Outcome from spontaneous CP violation for B decays

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In the aspon model solution of the strong CP problem, there is a gauged U(1) symmetry, spontaneously broken by the same vacuum expectation value which breaks CP, whose massive gauge boson provides an additional mechanism of weak CP violation. We calculate the CP asymmetries in B decays for the aspon model and show that they are typically smaller than those predicted from the standard model. A linear relation between the CP asymmetries of different decay processes is obtained.

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The violation of CP symmetry was a surprising experimental discovery made almost 30 years ago in the neutral kaon system [1]. In field theory, one profound question about CP is whether it is explicitly broken in the fundamental Lagrangian or only spontaneously broken by the vacuum. Within the standard model *explicit* CP violation can be accommodated in the flavor mixing of three families by the Kobayashi-Maskawa (KM) mechanism [2]. The experimental information regarding CP violation still comes *only* from the neutral kaon system and is inadequate to determine whether the KM mechanism is the correct underpinning of CP violation. In dedicated B studies, with more than 10^8 samples of B^0 ($\bar{B}^0$) decay, it will be possible [3] to test this assumption stringently by measuring the angles of the well-known unitarity triangle whose sides correspond to the complex terms of the equation

$$V_{ub}^* V_{ud} + V_{tb}^* V_{td} + V_{cb}^* V_{cd} = 0 \ . \tag{1}$$

If CP is *spontaneously* broken, the outcome of these measurements will be different from the standard model. It is the purpose of this Brief Report to illustrate this in the context of the aspon model [4,5].

The standard model contains 19 parameters, of which two, commonly denoted by $\bar{\theta}$ and δ, pertain to CP violation. The value of $\bar{\theta}$, the strong CP violation parameter, is restricted by the neutron electric dipole moment to be $\bar{\theta} \lesssim 2\times 10^{-10}$. The KM mechanism offers no solution of this fine-tuning, which is generally explained by an independent mechanism. In the aspon model which solves the strong CP problem, there is a new gauged U(1) symmetry which is spontaneously broken by the same vacuum expectation value that breaks CP. The resulting massive gauge boson, the aspon, provides an additional mechanism for weak CP violation.

The three angles of the unitarity triangle [conventionally defined as α, β, and γ between the first and second, second and third, and third and first sides in (1), respectively] can be separately measured for the standard model by the time-dependent CP asymmetry [6]

$$a_f(t) = \frac{\Gamma(B^0(t)\to f) - \Gamma(\bar{B}^0(t)\to f)}{\Gamma(B^0(t)\to f) + \Gamma(\bar{B}^0(t)\to f)} \ , \tag{2}$$

where the final state f is a CP eigenstate. We define q,p in B^--$\bar{B}^0$ mixing by the mass eigenstates $B_{1,2}$:

$$|B_{1,2}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle \tag{3}$$

and similarly for $K_{1,2}$ in the kaon system. $A, \bar{A}$ are the decay amplitudes

$$A, \bar{A} = \langle f|H|B^0, \bar{B}^0\rangle \ . \tag{4}$$

Let us consider the specific cases of $f = \pi^+\pi^-, \psi K_S$ from B_d decay and $f = \rho K_S$ from B_s decay. We define $\lambda(f)$ by

$$\lambda(\pi^+\pi^-) = \left|\frac{q}{p}\right|_{B_d} \left|\frac{\bar{A}}{A}\right|_{B_d\to\pi^+\pi^-} \ , \tag{5}$$

$$\lambda(\psi K_S) = \left|\frac{q}{p}\right|_{B_d} \left|\frac{\bar{A}}{A}\right|_{B_d\to\psi K} \left|\frac{q}{p}\right|_{K} \ , \tag{6}$$

$$\lambda(\rho K_S) = \left|\frac{q}{p}\right|_{B_s} \left|\frac{\bar{A}}{A}\right|_{B_s\to\rho K} \left|\frac{q}{p}\right|_{K}^* \tag{7}$$

