Cabibbo-favored hadronic two-body B decays

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We have analyzed hadronic B decays into ψK, $D\pi$, $D^*\pi$, and $D\rho$ channels with a view to investigate if the uncertainties in the form factors and inclusion of final state interactions and annihilation terms would allow solutions with negative a_2/a_1. We have concluded that the new CLEO II data cannot be understood with negative a_2/a_1 even after allowing for the effects mentioned above.

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I. INTRODUCTION

It has been argued in the recent literature [1–3] that two-body hadronic decays of the B meson into $D\pi$, $D^*\pi$, $D\rho$, and $D^*\rho$ channels imply a positive sign for the Wilson coefficients a_2 [1,4]. The theoretical expectation is that the sign of a_2 is negative [4]. This expectation hinges on the validity of the limit $N\to\infty$ (N being the number of colors) in the b-mass region. Phenomenologically, the $N\to\infty$ limit appears [4] to work well in the c-mass region.

In this paper we have investigated the theoretical assumptions that have gone into the analysis of data leading to the conclusion that $a_2>0$. We have stretched these uncertainties to reasonable limits to see if a negative sign of a_2 can be accommodated by data. In addition to the theoretical uncertainties associated with the various form factors, we have considered effects of the final-state interactions (FSI) and the annihilation term. We have concluded that though the theory with negative values of a_2 could be stretched to allow agreement with the "old," pre-1992 data [5], the new CLEO II data [2,3] rule out a negative value of a_2.

II. ANALYSIS

The relevant part of the effective weak Hamiltonian is [1]

$$H_{\text{eff}}=\frac{G_F}{\sqrt{2}}V_{cb}^*V_{ud}[c_1Q_1+c_2Q_2]\ , \tag{1}$$

where c_i are the Wilson coefficients evaluated at the b mass and

$$Q_1=[(\bar{d}u)+(\bar{s}c)](\bar{c}b)\ , \tag{2}$$

$$Q_2=(\bar{c}u)(\bar{d}b)+(\bar{c}c)(\bar{s}b)\ . \tag{3}$$

The brackets around the Dirac bilinears indicate color-singlet $(V-A)$ currents. The Wilson coefficients c_i are related to the coefficients $c_\pm$ by

$$c_\pm=c_1\pm c_2\ , \tag{4}$$

where, in the leading-log approximation[1],

$$c_\pm(\mu)=\left[\frac{\alpha_s(m_W^2)}{\alpha_s(\mu^2)}\right]^{-6\gamma_\pm/33-2n_f}\ , \tag{5}$$

with $\gamma_-=-2\gamma_+=2$, and n_f, the number of "active" flavors, is 5 for b decays. From Eq. (5) one finds [1]

$$c_1(m_b)=1.12,\quad c_2(m_b)=-0.26\ . \tag{6}$$

Upon Fierz reordering in color, one can write H_{eff} in the notation of Ref. [4], as follows:

$$H_{\text{eff}}=\frac{G_F}{\sqrt{2}}V_{cb}^*V_{ud}[a_1Q_1^H+a_2Q_2^H]\ , \tag{7}$$

where the superscript H stands for "hadronic," implying that the Dirac bilinears of Eq. (3) should be treated as interpolating fields for the mesons and no further Fierz reordering need be done. a_1 and a_2 are related to c_1 and c_2 by

$$a_1=c_1+\frac{1}{N}c_2,\quad a_2=c_2+\frac{1}{N}c_1\ , \tag{8}$$

where N is the number of colors. One expects N to be 3. However, the phenomenology of charm decays appears to indicate that the $N\to\infty$ limit works well for D decays [4]. If this were also the case for B decays, one would have $a_1\simeq1.12$ and $a_2\simeq-0.26$. On the other hand, if $N=3$ is used in Eq. (8), one obtains $a_1\simeq1.03$ and $a_2\simeq0.11$. We will argue in this paper that this latter value of a_2 is not satisfactory in magnitude. B decay data require $|a_2|\approx0.2$ with about 30% error. However, our final conclusion is that a_2 is positive. In the Conclusion section we discuss its implications in greater detail. In Secs. II A–II D we consider $\bar{B}^0\to\psi K^0$, $D^-\pi^+$, $\bar{D}^0\pi^0$, and $B^+\to\bar{D}^0\pi^+$ in some detail.

