $N_{\rm gh}$ by one.

The antibrackets and the delta operator satisfy the three basic properties

$$(\Delta)^2 = 0 \text{ (nilpotency)}, \tag{2.5}$$

$$\Delta\{A,B\} = \{\Delta A, B\} + (-)^{A+1}\{A,\Delta B\} \text{ (Leibniz rule)}, \tag{2.6}$$

$$(-)^{(A+1)(C+1)}\{\{A,B\},C\} + \text{cyclic}(A,B,C) = 0, \text{ (Jacobi identity)}. \tag{2.7}$$

Equations (2.6) and (2.7) are consequences of $d\omega=0$, while Eq. (2.5) is a requirement on the density $\rho(z)$.[2] Other useful formulas concerning the antibrackets and the delta operator are [5,6]

$$(-)^A\{A,B\} = \Delta(AB) - \Delta A\cdot B - (-)^A A\Delta B, \tag{2.8}$$

and

$$\{A,B\} = -(-)^{(A+1)(B+1)}\{B,A\}, \tag{2.9}$$

$$\{A,BC\} = \{A,B\}C + (-)^{(A+1)B}B\{A,C\}, \tag{2.10}$$

$$\{AB,C\} = A\{B,C\} + (-)^{B(C+1)}\{A,C\}B. \tag{2.11}$$

The master equation for $S(z)$ reads

$$M(S) \equiv \Delta S + \frac{1}{2}\{S,S\} = 0, \tag{2.12}$$

or equivalently

$$\Delta e^S = 0. \tag{2.13}$$

Given a $S(z)$ satisfying the master equation (2.12) the gauge-fixed quantum theory is defined by the path integral

$$\langle A\rangle = \int_L d\lambda\, A(z)\exp[S(z)], \tag{2.14}$$

where the integration is over the Lagrangian submani-

[1] $(-)^A = +1$ (-1) if A is Grassmann even (odd).

[2] As a matter of fact, $d\omega=0$ follows from $\Delta^2=0$.

fold L and $d\lambda$ is the associated integration measure. The Lagrangian submanifold L is a $(k, n-k)$-dimensional submanifold of $\mathcal{M}$, such that $\omega(v, \tilde{v}) = 0$ for any pair, v and $\tilde{v}$, of tangent vectors to L at $z \in L$ $(v, \tilde{v} \in T_z L)$. The corresponding volume element $d\lambda$ is defined by

$$d\lambda(e_1, \ldots, e_n) = d\mu(e_1, \ldots, e_n, f^1, \ldots, f^n)^{1/2}, \quad (2.15)$$

where $d\mu$ is the volume element in $\mathcal{M}$, Eq. (2.2), and $(e_1, \ldots, e_n, f^1, \ldots, f^n)$ is a basis of the tangent space $T_z\mathcal{M}$ such that $(e_1, \ldots, e_n)$ is a basis of $T_z L$ and the condition $\omega(e_i, f^j) = \delta_i^j$ is satisfied. The choice of L corresponds to the choice of gauge fixing. In order for the expectation value $\langle A \rangle$ (2.14) to be independent of the choice of the Lagrangian submanifold L, the operator $A(z)$ has to satisfy the condition (see below)

$$\Delta A + \{S, A\} = 0. \quad (2.16)$$

The solution $S(z)$ of the master equation (2.12) is not unique. Given a solution $S(z)$, we have a continuous family of solutions obtained by the infinitesimal "gauge transformation" δ_ϵ [5,6]:

$$\delta_\epsilon S = \Delta\epsilon + \{S, \epsilon\}, \quad (2.17)$$

where the transformation parameter $\epsilon(z)$ carries $N_{\rm gh}[\epsilon] = -1$. In fact, using Eqs. (2.5)–(2.7), the master equation $M(S)$ of Eq. (2.12) is shown to transform homogeneously under δ_ϵ: $\delta_\epsilon M(S) = \{M(S), \epsilon\}$. The same formulas tell that the transformation δ_ϵ forms a closed algebra:

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \delta_{\{\epsilon_1, \epsilon_2\}}. \quad (2.18)$$

The finite transformation which has δ_ϵ of Eq. (2.17) as its infinitesimal expression is given by considering a general canonical transformation $g : \mathcal{M} \to \mathcal{M}$ satisfying $g^*\omega = \omega$ (see Ref. [7]).

The relationship between the master equation [plus the requirement (2.16) on A] and the independence of Eq. (2.14) on the choice of L is understood as follows. A general infinitesimal deformation of L may be realized by a canonical transformation: $z^I \to z^I + \{z^I, \epsilon\}$ for some $\epsilon(z)$. Therefore we have

$$\begin{aligned}\langle A \rangle_{L+\delta L} - \langle A \rangle_L &= \int_L d\lambda \left[\Delta\epsilon A e^S + \{A e^S, \epsilon\}\right] \\ &= \int_L d\lambda \left[\Delta\left(\epsilon A e^S\right) + \epsilon\,\Delta\left(A e^S\right)\right],\end{aligned} \quad (2.19)$$

where the $\Delta\epsilon$ term in the first expression originates from the change of the measure $d\lambda$, while the $\{Ae^S, \epsilon\}$ term expresses the coordinate transformation on the integrand $A(z)e^{S(z)}$ (see Sec. 3.3 of Ref. [7]). The last expression of Eq. (2.19) is obtained by using the formula (2.8). The $\Delta(\epsilon A e^S)$ term in the last expression of Eq. (2.19) vanishes due to the general formula [10]

$$\int_L d\lambda\, \Delta A = 0, \quad (2.20)$$

which holds for arbitrary $A(z)$. The master equation and the condition (2.16) ensure the vanishing of $\Delta(Ae^S) = (\Delta A + \{S, A\})\, e^S + A\Delta e^S$.

