

Higher order correction to the Garfinkle-Horowitz-Strominger string black hole

Makoto Natsuume*

Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030

(Received 4 April 1994; revised manuscript received 13 June 1994)

We study the order α' correction to the string black hole found by Garfinkle, Horowitz, and Strominger (GHS). We include all operators of dimension up to four in the Lagrangian, and use the field redefinition technique which facilitates the analysis. A mass correction, which is implied by the work of Giddings, Polchinski, and Strominger, is found for the extremal GHS black hole.

PACS number(s): 04.70.Bw, 11.25.Db

In recent years, the string-black-hole solution found by Garfinkle, Horowitz, and Strominger (GHS) has attracted much attention [1–3]. They solved the leading α' action of the low-energy effective theory for the heterotic string. The solution describes a four-dimensional magnetically charged black hole coupled with the dilaton.

On the other hand, Giddings, Polchinski, and Strominger (GPS) [4] have obtained the exact solution for the black hole in the extremal limit. GPS construct an exact conformal field theory (CFT) (so, the solution includes the effects of all higher order α' terms), which describes the throat region of the extremal GHS black hole. One new feature of the GPS solution compared with the GHS solution is the existence of a neutral throat. The neutral throat is possible for the GPS solution, since $R = |Q^2 - 1|^{1/2}$, where R is the throat radius and Q is the monopole charge. However, the GHS solution does not allow the neutral throat, since $R = Q (= 2M)$. The GPS solution is a solution of the full effective theory, but the GHS solution is only a leading-order solution; so any difference between these two solutions should come from the higher order terms in the effective theory. Thus, the GPS solution suggests that there exists a correction to the black-hole mass due to the higher order α' terms to make the neutral throat possible. Unfortunately, the GPS solution does not connect to the asymptotic region; so, they cannot get the mass correction. The purpose of this paper is to obtain the mass correction by a different approach.

To obtain the correction, we shall keep $O(\alpha')$ terms as well in the effective action and perform α' perturbative expansion around the GHS background. Without the detailed knowledge of the effective action, we will show that the extremal GHS black hole gets a correction in mass given by $M = Q/2 - \alpha'/40Q$.

In order to show this result, we will thoroughly employ the field redefinition technique for analysis. This is a useful technique to simplify higher order effective actions. Even though the field redefinitions change the action and fields like the metric, it does not change the physics; for

instance, the S matrix is invariant under the field redefinitions by the equivalence theorem [5,6]. Therefore, when one works with higher orders in the effective theory, one can simplify the effective theory by transforming the original action into a simpler one by the field redefinitions. Although this technique is illustrated for the magnetically charged black hole, the technique itself is general and is useful for the other “dirty” black holes.

In the extremal limit, the GHS solution is given by [1]

$$\begin{aligned} d^2 s_{\text{string}} &= -dt^2 + \left(1 - \frac{Q}{r}\right)^{-2} dr^2 + r^2 d\Omega, \\ e^{-2\phi} &= 1 - \frac{Q}{r}, \\ F &= Q \sin \theta d\theta \wedge d\varphi \end{aligned} \quad (1)$$

in the string metric, and

$$d^2 s = -\left(1 - \frac{Q}{r}\right) dt^2 + \left(1 - \frac{Q}{r}\right)^{-1} dr^2 + r(r - Q) d\Omega \quad (2)$$

in the Einstein metric. Here, the string metric $g_{\mu\nu}$ and the Einstein metric $\tilde{g}_{\mu\nu}$ are related by $g_{\mu\nu} = e^{2\phi} \tilde{g}_{\mu\nu}$. ϕ_0 , the asymptotic value of the dilaton, is set to zero for simplicity. The mass of the black hole is given by $M = Q/2$.