The complex conjugate appears in (7) because $B_s^0 \to \bar{K}^0$ while $B_d^0 \to K^0$. If to a sufficiently good approximation $|q/p| = 1$ and $|\bar{A}/A| = 1$ as we shall show for the aspon model below, then $\lambda(f)$ is related to the CP asymmetry through the B_1-B_2 mass difference ΔM by

$$a_f(t) = -\text{Im}\lambda(f)\sin(\Delta M t) \ . \tag{8}$$

In the standard model the angles of the unitarity triangle are related to the $\lambda(f)$ by [6] $\sin 2\alpha = \text{Im}\lambda(\pi^+\pi^-)$, $\sin 2\beta = -\text{Im}\lambda(\psi K_S)$, and $\sin 2\gamma = -\text{Im}\lambda(\rho K_S)$. Such relations are no longer valid in the aspon model because $\mathrm{Im}(q/p)_{B_d}$ has a major contribution from aspon exchange and $\mathrm{Im}(q/p)_K$ is dominated by aspon exchange.

To evaluate the CP asymmetries in B decays for the aspon model we need to evaluate the different factors in the $\lambda(f)$ given in Eqs. (5)–(7) above. More precisely we need, from Eq. (8), the imaginary part of the $\lambda(f)$. The aspon model adds new Feynman diagrams involving aspon exchange to those involving W exchange already present in the standard model. Because CP is only spontaneously broken, the W-exchange amplitudes are predominantly real and have very small phases while the aspon exchange has a much smaller magnitude but an unpredicted arbitrary phase. As a result, the $|\mathrm{Im}\lambda(f)|$ appearing in Eq. (8) are of order 0.002 or less, compared to the standard model expectation that $|\mathrm{Im}\lambda(f)|$ be generally of order of, although less than, unity. Thus CP asymmetries in B decays are predicted to be correspondingly smaller than in the standard model. This is our principal result. The remainder of this Brief Report provides more technical details.

First, we examine the three mixing factors (q/p) for the B_d, B_s, and K neutral meson systems. The definition is

$$\left[\frac{q}{p}\right]_{\xi}=\left[\frac{M_{21}(\xi)}{M_{12}(\xi)}\right]^{1/2}, \tag{9}$$

where $M_{12}(\xi)$ and $M_{21}(\xi)$ are the amplitudes for $\bar{\xi}\to\xi$ and $\xi\to\bar{\xi}$, respectively. The quantity $M_{12}(\xi)$ is the sum of one-loop W^+W^- exchange and tree-level aspon exchange amplitudes. For example,

$$M_{12}(B_d)\propto\frac{G_F m_t^2}{32\pi^2}f(z_t)(V_{td}^*V_{tb})^2\frac{g^2}{M_W^2}+(x_3^*x_1)^2\frac{g_A^2}{M_A^2}\ . \tag{10}$$

Similarly,

$$M_{12}(B_s)\propto\frac{G_F m_t^2}{32\pi^2}f(z_t)(V_{ts}^*V_{tb})^2\frac{g^2}{M_W^2}+(x_3^*x_2)^2\frac{g_A^2}{M_A^2}\ , \tag{11}$$

$$M_{12}(K)\propto\frac{G_F m_c^2}{32\pi^2}(V_{cd}^*V_{cs})^2\frac{g^2}{M_W^2}+(x_2^*x_1)^2\frac{g_A^2}{M_A^2}\ , \tag{12}$$

where m_t (m_c) is the mass of the top (charm) quark, $z_t=m_t^2/M_W^2$, and

$$f(z)=\frac{1}{4}\left[1+\frac{9}{1-z}-\frac{6}{(1-z)^2}-\frac{6z^2\ln z}{(1-z)^3}\right]\ . \tag{13}$$

