Exclusive semileptonic decays of B mesons into light mesons

R. Akhoury
Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan 48109

George Sterman
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11794-3840

Y.-P. Yao
Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan 48109
(Received 30 December 1993)

Using heavy quark effective field theory, exponentiation of Sudakov double logarithms, and perturbative factorization theorems for exclusive processes, we calculate the amplitude for the semileptonic decay of mesons with a single, very heavy quark into π and ρ for a large hadronic recoil momentum. A formula for these large-recoil widths is obtained in terms of two-particle wave functions of the heavy and light mesons and applied to semileptonic decays of the B.

PACS number(s): 13.20.He, 12.39.Hg

I. INTRODUCTION

The semileptonic decays of B mesons are of special interest for the measurement of flavor-changing couplings in the standard model, and by implication for tests of the standard model [1]. The evaluation of these decays in terms of standard model parameters, however, is not straightforward. In contrast to, say, the W and Z vector bosons, the essential presence of strong interactions in B meson bound states complicates the situation. On the other hand, the b quark is moderately heavy [2], which suggests that its decays might be perturbatively computable [3], with the mass of the b quark acting as the large scale, at least when the outgoing hadrons are very energetic. The main obstacle is the treatment of light degrees of freedom. In this paper, we combine the ideas of heavy-quark effective field theory [4], the exponentiation of Sudakov double logarithms [5], and the perturbative treatment of elastic scattering processes based on factorization theorems [6] to study semileptonic decays of the type $B \rightarrow (l\nu)+\pi$. We argue that these decay amplitudes are computable in perturbative QCD in terms of pion and B valence wave functions. Thus, the light degrees of freedom in the B are represented only by its two-particle wave function [7]. This is a striking simplification, given that the b quark generally decays in a complex environment, which is not describable in terms of a single light quark.

The methods we shall describe have a formal relation to the perturbative description of the transverse momentum distribution of Drell-Yan pairs [8]. As in that case, the approach to a perturbative-dominated result is relatively slow, and proceeds as a fractional power of the heavy scale, in our case m_Q, the mass of the heavy quark. Typical corrections will be of the relative size $\alpha_s(C\Lambda m_Q)$, with C a number generally larger than, but of order, unity. For $Q=b$, this is not such a small number. We believe, however, that it is useful to identify the true asymptotic behavior for large m_Q, even if for $m_Q=m_b$ corrections may be significant. In this spirit, we shall present numerical results that can be compared to experiment [9]. Perhaps not surprisingly, the perturbative width is small at large recoil, reflecting the usual suppression of exclusive amplitudes with large momentum transfer.

Our method will apply in the kinematic region where the produced hadron of momentum p' is very energetic in the rest frame of the heavy meson, of momentum p. To make this condition explicit, we introduce a variable x through the invariant mass of the lepton-neutrino pair (neglecting the mass of the final state hadron):

$$m_l^2=(p'-p)^2=(1-x)m_b^2 \quad \text{with } x=\frac{2p\cdot p'}{m_b^2} . \qquad (1)$$

We will be interested in x fixed away from zero, for large $m_Q^2=m_b^2 \sim p^2$. In the region $x \sim 0$, the leptons carry away most of the energy, and the produced hadron is soft. This kinematic region may be treated by soft-pion theorems and related methods [10].

The physical basis for the simplifications discussed above is relatively straightforward. Let us suppose that the b quark decays into a nearly on-shell u quark. This u quark propagates through the remaining hadronic medium, picking up a light antiquark, to form an outgoing pion, after some distance y. For an exclusive final state, we demand that there be no radiation of gluons in this process. But if y is large, of the order of hadronic sizes, i.e., $O(1/\Lambda)$, then QCD corrections *will* be large, like those in the "Sudakov" form factor, which vanishes on shell [6]. We thus expect exclusive processes to be associated with limited ($y^- < 1/\Lambda$) distances of propagation before the light antiquark is picked up, forming a pair which evolves into the pion. But in this situation, the outgoing gluon is typically off shell by order $(m_Q/y^-) \sim \Lambda m_Q$, which is substantial. This suggests the applicability of factorization theorems [6].

The arguments given here will also have applications to nonleptonic decays of B mesons [7,12]. We shall pursue these extensions in a future publication.

II. SUMMARY OF THE ARGUMENT

Because the argument draws on a number of techniques that are not usually combined, it may be worthwhile to outline our reasoning in some detail. This will also serve to organize the specific calculations that follow, and to show why they have implications for arbitrary orders in perturbation theory.

Structure of perturbative B decay. Our discussion begins with the diagrammatic description of B decay. We identify the sources of long-distance behavior, which cannot be successfully calculated in perturbation theory. We shall see that they may be identified with (i) collinear divergences due to virtual partons moving parallel to the outgoing pion, and (ii) infrared divergences due to soft gluons that interact with the incoming $\bar{b}$ antiquark and the fast-moving partons of the outgoing pion. Our aim will be to show that these infrared and collinear divergences may be separated (factorized) from calculable short-distance contributions, and organized into phenomenologically accessible nonperturbative functions.

Heavy-quark effective field theory. As a next step, we apply the ideas of heavy-quark effective field theory, recognizing that the $\bar{b}$ quark carries nearly all the momentum in the initial state. By introducing an "eikonal" description of $\bar{b}$-gluon interactions, we can systematically separate infrared divergent couplings of gluons to the $\bar{b}$ quark in peturbation theory from short-distance contributions. This procedure, however, leaves untreated the collinear divergences found at each order of perturbation theory, so that by itself it is not an adequate factorization.

Higher-order corrections. We next show how higher-order corrections regulate the behavior of perturbation theory in momentum space regions that give rise to collinear divergences. We show that a gluon of virtuality k^2, with energy $2^{-1/2}xm_b$, moving collinear to the outgoing pion, couples at higher order to softer gluons that produce corrections of the form $\alpha_s\ln^2[(xm_b)^2/k^2]$. These leading logarithms exponentiate, and act to regulate collinear logarithms associated with the propagation of color nonsinglet degrees of freedom at high energy.

Role of the valence states. The exponentiation of "Sudakov" double logarithms in QCD strongly suppresses the propagation of the virtual gluon, of momentum k unless its invariant mass $|k^2|$ is substantial. In fact, for $k^2<xm_b\Lambda$, the perturbative amplitude *vanishes* due to a pole in the running coupling. In the resummed perturbative amplitude, then, this gluon is generally far off-shell. It will, however, typically propagate without radiating for a distance of order $1/\Lambda$, the lifetime of a state that is off-shell by order $m_b\Lambda$ with energy m_b [13]. Standard parton-model and factorization considerations [11], however, show that such an off-shell particle interacts with a minimum number of physical partons, up to corrections that are suppressed by a power of its mass. This parton must be the valence d quark that is necessary to form the pion. Corrections due to states with more partons are suppressed by order Λ/m_b. Once the d quark picks up the gluon's momentum, it forms a gauge singlet with the outgoing $\bar{u}$, produced at the decay vertex, which propagates into the final state. Soft interactions of this combination are suppressed by its color singlet nature.

The decay amplitude. The reasoning described above suggests an expression for the form factor as an integral of the B valence wave function with an exponentiated Sudakov factor. If the wave function is very soft (corresponding to a "large" B meson), the Sudakov exponential dominates the resulting integral, which, once it is transformed to position space, depends only on the value of the valence wave function at the origin. The resulting expression is then proportional to f_B. In fact, we shall show that this expression holds only for a *very* heavy quark, due to the slow evolution of the Sudakov exponent. We give a more realistic result as an integral over the B two-particle wave function.

We now begin our description of the details of our argument, starting with our analysis of perturbation theory.

III. PERTURBATIVE DESCRIPTION OF B DECAY

The general process of $B^0\to\pi^-(l^+,\nu_l)$ decay is represented in Fig. 1. To begin with, we may regard the figure as a space-time picture of this decay. We think of this process in the B rest frame. In the figure, B_n represents the wave function for the n-particle virtual state of the B meson. The case $n=2$ is the "valence" state, consisting of the $\bar{b}$ quark and a light (d) quark only. At the point labeled h, the B quark decays, producing a virtual W^+, a $\bar{u}$ quark, and possibly some number of other partons, labeled G. As these partons propagate away from the decay, they interact with the remaining $n-1$ partons of the B. For the exclusive decay at hand, *all* of the latter must be "picked up" on the way out, or they will appear in the final state. The details of this process are absorbed into the blob labeled J_π in the figure. Finally, after all the remaining partons are absorbed, the pion itself is formed, and emerges into the final state.

Our aim is to use perturbative methods to analyze heavy-quark decay. Of course, any purely perturbative picture of an exclusive process includes quarks and gluons that are on or near the mass shell. For such mo-

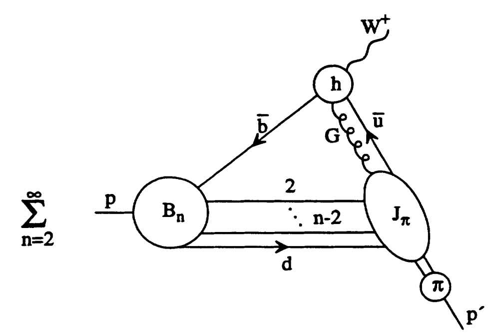

FIG. 1. Space-time picture for the decay $B\to(l\nu)+\pi$.

menta, higher-order corrections are guaranteed to be large and uncontrollable. We must therefore succeed in factorizing perturbatively incalculable long-distance information into universal functions or parameters. The specific functions we will need are the wave functions of the pion and the B meson.

The factorization process begins with purely perturbative scattering amplitudes, suitably regularized in the infrared. Factorization consists of separating amplitudes of this sort into long- and short-distance contributions, in such a way that the long-distance contributions are formally identical with definite nonperturbative quantities, such as the wave functions mentioned above. The remaining, short-distance ("infrared safe") functions are constructed to be calculable in perturbation theory, either order by order, or, as we shall do below, after a suitable reorganization ("resummation") of perturbation theory. Once the short-distance functions are calculated, they may be combined with phenomenologically determined long-distance quantities to derive predictions of the theory.

Given this framework, we now turn to the case at hand, and ask how long- and short-distance contributions occur in a perturbative description of B decay. We therefore reinterpret Fig. 1 as a perturbation theory diagram, and ask at what momenta the corresponding perturbative amplitude receives contributions from long distances, that is, from lines near the mass shell, which must be factorized.