A. $\bar{B}^0\to\psi K^0$ decay

This is a color-suppressed process whose amplitude is proportional to a_2 in the factorization approximation. Indeed, this decay is used [1–3] to determine $|a_2|$. The decay amplitude is (there are no annihilation terms allowed for this decay mode)

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$$A(\bar{B}^0 \to \psi \bar{K}^0) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} a_2 (2m_\psi) f_\psi F_1^{BK}(m_\psi^2) \epsilon^* \cdot p \ , \tag{9}$$

where p is the B meson four-momentum.

The form factor F_1 is defined in Refs. [1,4] and we take $f_\psi = 0.384$ GeV [1]. In Ref. [1], and in subsequent analyses of data in [2,3] the authors have used $F_1^{BK}(m_\psi^2) = 0.834$. As none of these authors has explained what form factors have been used in their analyses, and as we are interested in exploring the consequences of the uncertainties in these form factors, we take some pains to explain how the numerical values for these form factors are arrived at.

As the weak decay involves the $b \to s$ transition, heavy quark effective theory (HQET) cannot be used to constrain these heavy-to-light quark transition form factors. In Ref. [1], the authors have used the model calculation of Ref. [4]: $F_1^{BK}(0) = 0.379$, and extrapolated it to $q^2 = m_\psi^2$ using a dipole form factor with a mass parameter $m_{1^-} = 5.43$ GeV. The result is $F_1^{BK}(m_\psi^2) = 0.834$. There are two kinds of theoretical uncertainties here: one, a *model* calculation of $F_1^{BK}(0)$, and the other associated with the extrapolation to rather a large value of q^2. We keep $F_1^{BK}(m_\psi^2)$ in our expression so that we can trace uncertainties in its value to errors in the calculated value of $|a_2|$.

Following Refs. [1,6], if we use

$$\left[\frac{\tau_{B^0}}{1.18 \text{ ps}} \right]^{1/2} V_{cb} = 0.045 \ , \tag{10}$$

the formula for $B(\bar{B}^0 \to \psi \bar{K}^0)$ in percent is

$$B(\bar{B}^0 \to \psi \bar{K}^0) = 1.817 \omega^2 a_2^2 \% \ , \tag{11}$$

where

$$\omega = \frac{F_1^{BK}(m_\psi^2)}{0.834} \ . \tag{12}$$

Using the experimental branching ratio of $(0.07 \pm 0.03)\%$ [5], we find

$$|a_2| = 0.20 \pm 0.04 \pm 0.04 \ , \tag{13}$$

where the last error is theoretical and arises from allowing a 20% uncertainty in $F_1^{BK}(m_\psi^2)$ centered around 0.834. The CLEO II determination [2,3]

$$|a_2| = 0.27 \pm 0.014 \pm 0.006 \pm 0.02$$

is somewhat larger because they have used their own data for all the two-body decays of B^0 and B^- involving ψ in the final state to extract $|a_2|$.

B. $B^+ \to \bar{D}^0 \pi^+$ decay

The annihilation amplitude is not allowed to order $V_{cb}^* V_{ud}$. The decay amplitude in the factorization scheme is given by

$$A(B^+ \to \bar{D}^0 \pi^+) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} a_1 f_\pi (m_B^2 - m_D^2) F_0^{BD}(0) \times \left[1 + \frac{a_2}{a_1} \frac{f_D}{f_\pi} \frac{m_B^2}{m_B^2 - m_D^2} \frac{F_0^{B\pi}(m_D^2)}{F_0^{BD}(0)} \right] . \tag{14}$$

The form factor $F_0(q^2)$ is defined in Refs. [1,4]. In Ref. [1], F_0^{BD} is calculated at maximum momentum transfer where HQET can be used and is then extrapolated down to $q^2 = 0$, resulting in $F_0^{BD}(0) = 0.58$. By contrast, a model calculation of $F_0^{BD}(0)$ in Ref. [4] yielded 0.69. We believe that there is a theoretical uncertainty associated with the extrapolation of $F_0^{BD}(q^2)$ through this rather large region of q^2.