The function $f(z_t)$ takes values between 0.64 and 0.51 for m_t between 120 and 200 GeV. In Eqs. (10)–(12), g is the coupling constant for electroweak SU(2), g_A is the coupling constant for the new gauged U(1), M_A is the mass of the aspon, and the x_i are given in the notation of [5] as $x_i=F_i/M$. The expression for K^0-$\bar{K}^0$ mixing uses the fact that $|V_{cd}^*V_{cs}m_c|\gg|V_{td}^*V_{ts}m_t|$ and hence among possible internal quarks the charm quark dominates the top quark, unlike the case for B^0-$\bar{B}^0$ mixing where the top quark dominates. This is important because, as a consequence, aspon exchange dominates the imaginary part of $M_{12}(K)$. This allowed upper limits on the aspon mass (and the CP-violating scale) to be arrived at in the original aspon papers [4,5]. The aspon mass was made sufficiently small so that the $K^0-\bar{K}^0$ mixing phase compensated the smallness of the phase in the decay amplitude for $s\to u\bar{u}d$ (W exchange). The difference in physics between the neutral B and K systems depends further on the inequalities $|x_3|\ll|x_2|\ll|x_1|$.

In the neutral B system the phase of the amplitude for the decay $b\to u\bar{u}d$ (W exchange) is again smaller in the aspon model than in the standard model in a suitable phase convention. But in this case the phase of the B^0-$\bar{B}^0$ mixing has significant contributions from W^+W^- exchange. As can be gleaned from Eqs. (10)–(12), there are two numerical reasons, both acting in the same direction, why the imaginary part of the one-loop W^+W^- exchange is comparable to that of the tree-level aspon exchange in B^0-$\bar{B}^0$ mixing . First, the W^+W^- exchange amplitude in B^0-$\bar{B}^0$ mixing is enhanced by a relative factor $(m_t/m_c)^2$; second, there is a suppression $|x_3/x_2|^2$ (for B_d) in the aspon exchange amplitude relative to the neutral kaon system.

To evaluate the mixing we need the aspon model values of the V_{ij}. These are given by [5]

$$V_{ij}=(j_L^\dagger)_{il}C_{ln}(k_L)_{nj}+\tilde{x}_i x_j^*\ , \tag{14}$$

where C_{ln} is a real matrix. Up to quadratic order in the $x_i,\tilde{x}_i$, and using $m_d\ll m_s\ll m_b, m_u\ll m_c\ll m_t$, the matrices $j_L^\dagger$ and k_L are given by

$$j_L^\dagger\simeq\begin{bmatrix}\tilde{a}_1 & 0 & 0\\ -\tilde{x}_1^*\tilde{x}_2 & \tilde{a}_2 & 0\\ -\tilde{x}_1^*\tilde{x}_3 & -\tilde{x}_2^*\tilde{x}_3 & \tilde{a}_3\end{bmatrix} \tag{15}$$

and

$$k_L\simeq\begin{bmatrix}a_1 & -x_1x_2^* & -x_1x_3^*\\ 0 & a_2 & -x_2x_3^*\\ 0 & 0 & a_3\end{bmatrix}\ . \tag{16}$$

In these expressions $a_i=1-\frac{1}{2}|x_i|^2$, $\tilde{a}_i=1-\frac{1}{2}|\tilde{x}_i|^2$, and $\tilde{x}_i=C_{ij}x_j$, where the tilde refers to variables in the charge $(+\frac{2}{3})$ sector as opposed to the charge $(-\frac{1}{3})$ sector.

The $\bar{B}^0$ decay amplitudes are given by

$$\bar{A}_{B_d\to\pi^+\pi^-}\propto 3(V_{ud}^*V_{ub})\frac{g^2}{M_W^2}+x_3^*x_1|\tilde{x}_1|^2\frac{g_A^2}{M_A^2}\zeta_{\pi\pi}\ , \tag{17}$$

$$\bar{A}_{B_d\to\psi K}\propto(V_{cs}^*V_{cb})\frac{g^2}{M_W^2}+3x_3^*x_2|\tilde{x}_2|^2\frac{g_A^2}{M_A^2}\zeta_{\psi K}\ , \tag{18}$$