According to the analysis described, for instance, in Ref. [14], infrared-sensitive contributions arise only when the on-shell lines of the diagram actually describe a physical scattering process, with free propagation between the initial and final states, and with every vertex represented by a point in space-time. In this process, off-shell lines are shrunk to points, so that the vertices of the resulting "reduced" diagram may connect more lines than the elementary vertices of the theory. Let us thus again consider Fig. 1 as a physical process, for the scattering of a $\bar{b}$ and a d quark into a lepton pair, and a $\bar{u}d$ pair of light quarks moving parallel into the final state. We consider the (physically interesting) case where the $\bar{b}$ quark carries essentially all the incoming momentum.

In the relevant physical process, the lines internal to the subdiagram J_π, as well as the virtual $\bar{u}$ quark and the set G of lines that attach to the decay vertex, are all on-shell and collinear to the outgoing pion. In addition, the $n-1$ partons coupled to the incoming $\bar{b}$ quark and to J_π are all "soft," that is, carry momenta that are small in all four components compared to the mass of the $\bar{b}$ quark. As such, they may be thought of as "long wavelength" lines, that connect to vertices widely separated in space-time. Thus, they describe the production of partons by the B meson far in the past. In perturbation theory, with massless gluons and light quarks, these lines will be associated with infrared divergences, while collinear lines will be associated with collinear divergences.

The subdiagrams that give infrared and collinear divergences may be quite complex with increasing powers of the coupling. Nevertheless, the processes associated with these divergences have a universal underlying structure, which will be the basis of the remaining analysis. The most important point is that, to any order in perturbation theory, the pion is formed from a connected set of nearly on-shell energetic collinear lines. Until all the $n-1$ partons are absorbed, this "jet" of virtual particles carries a net color. Collinear divergences come only from such lines. As we shall see, it is just this property that will allow us to reorganize perturbation theory in a useful manner. We shall argue that Sudakov effects dynamically produce an off-shell process, which is not present at lowest order in perturbation theory. First, however, we shall discuss the immediate consequences of having a very heavy quark in the initial state.

IV. IMPLEMENTING HEAVY-QUARK EFFECTIVE FIELD THEORY

We now appeal to the basic tenets of heavy-quark effective field theories [4]. To be specific, we split the QCD Lagrange density $\mathcal{L}_{\rm QCD}$ into terms involving the heavy quarks (in this case the $\bar{b}$ quark only), and the remaining terms, which involve the light quarks, denoted collectively as q:

$$\mathcal{L}_{\rm QCD}=\mathcal{L}(q)+\mathcal{L}_0(b)-g\bar{b}\gamma A b \ , \qquad (2)$$

where A^μ is the gluon field (treated as a matrix in color space), and $\mathcal{L}_0(b)$ is the b-quark free Lagrange density. Here and below g will refer to the strong coupling. We shall neglect the charmed quark, except as it appears in the running coupling.

Consider the amplitude for the semileptonic decay of a bound state of quantum numbers ($\bar{b}q$). Corrections to form factors and decay amplitudes that are due to heavy quark *loops* enter only as small (relative order Λ/m_b) corrections. We express this decoupling assumption [15] in the form (J_μ^h is the hadronic weak current)

$$\begin{aligned} M_\mu(p',p)&=\langle \pi(p')|J_\mu^h(0)|B(p)\rangle_{\mathcal{L}_{\rm QCD}} \\ &=\langle \pi(p')|T[J_\mu^h(0)V_b(A^\beta)]|B(p)\rangle_{\mathcal{L}(q)} \ , \end{aligned} \qquad (3)$$

where the operator $V_b(A^\mu)$ represents a single b-quark line, connected to the weak vertex. $V_b(A^\mu)$ consists of a string of b-quark propagators connecting gluon fields $A^\mu(x)$, and thus, V represents the full effect of the terms $\mathcal{L}_0(b)-g\bar{b}\gamma A b$ in Eq. (2), up to loop corrections. The resulting matrix element is then computed with the light-quark Lagrange density $\mathcal{L}(q)$, as indicated by the suffix.

Once the dependence on the single b-quark line is made explicit as in Eq. (3), it is relatively simple to separate the part of the matrix element that is described by heavy-quark effective field theory. To this end, we go to the rest frame of the B meson (approximately the same as the rest frame for the b quark) and introduce the non-Abelian path-ordered eikonal phase:

$$U(A^0)=\mathcal{P}\exp\left[-ig\int_{-\infty}^{0} A^0(\lambda n_0)d\lambda\right] . \qquad (4)$$

A^0 is the component of the gluon field (treated as a matrix in color space) that couples to the b quark at rest. n_0 is a unit vector in the time direction, $n_0^\mu=\delta_{\mu 0}$. In Ref.

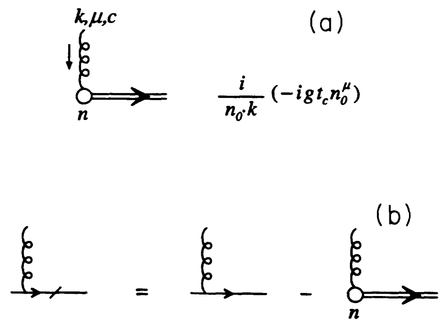

FIG. 2. (a) Feynman rules for the eikonal phase. (b) Definition of subtracted b-quark line and gluon vertex.

[16] phases like Eq. (4) were employed to study the energy and mass dependence of massive quark form factors. The Feynman rules for the eikonal phase are shown in Fig. 2(a), where t_c is the usual color matrix generator in the quark representation (defined here to be Hermitian). Note that the eikonal propagator depends only on the four-velocity n_0^μ of the heavy quark at rest [2,4]. We also define a "subtracted" b-quark line and gluon vertex, as shown in Fig. 2(b), and easily verify that infrared divergences due to the divergence of the original propagator at zero gluon momentum are canceled by subtracting the eikonal line. This cancellation generalizes to arbitrary order (see Appendix) and, as a result, we may define an infrared-finite b-quark line by

$$V_b^{(\mathrm{fin})}(A^\beta)\equiv V_b(A^\beta)U^{-1}(A^0)\ . \tag{5}$$

A sum over color indices is implicit in this product.

We now insert unity in the form $U^{-1}U$ into Eq. (3), and sum over the contributions of differing virtual states n, to get

$$M_\mu(p',p)=\sum_{n=2}^{\infty}\langle \pi(p')|T[J_\mu^h(0)V_b^{(\mathrm{fin})}(A^\beta)\otimes_n U(A^0)]|B(p)\rangle_{\mathcal{L}(q)}\ . \tag{6}$$

The notation "$\otimes_n$" denotes integration over momenta and summation of discrete indices for states with a b quark and $n-1$ light partons, as in Fig. 3, which is related to Fig. 1 by the replacement of the b-quark line by its corresponding eikonal approximation. The original b-quark propagators are absorbed into the modified decay vertex, labelled $\hat{h}$ in Fig. 3, which is now free of infrared divergences. We shall use this result below.

For the case $n=2$ we will find it useful to define a corresponding position-space wave function, which we express here as a matrix with Dirac indices α and β:

$$\tilde{\Psi}_B(x)_{\alpha\beta}=\langle 0|T(\bar{b}^{\,\mathrm{free}}(0)_\alpha U(A^0)q(x)_\beta)|B(p)\rangle_{\mathcal{L}(q)}\ , \tag{7}$$

with q_β and $b_\alpha^{(\mathrm{free})}$ the light-quark and free b fields, respectively. All interactions of the b field have been absorbed into the non-Abelian eikonal phase, as shown. The organization of the amplitude in Eq. (6) is analogous to the usual organization according to "Fock states" in the development of factorization theorems for elastic scattering [6]. In this and the following, we shall assume a trace over color indices. $\tilde{\Psi}$ and similar "wave functions" can be defined in a gauge invariant manner by connecting the color indices of the quark fields with an additional ordered exponential of the gauge field in the color trace. We shall suppress this standard factor below.

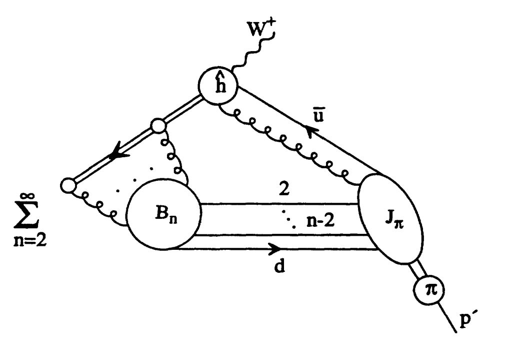

FIG. 3. Same as Fig. 1, with the replacement of the b-quark line by its eikonal approximation.

As it stands, Eq. (6) includes in its sum states of arbitrary particle content, including extra gluons and light-quark pairs, relative to the "valence" states (b,q). We can, however, reorganize the perturbative sum in such a way that the convolution $\otimes_n$ describes the final two-particle state before the b-quark decay. Such a reorganization leads to an expression of the form

$$M_\mu(p',p)=\frac{1}{(2\pi)^4}\int d^4k\,\mathcal{A}_\mu(p',k,p)\Psi_B(k)\ , \tag{8}$$

where $\Psi_B(k)$ is the wave function (7) in momentum space, and $\mathcal{A}_\mu(p',k,p)$ is the rest of the amplitude. We note that, in general, it includes interactions of gluons from the eikonal phase. So far, this reorganization is purely formal, but we shall show how the momentum transfer in $\mathcal{A}_\mu$ is dynamically short-distance dominated, for the physical reasons described above. The two-particle states of the B meson will then dominate the amplitude.