The simplest coordinate system for $\mathcal{M}$ is the Darboux frame $(\phi^i, \phi_i^*)_{i=1,\ldots,n}$ with $\omega = -2\sum_i d\phi^i \wedge d\phi_i^*$. The coordinates ϕ^i and ϕ_i^* are called fields and antifields, respectively, and they satisfy $N_{\rm gh}[\phi^i] + N_{\rm gh}[\phi_i^*] = -1$. The antibracket and the delta operator in the Darboux frame (with $\rho(z) = 1$) are given by

$$\{A, B\} = \frac{\partial_r A}{\partial\phi^i}\frac{\partial_l B}{\partial\phi_i^*} - \frac{\partial_r A}{\partial\phi_i^*}\frac{\partial_l B}{\partial\phi^i}, \quad (2.21)$$

$$\Delta = (-)^{\phi^i}\frac{\partial_l^2}{\partial\phi^i\partial\phi_i^*}. \quad (2.22)$$

In the Darboux frame the Lagrangian submanifold L is specified by

$$L : \phi_i^* = \frac{\partial\Upsilon(\phi)}{\partial\phi^i}, \quad (2.23)$$

where $\Upsilon(\phi)$ is called gauge fermion ($N_{\rm gh}[\Upsilon] = -1$). The expectation value of an observable A, Eq. (2.14), now reads

$$\langle A \rangle = \int d\phi\, A(\phi, \phi^*) \exp[S(\phi, \phi^*)]\Big|_{\phi_i^* = \partial\Upsilon(\phi)/\partial\phi^i}. \quad (2.24)$$

Finally, we shall explain the BRST transformation in the BV quantized theory (2.24) in the Darboux frame. First we define the pre-BRST transformation δ_B on a general $A(z)$ by

$$\delta_B A = \{S, A\}. \quad (2.25)$$

Then the BRST transformation in the gauge-fixed theory $\widehat{\delta}_B$ is defined by restricting δ_B to the Lagrangian submanifold:

$$\widehat{\delta}_B\phi^i = \delta_B\phi^i|_L = \frac{\partial S(\phi, \phi^*)}{\partial\phi_i^*}\bigg|_{\phi_i^* = \partial\Upsilon(\phi)/\partial\phi^i}. \quad (2.26)$$

The master equation ensures the quantum BRST invariance and the on-shell nilpotency of $\widehat{\delta}_B$:

$$\widehat{\delta}_B\left(\widehat{S}(\phi) + \ln\prod_i d\phi^i\right) = 0, \quad (2.27)$$

$$\left(\widehat{\delta}_B\right)^2\phi \propto \frac{\delta\widehat{S}(\phi)}{\delta\phi}, \quad (2.28)$$

where $\widehat{S}(\phi) \equiv S(\phi, \phi^* = \partial\Upsilon/\partial\phi)$.

III. THEORY OF THEORIES

As seen above, the master equation has a local gauge symmetry given by δ_ϵ of Eq. (2.17). In this section, we shall construct the TT, namely, a gauge theory which has $S(z)$ as its dynamical variable and has an invariance under δ_ϵ. We shall then study the quantization of the TT based on the BV formalism and the relationship to the ordinary formulation of gauge theories.

A. Actional

First we need the actional $I(S)$ for the TT. As stated in Sec. I, we demand that $I(S)$ has an invariance under

δ_ϵ and that the equation of motion, $\delta I(S)/\delta S = 0$, gives the master equation (2.12). Such an actional has been proposed in Ref. [7]. It takes a fairly simple form

$$I_{hz}(S) = \int_{\mathcal{M}} d\mu H(z)\Delta H(z), \tag{3.1}$$

where $H(z) \equiv \exp[S(z)]$ [see Sec. 3.2 of Ref. [7], where we denoted $I_{hz}(S)$ by $\mathcal{A}(S)$]. In this paper we adopt, instead of (3.1), the slightly modified actional $I(S)$:

$$\begin{aligned} I(S) &= \frac{1}{2}\int_{\mathcal{M}} d\mu(z)H(\overline{z})\Delta H(z) \\ &= \frac{1}{2}\int_{\mathcal{M}} d\mu(z)H(z)\Delta H(\overline{z}), \end{aligned} \tag{3.2}$$

where the coordinate $\overline{z}^I$ is related to the original one, z^I, by the "inversion" of the Grassmann odd components:

$$\overline{z}^I \equiv (-)^{z^I} z^I. \tag{3.3}$$

The two expressions of Eq. (3.2) are equivalent on using the partial integration formula

$$\int_{\mathcal{M}} d\mu A\Delta B = -\frac{1}{2}\int_{\mathcal{M}} d\mu \{A, B\} = (-)^A \int_{\mathcal{M}} d\mu\, (\Delta A)\, B. \tag{3.4}$$

Since we have $N_{\rm gh}[\Delta] = +1$ and $N_{\rm gh}[S(z)] = 0$, the requirement that the actional $I(S)$ carries no ghost number $N_{\rm gh}$ leads to the following restriction on the ghost numbers of z^I:

$$N_{\rm gh}[d\mu(z)] \equiv \sum_{I=1}^{2n} (-)^{z^I} N_{\rm gh}[z^I] = -1. \tag{3.5}$$

Since $N_{\rm gh}[z^I]$ = even (odd) if z^I is Grassmann even (odd), the requirement (3.5) tells that the dimension n of our (n,n) supermanifold $\mathcal{M}$ must be an odd integer [this fact follows immediately from the requirement that $I(S)$ be a bosonic quantity since Δ is fermionic]. It is a delicate matter whether condition (3.5) is satisfied for a concrete system, such as string field theory, since the index I is in fact a continuous parameter. Here we simply assume that the condition (3.5) is satisfied.