$$\bar{A}_{B_s\to\rho K}\propto(V_{ud}^*V_{ub})\frac{g^2}{M_W^2}+x_3^*x_1|\tilde{x}_1|^2\frac{g_A^2}{M_A^2}\zeta_{\rho K}\ , \tag{19}$$

where the first and second terms originate from W exchange and aspon exchange, respectively. The ζ factors, which are of order 1, account for the fact that the aspon exchange amplitudes involve different strong interaction dynamics from the W-exchange amplitudes. The factors of 3 in Eqs. (17) and (18) arise from color SU(3); the absence of a relative color weighting in Eq. (19) follows from considerations of isospin. In all cases, the real and imaginary parts are dominated by the W-exchange amplitudes and so $|\bar{A}/A|=1$. This confirms the validity of Eq. (8).

We may parametrize $x_j=|x_j|\exp(i\phi_j)$. The phases ϕ_j are unknown. If we focus on the imaginary parts of the quantities in Eqs. (5)–(7) and assume that $g_A=g$, we find

$$\mathrm{Im}\left[\frac{q}{p}\right]_K=\frac{2}{V_{cd}V_{cs}}\{[(V_{ud}V_{cs}-V_{us}V_{cd})^2-V_{cs}^2]|x_1x_2|\sin(\phi_1-\phi_2)+V_{us}V_{ub}|x_1x_3|\sin(\phi_1-\phi_3)$$
$$-V_{ud}V_{ub}|x_2x_3|\sin(\phi_2-\phi_3)\}-\frac{32\pi^2}{G_Fm_c^2}\frac{M_W^2}{M_A^2}\frac{|x_1x_2|^2}{(V_{cd}V_{cs})^2}\sin2(\phi_1-\phi_2)\ , \tag{20}$$

$$\mathrm{Im}\left[\frac{q}{p}\right]_{B_d}=2\frac{V_{td}}{V_{tb}}|x_1x_3|\sin(\phi_1-\phi_3)+2\frac{V_{ts}}{V_{tb}}|x_2x_3|\sin(\phi_2-\phi_3)-\frac{32\pi^2}{G_Fm_t^2f(z_t)}\frac{M_W^2}{M_A^2}\frac{|x_1x_3|^2}{(V_{td}V_{tb})^2}\sin2(\phi_1-\phi_3)\ , \tag{21}$$

$$\mathrm{Im}\left[\frac{q}{p}\right]_{B_s}=-2\frac{V_{td}}{V_{ts}}|x_1x_2|\sin(\phi_1-\phi_2)+2\frac{V_{td}}{V_{tb}}|x_1x_3|\sin(\phi_1-\phi_3)+2\frac{V_{ts}}{V_{tb}}|x_2x_3|\sin(\phi_2-\phi_3)$$
$$-\frac{32\pi^2}{G_Fm_t^2f(z_t)}\frac{M_W^2}{M_A^2}\frac{|x_2x_3|^2}{(V_{ts}V_{tb})^2}\sin2(\phi_2-\phi_3)\ , \tag{22}$$

$$\mathrm{Im}\left[\frac{\bar{A}}{A}\right]_{B_d\to\pi^+\pi^-}=\mathrm{Im}\left[\frac{\bar{A}}{A}\right]_{B_s\to\rho K}$$
$$=2\frac{V_{us}}{V_{ud}}|x_1x_2|\sin(\phi_1-\phi_2)+2\frac{V_{ub}}{V_{ud}}|x_1x_3|\sin(\phi_1-\phi_3)\ , \tag{23}$$

$$\mathrm{Im}\left[\frac{\bar{A}}{A}\right]_{B_d\to\psi K}=\frac{2}{V_{cs}V_{cb}}\{-V_{td}V_{tb}|x_1x_2|\sin(\phi_1-\phi_2)+V_{td}V_{ts}|x_1x_3|\sin(\phi_1-\phi_3)$$
$$+[(V_{cs}V_{tb}-V_{cb}V_{ts})^2-V_{cs}^2]|x_2x_3|\sin(\phi_2-\phi_3)\}\ . \tag{24}$$