V. WAVE FUNCTIONS

To proceed further, we must develop a few relevant properties of the B and pion valence wave functions. We will work in a frame in which the B meson is at rest, and the pion momentum is in the $+$ direction. In this frame, as we shall see, the only nontrivial dependence of the amplitude $\mathcal{A}$ on k, the internal momentum of the B meson, is through k^- $[V^\pm=2^{-1/2}(V^0\pm V^3)]$. Note that, according to heavy-quark effective field theory,

$$k^\mu \ll m_b\ . \tag{9}$$

Taking this result into account, we define a scalar wave function with k^+ and $k_\perp$ integrated over, by

$$(2\sqrt{2})^{-1}[\gamma_5(\not{p}+m_b)]_{\alpha\beta}\phi_B(k^-)=\frac{1}{2\sqrt{3}}\int\frac{d^2k_\perp dk^+}{(2\pi)^4}\Psi_B(k)_{\alpha\beta}$$
$$=\int\frac{dy^+e^{-ik^-y^+}}{4\pi\sqrt{3}}\langle 0|T[\bar{b}^{\,\mathrm{free}}(0)_\alpha U(A^0)q(y^+)_\beta]|B(p)\rangle_{\mathcal{L}(q)}\ . \tag{10}$$

Denoting by b^+ the coordinate conjugate to k^-, we denote the configuration space version of the scalar wave function as

$$\tilde{\phi}_B(b^+)=\int dk^- e^{-ik^-b^+}\phi_B(k^-)\ . \tag{11}$$

With the matrix structure shown in Eq. (10), the wave function obeys the usual Ward identities at any vector current to which the b quark attaches. This is required by current conservation in the electromagnetic form factor of the B meson. In the limit that k^- is small compared to m_b, amplitudes formed with this wave function are also color gauge invariant, up to corrections of an inverse power of the heavy-quark mass.

We shall also make use of the standard pion wave function, which we may define, analogously to Eq. (10),

$$(1/2\sqrt{2}p'^+)(\gamma_5\not{p}')_{\alpha\beta}\phi_\pi(\xi,Q^2)=\int\frac{dy^-}{4\pi\sqrt{3}}e^{-i\xi p'^+y^-}\langle 0|T[\bar{q}(y^-)_\alpha q(0)_\beta]|\pi(p')\rangle\ . \tag{12}$$

Here Q^2 sets the scale dependence due to the lightlike separation of the two fields. Note that ϕ_π has dimensions of mass, while ϕ_B in (10) is dimensionless. The two wave functions satisfy

$$\int_0^1 d\xi\,\phi_\pi(\xi,Q^2)=\frac{1}{2\sqrt{3}}f_\pi\quad(f_\pi\cong 93\ \mathrm{MeV})\ , \tag{13}$$

$$\int dk^-\phi_B(k^-)=\tilde{\phi}_B(0)=\frac{1}{2\sqrt{3}}f_B\ , \tag{14}$$

and f_B is the B-meson decay constant defined through the matrix element of the appropriate electroweak current between the B state and the vacuum:

$$\langle 0|\bar{b}\gamma_\mu(1-\gamma_5)d|B^0(p)\rangle=-\sqrt{2}f_Bp_\mu\ . \tag{15}$$

VI. LOWEST-ORDER AMPLITUDES

We now return to the analysis of the amplitude for our heavy-quark semileptonic decay, $B^0\to\pi^-l^+\nu_l$, where l denotes the lepton. In accordance with our comments in Sec. II, we will want to study behavior of the decay process for the valence state of the B. Once we find (in the next section) a Sudakov suppression of collinear divergences, our concentration on the two-particle state will be justified. Throughout, we work to leading order in $1/m_b$ and neglect the light-quark masses.

The decay is governed by the four-Fermi interaction (G_F is the Fermi coupling constant),

$$\frac{G_F}{\sqrt{2}}V_{ub}[\bar{b}\gamma_\mu(1-\gamma_5)u][\bar{\nu}_l\gamma^\mu(1-\gamma_5)l]\ , \tag{16}$$

and the effective amplitude is, in the notation of Eq. (3),

$$A(p',p)=\frac{G_F}{\sqrt{2}}V_{ub}\bar{\nu}_l\gamma^\mu(1-\gamma_5)l\langle\pi(p')|\bar{b}\gamma_\mu u|B(p)\rangle=L^\mu M_\mu\ , \tag{17}$$

with L^μ the leptonic and M_μ the hadronic amplitudes. We are finally ready to state the full factorized form of the $B\to\pi$ transition form factor, $M_\mu=\langle\pi|\bar{b}\gamma_\mu u|B\rangle$. Assuming, as we shall verify below explicitly, that the pion emerges from a short-distance region (in the B-π center of mass), we may write the amplitude as

$$M_\mu(p',p)=\int_0^1 d\xi\int dk^-[\phi_\pi(\xi,Q^2)H_\mu(p',k,p,\xi)\tilde{\phi}_B(k^-)]\ . \tag{18}$$

In this expression, $(1-\xi)$ is the longitudinal fraction of the pion momenta (p') carried by the $\bar{u}$ quark in the pion, whose wave function is $\phi_\pi(\xi,Q^2)$. Q^2 will be taken as a typical momentum transfer in the problem, to be specified later. With the Dirac structure of the wave functions shown explicitly, the hadronic amplitude becomes

$$M_\mu(p',p)=\int_0^1 d\xi\int dk^-\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}(\gamma_5\not{p}')_{\beta\beta'}\phi_\pi(\xi,Q^2)H_{\mu;\beta'\alpha;\beta\alpha'}(p',k^-,p,\xi)\phi_B(k^-)\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}[\gamma_5(\not{p}+m_b)]_{\alpha'\alpha}\ . \tag{19}$$

We first discuss this expression at lowest order in H, and then demonstrate the role of corrections. Our results will be seen to be valid in the kinematic regime in which xm_b is large, with x defined in Eq. (1).

In the frame chosen above, $p=2^{-1/2}(m_b,m_b,0)$, $p'=2^{-1/2}(xm_b,0,0)$ in the notation $V^\mu=(V^+,V^-,\mathbf{V}_\perp)$. At the lowest order, there are two diagrams [Figs. 4(a) and 4(b)] that contribute to the hard decay amplitude. Consider Fig. 4(a) first. Consistent with the definition of the hard scattering amplitude using Eq. (5), we must perform a subtraction on the b-antiquark propagator after the hard gluon exchange. This is represented by a slash on the propagator, as in Fig. 2(b). This removes the infrared divergence at $\xi=0$ and gives the following expression for the amplitude (throughout, we use the Feynman gauge):

$$H^{(a)}_{\mu;\beta'\alpha;\beta\alpha'}(p',k^-,p,\xi)=\frac{(g^2)(\gamma_\nu)_{\beta'\alpha}\{\gamma_\nu(\xi\not{p}')\gamma_\mu\}_{\alpha'\beta}N_cC_2(F)}{(-2\xi p'^+k^-)(x\xi m_b^2)}\ . \tag{20}$$

In the above, a sum over external color indices has been performed. $N_c=3$ is the number of colors. At this order, the subtraction is equivalent to simply deleting the term $-\not{p}+m_b$ from the numerator of the propagator that carries momentum $p-\xi p'$. As mentioned earlier, the only significant dependence of H on k is through k^-: k^- is multiplied by terms of $O(m_b)$ while k^+ is added to them, and hence may be neglected in view of (14). The expression for Fig. 4(b) is similar, except that there is no subtraction for this graph, because the gluon cannot be considered part of the B bound state. Thus, we get

$$H^{(b)}_{\mu;\beta'\alpha;\beta\alpha'}=\frac{-(g^2)(\gamma_\nu)_{\beta'\alpha}\{\gamma_\mu(\not{p}'+\not{k})\gamma_\nu\}_{\beta\alpha'}}{(2\xi p'^+k^-)(2p'^+k^-)}\ . \tag{21}$$

As mentioned above, these results, although IR subtracted, remain sensitive to the collinear ($k^-\to 0$) region, through radiative corrections.

The leading (double-logarithmic) one-loop corrections to Fig. 4 for large xm_b are shown, respectively, in Figs. 5(a) and 5(b). They involve interactions of the eikonal phase, which generates soft divergences, with the hard gluon, of momentum $\xi p'+k$. We recall that such interactions have been separated from ϕ_B, since they occur after the final two-particle state that preceeds the decay [see the discussion Eq. (8) above]. Other, nonleading, corrections will cause the coupling g^2 in Eqs. (20) and (21) to run, with a scale set by the "off shellness" of the exchanged hard gluon, i.e., $(\xi p'+k)^2$. We shall evaluate the coupling simply at $\xi=1$; a more complete treatment of these nonleading contributions in a related context may be found in Ref. [21].

The leading behavior of Fig. 5(a) may now be easily evaluated with the result

$$\frac{(g^2)(\gamma_\nu)_{\beta'\alpha}\{\gamma_\nu(-\xi\not{p}')\gamma_\mu\}_{\beta\alpha'}N_cC_2(F)}{(2\xi p'^+k^-)(x\xi m_b^2)}\left\{-\frac{\alpha_s}{8\pi}C_A\ln^2\frac{(2\hat{\mathbf{k}}\cdot n)^2}{-\hat{\mathbf{k}}^2}\right\}, \tag{22}$$

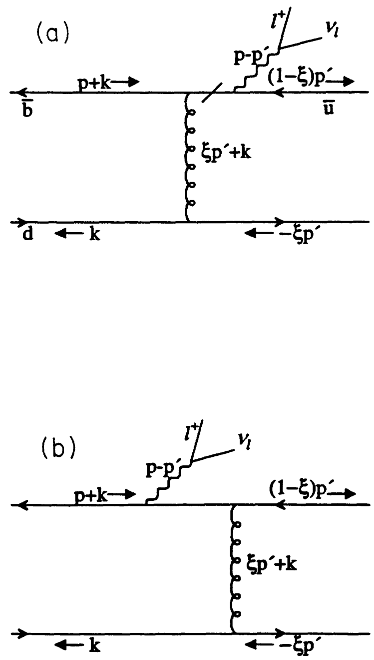

FIG. 4. Lowest-order diagrams contributing to the hard scattering amplitude for the decay $B\to(l\nu)+\pi$.