The value of $F_0^{B\pi}(m_D^2)$ used in all the previous analyses is arrived at by using $F_0^{B\pi}(0) = 0.333$, as given in the model calculation of Ref. [4], and extrapolating it to $q^2 = m_D^2$ by using a monopole formula with mass $m_{0^+} = 5.78$ GeV. One obtains $F_0^{B\pi}(m_D^2) = 0.37$. There is a theoretical uncertainty here also. For example, with the universal β functions, Isgur, Scora, Grinstein, and Wise (ISGW) [7] would calculate at maximum momentum transfer,

$$F_0^{B\pi}(q_{\max}^2) \approx \frac{2(\bar{m}_B \bar{m}_\pi)^{1/2}}{(\bar{m}_B + \bar{m}_\pi)} \ , \tag{15}$$

where $\bar{m}$ is the relevant mock mass, i.e., sum of the constituent quark masses. Equation (15) yields $F_0^{B\pi}(q_{\max}^2) \approx 0.68$. If this is extrapolated to $q^2 = 0$ using a monopole formula, one gets $F_0^{B\pi}(0) \approx 0.14$—a value much smaller than that given in Ref. [4]. We make this point only to emphasize that there are theoretical uncertainties associated with $F_0^{B\pi}(m_D^2)$. The branching ratio $B(B^+ \to \bar{D}^0 \pi^+)$, in percent, is given by [here we have used $\tau_{B^0} = \tau_{B^+}$ and Eq. (10)]

$$B(B^+ \to \bar{D}^0 \pi^+) = 0.264 a_1^2 X^2 \left[1 + 1.22 \frac{a_2}{a_1} \frac{YZ}{X} \right]^2 \% \ , \tag{16}$$

where we have introduced the following definitions

$$X = \frac{F_0^{BD}(0)}{0.58}, \quad Y = \frac{F_0^{B\pi}(m_D^2)}{0.37}, \quad Z = \frac{f_D}{0.22 \text{ GeV}} \ . \tag{17}$$

If we set $X = Y = Z = 1$, we recover the formulas used in Refs. [1–3]. If we use the pre-1992 average value [5],

$$B(B^+ \to \bar{D}^0 \pi^+) = (0.38 \pm 0.11)\%$$

[the new CLEO II value [2,3] is somewhat higher: $B(B^+ \to \bar{D}^0 \pi^+) = (0.470 \pm 0.034 \pm 0.047 \pm 0.020)\%$], we can solve Eq. (14) for a_1 for chosen values of X, Y, Z, and the ratio a_2/a_1. As fitting data with *positive* values of a_2/a_1 does not pose a problem [1–3], we deliberately choose a_2/a_1 to be negative to see if the data allow such solutions. We set $Z = 1$ and allow up to about 20% uncertainty in X and Y. In Table I, we have shown some solutions for a_1 in the expected range.

TABLE I. Values of a_1 from Eq. (16) for $B(B^+\to\bar{D}^0\pi^+)$ for some selected values of X, Y, and a_2/a_1. Z is set equal to 1. Data used pre-1992 [5]: $B(B^+\to\bar{D}^0\pi^+)=(0.38\pm0.11)\%$.

	Value of a_1		
a_2/a_1	$X=1.1,\ Y=0.9$	$X=1.15,\ Y=0.85$	$X=1,2,\ Y=0.8$
$-\frac{1}{5}$	1.36 ± 0.20	1.27 ± 0.18	1.19 ± 0.17
$-\frac{1}{6}$	1.31 ± 0.19	1.23 ± 0.18	1.16 ± 0.16
$-\frac{1}{7}$	1.27 ± 0.18	1.20 ± 0.17	1.13 ± 0.16
$-\frac{1}{8}$	1.25 ± 0.18	1.18 ± 0.17	1.11 ± 0.16

It is important to see what solutions one is led to if the new CLEO II data [2,3] are used. With $a_2/a_1>0$, there is no problem fitting data with anticipated values of a_1 and a_2 [2,3]. With $a_2/a_1<0$, however, to fit a larger branching ratio, X would have to be larger than 1 to pick up the slack. We will return to this point as we discuss $B(\bar{B}^0\to D^-\pi^+)$ in Sec. II C.

C. $\bar{B}^0\to D^-\pi^+$ decay

In all the analyses of data to date [1–3], only the spectator amplitude is considered. In fact, the annihilation process (W exchange) is allowed for this mode, as it is also for the $\bar{B}^0\to\bar{D}^0\pi^0$ mode, as we shall see later. For two pseudoscalar mesons in the final state, one invokes the conserved vector current hypothesis (CVC) to rule out any significant contribution from the annihilation process to D decays. This is because the vector currents involved in charm decays are composed of light quarks u,d,s. Within the SU(3) limit, CVC works well in D decays. Such is not the case in B decays as the relevant current involves an anti-charmed quark and an up quark. One might, therefore, expect a significant annihilation term; however, the annihilation term is suppressed for a different reason—the relevant form factor is needed at $q^2=m_B^2$ where we expect it to be small.