In Eqs. (20)–(22), the terms of order $|x_ix_j|$ are from the one-loop W^+W^- exchange and the terms of order $|x_ix_j|^2$ are from tree-level aspon exchange. The order $|x_ix_j|^2$ contributions from the box diagrams have been ignored because in general these are much smaller than the aspon contributions displayed. In Eqs. (23) and (24), the displayed terms are from W exchange only because, as mentioned earlier, contributions from aspon exchange are negligible. To the order of accuracy shown in Eqs. (20)–(24), the V_{ij} can be regarded as real and are given by the corresponding C_{ij}.

Combining Eqs. (20)–(24) with Eqs. (5)–(7) yields the results

$$|\mathrm{Im}\lambda(\pi^+\pi^-)|\lesssim 1\times10^{-5}\ , \tag{25a}$$

$$|\mathrm{Im}\lambda(\psi K_S)|\lesssim 2\times10^{-3}\ , \tag{25b}$$

$$|\mathrm{Im}\lambda(\rho K_S)|\lesssim 2\times10^{-3}\ . \tag{25c}$$

The numerical values were obtained by using the central values of the V_{ij} listed in the most recent Particle Data Group paper [7] and the value 300 GeV for the aspon mass, and the magnitudes of the x_i have been restricted by the upper limit on the strong CP parameter $\bar{\theta}$ to be [5] $|x_1|\lesssim10^{-2}$, $|x_2|\lesssim10^{-3}$, and $|x_3|\lesssim10^{-4}$. The resulting CP asymmetries $a_f(t)$ are in general smaller than those predicted by the standard model. We note that the final states with a kaon typically have larger asymmetries. If CP violation is not detected in the planned B studies as initially envisaged, it may be a signal that spontaneous CP violation is at work.

It is also interesting to observe from Eqs. (5)–(7) that, since the aspon exchange terms in Eqs. (17)–(19) are negligible,

$$\frac{\lambda(\psi K_S)\lambda(\rho K_S)}{\lambda(\pi^+\pi^-)}=\left[\frac{q}{p}\right]_{B_s}\left[\frac{\bar{A}}{A}\right]_{B_d\to\psi K}$$
$$=\left[\frac{q}{p}\right]_{B_s}\left[\frac{\bar{A}}{A}\right]_{B_s\to D_s^+D_s^-}$$
$$=\lambda(D_s^+D_s^-)\ . \tag{26}$$

In the aspon model where the $\lambda(f)$ have unit moduli and $|\mathrm{Im}\lambda(f)|\ll1$, this relation implies a linear relation for the imaginary parts:

$$\mathrm{Im}\lambda(\psi K_S)+\mathrm{Im}\lambda(\rho K_S)-\mathrm{Im}\lambda(\pi^+\pi^-)$$
$$-\mathrm{Im}\lambda(D_s^+D_s^-)=0\ , \tag{27}$$

which provides an additional test of the aspon model.

We may also infer from Eqs. (25) and (27) that

$$|\mathrm{Im}\lambda(D_s^+ D_s^-)| \lesssim 4\times 10^{-3} . \tag{28}$$

In conclusion, our result is that, if the aspon model were correct, *CP* asymmetries in *B* decays would be much smaller than predicted by the standard model and the relation (27) would be satisfied. Although we have considered only final states which are *CP* eigenstates, it is expected that the *CP* violation effects will likewise be small for the *CP* noneigenstates.

If *CP* violation in the neutral *B* meson decays shows up at the level expected from the KM mechanism, it will disfavor spontaneous *CP* violation. On the other hand, if this is *not* observed, spontaneous *CP* violation, as exemplified by the aspon model, will become a viable alternative to the KM mechanism. To verify then the aspon model would require the ability to measure very small *CP* asymmetries in the *B* system as well as detection of the aspon in a hadron collider [8].

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