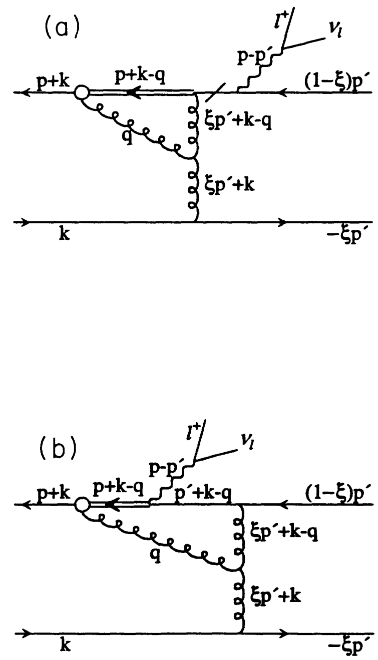

FIG. 5. The leading one-loop corrections to Figs. 4(a) and 4(b).

where we introduce the notation

$$\hat{\mathbf{k}} \equiv \xi p' + k \ . \tag{23}$$

The one-loop correction term in curly brackets is the same for Fig. 5(b). Of course, there are a large set of corrections that we have not considered here. However, for $x \neq 0$, in the double-logarithmic approximation, Fig. 5 gives the dominant effect. For example, double-logarithmic contributions from diagrams in which the gluon of momentum q attaches to the light quark or antiquark in the pion, instead of to the hard gluon in Fig. 5, cancel pairwise. Other vertex and self-energy corrections will not give double logarithms in the Feynman gauge, except near $\xi = 0$, where they contribute to an exponential sum that suppresses "end-point" contributions [21,22]. Further, as we will now show, double-logarithmic corrections force the gluon off-shell, while the leading effect of the remaining corrections will be to cause the coupling g to run.

VII. EXPONENTIATION

We now discuss the exponentiation of Sudakov double logarithms [5,17], which organizes the large corrections just identified. We can organize the logarithmic corrections to all orders to the hard scattering amplitude shown in Fig. 4(a) into a scalar form factor $\Gamma(\hat{\mathbf{k}}^2/\mu^2, (2\hat{\mathbf{k}}\cdot n)^2/\mu^2, \alpha_s(\mu))$, where μ is the renormalization scale, which we will fix later, and where, as before, $\hat{\mathbf{k}} = \xi p' + k$ is the momentum of the energetic gluon line in the decay. In general, Γ contains large logarithms in ratios of both $\hat{\mathbf{k}}\cdot n = \frac{1}{2} x \xi m_b^2$ and $\hat{\mathbf{k}}^2 = x \xi m_b k^-$ to μ. It is these logarithms that we would like to resum.

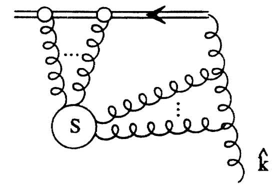

FIG. 6. The eikon form factor with external off-shell gluon line.

Sets of diagrams such as Fig. 6 have been studied for external on-shell particles in discussions of the Sudakov form factor based on factorization theorems [17]. The discussion for an external, off-shell gluon is nearly identical. We first note that we may represent the set of diagrams in Fig. 6 as the perturbative expansion of the matrix element

$$t_b \epsilon\cdot\gamma \Gamma(\hat{\mathbf{k}}) = \int dy e^{-i\hat{\mathbf{k}}\cdot y} \left\langle 0 \left| t_a P \exp\left[\int_{-\infty}^{0} dz n\cdot A(zn) \right] \gamma\cdot A_a(0) \epsilon\cdot A_b(y) \right| 0 \right\rangle_{\mathrm{tr}} \times (\mathrm{UV}) \ , \tag{24}$$

where t_b is a group generator in the quark representation, where the factor "UV" is a renormalization factor, and where the subscript "tr" indicates that self-energy diagrams have been truncated in both the external gluon and the eikonal line in the n^μ direction. The vector ϵ^ν represents an external polarization for the gluon field of momentum $\hat{\mathbf{k}}$.

We now appeal to the reasoning of Ref. [17] to recognize two properties of the scalar function Γ: first, that it is multiplicatively renormalizable, and second, that it obeys an evolution equation in $2\hat{\mathbf{k}}\cdot n$ of the Sudakov type [5]

$$\frac{d \ln \Gamma(\hat{\mathbf{k}}^2, 2\hat{\mathbf{k}}\cdot n, \mu, \alpha_s(\mu))}{d(\ln 2\hat{\mathbf{k}}\cdot n)} = \tilde{K}\left[\frac{\hat{\mathbf{k}}^2}{\mu(2\hat{\mathbf{k}}\cdot n)}, \alpha_s(\mu^2) \right] + \tilde{G}\left[\frac{2\hat{\mathbf{k}}\cdot n}{\mu}, \alpha_s(\mu^2) \right] , \tag{25}$$

where the dimensionless functions $\tilde{K}$ and $\tilde{G}$ depend separately on the ratios $\hat{\mathbf{k}}^2/\mu(2\hat{\mathbf{k}}\cdot n)$ and $2\hat{\mathbf{k}}\cdot n/\mu$, respectively. Assuming that Γ is multiplicatively renormalizable, the combination $\tilde{K} + \tilde{G}$ must be a renormalization group invariant:

$$\mu \frac{d(\tilde{K} + \tilde{G})}{d\mu} = 0 \ . \tag{26}$$

Now, because $\tilde{K}$ and $\tilde{G}$ have only the dimensionless argument $\alpha_s(\mu)$ in common, their μ dependence is determined by the relations

$$\mu \frac{d\tilde{K}}{d\mu} = -\gamma_{\tilde{K}}(\alpha_s) = -\mu \frac{d(\tilde{G})}{d\mu} \ , \tag{27}$$

with $\gamma_{\tilde{K}}$ a power series in α_s. To one loop $\tilde{K}$, $\tilde{G}$, and $\gamma_{\tilde{K}}$ can be found directly from the explicit one-loop results, Eq. (22):

$$\tilde{K} = \frac{\alpha_s}{\pi} \tilde{K}^{(1)} = \frac{\alpha_s}{\pi} \frac{C_A}{2} \ln \frac{\hat{\mathbf{k}}^2}{\mu(2\hat{\mathbf{k}}\cdot n)} \ , \tag{28}$$

$$\tilde{G} = \frac{\alpha_s}{\pi} \tilde{G}^{(1)} = -\frac{\alpha_s}{\pi} \frac{C_A}{2} \ln \frac{2\hat{\mathbf{k}}\cdot n}{\mu} \ , \tag{29}$$

$$\gamma_{\tilde{K}} = \frac{\alpha_s}{\pi} \gamma_{\tilde{K}}^{(1)} + \left[\frac{\alpha_s}{\pi} \right]^2 \gamma_{\tilde{K}}^{(2)} \ . \tag{30}$$

From Eq. (27), we find

$$\gamma_{\tilde{K}}^{(1)} = \frac{C_A}{2} \ . \tag{31}$$

We may note, however, another way of deriving this quantity. As noted by Mueller [18], and by Kodiara and Trentadue [19], the coefficient of the $\ln N$ term in the quark-quark entry of the anomalous dimension matrix for deeply inelastic scattering gives the corresponding anomalous dimension γ_K for the quark Sudakov form factor. Here we have essentially "half" of a gluonic form factor, with logarithms associated with a single gluon which, although off shell, is much closer to the mass shell than the scale of momentum transfer (whose role is played here by $2\hat{\mathbf{k}}\cdot n$). Because we do not have a singlet form factor, we cannot simply read off our anomalous dimension in the same manner. We shall assume, however, that the ratio of the one- to two-loop contributions to $\gamma_{\tilde{K}}$ are given by the $\ln N$ terms in the gluon-gluon anomalous dimension. By inspection of one- and two-loop results [20], we then find for $\gamma_{\tilde{K}}^{(2)}$ the corresponding result

$$\gamma_{\tilde{K}}^{(2)}=\frac{C_A}{4}\left[\left[\frac{67}{18}-\frac{\pi^2}{6}\right]C_A-\frac{10}{9}\frac{N_f}{2}\right], \tag{32}$$

where N_f is the number of flavors.

The solution to Eq. (25) is simple:

$$\ln\Gamma\left[\frac{(\hat{\mathbf{k}}^2)^2}{\mu^2(2\hat{\mathbf{k}}\cdot n)^2},\frac{(2\hat{\mathbf{k}}\cdot n)^2}{\mu^2},\alpha_s(\mu^2)\right]=\ln\Gamma(\hat{\mathbf{k}}^2/\mu^2,\hat{\mathbf{k}}^2/\mu^2,\alpha_s(\mu^2))+\frac{1}{2}\int_{\hat{\mathbf{k}}^2}^{(2\hat{\mathbf{k}}\cdot n)^2}\frac{d\eta^2}{\eta^2}\left[\tilde{K}\left[\frac{(\hat{\mathbf{k}}^2)^2}{\eta^2\mu^2},\alpha_s(\mu)\right]+\tilde{G}(\eta^2/\mu^2,\alpha_s(\mu))\right]. \tag{33}$$

If we set μ^2 to $\hat{\mathbf{k}}^2$ on both sides of this solution, use the renormalization group equation, (27), and the fact that from Eq. (28) $\tilde{K}(1,\alpha_s[(\hat{\mathbf{k}}^2)^2/\eta^2])$ vanishes, we get

$$\ln\Gamma\left[\frac{(\hat{\mathbf{k}}^2)^2}{(2\hat{\mathbf{k}}\cdot n)^2},\frac{(2\hat{\mathbf{k}}\cdot n)^2}{\hat{\mathbf{k}}^2},\alpha_s(\hat{\mathbf{k}}^2)\right]=\ln\Gamma(1,1,\alpha_s(\hat{\mathbf{k}}^2))+\frac{1}{2}\int_{\hat{\mathbf{k}}^2}^{(2\hat{\mathbf{k}}\cdot n)^2}\frac{d\eta^2}{\eta^2}\left[\tilde{G}(1,\alpha_s(\eta^2))-\frac{1}{2}\int_{(\hat{\mathbf{k}}^2)^2/\eta^2}^{\eta^2}\frac{d\lambda^2}{\lambda^2}\gamma_{\tilde{K}}[\alpha_s(\lambda^2)]\right]. \tag{34}$$

To derive leading and next-to-leading logarithms, we substitute the one- and two-loop coefficients of $\gamma_{\tilde{K}}$ and the running coupling, and the one-loop coefficient of $\tilde{G}$. This gives

$$\Gamma(1,(\hat{\mathbf{k}}^2)^2/\hat{\mathbf{k}}^2,\alpha_s(\mu))=\Gamma(1,1,\alpha_s(\hat{\mathbf{k}}^2))\exp[-I(\hat{\mathbf{k}})], \tag{35}$$

where

$$I(\hat{\mathbf{k}})=\frac{C_A}{-2\beta_1}\left\{2C_1\ln 2C_1+2\rho\ln 2\rho-2(\rho+C_1)\ln(\rho+C_1)-\left[\frac{\beta_2}{\beta_1^2}\right][\tfrac{1}{2}\ln^2 2C_1e+\tfrac{1}{2}\ln^2 2\rho e-\ln^2(\rho+C_1)e]\right\}-\frac{4\gamma_{\tilde{K}}^{(2)}}{\beta_1^2}\{\ln 2C_1+\ln 2\rho-2\ln(\rho+C_1)\}. \tag{36}$$

In the above, we define

$$\rho=\ln\frac{k^-}{\Lambda},\quad C_1=\ln\frac{x\xi m_b}{\Lambda}, \tag{37}$$

and we take

$$\beta_1=-(11-\tfrac{2}{3}N_f),\quad \beta_2=-(102-\tfrac{38}{3}N_f). \tag{38}$$

In view of our decoupling assumption, we have $N_f=4$.