The decay amplitude with the inclusion of the annihilation term is

$$A(\bar{B}^0\to D^-\pi^+)=\frac{G_F}{\sqrt{2}}V_{cb}^*V_{ud}a_1f_\pi(m_B^2-m_D^2)F_0^{BD}(0)\times\left[1+\frac{a_2}{a_1}\frac{f_B}{f_\pi}\frac{m_D^2}{m_B^2-m_D^2}\frac{F_0^{D\pi}(m_B^2)}{F_0^{BD}(0)}\right]. \tag{18}$$

In the above expression, the annihilation term proportional to f_B has been written down in the factorization approximation with vacuum intermediate state. Note that $F_0^{D\pi}(q^2)$ is needed at $q^2=m_B^2$, which, being in the physical region for πD scattering, would require $F_0^{D\pi}(m_B^2)$ to be complex. A complete analysis must take this fact into account. However, for our present purpose, we treat $F_0^{D\pi}(m_B^2)$ to be essentially real and argue in the following as to the size of the annihilation term in $\bar{B}^0\to D^-\pi^+$ decay. Let us define a parameter

$$\xi=\frac{f_B}{f_\pi}\frac{m_D^2}{m_B^2-m_D^2}\frac{F_0^{D\pi}(m_B^2)}{F_0^{BD}(0)}. \tag{19}$$

Notice that ξ is mass suppressed by the factor $m_D^2/(m_B^2-m_D^2)$. If we assume

$$\frac{f_B}{f_D}=\sqrt{(m_D/m_B)}, \tag{20}$$

then, with $f_D\approx0.22$ GeV, one finds $f_B\approx f_\pi$ and

$$\xi=0.246\frac{F_0^{D\pi}(m_B^2)}{X}. \tag{21}$$

With $X\approx1$ and $F_0^{D\pi}(m_B^2)\approx\frac{1}{3}$ (which might be generous), $\xi\approx\frac{1}{10}$. As ξ appears multiplied by a_2/a_1 in the decay amplitude (18), we do not expect annihilation amplitude to play an important role in $\bar{B}^0\to D^-\pi^+$ decay. Ignoring FSI, the branching ratio for $\bar{B}^0\to D^-\pi^+$, in percent, is given by

$$B(\bar{B}^0\to D^-\pi^+)=0.264a_1^2X^2\left[1+\frac{a_2}{a_1}\xi\right]^2\%. \tag{22}$$

Because of the anticipated small value of ξ, this rate is not very sensitive to the size or the sign of a_2/a_1. However, for $\xi=0.1$, and $-\frac{1}{8}<a_2/a_1<-\frac{1}{5}$, the pre-1992 world average,

$$B(\bar{B}^0\to D^-\pi^+)=(0.32\pm0.07)\%$$

[5], results in $a_1=1.02\pm0.11$, 0.97 ± 0.11, and 0.93 ± 0.10 for $X=1.1$, 1.15, and 1.20, respectively. Within errors, these solutions are compatible with those determined from the pre-1992 world averaged $B(B^+\to\bar{D}^0\pi^+)$ given in Table I.

The new CLEO II measurement of $B(\bar{B}^0\to D^-\pi^+)$, however, yields a much smaller branching ratio [2,3]:

$$B(\bar{B}^0\to D^-\pi^+)=(0.221\pm0.029\pm0.019\pm0.031)\%. \tag{23}$$

As the rate in Eq. (22) is largely insensitive to the size or the sign of a_2/a_1, in order to reproduce this smaller rate with $a_1\approx1$, one would have to choose $X<1$. This is in contrast to the discussion towards the end of Sec. II B where it was concluded that, with $a_2/a_1<0$, in confronting the new CLEO II measurement of $B(B^+\to\bar{D}^0\pi^+)$ one required $X>1$.

This is the first hint of troubles which we will delve into further with the theory with $a_2/a_1<0$ in confronting the new CLEO II data.