It is obvious that the equation of motion, $\delta I(S)/\delta S(z) = 0$, gives the master equation (2.13). The invariance of $I(S)$ (3.2) under the gauge transformation δ_ϵ of Eq. (2.17), which is expressed on $H(z)$ as

$$\delta_\epsilon H = \Delta(H\epsilon) - (\Delta H)\,\epsilon, \tag{3.6}$$

is shown as

$$\begin{aligned} \delta_\epsilon I(S) &= \int d\mu \Big[\Delta(H\epsilon) - \Delta H\epsilon\Big]\Delta\overline{H} \\ &= \int d\mu\, \left(-H\Delta^2\overline{H} + \Delta H\Delta\overline{H}\right)\epsilon = 0, \end{aligned} \tag{3.7}$$

where $\overline{H}$ is short for $H(\overline{z})$, and we have used Eq. (3.4) and the nilpotency of Δ. The vanishing of the $\Delta H\Delta\overline{H}\epsilon$ term is understood by making the change of variables from z to $\overline{z}$:

$$\int d\mu\Delta H\Delta\overline{H}\epsilon = \int (-d\mu)\,\Delta\overline{H}\Delta H(-\epsilon) = 0, \tag{3.8}$$

where we have used the properties

$$d\mu(\overline{z}) = -d\mu(z), \tag{3.9}$$

$$\Delta(\overline{z}) = -\Delta(z), \tag{3.10}$$

$$\epsilon(\overline{z}) = -\epsilon(z). \tag{3.11}$$

Equation (3.11) is the restriction on $\epsilon(z)$ that it should not contain Grassmann odd "constants." Note that $f(\overline{z}) = \pm f(z)$ depending on whether the function $f(z) = \omega^{IJ}(z)$, $\rho(z)$, etc., is Grassmann even (upper sign) or odd (lower sign). For example, we have $\rho(\overline{z}) = \rho(z)$.

B. Master equation for the TT

Having presented the gauge-invariant actional of the TT, our next task is to quantize it. We shall carry out this quantization using the BV formalism. For this purpose we have to first define the antibracket and delta operator for the TT, which we denote by $(*,*)$ and $\square$, respectively. Here we shall adopt the following ones:

$$\begin{aligned} (\mathcal{A},\mathcal{B}) &= \mathcal{A}(S)\frac{\overleftarrow{\delta}}{\delta H(z)}\int\frac{d\mu(z)}{[\rho(z)]^2}\frac{\overrightarrow{\delta}}{\delta H(\overline{z})}\mathcal{B}(S) \\ &= \int\frac{d\mu(z)}{[\rho(z)]^2}\frac{\overrightarrow{\delta}}{\delta H(z)}\mathcal{A}(S)\frac{\overrightarrow{\delta}}{\delta H(\overline{z})}\mathcal{B}(S), \end{aligned} \tag{3.12}$$

$$\square\mathcal{A} = \frac{1}{2}\int\frac{d\mu(z)}{[\rho(z)]^2}\frac{\overrightarrow{\delta}}{\delta H(z)}\frac{\overrightarrow{\delta}}{\delta H(\overline{z})}\mathcal{A}(S), \tag{3.13}$$

where $\mathcal{A}(S)$ and $\mathcal{B}(S)$ are arbitrary functionals of $S(z)$. Note that the differential operator $\delta/\delta H(z)$ is Grassmann odd since we have

$$\frac{\overrightarrow{\delta}}{\delta H(z)}H(z') = -H(z')\frac{\overleftarrow{\delta}}{\delta H(z)} = \delta(z-z') = -\delta(z'-z), \tag{3.14}$$

and $\delta(z-z')$ is Grassmann odd (recall that n is odd). In obtaining the second expression of Eq. (3.12) we have used the formula

$$\mathcal{A}(S)\frac{\overleftarrow{\delta}}{\delta H(z)} = (-)^{\mathcal{A}+1}\frac{\overrightarrow{\delta}}{\delta H(z)}\mathcal{A}(S). \tag{3.15}$$

It is easily seen that the antibracket $(*,*)$ and the "delta" operator $\square$ satisfy the basic properties of Eqs. (2.5) – (2.11) with $\{*,*\}$ and Δ replaced with $(*,*)$ and $\square$, respectively. Since $N_{\rm gh}[\delta/\delta H(z)] = N_{\rm gh}[\delta(z)] = -N_{\rm gh}[d\mu(z)]$, the condition (3.5) ensures that both $(*,*)$ and $\square$ raise $N_{\rm gh}$ by one.

Here we should add a comment on the ghost number restriction on $S(z)$ as the argument of $I(S)$. Let us suppose a (formal) expansion of $S(z)$ in terms of a "complete set of interactions" $\{f_n(z)\}$:

$$S(z) = \sum_n f_n(z) s_n, \tag{3.16}$$

where the coupling constants s_n are now the dynamical variables. When we consider the gauge-invariant actional $I(S)$ (3.2) and the gauge transformation (2.17), we can consistently restrict the summation (3.16) to those n with $N_{\rm gh}[s_n] = 0$ by restricting also the expansion of the transformation parameter $\epsilon(z) = \sum_n f_n(z)\epsilon_n$ to $N_{\rm gh}[\epsilon_n] = 0$ (and therefore $N_{\rm gh}[f_n] = -1$). In other words, we can impose the condition

$$S(z_\theta) = S(z), \quad \epsilon(z_\theta) = e^{-\theta}\epsilon(z), \quad (z^I_\theta \equiv e^{\theta N_{\rm gh}[z^I]} z^I), \tag{3.17}$$

for an arbitrary θ. This restriction looks natural if we regard $I(S)$ as a "classical" actional before introducing the FP ghosts for quantization. However, when we discuss the BV formalism, we have to relax this ghost number restriction on $S(z)$. Namely, in order for the antibracket and the delta operator of Eqs. (3.12) and (3.13) to be nonvanishing and make sense, we have to allow the existence in $S(z)$ of the couplings s_n of any ghost number. The situation is the same as in string field theory [11,4].