We now observe that for $\hat{\mathbf{k}}^2$ large, $\Gamma(1,1,\alpha_s(\hat{\mathbf{k}}^2))$ may be approximated by unity, and we derive the generalization, within the above-mentioned approximations, of Eq. (22), for the resumed double-logarithmic corrections to Fig. 4(a):

$$H^{(a)}_{\mu;\beta'\alpha;\beta\alpha'}=\frac{g^2(\gamma_\nu)_{\beta'\alpha}\{\gamma_\nu\xi\not{p}'\gamma_\mu\}_{\beta\alpha'}N_cC_2(F)}{(2\xi p'^+k^-)(-x\xi m_b^2)}e^{-I(k^-,Q^2)}, \tag{39}$$

with $I(k^-,Q^2)$ given by Eq. (36). Similarly the leading double-logarithmic corrections to Fig. 4(b) are

$$H^{(b)}_{\mu;\beta'\alpha;\beta\alpha'}=\frac{-(g^2)(\gamma_\nu)_{\beta'\alpha}\{\gamma_\mu(\not{p}'+\not{k})\gamma_\nu\}_{\beta\alpha'}N_cC_2(F)}{(2\xi p'^+k^-)(2p'+k^-)}e^{-I(k^-,Q^2)}. \tag{40}$$

Then, for the complete hard scattering amplitude $H=H^{(a)}+H^{(b)}$, we can write

$$\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}(\gamma_5\not{p}')_{\beta\beta'}H_{\mu;\beta'\alpha;\beta\alpha'}(p',k^-,p,\xi)\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}[\gamma_5(\not{p}+m_b)]_{\alpha'\alpha}=h_\mu(\xi,x)S(k^-,Q^2,x), \tag{41}$$

with

$$h_\mu(\xi,x)=\frac{4g^2C_2(F)}{x^2\xi m_b}(x-1)p'_\mu+\frac{4g^2}{x\xi m_b}C_2(F)p_\mu\ , \tag{42}$$

$$S(k^-,Q^2)=\frac{e^{-I(k^-,Q^2)}}{k^-}\ . \tag{43}$$

In the above, we denote xm_b by Q. When substituted into Eq. (19) this gives an expression for the amplitude in terms of the B and pion wave functions:

$$M_\mu(p',p)=\int_0^1 d\xi\int dk^-[\phi_\pi(\xi,Q^2)h_\mu(\xi,x)S(k^-,Q^2)\phi_B(k^-)]\ . \tag{44}$$

We next discuss the evaluation of this integral.

VIII. LEADING BEHAVIOR

To proceed further, we need to make some assumptions on the nature of the B-meson wave function $\phi_B(k^-)$. To this end, two approaches will be employed in this section. In the first, which would be completely justified for asymptotically large values of the heavy-quark mass, it will be argued that one can replace $\phi_B(-b^+)$, Eq. (11), by $(1/2\sqrt{3})f_B$. The short-distance nature of the b^+ integration which allows this will be discussed, and it will be seen that the approach to asymptotia is slow. However, even for realistic values of m_b this method serves to provide a test for our second, model-dependent approach. In this second approach we employ model wave functions $\phi_B(k^-)$ to evaluate Eq. (43) and hence the decay rate. Two such model wave functions are used below. We will begin with a discussion of the first approach, which employs only heavy-quark field theory in combination with the perturbative resummation.

A. Asymptotic behavior

We begin by rewriting Eq. (44) as an integral over b^+, conjugate to k^- as in Eq. (11):

$$M_\mu(p',p)=\int_0^1 d\xi\int\frac{db^+}{2\pi}\phi_\pi(\xi,Q^2)h_\mu(\xi,x)\times\tilde{S}(b^+,Q^2)\tilde{\phi}_B(-b^+)\ , \tag{45}$$

with

$$\tilde{S}(b^+,Q^2)=\int dk^-e^{-ik^-b^+}S(k^-,Q^2)\ . \tag{46}$$

Examination of Fig. 4 shows that at this order (and all higher orders for positive x), the singularities of $S(k^-,Q^2,x)$ In the k^- plane are all in the lower half k^- plane. This implies

$$\tilde{S}(b^+,Q^2)=0\ ,\quad b^+<0\ . \tag{47}$$

To estimate the integral in Eq. (45), we appeal to heavy quark effective field theory, and assume that the configuration space wave function $\tilde{\phi}(-b^+)$, Eq. (11), is essentially constant on scales $b\ll 1/\Lambda$. If this were not the case, the light quark would carry more than a fraction of order Λ/m_B of the heavy meson's momentum. Then in Eq. (45) we may replace $\tilde{\phi}_B(-b^+)$ by $(1/2\sqrt{3})f_B$ [Eq. (14)] and we get

$$M_\mu(p',p)=\frac{1}{2\sqrt{3}}f_B\int_0^1 d\xi\phi_\pi(\xi,Q^2)h_\mu(\xi,x)\times\int\frac{db^+}{2\pi}\tilde{S}(b^+,Q^2)\ . \tag{48}$$

Of course, this replacement is possible only to the extent that $\tilde{S}$ falls off on a scale much smaller than $1/\Lambda$. In fact, this is not generally the case, as we shall see below. It will be true, however, that the Sudakov resummation stabilizes the perturbative contribution, making it unnecessary to introduce an extra infrared cutoff.

From Eqs. (46) and (43) for S, we have

$$\tilde{S}(b^+,Q^2)=\int dk^-e^{-ik^-b^+}\frac{1}{k^-}e^{-I(k^-,Q^2)}\ , \tag{49}$$

with $I(k^-,Q^2)$ given by Eq. (36). We shall be concerned only with the range

$$\Lambda\le k^-\le Q\ , \tag{50}$$

in which the Sudakov resummation makes sense. At $k^-=\Lambda$, $I(k^-,Q^2)$ diverges, and we assume the contribution from smaller k^- is negligible (in any case, it is nonperturbative). Once k^- is larger than Q, on the other hand, we are in the truly ultraviolet region, where higher-order corrections are well behaved and relatively small.

Using the vanishing of $\tilde{S}$ for negative b^+, Eq. (47), and Eq. (43), we find

$$\int_{-\infty}^{\infty}\frac{db^+}{2\pi}\tilde{S}(b^+,Q^2)=\left[\frac{i}{2\pi}\right]\int_0^\infty\frac{dk^-}{k^-}S(k^-,Q^2)=\left[\frac{i}{2\pi}\right]\int_0^\infty\frac{dk^-}{(k^-)^2}e^{-I(k^-,Q^2)}\ . \tag{51}$$

Let us first obtain estimates for $I(k^-,Q^2)$ using only one loop in the running coupling and $\tilde{\gamma}_{\bar{k}}$. Then it is easy to see that

$$\int\frac{dk^-}{k^{-2}}e^{-I(k^-)}$$

has a saddle point at

$$k^-=\Lambda\left[\frac{Q}{\Lambda}\right]^\kappa\ ,\quad \kappa=\frac{1}{2e^{\beta_1/3}-1}\ . \tag{52}$$

For $x\cong 1$, $\Lambda=100$ MeV, and $m_b=5$ GeV, k^- turns out to be 1.2 Λ and κ about 0.03. Thus, the typical mass of the gluon actually increases, and the distance covered by the gluon decreases as a fractional power of the heavy-quark mass. The power κ, however, is far too small for this to be a significant effect at the B mass. Nevertheless, the integral is cut off by the Sudakov factor at $k^-=\Lambda$, so that the remaining, perturbative expression is finite. To derive the asymptotic normalization of the integral over S, it is, however, necessary to include two-loop corrections.

For the full $I(k^-,Q^2)$ given in Eq. (36), the saddle point can be obtained numerically from the solution to

$$1=\frac{C_A}{-\beta_1}\left\{\ln(\rho_0+C_1)-\ln 2\rho_0 + \frac{\beta_2}{\beta_1^2}\left[\frac{1}{2\rho_0}\ln 2\rho_0 e-\frac{1}{\rho_0+C_1}\ln(\rho_0+C_1)e\right]\right\} + \frac{8\gamma_{\tilde{K}}^{(2)}}{\beta_1^2}\left[\frac{1}{2\rho_0}-\frac{1}{\rho_0+C_1}\right], \tag{53}$$

where ρ_0 is the saddle point value of $\rho=\ln(k^-/\Lambda)$ [Eq. (37)], which for $x\sim 1$, gives $k^-\approx(1.4)\Lambda_{\rm QCD}$ at the saddle point. More significantly, the running coupling at this scale is

$$\alpha_s(m_b\Lambda e^{\rho_0})\cong 0.35\ . \tag{54}$$

Thus, even the modest values of k^- which dominate the integrals in Eq. (51) are sufficient to take the gluon substantially off shell. Because this gluon is moving rapidly in the rest frame of the B meson, it acts much like an incident parton in hadron-hadron scattering [11], and its interactions with more than one parton from the B meson are suppressed by a power of its invariant mass, and thus by a power of m_b.

The above considerations again show that Sudakov effects in B decay dynamically produce an off-shell process for large recoil energy, as argued earlier from physical considerations. In fact, in the center-of-mass frame the entire process is short distance, thus justifying in principle our use of two-particle wave functions for the B and the pion. Even though the effect for realistic values of the b-quark mass is not completely satisfying, it is a step in the right direction. Note also that the higher-order corrections tend to increase this effect. In any case, we have been able to identify the asymptotic behavior for large m_Q even though corrections for the realistic case of $m_Q=m_b$ are not small. Alternately, this result describes the decay of a hypothetical "large" B meson, whose radius is much larger than $1/\Lambda$.