Our starting point is the most general action to the order α' with all possible independent terms:

$$S = \int d^4x \sqrt{-g} e^{-2\phi} \left\{ \mathcal{L}_2 + \sqrt{\alpha'} \mathcal{L}_3 + \alpha' \mathcal{L}_4 \right\}, \quad (3)$$

where \mathcal{L}_i denotes the contributions of i derivative operators. \mathcal{L}_2 is the leading order Lagrangian given by

$$\mathcal{L}_2 = R + 4(\nabla\phi)^2 - \frac{1}{2}F^2. \quad (4)$$

We need to include all operators, which contain at most four derivatives because the effective theory expansion is simply a derivative expansion for the above choice of the field normalizations [7]. The field normalization of A_μ indicates that the gauge field has zero mass dimension.

*Electronic address: makoto@sbitp.ucsb.edu

This is different from the conventional normalization [8] where A_μ has one mass dimension [thus, F^2 is an $O(\alpha')$ term]. In other words, we implicitly made the α' rescaling of A_μ , which implies that the charge Q is not small; i.e., Q is of order 1 instead of $O(\sqrt{\alpha'})$. The rescaling is natural, since we consider a large mass black hole for the α' perturbation to be valid.

\mathcal{L}_3 and \mathcal{L}_4 are given by

$$\mathcal{L}_3 = a_1 F^{\mu\nu} F_{\nu\rho} F^\rho{}_\mu, \quad (5)$$

$$\begin{aligned} \mathcal{L}_4 = & a_2 R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} + a_3 R^{\mu\nu} R_{\mu\nu} + a_4 R^2 \\ & + a_5 R^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + a_6 R (\nabla \phi)^2 + a_7 R \nabla^2 \phi \\ & + a_8 (\nabla^2 \phi)^2 + a_9 (\nabla \phi)^2 \nabla^2 \phi + a_{10} (\nabla \phi)^4 \\ & + a_{11} (\nabla_\mu F^{\mu\rho}) (\nabla^\nu F_{\nu\rho}) \\ & + a_{12} F^{\mu\nu} F_{\nu\rho} F^{\rho\sigma} F_{\sigma\mu} + a_{13} (F^{\mu\nu} F_{\mu\nu})^2 \\ & + a_{14} R^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + a_{15} R^{\mu\nu} F_{\mu\rho} F_\nu{}^\rho + a_{16} R F^2 \\ & + a_{17} F^2 (\nabla \phi)^2 + a_{18} F^2 (\nabla^2 \phi) \\ & + a_{19} \nabla_\mu \phi \nabla_\nu \phi F^{\mu\rho} F^\nu{}_\rho. \end{aligned} \quad (6)$$

Using the leading order GHS solution, one can easily check that every operator has the same order of magnitude.

We did not include terms that are proportional to the three-form field strength $H_{\mu\nu\rho}$, where

$$H_{\mu\nu\rho} = \partial_{[\rho} B_{\mu\nu]} + \frac{1}{4} (\Omega_{\mu\nu\rho}^{(L)} - \Omega_{\mu\nu\rho}^{(Y)}). \quad (7)$$

$\Omega^{(L)}$ and $\Omega^{(Y)}$ are the Lorentz and Yang-Mills Chern-Simons forms, respectively. Since the Chern-Simons forms linearly couple with $B_{\mu\nu}$, the Chern-Simons forms act as sources for $H_{\mu\nu\rho}$; therefore, we may not be able to simply set $H = 0$. However, for the spherically symmetric metrics we examine below, the Lorentz Chern-Simons form can be expressed as the exterior derivative of a three form; thus we can absorb it into the definition of $B_{\mu\nu}$ [9]. Also, the Yang-Mills Chern-Simons form vanishes for the purely magnetic case. For these reasons, it is consistent to set $H_{\mu\nu\rho}$ to zero.

We assume spherical symmetry; the Bianchi identity then implies that $F_{\mu\nu}$ is unchanged. Thus, the a_1, a_{11} , and a_{19} terms vanish; also, the a_{12} term is no longer independent of the a_{13} term. We will not consider these terms further, so the above conditions leave fifteen $O(\alpha')$ terms (a_2, \dots, a_{10} and a_{13}, \dots, a_{18}).

