Late-time phase transition and the galactic halo as a Bose liquid

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We consider the ultralight pseudo Nambu-Goldstone boson appearing in the late-time cosmological phase transition theories as a major dark matter candidate. Since it is almost massless, its nature is more wavelike than particlelike. We study quantum mechanically how they form a galactic halo. Three predictions are made: (i) the mass profile $\rho \sim r^{-1.6}$; (ii) there are ripplelike fine structures in the rotation curve; (iii) the rotation velocity multiplied by the ripple's wavelength is galaxy independent.

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I. INTRODUCTION

Recently, motivated by the large scale structure, an ultralight Nambu-Goldstone boson was introduced in dark matter physics under the name of the "late time cosmological phase transition" [1]. The basic idea of the theory is that, if a phase transition happens *after* decoupling, one can avoid the constraint imposed by the isotropy of the microwave background. If the phase transition occurred so late when the Universe was already big, then the relevant particle, whose Compton wavelength provides the scale of interesting structure, must be very light. To be responsible for the structure formation this particle should be a major component of dark matter [2]

If the dark matter consists of this particle whose mass is, say, $m \sim 10^{-24}$ eV and dark matter density around us is $\sim 10^{-25}$ g/cm^3, then the interparticle distance is of the order of 10^{-13} cm, while the Compton wavelength is of 10 pc order. Hence the wave functions of the particles are heavily overlapping and the bosons must be in a condensation. Therefore, a galactic halo is a giant system of Bose liquid. In this context, we must consider the dark matter distribution quantum mechanically.

Previously, the Bose condensation in astrophysics was studied by many groups seeking a possibility of the Bose star [3,4]. They were looking for *ground state* condensation of Bose particles. For the axion case, however, the ground-state condensation was shown to be very unlikely [5]. The reason is very simple. The coupling of a Bose particle to the photon is too weak to be "cooled." The same reason is applied in our case. So, if the dark matter consists of bosons and the present temperature of the Universe is low enough, the halo must be in a higher condensation state. Therefore, we look for the condensation wave function for the galactic halo that has nodes. One should remember that a hydrogen atom in a excited state is stable *if* there is no coupling to the photon.

Our result will show that mass profile $\rho \sim r^{-1.6}$, leading to slightly increasing rotation curve. Because of the system's wavelike nature, the rotation curve has the ripplelike fine structure. Our theory also predicts that the rotation velocity times the ripple's wavelength is galaxy independent. The rest of this paper is as follows. In Sec. II, we set up equations. In Sec. III, properties of the solutions are explored, and three predictions are made. In Sec. IV, we compare the theory and the observation. In Sec. V, the limits of present work and future projects are discussed. In the Appendix, we argue the impossibility of ground state condensation by evaporation.

II. QUANTUM ASPECTS OF DARK MATTER

The self-gravitating many-body system has been studied extensively. For classical results, see the excellent text book by Binney and Tremaine and references therein [6]. Here, the dark matter is assumed to be a system of Bose particles with very small mass. Then the particles are in the Bose condensation state, as we mentioned above. So the entire system is described by one order parameter ψ [7] called the condensate wave function. This is a typical Landau-Ginzburg theory.

For simplicity, we make two assumptions: (i) the system has spherical symmetry; (ii) self-interaction is negligible. We consider that this simplification is not a bad starting point. Remember that we are considering the halo not just the visible part.

Now consider a particle moving inside a potential generated by the gravity of the other particles. The Newtonian potential V is given by

$$\nabla^2 V = 4\pi G(\rho_{\mathrm{dark}} + \rho_{\mathrm{visible}})$$

and the Schrödinger equation is

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi(r).$$

By identifying $\rho_{\mathrm{dark}} = GM_0m|\psi|^2$ and defining $\rho_{\mathrm{visible}} = GM_0m\rho_v$ we get

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + GmM_0\int_0^r dr'\frac{1}{r'^2} \times \int_0^{r'} dr''4\pi r''^2(|\psi|^2 + \rho_v)\psi(r). \qquad (1)$$

Here m is the individual particle mass and M_0 is a mass parameter introduced for dimensional reason. The effect of visible matter will be considered in a separate paper [8] and from now on we set $\rho_v = 0$. Notice that the system is described by a nonlinear equation. We emphasize that ψ describes the whole system, not just a particle. One might ask whether the nonrelativistic treatment is reasonable when the mass of the particle is so small. The virial theorem tells us that $v^2 = GM/R$. One can also show that the gravitational Bohr radius r_0 is equal to $1/GM\ m^2$ with R, M the system size and mass, respectively. (We set $\hbar = c = 1$.) Then $R \gg r_0$; therefore,

$$v^2 \ll (GMm)^2 = \left(\frac{Mm}{m_{\text{Planck}}^2}\right)^2. \qquad (2)$$

The nonrelativistic treatment is justified if $GmM \ll 1$. In our case the relevant mass scale is $M \sim 10^{12} M_\odot$ and $m \leq 10^{-24}$; hence, $v \ll 10^{-2}$.