It should be pointed out here that heavy-quark effective field theory ideas tell us that k^- is of order Λ, although the precise relation between the two is not specified. Our approach provides this missing information, at least for this process. It is also interesting that the scale $\sim$(few)$\times\Lambda$ seems to emerge in heavy meson systems with one heavy and one light quark. In view of the above, we feel encouraged in our use of perturbative QCD.

With short-distance nature of the b^+ integration in hand, at least asymptotically, we return to the evaluation of $M_\mu(p',p)$, Eq. (48), by a saddle point approximation. For this we need to evaluate the integral

$$G(\rho,Q^2)=\int\frac{dk^-}{k^{-2}}e^{-I(k^-,Q^2)} \tag{55}$$

at the saddle point. Let us define

$$f(\rho_0)\equiv\rho_0+I(\rho_0)\ , \tag{56}$$

where $I(\rho_0)$ denotes $I(k^-,Q^2)$ in Eq. (36) evaluated at the saddle point ρ_0 in the variable $\rho=\ln(k^-/\Lambda)$ [Eq. (37)]. In these terms, we have

$$|f''(\rho^0)|=\frac{C_A}{-\beta_1}\left\{\frac{1}{\rho_0+C_1}-\frac{1}{\rho_0}+\frac{\beta_2}{\beta_1^2}\left[\frac{1}{(\rho_0+C_1)^2}\ln(\rho_0+C_1)-\frac{1}{2\rho_0^2}\ln 2\rho_0\right]\right\}+\frac{8\gamma_{\tilde{K}}^{(2)}}{\beta_1^2}\left[\frac{1}{(\rho_0+C_1)^2}-\frac{1}{2\rho_0^2}\right], \tag{57}$$

and the saddle point approximation is

$$G(\rho_0,Q^2)=\frac{e^{-f(\rho_0)}}{\Lambda}\left[\frac{2\pi}{|f''(\rho_0)|}\right]^{1/2}. \tag{58}$$

In Eq. (48) for M_μ, we now use (58) for the integral over b^+ of $\tilde{S}$, and (42) for h_μ. The resulting expression for the $B\to\pi l\nu$ amplitude Eq. (17) is

$$A(p',p)=\frac{G_F}{\sqrt{2}}V_{ub}\bar{u}_\nu\gamma_\mu(1-\gamma_5)v_l\times\{f_1(x,m_b)p'^\mu+f_2(x,m_b)p^\mu\}\ , \tag{59}$$

where the quantity in curly brackets is the $B-\pi$ form factor:

$$f_i(x,m_b)=g_i(x)\frac{C_2(F)}{m_b}\left\{\left[\frac{4if_B}{\sqrt{3}}\right]\alpha_s(xm_b\Lambda e^{\rho_0})\times\int_0^1\frac{d\xi}{\xi}\phi_\pi(\xi,Q^2)G(\rho_0,Q^2)\right\},$$
$$g_1(x)=-\frac{1-x}{x^2}\ \text{and}\ g_2(x)=\frac{1}{x}\ . \tag{60}$$

We have chosen to evaluate α_s at the typical hard scattering scale of the problem. We should note, however, that the scale of the hard scattering vanishes with ξ (the fraction of the pion's momentum carried by the hard gluon to the d quark). Taking this into account would increase our estimates somewhat, since the running coupling at that scale would also increase. The singularity in the running coupling at $\xi=0$ would be regulated by Sudakov corrections associated with the $\bar{b}$-u vertex, which is on shell at $\xi=0$. We shall not attempt to include such refinements here.

Equation (60) is the basic result of our first approach, accurate up to corrections that decay as inverse powers of the logarithms of the heavy-quark mass when x is fixed. For the b quark, of course, such logarithms are not very large, and corrections are likely to be numerically significant. It is worthwhile, however, to consider the numerical implications of this asymptotic formula, which we shall exhibit in the next section.

B. Wave functions

We next turn to the second approach, i.e., evaluation of Eq. (45) using model wave functions for the B meson. To this end we write this equation as before, in the form

$$M_\mu(p',p)=p'_\mu f_1(x,m_b)+p_\mu f_2(x,m_b)\ , \tag{61}$$

where, in general,

$$f_i(x,m_b)=g_i(x)\frac{C_2(F)}{m_b}\left\{16\pi\int_0^1\frac{d\xi}{\xi}\phi_\pi(\xi,Q^2)\times\int dk^-\frac{e^{-I(k^-)}}{k^-}\alpha_s(xm_bk^-)\phi_B(k^-)\right\}, \tag{62}$$

and where the $g_i(x)$ are given in Eq. (60). As above, we have neglected the effect of ξ on the scale of the running coupling. For $\phi_B(k^-)$, we employ two models, to illustrate the range of possible predictions.

Model I. Here we use the oscillator wave function of Ref. [23]:

$$\phi_B^{(\mathrm{I})}(k^-)=\frac{N}{m_b}\sqrt{x(1-x)}\times\exp\left[-\frac{m^2}{2\omega^2}\left[x-\frac{1}{2}-\frac{m_b^2}{2m^2}\right]^2\right], \tag{63}$$

where x is the fraction of the b-quark momentum, and we identify $x=(1-k^-/m)$. As expected this wave function is peaked near $x\approx1$. Other parameters in this model wave function are $\omega=0.4$ GeV and $m=$ meson mass $=5.28$ GeV. N can be determined from the normalization condition, Eq. (14), to be $N=1.56$ GeV. Again, numerical results are given in the next section.

Model II. Following Ref. [24], we write

$$\phi_B^{(\mathrm{II})}(x,\mathbf{k}_\perp)=\frac{N'}{\left[C+\dfrac{m_b^2}{x}+\dfrac{k_\perp^2}{x(1-x)}\right]^2}\ , \tag{64}$$

and fix N',C by the conditions

$$\frac{N'}{16\pi^2}\int_0^1dx\int dk_\perp^2\frac{1}{\left[C+\dfrac{m_b^2}{x}+\dfrac{k_\perp^2}{x(1-x)}\right]^2}=\frac{1}{2\sqrt{3}}f_B\ ,$$
$$\frac{N'^2}{16\pi^2}\int_0^1dx\int dk_\perp^2\frac{1}{\left[C+\dfrac{m_b^2}{x}+\dfrac{k_\perp^2}{x(1-x)}\right]^4}=\frac{1}{2}\ . \tag{65}$$

The x distribution one then gets is

$$\phi_B^{(\mathrm{II})}(k^-)=\frac{60.2\ \mathrm{GeV}}{m_b\pi^2}\frac{x^2(1-x)}{(25-21.6x)}\ , \tag{66}$$

and once again we substitute $x=1-k^-/m$.

In the subsequent numerical evaluations, the coupling constant $\alpha_s(xm_bk^-)$ in Eq. (62) is frozen at its value dictated by the (sharp) maxima of the integrand in k^-. In both models this happens at $k^-\sim\text{few}\times\Lambda$.

IX. NUMERICAL ESTIMATES AND DISCUSSION

Neglecting the lepton masses, the differential decay rate becomes

$$\frac{d\Gamma}{dx}=|V_{ub}|^2G_F^2\frac{m_b^5x^3}{768\pi^3}|f_1+f_2|^2\equiv|V_{ub}|^2R_\pi(x)\ , \tag{67}$$

where the second relation defines $R_\pi(x)$.

To proceed further, we need a pion wave function. As a test case, we adopt the model of Ref. [25], neglecting evolution in Q^2:

$$\phi_\pi(\xi,Q^2)\simeq5\sqrt{3}f_\pi\xi(1-\xi)(1-2\xi)^2\ . \tag{68}$$

Then, for $B^0\to\pi^-l^+\nu_e$ we find, using our first ("asymptotic") approach,

$$R_\pi^{(\mathrm{asy})}=\frac{25}{972\pi^3}G_F^2f_B^2f_\pi^2m_b^3x\left[2-\frac{1}{x}\right]^2\alpha_s^2|G(\rho_0,Q^2)|^2\ . \tag{69}$$

Here, as mentioned earlier, we have frozen k^- to its value given by the maximum of Eq. (62), in $G(\rho,Q^2)$ and in $\alpha_s(xm_bk^-)$. Throughout we use $\Lambda=100$ MeV, and we take $f_B=160$ MeV, as indicated by lattice simulations [26]. $R_\pi^{(\mathrm{asy})}(x)$ is plotted against x in Fig. 7 (solid line).

For the model wave functions, Eqs. (63) and (64), we have evaluated the expression

$$R_\pi^{(\mathrm{I,II})}=G_F^2\frac{16m_b^3x}{27\pi}\left[2-\frac{1}{x}\right]^2\left|\int_0^1\frac{d\xi}{\xi}\phi_\pi(\xi,Q^2)\int dk^-\frac{e^{-I(k^-)}}{k^-}\alpha_s(xm_bk^-)\phi_B^{(\mathrm{I,II})}(k^-)\right|^2\ . \tag{70}$$

These $R_\pi(x)$'s for the two model-dependent wave functions are also plotted in Fig. 7 (dashed line for model I and dotted line for model II).

The calculation of the decay $B^0\to\rho l\nu$ proceeds in an analogous manner, for the helicity zero ($\lambda=0$) final state vector meson. In perturbative QCD, the decay to these "longitudinal" ρ mesons is the dominant mode. The final state longitudinal ρ is projected out by $(\not p')_{\beta'\alpha'}$ instead of

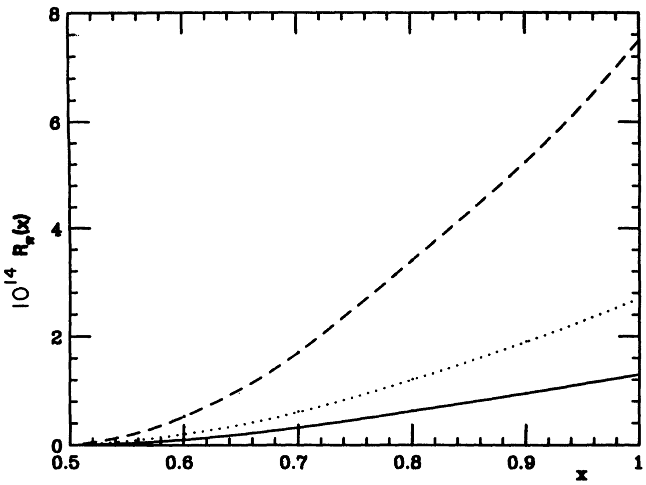

FIG. 7. Plot of $R_\pi(x)$, in units of GeV, with x.