The coefficients of higher order terms are free parameters in general due to field redefinition ambiguity. To see this, consider the most general field redefinitions:

$$\begin{aligned} g_{\mu\nu} &= g'_{\mu\nu} + \alpha' T_{\mu\nu}(g', \phi', F') + O(\alpha'^2), \\ \phi &= \phi' + \alpha' T(g', \phi', F') + O(\alpha'^2) a, \\ A_\mu &= A'_\mu + O(\alpha'^2), \end{aligned} \quad (8)$$

where

$$\begin{aligned} T_{\mu\nu} &= g_1 R_{\mu\nu} + g_2 \nabla_\mu \phi \nabla_\nu \phi + g_3 F_{\mu\rho} F_\nu{}^\rho \\ &\quad + g_{\mu\nu} \{ g_4 R + g_5 \nabla^2 \phi + g_6 (\nabla \phi)^2 + g_7 F^2 \}, \\ T &= d_1 R + d_2 \nabla^2 \phi + d_3 (\nabla \phi)^2 + d_4 F^2. \end{aligned} \quad (9)$$

Here, g_i and d_i are free parameters. We have used spherical symmetry to eliminate possible field redefinition for A_μ at $O(\alpha')$. Substituting (8) and (9) into (3), one finds that \mathcal{L}_4 retains its form but the coefficients a_i change in general. The explicit result can be found in the Appendix. (The coefficient changes for the gravity-dilaton part have been considered by Metsaev and Tseytlin [10].)

In the literature, one often claims that the ambiguity is resolved by choosing the Gauss-Bonnet scheme for curvature squared terms; i.e., by taking $a_2 = -1/2$ and $a_3 = 1/8$. The justification is the argument by Zwiebach [11]; he argued that only the Gauss-Bonnet combination gives a ghost-free theory in the weak field expansion. The argument is actually irrelevant [8], since our effective action is a perturbative expansion in powers of momentum; the perturbation itself is not valid at the energy of the apparent ghost.

While we are not able to resolve the ambiguity, this ambiguity does not matter as long as physical quantities measured at infinity are concerned. This is because the field redefinitions do not alter these quantities. In this sense, the only meaningful quantities to evaluate in higher order effective theories are relations of physical quantities, like mass-charge, mass-temperature relations, and so on. Moreover, because physical results are unchanged under the redefinitions, what one should do is to find the simplest Lagrangian one can reach by the redefinitions to simplify calculations.

As a check of the above statement, we show that mass and charge are unchanged under the field redefinitions (8). The monopole charge is of course invariant, since there is no possible field redefinition for A_μ at this order. The gravitational mass M_G and the inertial mass M_I are defined by the asymptotic behavior of the Einstein metric [12]:

$$\begin{aligned} \tilde{g}_{00} &= -1 + \frac{2M_G}{r} + O(r^{-2}), \\ \tilde{g}_{11} &= 1 + \frac{2M_I}{r} + O(r^{-2}). \end{aligned} \quad (10)$$

In the string metric, M_G depends on $O(r^{-1})$ terms in both g_{00} and ϕ (and similarly for M_I). Using the GHS solution (1), one can check that the field redefinitions affect the terms of order r^{-3} or higher for both the metric and the dilaton; therefore, black-hole mass is invariant under the field redefinitions.

We now simplify \mathcal{L}_4 using the field redefinitions. There are originally fifteen terms in \mathcal{L}_4 . From the explicit calculation, a_2 and a_{14} are invariant under the redefinitions. The coefficients of these terms are thus determined from a standard S -matrix calculation; for the heterotic string, $a_2 = 1/8$ and $a_{14} = 0$ [8]. This leaves thirteen field redefinition dependent terms (ambiguous terms). The field redefinitions have eleven free parameters, so one might expect that it is possible to remove all the ambiguous terms except two by appropriate field redefinitions. This

conclusion is, in fact, correct, but the reasoning is wrong. There is a subtlety in the counting because variations of a_i are not independent of each other (see the Appendix).