D. $\bar{B}^0\to\bar{D}^0\pi^0$ decay

Including the annihilation term, the decay amplitude is given by

$$A(\bar{B}^0\to\bar{D}^0\pi^0)=-\frac{G_F}{\sqrt{2}}V_{cb}^*V_{ud}\frac{a_2}{\sqrt{2}}f_Dm_B^2F_0^{B\pi}(m_D^2)\times\left[1-\frac{f_B}{f_D}\frac{m_D^2}{m_B^2}\frac{F_0^{D\pi}(m_B^2)}{F_0^{B\pi}(m_D^2)}\right]. \tag{24}$$

The resulting branching ratio, in percent, is given by

$$B(\bar{B}^0 \to \bar{D}^0\pi^0) = 0.2a_2^2X^2Z^2(1-\eta)^2\% , \tag{25}$$

where

$$\eta = \frac{f_B}{f_D}\frac{m_D^2}{m_B^2}\frac{F_0^{D\pi}(m_B^2)}{F_0^{B\pi}(m_D^2)} . \tag{26}$$

We expect η to be comparable to ξ, which we had assigned a generous value of 0.1. If we set $X = Z = 1$ and $\eta = 0$, we recover the formula for the branching ratio used in data analyses [1–3]. As X and Z are expected to be close to unity and $|\eta| \ll 1$, the branching ratio resulting from Eq. (25) stays well within the experimental upper limit of 0.035% [2,3].

III. FINAL-STATE INTERACTIONS

In this section we discuss the role of FSI. At the very outset, as all the neutral modes $\bar{B}^0 \to \bar{D}^0\pi^0$, $\bar{D}^0\rho^0$, $\bar{D}^{*0}\pi^0$, $\bar{D}^{*0}\rho^0$ are strongly suppressed [2,3], in contrast to, say, $\bar{D}^0 \to \bar{K}^0\pi^0$, one anticipates that FSI do not play an important role in these B decays. FSI has the tendency to "lift" the color suppression. We will use the upper limit on $B(\bar{B}^0 \to \bar{D}^0\pi^0)$ to set an upper limit on the FSI phase difference.

We introduce isospin amplitudes $A_{1/2}$ and $A_{3/2}$ as follows:

$$\begin{aligned} A(\bar{B}^0 \to D^-\pi^+) &= \frac{1}{\sqrt{3}}[\sqrt{2}A_{1/2}\exp(i\delta_{1/2}) \\ &\quad + A_{3/2}\exp(i\delta_{3/2})] \\ A(\bar{B}^0 \to \bar{D}^0\pi^0) &= \frac{1}{\sqrt{3}}[A_{1/2}\exp(i\delta_{1/2}) \\ &\quad - \sqrt{2}A_{3/2}\exp(i\delta_{3/2})] \\ A(B^+ \to \bar{D}^0\pi^+) &= \sqrt{3}A_{3/2}\exp(i\delta_{3/2}) . \end{aligned} \tag{27}$$

Setting the phases to zero, we can determine $A_{1/2}$ and $A_{3/2}$ from Eqs. (18), (24), and (27) with the result [an overall factor $(G_F/\sqrt{2})V_{cb}^*V_{ud}$ is suppressed]

$$\begin{aligned} A_{1/2} &= \sqrt{\tfrac{2}{3}}a_1f_\pi(m_B^2-m_D^2)F_0^{BD}(0) \\ &\quad \times \left[1-0.61\frac{a_2}{a_1}\frac{YZ}{X}+1.5\frac{a_2}{a_1}\xi\right] , \\ A_{3/2} &= \sqrt{\tfrac{1}{3}}a_1f_\pi(m_B^2-m_D^2)F_0^{BD}(0)\left[1+1.22\frac{a_2}{a_1}\frac{YZ}{X}\right] . \end{aligned} \tag{28}$$

The branching ratios for $\bar{B}^0 \to D^-\pi^+$ and $\bar{B}^0 \to \bar{D}^0\pi^0$, in percent, are given by

$$\begin{aligned} B(\bar{B}^0 \to D^-\pi^+) &= \frac{0.1176a_1^2X^2}{2r^2}\left[1+1.22\frac{a_2}{a_1}\frac{YZ}{X}\right]^2 \\ &\quad \times \left[1+\frac{r^2}{2}+\sqrt{2}r\cos\delta\right]\% , \\ B(\bar{B}^0 \to \bar{D}^0\pi^0) &= \frac{0.0588a_1^2X^2}{2r^2}\left[1+1.22\frac{a_2}{a_1}\frac{YZ}{X}\right]^2 \\ &\quad \times [1+2r^2-2\sqrt{2}r\cos\delta]\% , \end{aligned} \tag{29}$$

where $r = A_{3/2}/A_{1/2}$ and $\delta = \delta_{1/2} - \delta_{3/2}$.