Since the galactic halo is in a relaxed state, a stationary solution is the most natural candidate of the state of the galactic halo. The superposition of two such states is not a solution of of Eq. (1) since it is nonlinear. Therefore we look for a solution of the form $\psi(r,t) = e^{-iEt/\hbar}\psi(r)$. Inserting this into Eq. (1) and scaling by

$$r = r_0\hat{r},\ \psi = r_0^{-3/2}\hat{\psi},\ E = \frac{\hbar^2\epsilon}{2m} \ \text{ with } r_0 = \hbar^2/2GM_0m^2, \qquad (3)$$

we can rewrite (3) in terms of the radial wave function $u(r)$:

$$u''(\hat{r}) + \left(\epsilon - \int_0^{\hat{r}} d\hat{r}' \frac{1}{\hat{r}'^2} \int_0^{\hat{r}'} d\hat{r}'' u^2(\hat{r}'')\right) u(\hat{r}) = 0, \qquad (4)$$

where $\hat{\psi}(\hat{r}) = \frac{1}{\sqrt{4\pi}} \frac{u(\hat{r})}{\hat{r}}$.

III. PROPERTIES OF THE SOLUTIONS AND PREDICTIONS

We now solve Eq. (4) numerically and look for states with nodes. The equation (4) is nonlinear; therefore, we do not have the freedom to normalize the solution. However, as we shall show later, this equation has a scaling symmetry that allows us to resolve the ambiguity due to the choice of M_0. That is, the total mass of the system does not depend on the choice of M_0. This is a nontrivial but an expected property. Let n be the number of nodes plus 1 ($n = 1$ for the ground state.) For n greater than 4, the rotation curve resembles the observed ones and this feature continues as we increase n.

Figure 1 is a sample of a stationary solution and Figure 2 is the corresponding velocity curve given by

$$\hat{v}(r) = v(r)/v_0 = \left(\int_0^{\hat{r}} u^2 d\hat{r}'/\hat{r}\right)^{1/2},\ v_0 = \sqrt{GM_0/r_0}. \qquad (5)$$

The numerical study shows that the mass profile $\rho \sim r^{-1.56}$, $r \gg 1$ for $n = 6$, and the exponent depends on n very weakly. One can understand this behavior qualitatively by simple semiclassical analysis. Since $M(r) \sim rv^2$, the flatness of the rotation curve is equivalent to $M'(r) \sim u^2 \sim$const. The WKB method gives

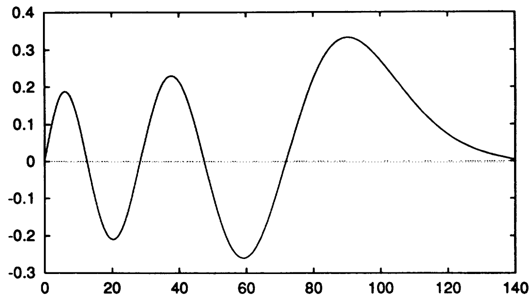

FIG. 1. Sample solution $u(r)$; $n = 5$, $\hat{\psi}_0 = 0.05$, $\hat{M} = 4.531\,63$, $e = 0.074\,53$.

$$u(r) \sim \frac{1}{\sqrt{p_r}} \cos\left(\int_0^r p_r dr - \pi/4\right), \qquad (6)$$

where $p_r = \sqrt{2m[\epsilon - V(u)]}$. Since the potential is increasing slowly as a function of r, so is $1/\sqrt{p_r}$. This explains the slowly increasing aspect of the rotation curve of our theory. The ripplelike structure is an inevitable phenomenon predicted in this theory since we use eigenfunctions of the wave equation. Apart from the mass profile and the ripple's existence, our theory has one quantitative prediction. The rotation velocity times the ripple wavelength is largely galaxy independent. This is nothing more than a rephrasing of the uncertainty principle. More precisely,

$$v_{\text{rotation}}\lambda_{\text{ripple}} = \frac{1}{\sqrt{2}}\frac{1}{m} f(\hat{r}), \qquad (7)$$

where $f(\hat{r})$ is a slowly increasing dimensionless function given by

$$f(\hat{r}) = \left(\int_0^{\hat{r}} d\hat{r} u^2(\hat{r})/\hat{r}[\hat{V}(\hat{r}_t) - \hat{V}(\hat{r})]\right)^{\frac{1}{2}},$$

$\hat{V}(\hat{r}) = V(r)/mv_0^2$, and r_t is the effective classical turning point.