$(\gamma_5 \not{p}')_{\beta'\alpha'}$, replacing f_π by $\sqrt{2}f_\rho$ ($f_\rho = 109$ MeV) and the pion wave function by that of the longitudinal ρ meson (see Ref. [6]). In the end, one arrives at the same formula as before [Eq. (60)] with the above replacements. A $\lambda=0$, ρ-meson wave function may also be found in Ref. [25]:

$$\phi_\rho(\xi,Q^2)=5\sqrt{3}(\sqrt{2}f_\rho)\xi(1-\xi)[\tfrac{1}{4}(1-2\xi)^2+0.15] \ . \quad (71)$$

Then, we find, for $B^0\to\rho^+ l\nu$, approximately,

$$\frac{d\Gamma}{dx}=\frac{1}{36\pi^3}G_F^2|V_{ub}|^2\alpha_s^2 f_B^2 f_\rho^2 m_b^3 x(2-1/x)^2|G(\rho_0,Q^2)|^2$$
$$\equiv|V_{ub}|^2R_\rho(x) \ . \quad (72)$$

In Fig. 8, $R_\rho(x)$ is plotted against x using the first approach only as an illustration of the order of magnitudes of the prediction. It should be noted that the expressions for $d\Gamma/dx$ diverge for small x where the assumption of large hadronic recoil fails. In Table I, results for the partial decay rates with x integrated from x_1 to x_2 are shown for the decays to a pion and a longitudinal ρ meson. The total width of the B^0 meson is $(0.51\pm0.02)\times10^{-9}$ MeV [27,28]. For $B\to\rho^+ l\nu$ and $B\to\pi l\nu$, current experimental upper limits on the branching ratios are, respectively, 1.6×10^{-4} [29] and 3.3×10^{-4} [30].

The rates for other charged B-meson decays may be obtained from the above by isospin symmetry and quark model considerations: for example,

$$\frac{d}{dx}\Gamma(B^+\to\pi^0 l^+\nu)=\frac{1}{2}\frac{d\Gamma}{dx}(B^0\to\pi^- l^+\nu) \quad (73)$$

and so on, with similar results for the ρ meson.

A striking feature of these formulas is the presence of a dynamical zero in the differential rate at $x=\frac{1}{2}$. It is unclear whether our calculation can be trusted at this kinematic point, and even if it is indicative, the next-to-leading corrections may remove this zero. The possibility of a dip in the differential decay rate remains, and this could have important consequences on the extraction of V_{ub} from the experimental data. In particular, not taking this dip into account, if actually present, could result in an underestimation of this Kobayashi-Moskawa (KM) matrix element.

As indicated after Eq. (60), many refinements of our results are possible, including the effect of the variable ξ of the pion wave function in the scale of the running coupling, and the inclusion of Sudakov effects near $\xi=0$, in addition to the transverse momentum of the pion wave function. Also, we must recognize that the Sudakov exponentiation cuts off the integrals for values of k^- that are too small to be reliably estimated by lowest-order calculation in the exponent. Because the cutoff is relatively sharp, however, we do not believe that the latter point has a large effect on the accuracy of our estimates.

Finally, let us comment on the size of our results. If we take, as in Ref. [27], $|V_{ub}|\sim3\times10^{-3}$, then the integrated widths given in Table I are of order 10^{-7} of the total B decay width quoted above. Thus, in the perturbative calculation, these large recoil decays are indeed rare.

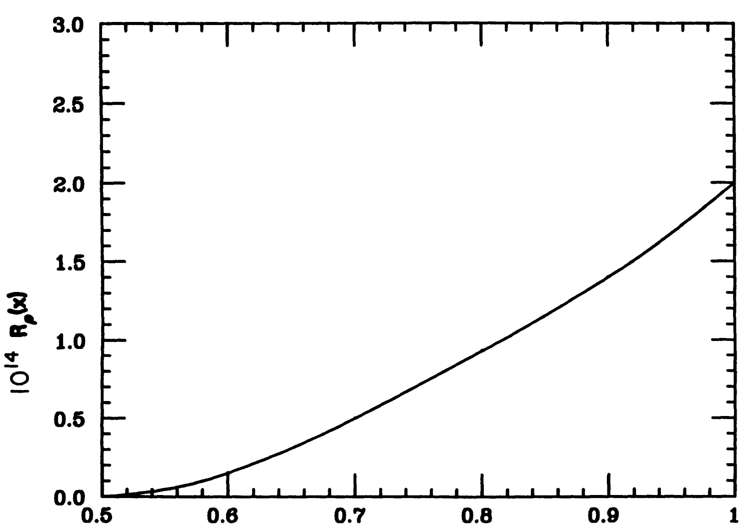

FIG. 8. Plot of $R_\rho(x)$, in units of GeV, with x.

TABLE I. Results for partial decay rates with x integrated from x_1 to x_2.

Process	x_1	x_2	$\Gamma^{\text{partial}}_{\text{theory}}$ (MeV) Method I	$\Gamma^{\text{partial}}_{\text{theory}}$ (MeV) Model I	$\Gamma^{\text{partial}}_{\text{theory}}$ (MeV) Model II
$B^0\to\pi l\nu$	0.6	1.0	$2.6\times10^{-12}\lvert V_{ub}\rvert^2$	$1.4\times10^{-11}\lvert V_{ub}\rvert^2$	$0.5\times10^{-11}\lvert V_{ub}\rvert^2$
	0.8	1.0	$2\times10^{-12}\lvert V_{ub}\rvert^2$	$1\times10^{-11}\lvert V_{ub}\rvert^2$	$0.35\times10^{-11}\lvert V_{ub}\rvert^2$
	0.6	0.9	$1.4\times10^{-12}\lvert V_{ub}\rvert^2$	$0.8\times10^{-11}\lvert V_{ub}\rvert^2$	$0.3\times10^{-11}\lvert V_{ub}\rvert^2$
$B^0\to\rho l\nu$	0.6	1.0	$4\times10^{-12}\lvert V_{ub}\rvert^2$	$2\times10^{-11}\lvert V_{ub}\rvert^2$	$0.75\times10^{-11}\lvert V_{ub}\rvert^2$
	0.8	1.0	$3\times10^{-12}\lvert V_{ub}\rvert^2$	$1.5\times10^{-11}\lvert V_{ub}\rvert^2$	$0.53\times10^{-11}\lvert V_{ub}\rvert^2$
	0.6	0.9	$2\times10^{-12}\lvert V_{ub}\rvert^2$	$1.2\times10^{-11}\lvert V_{ub}\rvert^2$	$0.44\times10^{-11}\lvert V_{ub}\rvert^2$

For comparison, the total semileptonic decay width for $B^0\to Dl\nu$ has a branching ratio of roughly 10^{-2}. Roughly speaking, this difference may be accounted for by the factor

$$\left[\left|\frac{V_{ub}}{V_{cb}}\right|\frac{f_\pi}{m_b}\right]^2\sim10^{-5}\ . \tag{74}$$

The scaling of the width with m_b^{-2} is a direct result of the cutoff at $k^->\Lambda$ in the basic expression for the amplitude, Eq. (44), which was dynamically induced by resumed double logarithmic corrections. It is possible, of course, that contributions that are formally suppressed by an additional power of m_b in perturbation theory may be as large as, or larger than, our results, because of nonpertubative effects. We consider it useful, however, to have in hand the perturbation estimates derived above.

ACKNOWLEDGMENTS

We thank Stan Brodsky, Ira Rothstein, and Mark Wise for helpful discussions. We would also like to thank Will Loinaz for help with numerical work. This work was supported in part by the Department of Energy, by the National Science Foundation under Contract No. PHY 9211367, and by the Texas National Research Foundation.

APPENDIX: THE EIKONAL PHASE AND INFRARED SUBTRACTIONS

In this appendix, we briefly describe the cancellation of infrared enhancements in the definition of $V^{(\text{fin})}$, Eq. (5). (In this appendix, we drop the subscript "b" on V.) We begin with the observation that infrared divergences from the couplings of gluons to the heavy quark are associated with times far in the initial state, and therefore with gluon attachments to the incoming $\bar{b}$-quark line in Fig. 1. Let us consider a propagator along the incoming b-antiquark line, carrying the momenta of j gluons, with which it has interacted following the initial state. In this situation, the power-counting analysis used in quantum electrodynamics [31,14] shows that we may identify all infrared divergences by working in the eikonal approximation for the fermion line. The eikonal approximation may be defined as the replacement

$$\frac{-\not{p}-\sum_{i=1}^{j}\not{k}_i+m}{\left[p+\sum_{i=1}^{j}k_i\right]^2-m^2}\to\frac{-\not{p}+m}{2p\cdot\left[\sum_{i=1}^{j}k_i\right]}\ . \tag{A1}$$

A further simplification comes from anticommuting the factors $-\not{p}+m$ from each quark line out to the external $\bar{b}$ spinor, $\bar{v}(p)$. This results in precisely the combination of eikonal vertices $-2p^\mu\times$(group matrix) and propagators, $2p\cdot(\sum k_i)$, shown in Fig. 2(a). Infrared divergences from the quark line occur when the first j gluons all have soft momenta, with $j\ge1$ [31]. In this momentum region, we may write

$$V(k_i)^{(j)}_{\mu_i,c_i}=\prod_{i=1}^{j}\frac{-2p_{\mu_i}t_{c_i}}{2K_{j,j-1,\dots,1}}\bar{V}\ , \tag{A2}$$

where the product is ordered according to group factors, where $\bar{V}$ involves only hard gluons, and where we define

$$K_{m,n,\dots,l}=p\cdot(k_m+k_n+\cdots+k_l)\ . \tag{A3}$$

At each order in perturbation theory, infrared divergences result from precisely the combination of eikonal lines and vertices that are generated from the eikonal phase, Eq. (4), which at nth order gives

$$U(k_i)^{(j)}_{\mu_i,c_i}=\prod_{i=1}^{j}\frac{-p_{\mu_i}t_{c_i}}{K_{j,j-1,\dots,1}}\ . \tag{A4}$$

Here and below we use the homogeneity of the combination of eikonal vertices and propagators to replace the vector n_0 of Eq. (4) by the antiquark momentum p. Since, by assumption, all b-quark lines in $\bar{V}$, Eq. (A2), are off-shell, we can neglect the soft momenta (the k_i) in $\bar{V}$. V can thus be factored into two separate sums, one for infrared divergent gluons, which is identical to the series generated by $U(A^0)$, and one generated by the hard gluons in $\bar{V}$. As a result, multiplication by $U^{-1}(A^0)$ is guaranteed to cancel the infrared divergences of V.