First, consider the gravity-dilaton part of \mathcal{L}_4 . There are eight ambiguous terms (a_3, \dots, a_{10}) and the field redefinitions have eight parameters as well. (F -dependent redefinitions do not affect the gravity-dilaton action.) However, the variations of these a_i satisfy (A1); one δa_i is fixed completely once the rest are chosen. Therefore, one term cannot be eliminated in the gravity-dilaton action. Fortunately, Metsaev and Tseytlin show that the remaining term vanishes after the field redefinitions [10]; thus, all ambiguous terms are removed. But it is actually useful to work with the Gauss-Bonnet scheme instead of keeping only the a_2 term. This scheme is useful because field equations are at most second order in derivatives, which is reminiscent of the claim that the scheme gives the “ghost-free” theory.

Next, consider the terms coupled with the gauge field. There are five ambiguous terms in the action ($a_{13}, a_{15}, \dots, a_{18}$) and three F -dependent field redefinition parameters (g_3, g_7 , and d_4) [13]. This leaves two ambiguous terms, but another relation (A2) partly determines which two a_i should be left. Since the variations of a_{13}, a_{15}, a_{17} , and a_{18} contribute to (A2), at least one of them should be left. We will keep the a_{13} and a_{17} terms, since the equations of motions become simple.

Consequently, we have reached the simple Lagrangian

$$\begin{aligned} \mathcal{L} \propto & -f^2 \Phi_2'^2 - x^3 f \{2 - (1 + 2c)x\} \Phi_2' \\ & + f^2 \phi_2'^2 + \frac{1}{5} x^4 \{-10c + (9c - b)x\} \phi_2' - 2f \left(1 - \frac{2}{x}\right) \phi_2' \Lambda_2 - \left(1 - \frac{2}{x^2}\right) \Lambda_2^2 + (b - c)x^4 \Lambda_2, \end{aligned} \quad (13)$$

where $f = 1 - Q/r$, $x = Q/r$, and the prime denotes a derivative with respect to x .

The only solution regular at the horizon $r = Q$ is given by

$$\begin{aligned} \Phi_2 = & -\frac{1 + 2c}{8} x^4 \\ & + \frac{1 - 2c}{2} \left\{ \frac{1}{3} x^3 + \frac{1}{2} x^2 + x + \ln(1 - x) \right\}, \end{aligned} \quad (14)$$

$$\begin{aligned} \Lambda_2 = & -\frac{c}{20} x(14x^3 + 9x^2 + 5x + 2) \\ & - \frac{b}{20} \frac{x}{1 - x} (6x^4 + 5x^3 + 4x^2 + 3x + 2), \end{aligned} \quad (15)$$

$$\begin{aligned} \phi_2 = & \frac{bx}{1 - x} + 2(b - c) \ln(1 - x) \\ & + \frac{11b - 19c}{10} x + \frac{b - 4c}{5} x^2 - \frac{b + 21c}{60} x^3 - \frac{b + c}{20} x^4. \end{aligned} \quad (16)$$

Even though $1/(1 - x)$ terms in Λ_2 and ϕ_2 look like singular perturbations, they are not, since a coordinate transformation $x \rightarrow x - b\hat{\alpha}(6x^4 + 5x^3 + 4x^2 + 3x + 2)/20$

$$\begin{aligned} \mathcal{L}_4 = & \frac{1}{8} (R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2) \\ & + b(F^2)^2 + cF^2(\nabla\phi)^2. \end{aligned} \quad (11)$$

b and c are the parameters that we can fix by an S -matrix calculation [8] and (A3). However, this step is unnecessary. The mass-charge relation we are interested in will not depend on b after a coordinate transformation and c will be fixed once we impose that the solution behaves like the GPS solution in the throat region.