Once we relax the ghost number restriction (3.17) on $S(z)$, the same $I(S)$ as Eq. (3.2) satisfies the master equation of the TT:

$$\Box \frac{1}{\lambda} I + \frac{1}{2}\left(\frac{1}{\lambda} I, \frac{1}{\lambda} I\right) = 0, \tag{3.18}$$

for an arbitrary coupling constant λ. Namely, each term of Eq. (3.18) vanishes separately:

$$(I, I) = \int d\mu\, \Delta\overline{H}\Delta H = \int d\mu\, \overline{H}\Delta^2 H = 0, \tag{3.19}$$

$$\Box I = \frac{1}{2}\mathrm{Tr}\Delta = \frac{1}{2}\int \prod_I dz^I \Delta\delta(z - z')|_{z'=z} = 0. \tag{3.20}$$

$\Box I$ vanishes because it is a Grassmann odd constant which we do not have.[3]

C. BV quantization of the TT

We now apply the BV quantization method to our TT described by the actional $(1/\lambda) I(S)$. The expectation value of an observable $\mathcal{O}(S)$ is given by [cf. Eq. (2.14)]

$$\langle\!\langle \mathcal{O} \rangle\!\rangle = \frac{1}{\mathcal{Z}(\lambda)} \int_{\mathcal{L}} \mathcal{D}\Lambda\, \mathcal{O}(S) \exp\left[\frac{1}{\lambda} I(S)\right], \tag{3.21}$$

where $\mathcal{L}$ is a Lagrangian submanifold of the supermanifold of $H(z)$, and $\mathcal{D}\Lambda$ is the volume element on $\mathcal{L}$ defined similarly to Eq. (2.15) on the basis of the volume element $\mathcal{D}H(z)$ on the total supermanifold of $H(z)$ (the explicit expression for $\mathcal{D}\Lambda$ will be given later). $\mathcal{Z}(\lambda)$ in Eq. (3.21) is the partition function of the TT. In order for the expectation value $\langle\!\langle \mathcal{O} \rangle\!\rangle$ to be independent of the choice of the Lagrangian submanifold $\mathcal{L}$, the observable $\mathcal{O}(S)$ has to satisfy the condition [recall Eq. (2.16)]

$$\Box \mathcal{O} + \frac{1}{\lambda}(I, \mathcal{O}) = 0. \tag{3.22}$$

As an example of an observable satisfying Eq. (3.22) we have the "partition function operator" $V_L(S)$:

$$V_L(S) = \int_L d\lambda\, H(z). \tag{3.23}$$

For this V_L each term of Eq. (3.22) vanishes separately. $\Box V_L = 0$ is obvious since V_L is linear in $H(z)$. As for the second term, (I, V_L), we have

$$(I, V_L) = \int_L d\lambda\, \Delta H(z) = 0, \tag{3.24}$$

where use has been made of the general property (2.20) of the integration over the Lagrangian submanifold L.

$\langle\!\langle V_L \rangle\!\rangle$ is not only independent of the choice of $\mathcal{L}$, but in fact is also independent of the choice of the Lagrangian submanifold L defining V_L. This may be seen as follows. First, under a small deformation δ_L of L corresponding to the canonical transformation $z^I \to z^I + \{z^I, \epsilon(z)\}$, the observable V_L transforms as [cf. Eq. (2.19)]

$$\begin{aligned} \delta_L V_L(S) &= \int_L d\lambda \Big[\Delta\epsilon H + \{H, \epsilon\}\Big] \\ &= \int_L d\lambda\, \epsilon\, \Delta H = (X, I), \end{aligned} \tag{3.25}$$

with

$$X = \int_L d\lambda\, \epsilon(z) H(z). \tag{3.26}$$

Then the variation of the expectation value $\langle\!\langle V_L \rangle\!\rangle$ under δ_L is

$$\begin{aligned} \delta_L \langle\!\langle V_L \rangle\!\rangle &= \lambda \int_{\mathcal{L}} \mathcal{D}\Lambda \left(X, e^{I/\lambda}\right) \\ &= \lambda \int_{\mathcal{L}} \mathcal{D}\Lambda \Big[-\Box\left(X e^{I/\lambda}\right) + \Box X e^{I/\lambda} \\ &\qquad - X \Box e^{I/\lambda}\Big] \\ &= 0, \end{aligned} \tag{3.27}$$

where the three terms in the last expression vanish separately upon using (i) the formula (2.20) applied to the TT, (ii) the fact that X is linear in $H(z)$, and (iii) the master equation for $I(S)$, Eq. (3.18) [in Eq. (3.27) we have omitted to divide the RHS's by $\mathcal{Z}(\lambda)$].

D. Choosing a Lagrangian submanifold

The quantum TT is now given by Eq. (3.21). In this subsection we shall choose a concrete Lagrangian submanifold $\mathcal{L}$ for the quantization of the TT. For this pur-

[3] In fact, a more careful analysis using a suitable regularization may prove necessary, and this might give a nonvanishing S-dependent "anomaly" to Eq. (3.20).

pose we shall work in the Darboux frame for $\mathcal{M}$ with $\rho(z) = 1$ and treat one pair of field and antifield variables in a special manner. We choose a frame

$$z^I = (\tau, \theta, \tilde{z}^I), \qquad \tilde{z}^I = (\phi^i, \phi_i^*)_{i=1,\ldots,n-1}, \qquad (3.28)$$

with

$$\Delta = \frac{\partial_l^2}{\partial\tau\partial\theta} + \tilde{\Delta}, \qquad \tilde{\Delta} = \sum_{i=1}^{n-1} (-)^{\phi^i} \frac{\partial_l^2}{\partial\phi^i\partial\phi_i^*}, \qquad (3.29)$$

where τ and θ are Grassmann even and odd, respectively, and the condition $N_{\rm gh}[\tau] + N_{\rm gh}[\theta] = -1$ is satisfied. Then we make explicit the dependence on the Grassmann odd coordinate θ by expressing $H(z)$ as

$$H(z) = h(\tau, \tilde{z}) + \theta\chi(\tau, \tilde{z}). \qquad (3.30)$$

In terms of the components h and χ of Eq. (3.30), the functional differentiation $\delta/\delta H(z)$ is given by

$$\frac{\overrightarrow{\delta}}{\delta H(z)} = \theta \frac{\overrightarrow{\delta}}{\delta h(\tau,\tilde{z})} + \frac{\overrightarrow{\delta}}{\delta\chi(\tau,\tilde{z})},$$
$$\frac{\overleftarrow{\delta}}{\delta H(z)} = -\frac{\overleftarrow{\delta}}{\delta h(\tau,\tilde{z})}\theta + \frac{\overleftarrow{\delta}}{\delta\chi(\tau,\tilde{z})}. \qquad (3.31)$$