We observed numerically that as we vary the initial condition $\hat{\psi}_0$, the scale-free quantities ϵ and $\hat{M} := \int u^2 dr$ have a simple dependence on it. Namely

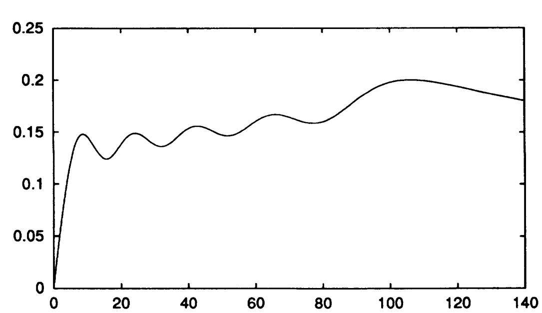

FIG. 2. Rotation curve obtained from Fig. 1.

$$\hat{M} = 20.31\sqrt{\hat{\psi}_0}, \ \epsilon = 1.49\hat{\psi}_0 \ \text{for } n = 5. \quad (8)$$

The coefficients in (9) depend on n; for n =6, they are given by 24.4408 and 1.5385, respectively. To understand this we have proved that Eq. (4) has the following symmetry.

If $\psi(r,\epsilon)$ is a solution, so is $\lambda^2\psi(\lambda r, \lambda^2\epsilon)$ Because of this symmetry, one may identify M_0 and M by choosing λ such that the norm of ψ is equal to one. However, this choice makes numerical integration very hard.

IV. COMPARISON WITH OBSERVATIONS

We now compare the rotation curves predicted in our theory and the actual observations. The dark matter distribution is most clearly imprinted in the rotation curves of disk galaxies. The "flatness" of a rotation curve (RC) is equivalent to mass profile $\rho \sim r^{-2}$. However, the observed rotation curves for the *field galaxies* are slightly increasing [9,10]. The mass profile predicted in our theory agrees with the observed rotation curve described by Rubin [10] $\rho \sim r^{-1.7}$ up to a few percent. One can make the agreements better by including the visible matter's effect. By including the visible matter's contribution, the rotation curve becomes flatter and in some case decreasing. This is expected because, by including the visible matter, the inner part is more enhanced by the presence of the visible matter and the outer part is attracted to inside slightly. The proper inclusion of the visible matter will be treated in a separate paper [8].

The confrontation of the theory with observation on ripple structure is more subtle. The data of Rubin *et al.* show that there are fluctuations that could indicate the fine structure in any galaxy. However, most of the data have bars that might wash out the signal of the fine structure. Among 63 galaxies listed in [9], there are few galaxies that have relatively small error bars and show ripples. The data of galaxy NGC2998 are particularly interesting for us and we plot the data and the theoretical curve in Fig. 3.

Comparing the velocity and wavelength of theory and observation we have the two relations $7r_0 = 8$ kpc, and $0.32v_0 = 204$ km/sec. Solving these two, we get $m = 3.3\times10^{-23}$ eV, and $M_0 = 0.69\times10^{12}M_\odot$. Using $\hat{M} = 8.74$ (numerically calculated value) for $\hat{\psi}_0 = 0.2$, we obtain $M = \hat{M}M_0 = 5.9\times10^{12}M_\odot$. The central density of dark matter is given by $\rho_c{=}1/4\pi|\hat{\psi}_0|^2(M_0/r_0^3){=}1.08\times10^{-22}$ g/cm^3. Therefore we can weigh the Galaxy by looking at the rotation curve only. In the model adopted, the visible matter's total mass is $2.7\times10^{11}M_\odot$. For later data analysis we give the general formula for m, M with $n = 5$:

$$m = \frac{8 \text{ kpc}}{\lambda}\frac{204 \text{ km/sec}}{v} \times 2.7\times10^{-23} \text{ eV}, \quad (9)$$

$$M = \frac{\lambda}{8 \text{ kpc}}\left(\frac{v}{204 \text{ km/sec}}\right)^2 \times 5.9\times10^{12}M_\odot. \quad (10)$$

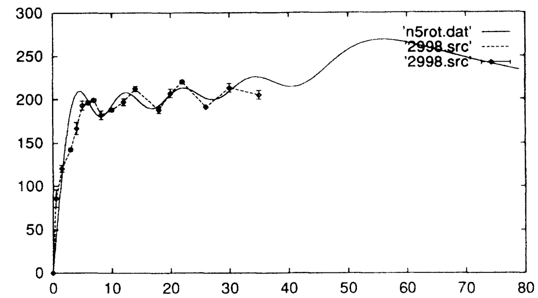

FIG. 3. Comparing theory and observation for NGC2998; velocity (km/sec) vs distance from the center (kpc). $n = 5$. The solid line is the theoretical curve and the dotted line with the error bar is the observed data.