We solve field equations by perturbing around the GHS background. Take the Schwarzschild gauge in the string metric

$$d^2 s_{\text{string}} = -e^{2\Phi} dt^2 + e^{2\Lambda} dr^2 + r^2 d\Omega \quad (12)$$

and expand the metric functions in $\hat{\alpha} \equiv \alpha'/Q^2$:

$$\begin{aligned} \Phi &= \hat{\alpha} \Phi_2 + \dots, \\ \Lambda &= \Lambda_1 + \hat{\alpha} \Lambda_2 + \dots, \\ \phi &= \phi_1 + \frac{\hat{\alpha}}{2} (\Phi_2 - \phi_2) + \dots, \end{aligned}$$

where Λ_1 and ϕ_1 are given by the GHS solution. (Note that $\Phi_1 = 0$.) The somewhat artificial choice of ϕ_2 is useful to simplify field equations. Then, the Lagrangian (11) becomes

removes the terms. Also, the above solution suggests $c = 1/2$; otherwise, Φ_2 contains a term proportional to $\ln(1 - x)$. Such a term should be absent in the light of the GPS solution. GPS claim that the extremal solution in the throat region is a product of an rt CFT (the linear dilaton theory) and an angular CFT. In particular, $g_{00} = -1$. If the $\ln(1 - x)$ term existed in Φ_2 , our solution would not give the linear dilaton theory in the throat region under any choice of field redefinition, i.e., g_{00} would not approach to -1 . On the other hand, $\ln(1 - x)$ in ϕ_2 is safe; it just shifts the dilaton gradient in the throat limit from $-1/2Q$ to $-(1 + \epsilon)/2Q$, where $\epsilon = (2b - 1)\hat{\alpha}$.

After the coordinate transformation, we get

$$\begin{aligned} d^2 s_{\text{string}} = & -\left(1 - \frac{\hat{\alpha}}{2} x^4\right) dt^2 \\ & + \left(1 - \frac{Q}{r}\right)^2 f_1(r) dr^2 + r^2 d\Omega, \\ e^{-2\phi} = & \left(1 - \frac{Q}{r}\right)^{1+\epsilon} f_4(r) \end{aligned} \quad (17)$$

in the string metric, and

$$d^2s = - \left(1 - \frac{Q}{r}\right)^{1+\epsilon} f_2(r) dt^2 + \left(1 - \frac{Q}{r}\right)^{-1+\epsilon} \times f_3(r) dr^2 + r^2 \left(1 - \frac{Q}{r}\right)^{1+\epsilon} f_4(r) d\Omega \quad (18)$$

in the Einstein metric, where

$$\begin{aligned} f_1(r) &= 1 - \frac{\hat{\alpha}}{20} x(14x^3 + 9x^2 + 5x + 2), \\ f_2(r) &= 1 - \frac{\hat{\alpha}}{40} x(11x^3 + 7x^2 + 16x + 38) + g(r), \\ f_3(r) &= 1 - \frac{\hat{\alpha}}{40} x(19x^3 + 25x^2 + 26x + 42) + g(r), \\ f_4(r) &= 1 - \frac{\hat{\alpha}}{40} x(-9x^3 + 7x^2 + 16x + 38) + g(r), \\ g(r) &= \frac{\hat{\alpha}}{60} b x(15x^3 + 32x^2 + 57x + 120). \end{aligned} \quad (19)$$

The solution describes an extremal black hole whose singularity and horizon coincide. However, the detailed form of the solution is not important because $O(1/r^3)$ terms do not have invariant meaning. What are important to us are invariant relations such as $M = M(Q)$. Using (10), we get that the gravitational and inertial masses are the same and given by

$$M = \frac{Q}{2} - \frac{\alpha'}{40Q}. \quad (20)$$

This is our main result.

It is not hard to see why $M(Q)$ is modified by higher dimensional operators. As is well known, the throat of the extremal GHS black hole results from the balance of curvature against the monopole magnetic field. However, by including $O(\alpha')$ terms, the curvature squared terms, e.g., $R^{\mu\nu}R_{\mu\nu}$, cancel part of the monopole term $F_{\mu\nu}F^{\mu\nu}$. Thus, for the balance to work at this order, the black-hole mass has to be slightly lighter than the leading value for a given charge Q .