Note that $\delta/\delta h$ and $\delta/\delta\chi$ are Grassmann even and odd, respectively. Using Eq. (3.31), the antibracket (3.12) of the TT is re-expressed as

$$(\mathcal{A}, \mathcal{B}) = \int d\tau d\tilde{z}\, \mathcal{A} \left(\frac{\overleftarrow{\delta}}{\delta h} \frac{\overrightarrow{\delta}}{\delta \overline{\chi}} - \frac{\overleftarrow{\delta}}{\delta \chi} \frac{\overrightarrow{\delta}}{\delta \overline{h}} \right) \mathcal{B}, \qquad (3.32)$$

where $d\tilde{z} \equiv \prod_{i=1}^{n-1} d\phi^i d\phi_i^*$, and $h = h(\tau,\tilde{z})$ and $\overline{\chi} = \chi(\tau, \overline{\tilde{z}})$, etc. Namely, we have chosen a Darboux coordinate for the supermanifold of $H(z)$.

The Lagrangian submanifold $\mathcal{L}$ in the space of functions $H(z)$ is specified by

$$\mathcal{L} : \Gamma(\tau, \tilde{z}) = 0, \qquad (3.33)$$

where $\Gamma(\tau,\tilde{z})$, which is defined for each $(\tau,\tilde{z})$, is a functional of h and χ, and it satisfies the condition

$$(\Gamma(\tau_1,\tilde{z}_1), \Gamma(\tau_2,\tilde{z}_2)) = 0, \qquad (3.34)$$

for any pair $(\tau_1,\tilde{z}_1)$ and $(\tau_2,\tilde{z}_2)$. A solution to Eq. (3.34) is obtained by assuming that

$$\Gamma(\tau,\tilde{z}) = \chi(\tau,\tilde{z}) + \widehat{\Gamma}(\tau,\tilde{z}), \qquad (3.35)$$

where $\widehat{\Gamma}$ depends only on h. Then Eq. (3.34) is rewritten as

$$(\Gamma(\tau_1,\tilde{z}_1), \Gamma(\tau_2,\tilde{z}_2)) = \frac{\delta\widehat{\Gamma}(\tau_1,\tilde{z}_1)}{\delta h(\tau_2,\overline{\tilde{z}}_2)} - \frac{\delta\widehat{\Gamma}(\tau_2,\tilde{z}_2)}{\delta h(\tau_1,\overline{\tilde{z}}_1)} = 0, \qquad (3.36)$$

and $\widehat{\Gamma}$ should be given as a gradient form

$$\widehat{\Gamma}(\tau,\tilde{z}) = \frac{\delta G[h]}{\delta h(\tau,\overline{\tilde{z}})} \equiv \frac{\delta G[h]}{\delta \overline{h}}, \qquad (3.37)$$

in terms of the gauge fermion $G[h]$ of TT [cf. Eq. (2.23)].

E. The TT as a topological theory

In this subsection we shall carry out a concrete study of the BV quantized TT of Eq. (3.21) using the Lagrangian submanifold of the previous subsection. First, $I(S)$ and the pre-BRST transformation δ_B for the TT,

$$\delta_B H = (I, H) = \Delta H, \qquad (3.38)$$

are expressed in terms of h and χ of Eq. (3.30) as

$$I(S) = -\frac{1}{2}\int d\tau d\tilde{z} \left(\overline{\chi}\frac{\partial}{\partial\tau}\chi + \overline{\chi}\tilde{\Delta}h + \overline{h}\tilde{\Delta}\chi \right), \qquad (3.39)$$

and

$$\delta_B h = \tilde{\Delta}h + \frac{\partial\chi}{\partial\tau},$$
$$\delta_B \chi = \tilde{\Delta}\chi. \qquad (3.40)$$

We shall consider the expansion around a classical solution $H_0(z)$ of the master equation:

$$H_0 = h_0 + \theta\chi_0 \quad \text{with} \quad \Delta H_0 = 0. \qquad (3.41)$$

The master equation for the components h_0 and χ_0 reads

$$\tilde{\Delta}h_0 + \frac{\partial\chi_0}{\partial\tau} = 0,$$
$$\tilde{\Delta}\chi_0 = 0. \qquad (3.42)$$

Defining the fluctuation f by

$$f(\tau,\tilde{z}) \equiv h(\tau,\tilde{z}) - h_0(\tau,\tilde{z}), \qquad (3.43)$$

the Lagrangian submanifold $\mathcal{L}$ is specified by

$$\mathcal{L} : \chi(\tau,\tilde{z}) = \chi_0(\tau,\tilde{z}) + \frac{\delta G[f]}{\delta f(\tau,\overline{\tilde{z}})}. \qquad (3.44)$$

Note that this is slightly modified compared to the form in the previous subsection because of the redefinition of G. We assume that $\delta G[f]/\delta\overline{f}|_{f=0} = 0$ and therefore $\chi|_{f=0} = \chi_0$ on $\mathcal{L}$.