The ratio of these two branching ratios is then

$$R = 0.5\frac{(1+2r^2-2\sqrt{2}r\cos\delta)}{(1+r^2/2+\sqrt{2}r\cos\delta)} , \tag{30}$$

where

$$R = \frac{B(\bar{B}^0 \to \bar{D}^0\pi^0)}{B(\bar{B}^0 \to D^-\pi^+)} . \tag{31}$$

If, using the central value of the pre-1992 data [5],

$$B(\bar{B}^0 \to D^-\pi^+) = (0.32\pm0.07)\% , \tag{32}$$

we set the ratio R given in Eq. (30) to be <0.1, we find a constraint on δ,

$$\cos\delta > \frac{(0.8\pm1.9r^2)}{3.11r} . \tag{33}$$

For reasonable values of r, $0.5 < r < 1.0$, obtained by varying a_2/a_1 in the range $-\frac{1}{5} < a_2/a_1 < \frac{1}{5}$ (note that in D decays, r is in the range 0.3–0.4 [8]), we find that

$$\delta < 35^\circ . \tag{34}$$

The new CLEO II data [2,3] allow a little larger value of δ: $\delta < 47^\circ$. As both the phases are reasonably small, we do not anticipate large FSI effects. In the following section we present a quantitative estimate of the FSI contribution.

IV. THE RELATIVE BRANCHING RATIOS

Data are often presented in the form of the relative branching ratios of the charged B decay rates with respect to the corresponding neutral B decay rates [2,3]:

$$R_1 = \frac{B(B^+ \to \bar{D}^0\pi^+)}{B(\bar{B}^0 \to D^-\pi^+)}, \quad R_2 = \frac{B(B^+ \to \bar{D}^0\rho^+)}{B(\bar{B}^0 \to D^-\rho^+)} \tag{35}$$

and

$$R_3 = \frac{B(B^+ \to \bar{D}^{*0}\pi^+)}{B(\bar{B}^0 \to D^{*-}\pi^+)}, \quad R_4 = \frac{B(B^+ \to \bar{D}^{*0}\rho^+)}{B(\bar{B}^0 \to D^{*-}\rho^+)} . \tag{36}$$

In the following we do not discuss R_4 as the decay of B into two vector particles involves more than one form factor and hence, theoretically, larger uncertainties.

Starting with the ratio R_1, in Table II we have shown the theoretical value of R_1 for $X = 1.2$, $Y = 0.8$, $Z = 1$ for different negative values of a_2/a_1. We remind the reader that our intention is to investigate how far one can go in understanding data with negative a_2/a_1. In Table II column 2 shows R_1 *without* FSI and annihilation terms. Column 3 shows R_1 with the inclusion of FSI with $\delta = 30^\circ$, but still without the annihilation terms. Column 4 shows R_1 with FSI ($\delta = 30^\circ$) *and* the annihilation term ($\xi = 0.1$). The effect of FSI and annihilation terms is to raise R_1 by about 10%. Finally, in column 5 we have shown the effect of a difference in the lifetimes if τ_{B^+}/τ_{B^0} were 1.2.

Clearly, R_1 can be raised to values somewhat larger

TABLE II. Ratio R_1 for $X=1.2$, $Y=0.8$, $Z=1$, and different negative values of a_2/a_1. Column 2 gives R_1 without FSI and annihilation terms (AT). Column 3 includes the effect of FSI (with $\delta=30°$), column 4 includes both FSI and annihilation ($\xi=0.1$), and column 5 is obtained by multiplying entries of column 4 by an assumed ratio $\tau_{B^+}/\tau_{B^0}=1.2$.

	Ratio R_1			
a_2/a_1	No FSI, No AT	With FSI, No AT	With FSI and AT	$\tau_{B^+}/\tau_{B^0}=1.2$
$-\frac{1}{5}$	0.70	0.74	0.77	0.92
$-\frac{1}{6}$	0.75	0.79	0.82	0.98
$-\frac{1}{7}$	0.78	0.83	0.85	1.02
$-\frac{1}{8}$	0.81	0.85	0.88	1.06

than unity even for negative values of a_2/a_1. The two most important contributors to the rise in R_1 (from a value of ≈ 0.5 for $X=Y=Z=1$ and $a_2/a_1=-0.24$) are (i) a larger X, say, by about 20%, and (ii) $\tau_{B^+}/\tau_{B^0}=1.2$. The role of Y and Z is diluted because they come multiplied by a_2/a_1. However, a smaller Y helps. FSI and annihilation terms together contribute about 10% of the increase.