An interesting possibility arises by letting Q get small. Obviously, our perturbation breaks down for small Q , but if the result were still valid, the solution might suggest violation of the positive-energy theorem [14].

I am grateful to J. Polchinski for his continuous help and interest throughout the work. I am also pleased to thank J. LaChapelle and W. Nelson for their comments on the draft, and thank S. Chaudhuri, G. Horowitz, T. Jacobson, and G. Kang for useful conversation. This research was supported in part by NSF Grant No. PHY 8904035 and 9116964.

APPENDIX A: FIELD REDEFINITION RESULT

Under the field redefinitions (8), the coefficients of $O(\alpha')$ change as follows:

$$\begin{aligned} a'_2 &= a_2, \\ a'_3 &= a_3 - g_1, \\ a'_4 &= a_4 + \frac{1}{2}g_1 + \frac{1}{2}(D-2)g_4 - 2d_1, \\ a'_5 &= a_5 - 4g_1 - g_2, \\ a'_6 &= a_6 + \frac{1}{2}g_2 - 2Dg_4 + \frac{1}{2}(D-2)g_6 + 8d_1 - 2d_3, \\ a'_7 &= a_7 + g_1 + 2(D-1)g_4 + \frac{1}{2}(D-2)g_5 - 8d_1 - 2d_2, \\ a'_8 &= a_8 + 2(D-1)g_5 - 8d_2, \\ a'_9 &= a_9 + 3g_2 - 2Dg_5 + 2(D-1)g_6 + 8d_2 - 8d_3, \\ a'_{10} &= a_{10} - 4g_2 - 2Dg_6 + 8d_3, \\ a'_{13} &= a_{13} + \frac{1}{4}g_3 - \frac{1}{4}(D-4)g_7 + d_4, \\ a'_{14} &= a_{14}, \\ a'_{15} &= a_{15} + g_1 - g_3, \\ a'_{16} &= a_{16} - \frac{1}{4}g_1 + \frac{1}{2}g_3 \\ &\quad - \frac{1}{4}(D-4)g_4 + \frac{1}{2}(D-2)g_7 + d_1 - 2d_4, \\ a'_{17} &= a_{17} - \frac{1}{4}g_2 - g_3 - \frac{1}{4}(D-4)g_6 - 2Dg_7 + d_3 + 8d_4, \\ a'_{18} &= a_{18} + \frac{3}{2}g_3 - \frac{1}{4}(D-4)g_5 + 2(D-1)g_7 + d_2 - 8d_4, \end{aligned}$$

where D is the dimension of space-time. Not all the variations $\delta a_i = a'_i - a_i$ are independent of each other because

$$16\delta a_4 - 4\delta a_6 - 8\delta a_7 + 4\delta a_8 + 2\delta a_9 + \delta a_{10} = 0, \quad (A1)$$

$$\begin{aligned} 16\delta a_3 - \frac{3}{2}\delta a_5 + 4\delta a_8 + 3\delta a_9 + \frac{9}{4}\delta a_{10} + 16\delta a_{13} \\ + 10\delta a_{15} + 6\delta a_{17} + 8\delta a_{18} = 0. \end{aligned} \quad (A2)$$

The action (3) is transformed into (11) by the field redefinitions (we set $D = 4$)