The gauge-fixed actional $\widehat{I}(f)$ and the corresponding BRST transformation $\widehat{\delta}_B$ are given by

$$\widehat{I}(f) \equiv I|_{\mathcal{L}} = -\int d\tau d\tilde{z} \left(\frac{1}{2}\frac{\delta G}{\delta f}\frac{\partial}{\partial\tau}\frac{\delta G}{\delta\overline{f}} + \frac{\delta G}{\delta f}\tilde{\Delta}f \right), \qquad (3.45)$$

$$\widehat{\delta}_B f \equiv \delta_B f|_{\mathcal{L}} = \tilde{\Delta}f + \frac{\partial}{\partial\tau}\frac{\delta G}{\delta\overline{f}}, \qquad (3.46)$$

where $|_{\mathcal{L}}$ means the restriction to the Lagrangian submanifold (3.44). It can be checked explicitly that $\widehat{I}(f)$ is invariant under $\widehat{\delta}_B$ and that the latter is on-shell nilpotent:

$$\widehat{\delta}_B \widehat{I}(f) = 0, \tag{3.47}$$

$$\left(\widehat{\delta}_B\right)^2 f = \frac{\partial}{\partial \tau}\left(\frac{\delta \widehat{I}}{\delta \overline{f}}\right). \tag{3.48}$$

In proving Eq. (3.47) we have used, in particular, the manipulation

$$\int d\tau d\widetilde{z} \frac{\delta G}{\delta f} \widetilde{\Delta}\left(\frac{\partial}{\partial \tau}\frac{\delta G}{\delta \overline{f}}\right) = \int d\tau d\widetilde{z}\, \widetilde{\Delta}\left(\frac{\partial}{\partial \tau}\frac{\delta G}{\delta f}\right)\frac{\delta G}{\delta \overline{f}}$$
$$= -\int d\tau d\overline{\widetilde{z}} \overline{\widetilde{\Delta}}\left(\frac{\partial}{\partial \tau}\frac{\delta G}{\delta \overline{f}}\right)\frac{\delta G}{\delta f} = 0, \tag{3.49}$$

where the first equality is due to partial integrations, and at the second equality we have made a change of integration variables $\widetilde{z} \to \overline{\widetilde{z}}$, under which we have $d\overline{\widetilde{z}} = d\widetilde{z}$ and $\overline{\widetilde{\Delta}} = -\widetilde{\Delta}$.

The partition function operator V_L of Eq. (3.23) on $\mathcal{L}$ is given by

$$\widehat{V}_L(f) = V_L|_{\mathcal{L}} = V_L^0 + \int_L d\lambda \left(f + \theta \frac{\delta G}{\delta \overline{f}}\right), \tag{3.50}$$

where $V_L^0 \equiv \int_L d\lambda H_0(z)$ is the partition function of a gauge theory described by the action $S_0(z) = \ln H_0(z)$. The expectation value $\langle\langle V_L \rangle\rangle$ of Eq. (3.21) may now be explicitly written as

$$\langle\langle V_L \rangle\rangle = \frac{1}{\mathcal{Z}(\lambda)} \int \mathcal{D}f\, \widehat{V}_L(f) \exp\left[\frac{1}{\lambda}\widehat{I}(f)\right]. \tag{3.51}$$

Some comments are in order. First, it should be noted that the range of path integration over f in Eq. (3.51) is nontrivial since the original variable $H(z)$ is in fact an exponential function $H(z) = \exp[S(z)]$. It is restricted to the region

$$f|_{\theta_i=0} > -h_0|_{\theta_i=0}, \tag{3.52}$$

where θ_i denotes all of the Grassmann odd coordinates in $\widetilde{z}^I$. Therefore, even if we adopt $G[f]$, which is quadratic in f and hence makes $\widehat{I}$ (3.45) quadratic in f, the TT is not truly a free field theory: $\widehat{I}$ expressed in terms of unrestricted variable contains interactions.

Second, the (formal) invariance of the path-integral measure $\mathcal{D}f$ under the BRST transformation $\widehat{\delta}_B$ of Eq. (3.46) may be checked as follows:

$$\widehat{\delta}_B \ln \mathcal{D}f = \int d\tau d\widetilde{z} \frac{\delta\left[\delta_B f(\tau, \widetilde{z})\right]}{\delta f(\tau, \widetilde{z})}$$
$$= \mathrm{Tr}\widetilde{\Delta} + \int d\tau d\widetilde{z} \frac{\delta}{\delta f(\tau, \widetilde{z})}\frac{\partial}{\partial \tau}\frac{\delta G}{\delta f(\tau, \overline{\widetilde{z}})} = 0, \tag{3.53}$$

where the vanishing of the last term can be understood by partially integrating with respect to τ and then changing the integration variables from $\widetilde{z}$ to $\overline{\widetilde{z}}$ to obtain minus the original expression.

The third comment is that $\widehat{V}_L(f)$ (3.50) is not exactly a $\widehat{\delta}_B$ invariant operator but $\widehat{\delta}_B \widehat{V}_L$ is proportional to the equation of motion:

$$\widehat{\delta}_B \widehat{V}_L(f) = -\int_L d\lambda\, \theta \frac{\delta \widehat{I}(f)}{\delta \overline{f}}. \tag{3.54}$$

The invariance of $\langle\langle V_L \rangle\rangle$ under an infinitesimal change of the gauge fermion G by $\delta_G G$ can be checked explicitly using Eq. (3.54) and $\delta_G \widehat{I} = \widehat{\delta}_B\left(\delta_G G\right)$, etc.

The gauge-fixed actional (3.45) consists solely of terms containing the gauge fermion $G[f]$ which specifies the gauge fixing. Therefore one may suspect that our TT is a topological theory which has no physical degrees of freedom as a quantum theory of S. We show in the following that this is the case: by introducing an auxiliary field the gauge-fixed actional $\widehat{I}$ can be reduced to a BRST exact form. For this purpose we multiply both the denominator and the numerator of Eq. (3.51) by a Gaussian integration over a new Grassmann odd variable $B(\tau, \widetilde{z})$,

$$\int \mathcal{D}B \exp\left\{\frac{1}{2\lambda}\int d\tau d\widetilde{z}\left(\overline{B} - \frac{\delta G}{\delta f}\right)\frac{\partial}{\partial \tau}\left(B - \frac{\delta G}{\delta \overline{f}}\right)\right\}, \tag{3.55}$$

and consider the system of f and B variables [note that (3.55) is independent of the old variable f]. Let us define the new BRST transformation $\widehat{\widehat{\delta}}_B$ for the (f, B) system by

$$\widehat{\widehat{\delta}}_B f = \widetilde{\Delta} f + \frac{\partial B}{\partial \tau},$$
$$\widehat{\widehat{\delta}}_B B = \widetilde{\Delta} B. \tag{3.56}$$

