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Two-loop renormalization group equations for soft supersymmetry-breaking scalar interactions: Supergraph method

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We obtain the one- and two-loop renormalization group equations for soft SUSY-breaking scalar interactions in a general, semisimple SUSY gauge model, by using the supergraph method. We find that the method simplifies the calculation significantly because of the nonrenormalization theorem and also because of the property that the relevant divergences are derived by simple algebra from those in the exact SUSY case. A disagreement with the existing result is found in $\beta^{(2)}(m^2)$ and its cause is briefly discussed.

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I. INTRODUCTION

The gauge theory with softly broken supersymmetry (SUSY) has been widely studied as one of the plausible extensions of the standard model. The model contains many new couplings, the soft SUSY-breaking interactions, which are arbitrary in the low-energy effective theory. When the model is embedded in some unified theories, such as the grand unified models and the supergravity models, the soft SUSY-breaking couplings are expressed in terms of a few parameters at the very high unification scale. To obtain experimental predictions, we should extrapolate the values of these couplings to the weak scale by using the renormalization group equations.

The one-loop renormalization group equations for soft SUSY-breaking interactions have been given in [1] for general cases. However, the two-loop contributions to the equations have not been known for a long time, although they can in principle be derived from the general β functions in Refs. [2,3]. Very recently, the two-loop β functions for the gaugino masses [4,5] and the scale interactions [6] were obtained. In this paper, we present an alternative method for calculating the β functions, the supergraph method, in case of soft SUSY-breaking scalar interactions.

The supergraph method [7,8], which is a very powerful tool for studying exact SUSY models, is also applicable to softly broken SUSY models by using the "spurion" external field [9,10]. This method has many nice features: manifest SUSY and divergence cancellation, reduction of number of graphs and Lorentz indices, and the nonrenormalization theorem [11]. In most previous works [1,4–6], however, the renormalization group equations for soft SUSY-breaking interactions have been calculated in the component field method in the Wess-Zumino gauge. Although Ref. [12] has given a supergraph calculation of the one-loop scalar interactions, unfortunately, their specific method works only for very simple models.

We find that by using the supergraph method, the calculation of the β functions for the soft SUSY-breaking scalar interactions becomes much simpler than that in the component field method, due to the nonrenormalization theorem. If the gauge group has no U(1) factor, we only need the divergent parts of two- and one-point effective vertex functions of chiral supermultiplets with the spurion insertions. Moreover, these vertex functions can be obtained by simple algebra from those in the exact SUSY case, at least to the two-loop order.

The paper is organized as follows. In Sec. II, we review the superfield expression of the gauge model with softly broken SUSY. A problem for the renormalization of soft SUSY-breaking terms is discussed. In Sec. III, we show that the renormalization of soft SUSY-breaking scalar interactions can be obtained from the divergent parts of two-point functions of chiral supermultiplets in the exact SUSY case, if the gauge group has no U(1) factor. In Sec. IV, we check that our method gives the correct one-loop β functions for soft SUSY-breaking scalar interactions. In Sec. V, we calculate the two-loop β functions for these scalar interactions and compare them with the results in Ref. [6] which are obtained by the component field method. A discrepancy is found in the β function for the scalar masses, and its possible cause is discussed. Finally, Sec. VI gives our conclusions.

II. SOFTLY BROKEN SUSY MODEL IN THE SUPERFIELD FORMALISM

In this section, we review the superfield expression of the Lagrangian for general gauge model with softly broken SUSY. We then discuss a problem in the renormalization of soft SUSY-breaking terms in the supergraph method and show that it is solved by a suitable redefinition of superfields.

We first write down the Lagrangian of the general gauge model with softly broken SUSY in the superfield formalism by using the spurion external field $\eta \equiv \theta^2$. Our notation and conventions for the superfield formalism are given in the Appendix. We consider a model with a semisimple gauge group $G=\prod_A G_A$, where G_A's are simple subgroups. In this paper, we assume that there is no U(1) factor in G, which is crucial for later discussion. The model contains chiral supermultiplets Φ_i in the representations $R_i{}^A$ for the subgroup G_A and vector supermultiplets V^A. Here we use the following abbreviations: the index A represents the subgroups G_A and their generators T^A, and i represents the irreducible gauge multiplets and their components.