Now we show that above quantities are independent of the choice of $\hat{\psi}_0$ or M_0. Recall that under the scaling, $\hat{\psi} \to \lambda^2\hat{\psi}$, $\hat{r} \to \hat{r}/\lambda$, and $\epsilon \to \lambda^2\epsilon$ must follow. Therefore had we chosen $\lambda^2\hat{\psi}_0$ instead of $\hat{\psi}_0$ above, we would get 7 kpc/λ = 8 km/sec and $0.32v_0\lambda = 200$ km/sec, so that we get the same m and $1/\lambda$ times M_0 above. On the other hand, $\hat{M} = \int d^3\hat{r}|\hat{\psi}|^2$ must scale by λ^4/λ^3; therefore, $M = \hat{M}M_0$ is invariant under the scaling. The same argument shows ρ_c is scale invariant. Hence we can choose large $\hat{\psi}_0$ to save computing time.

V. DISCUSSION AND CONCLUSION

In this paper we investigated the mass distribution of self-gravitating ultralight boson appearing in late-time phase transition. Galactic halos made of this species are highly correlated Bose liquids. We develop a Landau-Ginzburg-type theory describing the collective behavior of the system. Definite predictions are made on the mass profile $\rho \sim r^{-1.6}$, the existence of ripplelike fine structures in rotation curve, and its relation to the rotation velocity. The reader should note that since the mass is finite, this scaling must only be valid out to $r \sim 100$ in Fig. 2. One also should not forget that the spiral arms may also be responsible for these ripples.

Now we describe some limitations of our theory and future direction. There are four ripples in the NGC2998's data, but we suspect that outside the measured region there can still be density peaks and nodes that could lead to further ripples. However, for the fixed ripple wavelength and maximum rotation velocity determined by present observation, the total galaxy mass increases as n increases, hence for the sufficiently large n, Ω exceeds 1, violating the bias of present theoretical community. For example, if we take n bigger than 7, the total mass of the halo is larger than $10^{13}M_\odot$. Therefore, there are some restrictions on the value of n. Here we take, for simplicity, the minimal choice n=5 (four nodes). In fact, the issue of why the system will settle into a particular energy state n=6 or 5, not 100 or 1, for most systems is the most important issue remained in this approach to the dark matter. Here I will describe a preliminary answer that consists of a sequence of ideas and conjectures.

Step 1. The first and main idea is that large quantum numbers cannot exist. Consider the hydrogen atom case. The reason why $n = \infty$ is possible is that the Coulomb potential is long ranged; i.e., the potential well is infinitely wide. For any system with finite potential well, the number of bound states is finite. I am sure this must be true in our case also, though it is a nonlinear differential equation. But I did not prove this mathematically yet. Our system of dark matter is distributed for a finite range; therefore, the number of possible quantum numbers should be finite. What is the number? And what determines the number? I do not know yet.

Step 2. Now the question is why most of the galaxies have similar quantum numbers. Perhaps the answer should be that most of the galaxies were formed simultaneously (in astronomical time scale). In fact there must be a moment in the history of the Universe when the galaxy formation happened everywhere. That is simply the moment when the growth of the density perturbation inherited from the early Universe became nonlinear. The dark matter has been collapsed from more dispersed state with highest quantum number (say $n = 7$) to the present state. If most of the galaxies have similar age, it is natural that most of the galaxies must have the same or similar quantum number n.

Step 3. Now, why is the present quantum number 5 or 6 at this time? Why not smaller? The spectrum of the possible states are closer for higher states, while they are relatively separated between low lying states. (Notice that the spectrum of the hydrogen atom is very dense in higher quantum numbers while the separation between the energies of low quntum number is relatively greater. I expect that similar phenomena should happen in our case also.) Since we proved the inefficiency of the energy-losing mechanism, it is not very surprising that the evolution from n=5 or 6 (the present states) to lower states is much harder than from higher to the present states.

The above is the program to be carried out in future, rather than a concrete work. Nevertheless, I wish some readers are more comfortable with this argument (than with no argument).