This $\widehat{\widehat{\delta}}_B$ is apparently the same as the pre-BRST transformation δ_B of Eq. (3.40) with (h, χ) replaced with (f, B) and hence is off-shell nilpotent:

$$\left(\widehat{\widehat{\delta}}_B\right)^2 = 0. \tag{3.57}$$

The actional $\widehat{\widehat{I}}(f, B)$ for the new (f, B) system is obtained by summing $\widehat{I}(f)$ (3.45) and the contribution from Eq. (3.55), and it is written as a $\widehat{\widehat{\delta}}_B$ exact form:

$$\widehat{\widehat{I}}(f, B) = \widehat{I}(f) + \frac{1}{2}\int d\tau d\widetilde{z}\left(\overline{B} - \frac{\delta G}{\delta f}\right)\frac{\partial}{\partial \tau}\left(B - \frac{\delta G}{\delta \overline{f}}\right)$$
$$= \int d\tau d\widetilde{z}\left(\frac{1}{2}\overline{B}\frac{\partial}{\partial \tau}B - \frac{\delta G}{\delta f}\frac{\partial B}{\partial \tau} - \frac{\delta G}{\delta f}\widetilde{\Delta} f\right)$$
$$= \widehat{\widehat{\delta}}_B\left(G[f] - \frac{1}{2}\int d\tau d\widetilde{z}\overline{B} f\right). \tag{3.58}$$

In the (f, B) system described by the actional $\widehat{\widehat{I}}$, the partition function operator $\widehat{V}_L$ (3.50) is equivalent to a new operator $\widehat{\widehat{V}}_L$,

$$\widehat{\widehat{V}}_L(f, B) = V_L^0 + \int_L d\lambda \left(f + \theta B\right), \tag{3.59}$$

so long as we consider only the the "one-point" function:

$$\langle\langle V_L\rangle\rangle = \frac{1}{\mathcal{Z}(\lambda)}\int \mathcal{D}f\int \mathcal{D}B\,\widehat{\widehat{V}}_L(f,B)\exp\left(\frac{1}{\lambda}\widehat{\widehat{I}}(f,B)\right). \tag{3.60}$$

This is because we have

$$\widehat{V}_L(f) = \widehat{\widehat{V}}_L(f,B) - \int_L d\lambda\,\theta\left(B - \frac{\delta G}{\delta \overline{f}}\right), \tag{3.61}$$

and the expectation value of the last term of Eq. (3.61) vanishes since it is odd with respect to $B - \delta G/\delta\overline{f}$ [cf. Eq. (3.55)].[4] For a general multi-V_L function $\langle\langle\prod_i V_{L_i}\rangle\rangle$ (whose physical meaning is not clear at present), $\widehat{V}_L$ cannot be replaced by $\widehat{\widehat{V}}_L$. In distinction to the case of the $\widehat{V}_L$ operator in the f formalism [cf. Eq. (3.54)], the $\widehat{\widehat{V}}_L$ operator in the (f,B) formulation is simply a $\widehat{\widehat{\delta}}_B$ invariant operator:

$$\widehat{\widehat{\delta}}_B\widehat{\widehat{V}}_L(f,B) = \int_L d\lambda\,\Delta(f+\theta B) = 0. \tag{3.62}$$

This implies that $\langle\langle V_L\rangle\rangle$ is in fact independent of the coupling constant λ:

$$\frac{\partial}{\partial\lambda}\langle\langle V_L\rangle\rangle = 0, \tag{3.63}$$

and therefore $\langle\langle V_L\rangle\rangle$ may be calculated in the "classical" limit $\lambda\to 0$ [8,9]. In this limit the (f,B) fields are frozen, and we find that the expectation value of the operator V_L in the TT is equal to the partition function of the gauge theory corresponding to a classical solution $S_0(z)$ of the master equation (2.12):

$$\langle\langle V_L\rangle\rangle = \lim_{\lambda\to 0}\langle\langle V_L\rangle\rangle = V_L^0. \tag{3.64}$$

This implies the equivalence between our TT and the ordinary formulation of gauge theories, at least as far as the partition function is concerned.

In deriving Eq. (3.64), we have implicitly assumed that there are no stationary points of $\widehat{\widehat{I}}(f,B)$ other than the trivial one $(f,B)=(0,0)$. What will happen if there are other stationary points of $\widehat{\widehat{I}}$? In this case there would be two possibilities: one is that we have to sum all the contributions from the stationary points of $\widehat{\widehat{I}}$, and the other is that only one of the stationary points is chosen in the same manner as in the case of spontaneous symmetry breakdown. Which of the two is realized depends on the dynamics of the TT: whether the TT with a nonvanishing coupling constant λ is in a disordered phase or in an ordered perturbative phase.

If all the stationary points of $\widehat{\widehat{I}}$ have to be summed over, the TT would be a very peculiar theory which could hardly reproduce an ordinary gauge theory. To discuss the latter case of choosing one stationary point, let us regard the variable f as the original h of Eq. (3.30), that is, let us put $H_0 = 0$ in the above equations. For simplicity we consider the stationary points of $\widehat{I}(h)$ instead of those of $\widehat{\widehat{I}}(h,B)$ (they give the same result for h). The stationary condition of $\widehat{I}(h)$ reads

$$\frac{\delta\widehat{I}(h)}{\delta h} = \left(\widetilde{\Delta}+\widehat{\delta}_B\right)\frac{\delta G[h]}{\delta h} = 0. \tag{3.65}$$

From Eqs. (3.65), (3.42), and (3.46) we see that, if we have a solution H_0 of the master equation $\Delta H_0 = 0$ on the Lagrangian submanifold $\mathcal{L}$ of the form

$$H_0 = h_0 + \theta\left.\frac{\delta G[h]}{\delta h}\right|_{h=h_0}, \tag{3.66}$$

then $h=h_0$ is automatically a stationary point of $\widehat{I}(h)$. However, not all solutions of Eq. (3.65) correspond to solutions of the master equation. If only one stationary point is chosen in evaluating $\langle\langle V_L\rangle\rangle$ in the TT, a stationary point h_0 which does not correspond to a solution of the master equation should not be selected, since in that case we would have $\widehat{\delta}_B h|_{h=h_0}\neq 0$ and the BRST symmetry in the TT would be spontaneously broken so that the above argument leading to the λ independence of $\langle\langle V_L\rangle\rangle$ would break down. If we have many stationary points which have corresponding solutions (3.66) of the master equation, we do not yet know how one of them might be singled out. Note that $\widehat{I}(h)$ vanishes at every one of these BRST-invariant stationary points.