Although it is not strictly necessary, it may be helpful to show explicitly how higher orders cancel in the combination $UU^{-1}=1$, since the same combinatorics cancels infrared divergences in VU^{-1}. Let us consider order n in the expansion of UU^{-1}, in which the n gluon fields have been contracted with other fields in a diagram, and are hence represented by gluon propagators of momentum k_i^μ. For simplicity, we shall not consider self-energies on the eikonal lines. We now sum over partitions of the n gluons between U and U^{-1}, keeping their relative order fixed: $m=1,\ldots,n$, with m the number of gluons from U^{-1} and $n-m$ the number from U. In U^{-1}, the order of momentum flow is reversed,

$$U(k_i)^{-1(m)}_{\mu_i,c_i}=\prod_{i=1}^{m}\frac{p_{\mu_i}t_{c_i}}{-K_{i,i+1,\ldots,m}}\ . \tag{A5}$$

The complete nth order contribution is illustrated in Fig. 9, and is given by

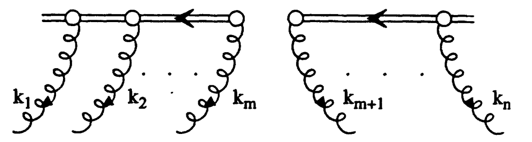

FIG. 9. Diagrammatic representation of the nth order contribution to UU^{-1}.

$$E^{(n)}_{\{\mu_i,c_i\}}=\sum_{m=0}^{n}\left[\prod_{i=1}^{n}p_{\mu_i}t_{c_i}\right]\frac{1}{(-K_{1,2,\ldots,m})\cdots(-K_{m,m-1})(-K_m)}\,\frac{1}{K_{m+1}K_{m+1,m+2}\cdots K_{m+1,m+2,\ldots,n}}$$
$$=0\ . \tag{A6}$$

Note that the product over vectors and color matrices is implicitly ordered, and is independent of the sum over eikonal propagators. The vanishing of the sum follows from the identity [32]

$$2\pi i E^{(n)}_{\{\mu_i,c_i\}}=\int_{-\infty}^{\infty}dx\frac{1}{x+i\epsilon}\prod_{i=1}^{n}\frac{1}{x+K_{i,i-1,\ldots,1}+i\epsilon}=0\ , \tag{A7}$$

which is zero as an immediate consequence of Cauchy's theorem.

In a practical sense, the advantage of inserting by UU^{-1} in Eq. (3) to get (6) is simply to make the infrared-finite decay process independent of the momenta of soft gluons that are infrared divergent in perturbation theory, and which correspond to the formation of the B-meson bound state. In this manner, the computation of the hard scattering amplitude H of Eq. (19) becomes unique, including the subtraction on the $\bar{b}$-quark propagator, and it is multiplied uniquely by the eikonalized B-meson wave function.

---

[1] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **29**, 637 (1985); B. Grinstein, N. Isgur, and B. Wise, Phys. Rev. Lett. **56**, 298 (1986); M. Bauer and M. Wirbel, Z. Phys. C **42**, 671 (1989); B. Grinstein, N. Isgur, D. Scorba, and M. B. Wise, Phys. Rev. D **39**, 797 (1989).

[2] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990); H. D. Politzer and M. B. Wise, *ibid.* **206**, 681 (1988); **208**, 504 (1988); M. B. Voloshin and M. A. Shifman, Yad. Fiz. **45**, 463 (1987) [Sov. J. Nucl. Phys. **45**, 511 (1987)]; E. Eichten and B. Hill, Phys. Lett. B **234**, 511 (1990); **243**, 427 (1990); B. Grinstein, Nucl. Phys. **B339**, 253 (1990): A. F. Falk and B. Grinstein, Phys. Lett. B **247**, 406 (1990); **249**, 314 (1990).

[3] G. Altarelli, N. Cabibbo, G. Corbò, L. Maiani, and G. Martinelli, Nucl. Phys. **B208**, 365 (1982); J. L. Rosner, Phys. Rev. D **42**, 3732 (1990).

[4] H. Georgi, Phys. Lett. B **240**, 447 (1990); A. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Nucl. Phys. **B343**, 1 (1990).

[5] A. H. Mueller, Phys. Rev. D **20**, 2037 (1979); J. C. Collins, *ibid.* **22**, 1478 (1980); A. Sen, *ibid.* **24**, 2181 (1981); G. P. Korchemskii and A. V. Radyushkin, Yad. Fiz. **45**, 1466 (1987) [Sov. J. Nucl. Phys. **45**, 910 (1987)].

[6] A. V. Efremov and A. V. Radyushkin, Phys. Lett. **94B**, 245 (1980); G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980); Phys. Lett. **87B**, 359 (1979); Phys. Rev. Lett. **43**, 545 (1979).

[7] A. Szczepaniak, E. M. Henley, and S. J. Brodsky, Phys. Lett. B **243**, 287 (1990); C. E. Carlson and J. Milana, *ibid.* **301**, 234 (1993).

[8] Yu L. Dokshitzer, D. I. Dyakonov, and S. I. Troyan, Phys. Lett. **78B**, 290 (1978); G. Parisi and R. Petronzio, Nucl. Phys. **B154**, 427 (1979); J. C. Collins and D. E. Soper, *ibid.* **B193**, 381 (1981); G. Altarelli, R. K. Ellis, M. Greco, and G. Martinelli, *ibid.* **B246**, 12 (1984); J. C. Collins, D. E. Soper, and G. Sterman, *ibid.* **B250**, 199 (1985); E. Levin, *ibid.* **B262**, 1 (1985); P. B. Arnold and R. P. Kauffman, *ibid.* **B349**, 381 (1991).

[9] R. Fulton *et al.*, Phys. Rev. Lett. **64**, 16 (1990); H. Albrecht *et al.*, Phys. Lett. B **255**, 297 (1991).

[10] C. Ramirez, J. Donoghue, and G. Bundman, Phys. Rev. D **41**, 1496 (1990); L. Wolfenstein, Phys. Lett. B **291**, 177 (1992).

[11] J. C. Collins, D. E. Soper, and G. Sterman, in *Perturbative Quantum Chromodynamics,* edited by A. H. Mueller (World Scientific, Singapore, 1989); S. J. Brodsky and G. P. Lepage, in *ibid.*

[12] Rosner [3]; M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **34**, 103 (1987); M. Tanimoto, K. Goda, and K. Senba, Phys. Rev. D **42**, 3741 (1990); H. D. Politzer and M. B. Wise, Phys. Lett. B **257**, 399 (1991).

[13] S. J. Brodsky and P. Hoyer, Phys. Lett. B **298**, 165 (1993); M. L. Good and R. Walker, Phys. Rev. **120**, 1855 (1960); E. L. Feinberg and I. Ya. Pomerancuk, Suppl. Nuovo Cimento **III**, Ser. X, 652 (1956).

[14] G. Sterman, Phys. Rev. D **17**, 2773 (1978).

[15] K. Symanzik, Commun. Math. Phys. **34**, 7 (1973); T. Appelquist and J. Carrazone, Phys. Rev. D **11**, 2856 (1975).

[16] A. V. Radyushkin, Phys. Lett. B **271**, 218 (1991); G. P. Korchemsky and A. V. Radyushkin, *ibid.* **279**, 359 (1992).

[17] J. C. Collins, in *Perturbative Quantum Chromodynamics* [11].

[18] Muller [5].
[19] J. Kodaira and L. Trentedue, Phys. Lett. **112B**, 66 (1982).
[20] E. G. Forates, D. A. Ross, and C. T. Sachrajda, Nucl. Phys. **B152**, 493 (1979); A. González-Arroyo and C. Lopez, *ibid.* **B174**, 474 (1980); G. Curci, G. Furmanski, and R. Petronzio, *ibid.* **B175**, 27 (1980).
[21] H.-N. Li and G. Sterman, Nucl. Phys. **B381**, 129 (1992).
[22] A. V. Radyushkin, Acta Phys. Pol. **B15**, 403 (1984); N. Isgur and C. H. Llewellyn Smith, Phys. Rev. Lett. **52**, 1080 (1984); Nucl. Phys. **B317**, 526 (1989).
[23] M. Bauer and M. Wirbel, Z. Phys. C **42**, 671 (1989).
[24] F. Schlumpf, Report No. SLAC-PUB-6335, 1993 (unpublished).
[25] A. R. Zhitnitskii, I. R. Zhitnitskii, and V. L. Chernyak, Yad. Fiz. **38**, 1277 (1983) [Sov. J. Nucl. Phys. **38**, 775 (1983)].
[26] C. Bernard, J. Labrenz, and A. Soni, in *Lattice '92,* Proceedings of the International Symposium, Amsterdam, The Netherlands, edited by J. Smit and P. van Baal [Nucl. Phys. B (Proc.) **30**, 465 (1993)].
[27] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).
[28] H. Kroha, in *Proceedings of the XXVIth International Conference on High Energy Physics,* Dallas, Texas, 1992, edited by J. Sanford, AIP Conf. Proc. No. 272 (AIP, New York, 1993), p. 472.
[29] CLEO Collaboration, A. Bean *et al.*, Phys. Rev. Lett. **70**, 18 (1993).
[30] CLEO Collaboration, B. Ong *et al.*, contributed paper to the XVI International Symposium on Lepton Photon Interactions, 1993 (unpublished).
[31] D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. (N.Y.) **13**, 379 (1961); G. Grammer, Jr. and D. R. Yennie, Phys. Rev. D **8**, 4332 (1973).
[32] S. B. Libby and G. Sterman, Phys. Rev. D **19**, 2468 (1979).