For the galaxies which are exceptionally compact and bright, the rotation curves are slightly decreasing [11]. We believe that for these galaxies, the ratio of the visible mass to dark mass is bigger than usual case. In fact, the study of the visible matter effect [8] shows that we can have decreasing rotaion curves for that case.

Another aspect we did not discuss in this paper is the angular momentum. There is a very interesting phenomenon associated with the angular momentum. The vortices are associated with angular momentum of the condensation, which in turn can generate a scale of prestar. This issue is reported separately [12].

There are many aspects in this theory that should be further clarified; nevertheless, I think that it is worthwhile using Bose condensation as the galactic halo model. I also think that a specialist should look for recent data, which I do not have, to see whether the predictions of this work are truely relevant to the reality.

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APPENDIX: IMPOSSIBILITY OF THE GROUND STATE CONDENSATION

There are two mechanisms by which a self-gravitating system can reduce its energy. One is emitting the massless particle (cooling) and the other is emitting a part of the system (evaporation and/or collapse procedure) [6]. For the axion case, Fukugita, Takasugi, and Yosimura [5] showed that it is impossible to reach the ground state by cooling. The basic reason is that cooling is far from sufficient since the coupling with the photon is too small. In our case, both the self-coupling and the coupling of the particle to the photon are much weaker than the axion case, therefore the same conclusion is derived in this case.

We now show that the evaporation is not efficient enough either. We can estimate the order of magnitude for the evaporation time by calculating the time required for a particle to get an appreciable fraction of initial velocity by the scattering through the particles. One gets

$$t_{\rm evap} \sim \frac{v^3_{\rm escape}}{12\pi\rho m G^2} \sim 10^{114}\ \text{sec},$$

where we took $\rho \sim 10^{-25}$ g/cc, $m = 10^{-24}$, and $v_{\rm escape} \sim$ 100 km/sec. Since the number density $n \sim 10^{40}$, there are about 10^{110} particles inside the volume of $(100\ \text{kpc})^3$. Therefore, during the whole history of the galaxy, only 10^{13} particles whose total mass is less than 10^{-44} g evaporated. We conclude that it is unlikely that the particle evaporation can lead to the ground state condensation. In this evaluation we consider only gravity and assume that there are no other kind of particle is present. Including these facts does not change our evaluation by many factors of 10. However, our calculations are based on the particle nature of the dark matter. Quantum consideration will give even further reason for the inefficiency of the evaporation, since bosons want to be together.

[1] C.T. Hill, D.N. Schramm, and J. Fry, Comments Nucl. Part. Phys. **19**, 25 (1990).
[2] J. Frieman, C. Hill, and R. Watkins, Phys. Rev. D **46**, 1226 (1992).
[3] R. Ruffini and S. Bonazzola, Phys. Rev. **187**, 1767 (1969); J.D. Breit, S. Gupta, and A. Zaks, Phys. Lett. B **140**, 329 (1984); R. Friedberg, T.D. Lee, and Y. Pang, Phys. Rev. D **35**, 3640 (1987); J.J. Van Der Bij and M.

Gleiser, Phys. Lett. B **194**, 482 (1987); E. Seidel and W. Suen, Phys. Rev. D **42**, 384 (1990).

[4] M. Colpi, S.L. Shapiro, and I. Wasserman, Phys. Rev. Lett. **57**, 2485 (1986).

[5] M. Fukugita, E. Takasugi, and M. Yosimura, Z. Phys. C **27**, 373 (1985).

[6] J. Binney and S. Tremaine, in *Galactic Dynamics* (Princeton University Press, Princeton, 1987).

[7] E.M. Lifshitz and L.P. Pitaevskii, Landau Lifshitz Course of Theoretical Physics (Pergamon, Oxford, 1980), Vol. 9.

[8] S.U. Ji and S.J. Sin, following paper, Phys. Rev. D **50**, 3655 (1994).

[9] V. Rubin *et al.*, Astrophys. J. **289**, 81 (1985); **261**, 439 (1982); **238**, 471 (1980).

[10] V. Rubin, in *Dark Matter in the Universe*, Proceedings of the IAU Symposium, Princeton, New Jersey, 1985, edited by J. Kormendy and G. Knapp, IAU Symposium No. 117 (Reidel, Boston, 1987).

[11] S. Casertano and T.S. van Alberta, in *Baryonic Dark Matter*, edited by D. Lynden-Bell and Gilmore, NATO ASI Series C, Vol. 306 (Kluwer Academic, Dordrecht, 1990).

[12] C.H. Lee and S.J. Sin, "The origin of the Hubble sequence and Quantum aspect of dark matter" (unpublished).