The pre-1992 world averaged data,

$$B(B^+\to\bar{D}^0\pi^+)=(0.38\pm0.11)\% ,$$
$$B(\bar{B}^0\to D^-\pi^+)=(0.32\pm0.07)\% ,$$

[5], lead to R_1 in the region of unity. The theory could conceivably be stretched to accommodate these data with negative values of a_2/a_1. However, the new CLEO II data [2,3]

$$R_1=2.13\pm0.32\pm0.39 ,\quad R_2=1.73\pm0.27\pm0.29 ,\quad R_3=1.89\pm0.19\pm0.14 , \tag{37}$$

cannot be understood with $a_2/a_1<0$. In contrast, simply using $a_2/a_1=0.24$ with no assist from the parameter X, FSI and annihilation terms achieve a value of $R_1=1.68$ [3]. FSI and annihilation terms could boost this ratio by about 10%, making the theory agree very well with the data.

In $B\to D\pi$ decays, the annihilation term is suppressed by the kinematic factor $m_D^2/(m_B^2-m_D^2)$ [see Eq. (19)]. In decays such as $B\to D\rho$ and $B\to D^*\pi$, this kinematic factor does not occur, making the annihilation term somewhat more significant. Hence, in discussing the ratios R_2 and R_3, we retain the annihilation term but ignore FSI effects. The smallness of the upper limits on $B(\bar{B}^0\to\bar{D}^0\rho^0)$ and $B(\bar{B}^0\to\bar{D}^{*0}\pi^0)$ [3] justifies the last step. We find

$$R_2=\frac{\left[1+0.662(a_2/a_1)\frac{Y_1Z}{X_1}\right]^2}{[1+(a_2/a_1)\xi_1]^2} , \tag{38}$$

where

$$X_1=\frac{F_1^{BD}(m_\rho^2)}{0.60},\quad Y_1=\frac{A_0^{B\rho}(m_D^2)}{0.37},\quad \xi_1=\frac{f_B}{f_\rho}\frac{A_0^{D\rho}(m_B^2)}{F_1^{BD}(m_\rho^2)} , \tag{39}$$

and

$$R_3=\frac{\left[1+1.29(a_2/a_1)\frac{Y_2Z_2}{X_2}\right]^2}{[1+(a_2/a_1)\xi_2]^2} , \tag{40}$$

where

$$X_2=\frac{A_0^{BD^*}(0)}{0.59},\quad Y_2=\frac{F_1^{B\pi}(m_D^2)}{0.45},\quad Z_2=\frac{f_{D^*}}{0.22\text{ GeV}} , \tag{41}$$

and

$$\xi_2=\frac{f_B}{f_\pi}\frac{A_0^{D^*\pi}(m_B^2)}{A_0^{BD^*}(0)} . \tag{42}$$

In Table III we have shown R_2 and R_3 for different negative values of a_2/a_1. We have set $X_1=X_2=1.2$, $Y_1=Y_2=0.8$, and $Z_2=1.0$. We have also set the annihilation parameters $\xi_1=\xi_2=0.3$, a generous assignment we believe, threefold larger than ξ because of the absence of the kinematic suppression. Columns 3 and 5 in Table III use an assumed value $\tau_{B^+}/\tau_{B^0}=1.2$.

Theoretically, again, it is very difficult to push R_2 and R_3 well past unity with $a_2/a_1<0$. By contrast, with $a_2/a_1=0.24$ alone, without any help from the form factor or the annihilation terms, one gets [3] $R_2=1.34$ and $R_3=1.72$. The effects we have considered could push these ratios a little higher, making theory agree with the new CLEO II data reproduced in Eq. (37).

Finally, consider the ratio

$$R_+=\left[\frac{X_1}{X}\right]^2\frac{\left[1+0.662(a_2/a_1)\frac{Y_1Z}{X_1}\right]^2}{\left[1+1.23(a_2/a_1)\frac{YZ}{X}\right]^2} , \tag{43}$$

where

TABLE III. Ratios R_2 and R_3 for $X_1=X_2=1.2$, $Y_1=Y_2=0.8$, and $Z_2=1$. Annihilation parameters $\xi_1=\xi_2=0.3$.