$$\begin{aligned} g_1 &= \frac{1}{2} + a_3, \\ g_2 &= -2 - 4a_3 + a_5, \\ g_3 &= \frac{1}{2} + a_3 + a_{15}, \\ g_4 &= \frac{1}{16}(3 + 8a_3 + 8a_4 + 6a_6 + 4a_7 - 2a_8 - a_9 - 4d_3), \\ g_5 &= \frac{3}{4} - 6a_4 + \frac{3}{2}a_6 + 3a_7 - a_8 - \frac{1}{4}a_9 - d_3, \\ g_6 &= \frac{5}{4} + 2a_3 - 2a_4 - \frac{1}{2}a_5 + \frac{1}{2}a_6 \\ &\quad + a_7 - \frac{1}{2}a_8 - \frac{1}{4}a_9 + d_3, \\ g_7 &= \frac{1}{32}(-3 + 24a_3 + 120a_4 - 6a_6 - 28a_7 + 6a_8 \\ &\quad + a_9 + 8a_{15} + 64a_{16} - 16a_{18} + 4d_3), \\ d_1 &= \frac{1}{32}(5 + 16a_3 + 24a_4 + 6a_6 + 4a_7 - 2a_8 - a_9 - 4d_3), \end{aligned}$$

$$d_2 = \frac{1}{16}(9 - 72a_4 + 18a_6 + 36a_7 - 10a_8 - 3a_9 - 12d_3),$$

$$d_4 = \frac{1}{32}(3 + 24a_3 + 72a_4 - 12a_7$$

$$+ 2a_8 + 12a_{15} + 48a_{16} - 8a_{18}).$$

After the redefinitions, the coefficients of $b (= a'_{13})$ and $c (= a'_{18})$ are given by

$$b = \frac{7}{32} + a_3 + \frac{9}{4}a_4 - \frac{3}{8}a_7 + \frac{1}{16}a_8$$

$$+ a_{13} + \frac{5}{8}a_{15} + \frac{3}{2}a_{16} - \frac{1}{4}a_{18},$$

$$c = \frac{3}{2} - 12a_4 - \frac{1}{4}a_5 + \frac{3}{2}a_6 + 4a_7$$

$$- a_8 - \frac{1}{4}a_9 - 4a_{16} + a_{17} + 2a_{18}. \quad (\text{A3})$$

-
- [1] D. Garfinkle, G. Horowitz, and A. Strominger, Phys. Rev. D **43**, 3140 (1991); **45**, 3888(E) (1992).
 - [2] G. W. Gibbons, Nucl. Phys. **B207**, 337 (1982); G.W. Gibbons and K. Maeda, *ibid.* **B298**, 741 (1988).
 - [3] G. Horowitz, "The Dark Side of String Theory: Black Holes and Black Strings," UCSB Report No. UCSBTH-92-32 (hep-th/9210119), in *String Theory and Quantum Gravity '92*, edited by J. Harvey, R. Iengo, K. Narain, S. Randjbar-Daemi, and H. Verlinde (World Scientific, Singapore, 1993).
 - [4] S. B. Giddings, J. Polchinski, and A. Strominger, Phys. Rev. D **48**, 5784 (1993).
 - [5] C. Itzykson and J. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).
 - [6] A. A. Tseytlin, Phys. Lett. B **176**, 92 (1986); Nucl. Phys. **B276**, 391 (1986).
 - [7] J. Polchinski, in *Recent Directions in Particle Theory—From Superstrings and Black Holes to the Standard Model*, Proceedings of the Theoretical Advanced Study Institute in Elementary Particle Physics, Boulder, Colorado, 1992, edited by J. Harvey and J. Polchinski (World Scientific, Singapore, 1993).
 - [8] D. J. Gross and J. H. Sloan, Nucl. Phys. **B291**, 41 (1987).
 - [9] B. A. Campbell, M. J. Duncan, N. Kaloper, and K. A. Olive, Nucl. Phys. **B351**, 778 (1991).
 - [10] R. R. Metsaev and A. A. Tseytlin, Phys. Lett. B **191**, 354 (1987); Nucl. Phys. **B293**, 385 (1987).
 - [11] B. Zwiebach, Phys. Lett. **156B**, 315 (1985).
 - [12] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).
 - [13] In the above discussion of the gravity-dilaton action, one field redefinition parameter is not used, since we fix only seven a_i in the action. However, the dependence on this parameter disappears after the F independent redefinitions.
 - [14] R. M. Wald, *General Relativity* (The University of Chicago Press, Chicago, 1984).