IV. SUMMARY AND DISCUSSION

In this paper we have proposed a new approach to gauge theory. It is formulated as a gauge theory having the action S of ordinary gauge theories as its dynamical variable. Applying the BV quantization method, we have found that our TT is essentially a topological theory described by a BRST exact actional and can reproduce an ordinary gauge theory corresponding to an action satisfying the master equation (1.2). Since this new formulation does not refer to any concrete solution of the master equation (1.2) in its basic formulation, our TT is expected to be useful in its application to closed-string field theory whose quantum action takes a very complicated form.

There are many questions left unanswered. Most of them are mentioned in the text. We finish this paper by summarizing them (not necessarily in the order of importance).

(i) Observables in the TT. At present we have only the partition function operator (3.23) as an example of an observable satisfying the condition (3.22). In order to make clearer the connection between the present TT and the ordinary formulation of gauge theory, we have to prepare more observables. An "on-shell amplitude operator" would be an interesting candidate, however, we do not know the explicit form of such an operator in the TT.

In relation to the problem of observables in the TT,

[4]Here we are assuming that the discrete symmetry $B\to -B+2\delta G/\delta\overline{f}$ is not spontaneously broken.

we comment that the product of the partition function operators, $\mathcal{O} = \prod_i V_{L_i}$, is also an observable satisfying the condition (3.22). We have $(I, \mathcal{O}) = \sum_i (I, V_{L_i}) \prod_{j\neq i} V_{L_j} = 0$ since $(I, V_{L_i}) = 0$. As for $\square\mathcal{O}$, it is expressed using Eq. (2.8) as a sum of terms which contain either $\square V_{L_i} (= 0)$ or

$$(V_{L_i}, V_{L_j}) = \int_{L_i} d\lambda(z) \int_{L_j} d\lambda(w) \frac{1}{\rho(z)} \delta(z - \overline{w}). \quad (4.1)$$

The quantity (4.1) vanishes since it is a constant with $N_{\rm gh} = 1$. However, we do not know whether there is any interesting meaning to the expectation value $\langle\langle \prod_i V_{L_i} \rangle\rangle$. Note that $(\partial/\partial\lambda)\langle\langle \prod_{i=1}^N V_{L_i} \rangle\rangle = 0$ does not hold for $N \geq 2$.

(ii) $N_{\rm gh}[I(S)] = 0$? We have left unanswered the question of whether the actional $I(S)$ of Eq. (3.2) carries no ghost number, that is, whether the condition (3.5) is satisfied in the system we are interested in, for example, closed-string field theory. As stated in the text, this is not an easy problem since the index I is in fact a continuous parameter.

(iii) We do not yet know how to treat the case where the actional $\widehat{\widehat{I}}$ has many stationary points, should this arise.

(iv) It is an interesting question whether the present TT formulation applied to closed-string field theory gives a space-time background independent formulation (see Refs. [12,13] for recent studies on the background independence of string field theory).

(v) We have no intuitive understanding of why the TT, which has a nontrivial classical actional, $I(S)$ of Eq. (3.2) with the restriction (3.17), is reduced upon BV quantization to a topological theory, at least in the evaluation of $\langle\langle V_L \rangle\rangle$.

(vi) Most of the manipulations in this paper are very formal. It is desirable to study the validity of the arguments for the TT using a simple model.

The last and the most important problem is how the present TT formulation is useful in the study of gauge theories, and, in particular, of string theory. This problem is currently under investigation.

ACKNOWLEDGMENTS

I would like to thank K.-I. Izawa for valuable discussions. I also wish to acknowledge M.G. Mitchard for carefully reading the manuscript. This work was supported in part by Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture (No. 05230037).

[1] I. A. Batalin and G. A. Vilkovisky, Phys. Lett. **102B**, 27 (1981); Phys. Rev. D **28**, 2567 (1983).

[2] H. Hata, Phys. Lett. B **217**, 445 (1989); Nucl. Phys. **B329**, 698 (1990); **B339**, 663 (1990).

[3] B. Zwiebach, Mod. Phys. Lett. A **5**, 2753 (1990); Commun. Math. Phys. **136**, 83 (1991); Phys. Lett. B **241**, 343 (1990).

[4] B. Zwiebach, Nucl. Phys. **B390**, 33 (1993).

[5] E. Witten, Mod. Phys. Lett. A **5**, 487 (1990).

[6] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems* (Princeton University Press, Princeton, N.J., 1992).

[7] H. Hata and B. Zwiebach, Ann. Phys. (N.Y.) **229**, 177 (1994).

[8] E. Witten, Commun. Math. Phys. **117**, 353 (1988); **118**, 411 (1988).

[9] D. Birmingham, M. Blau, M. Rakowski, and G. Thompson, Phys. Rep. **209**, 129 (1991).

[10] A. Schwarz, Commun. Math. Phys. **155**, 249 (1993).

[11] H. Hata, K. Itoh, T. Kugo, H. Kunitomo, and K. Ogawa, Phys. Rev. D **34**, 2360 (1986); **35**, 1318 (1987).

[12] E. Witten, Phys. Rev. D **46**, 5467 (1992); D **47**, 3405 (1993).

[13] A. Sen and B. Zwiebach, Nucl. Phys. **B414**, 649 (1994).