	R_2		R_3	
a_2/a_1	$\tau_{B^+}/\tau_{B^0}=1$	$\tau_{B^+}/\tau_{B^0}=1.2$	$\tau_{B^+}/\tau_{B^0}=1$	$\tau_{B^+}/\tau_{B^0}=1.2$
$-\frac{1}{5}$	0.94	1.13	0.78	0.94
$-\frac{1}{6}$	0.95	1.14	0.81	0.97
$-\frac{1}{7}$	0.96	1.15	0.84	1.01
$-\frac{1}{8}$	0.96	1.15	0.86	1.03

$$R_+=\frac{B(B^+\to\bar{D}^0\rho^+)}{B(B^+\to\bar{D}^0\pi^+)} . \tag{44}$$

With X, Y, Z, X_1, and Y_1 all set equal to 1, and $a_2/a_1=+\frac{1}{5}$, this ratio is 1.94. The new CLEO II data [2,3] yield 2.27 ± 0.22 (we have combined all the errors in quadrature), whereas $a_2/a_1=-\frac{1}{5}$ gives 3.12 for this ratio. Clearly, again, a positive value for a_2 is favored. We point out that the rates in Eq. (43) are free of annihilation terms and FSI interference effects.

V. CONCLUSION

In this paper we have analyzed hadronic B decays into ψK, $D\pi$, $D^*\pi$, and $D\rho$ channels with a view to investigate if the data could be understood with $a_2<0$. We conclude that this is not possible.

The CLEO II collaboration [2,3] has used all two-body decays of B^0 and B^- involving ψ in the final state to estimate $|a_2|=27\pm0.025$ (we have added their errors in quadrature). They have also concluded that $a_1=1.07\pm0.1$ (we have added their errors in quadrature). They conclude that $a_2/a_1>0$.

From $\bar{B}^0\to\psi K^0$ alone we find

$$|a_2|=0.20\pm0.04\pm0.04 .$$

If we also include CLEO II measurement [2,3]

$$B(B^-\to\psi K^-)=(0.112\pm0.015\pm0.007)\% ,$$

our estimate of $|a_2|$ would be higher by 17%.

We have studied the effects of (i) theoretical uncertainties in the form factors that enter through the hadronic matrix elements of weak currents, (ii) final-state interactions, and (iii) annihilation terms. First, by using the upper limit on the branching ratio for $\bar{B}^0\to\bar{D}^0\pi^0$, we have set an upper limit on strong interaction phase $\delta=\delta_{1/2}-\delta_{3/2}$. We found this phase to have a reasonably small upper bound. We have used $\delta=30°$ in our calculations. We have also argued that CVC does not forbid an annihilation term in B decays; however, this term is small simply because it is needed at a rather larger value of momentum transfer, $q^2=m_B^2$. Taking all these effects and uncertainties into account and, further, even allowing $\tau_{B^+}=1.2\tau_{B^0}$, we find it impossible to account for the new CLEO II data [2,3] with $a_2<0$. In contrast, the new CLEO II data are easily accounted for by using $a_2>0$, as is well known [1–3].

A model by Reader and Isgur [9] which uses $N=3$ in Eq. (8) has been widely quoted by experimentalists in confronting data on $B\to D\pi$, $D^*\pi$, $D\rho$, and $D^*\rho$. Use of $N=3$ in Eq. (8) generates a rather low but positive value for a_2, $a_2=0.11$. This results in a further lowering of the branching ratios of all the color-suppressed modes. As only upper limits exist for these modes, this does not pose a problem. However, the $B\to\psi K$ branching ratio does not allow such a low value of a_2.

Further, in the Reader and Isgur model [9], the ratios R_i ($i=1,\ldots,4$) [Eqs. (35) and (36)] stay close to 1, whereas the new CLEO II data have pushed all these ratios close to 2. The values of R_i obtained in Ref. [9] are really a consequence of using $a_2/a_1\approx0.1$ rather than any subtleties of their model.

In D decays it is well known that the $N\to\infty$ limit works well [4], though for obscure reasons. As experiments have ruled out negative a_2/a_1 in B decays, one may ask for an "effective" N that describes B decays. For equally obscure reasons, if one chooses $N_{\text{eff}}=2.4$ in Eq. (8), one gets $a_1=1.01$ and $a_2=0.20$, which are allowed by the new data.

Lastly, we are aware of a work by Deandrea *et al.* [10], who have also concluded that a_2/a_1 is positive. Their approach is different from ours and they do not investigate the role of FSI or the annihilation terms.

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