The Lagrangian is written as [13,9]

$$\mathcal{L}=\mathcal{L}_{\rm SUSY}+\mathcal{L}_{\rm soft}+\mathcal{L}_{\rm GF}+\mathcal{L}_{\rm FP}\ , \tag{2.1}$$

where the SUSY part is

$$\begin{aligned}\mathcal{L}_{\rm SUSY}=&\int d^4\theta\bar{\Phi}^i(e^{2gV})_i^j\Phi_j+\tfrac{1}{4}\int d^2\theta W^{A\alpha}W_\alpha^A\\&+\tfrac{1}{4}\int d^2\bar\theta\,\bar W_{\dot\alpha}^A\bar W^{A\dot\alpha}\\&+\int d^2\theta(\tfrac{1}{6}\lambda^{ijk}\Phi_i\Phi_j\Phi_k+\tfrac{1}{2}M^{ij}\Phi_i\Phi_j+L^i\Phi_i)\\&+\text{H.c.}\ ,\end{aligned}$$

$$W_\alpha^A=\frac{1}{8g_A}\bar D^2[e^{-2gV}D_\alpha e^{2gV}]^A\ , \tag{2.2}$$

$$gV=g_AV^AT^A\ ,$$

and the soft SUSY-breaking part is[1]

$$\begin{aligned}\mathcal{L}_{\rm soft}=&-\int d^2\theta\eta\left[\frac{1}{6}A^{ijk}\Phi_i\Phi_j\Phi_k+\frac{1}{2}B^{ij}\Phi_i\Phi_j+C^i\Phi_i\right.\\&\left.+\frac{m_A}{2}W^{A\alpha}W_\alpha^A\right]+\text{H.c.}\\&-\int d^4\theta\bar\eta\eta\bar\Phi^i(m^2)_i^j(e^{2gV})_j^k\Phi_k\ .\end{aligned} \tag{2.3}$$

An appropriate trace for group generators is understood in all the Lagrangians in this paper. $\mathcal{L}_{\rm soft}$ contains the scalar masses m^2, the gaugino masses m_A, and the scalar interactions A,B,C. The factor e^{2gV} in (2.3) is necessary to make $\mathcal{L}_{\rm soft}$ invariant under the supergauge transformation. If G has no U(1) factor, the Fayet-Iliopoulos term $\int d^4\theta V$ is not generated. The explicit forms of the gauge-fixing term $\mathcal{L}_{\rm GF}$ and Faddeev-Popov ghost term $\mathcal{L}_{\rm FP}$ are given in Sec. III.

We encounter one problem in the renormalization of (2.3). The Lagrangian (2.3) does not contain all possible divergent terms involving $\eta,\bar\eta$. In fact, loop correction produces the following types of divergences [9,12], in addition to those in (2.3):

$$\int d^4\theta[\eta\bar\Phi\Phi,\bar\eta\Phi,\bar\eta\eta\Phi,\bar\eta\eta D^\alpha W_\alpha]\ . \tag{2.4}$$

The last term in (2.4) can appear only if G has a U(1) factor.

In the component field formalism, these terms can be expressed as linear combinations of the usual terms in (2.3) by eliminating auxiliary components. But in the superfield formalism, this procedure means the substitution of the equation of motion into (2.4) and is not easy to justify. So, to obtain manifestly finite effective action in the superfield formalism, we should include the first three terms of (2.4) to the original Lagrangian:

$$\begin{aligned}\mathcal{L}=&\mathcal{L}_{\rm SUSY}+\mathcal{L}'_{\rm soft}+\mathcal{L}_{\rm GF}+\mathcal{L}_{\rm FP}\ ,\\ \mathcal{L}'_{\rm soft}=&-\int d^2\theta\,\eta\left[\frac{1}{6}\hat A^{ijk}\Phi_i\Phi_j\Phi_k+\frac{1}{2}\hat B^{ij}\Phi_i\Phi_j+\hat C^i\Phi_i+\frac{m_A}{2}W^{A\alpha}W_\alpha^A\right]+\text{H.c.}\\&+\int d^4\theta[-\bar\eta\eta\bar\Phi^i(\hat m^2)_i^j(e^{2gV})_j^k\Phi_k+\bar\Phi^i(\eta\kappa_i^j+\bar\eta\kappa_i^{*j})(e^{2gV})_j^k\Phi_k+\bar\eta\eta(\rho_1^i\Phi_i+\rho_{1i}^*\bar\Phi^i)+(\bar\eta\rho_2^i\Phi_i+\eta\rho_{2i}^*\bar\Phi^i)]\ .\end{aligned} \tag{2.5}$$

Here we have introduced new couplings κ, ρ_1 and ρ_2, whose dimensions are 1, 3, and 2, respectively.

Fortunately, the Lagrangian (2.5) can be transformed into the conventional form (2.3) by the following θ-dependent field redefinition:

$$\Phi_i=\Phi'_i-\eta(\kappa_i^j\Phi'_j+\rho_{2i}^*)\ . \tag{2.6}$$

Then the relations between the coupling constants in $\mathcal{L}_{\rm soft}$ and those in $\mathcal{L}'_{\rm soft}$ are

$$A^{ijk}=\hat A^{ijk}+\lambda^{ljk}\kappa_l^i+\lambda^{ilk}\kappa_l^j+\lambda^{ijl}\kappa_l^k\ , \tag{2.7}$$

$$B^{ij}=\hat B^{ij}+M^{lj}\kappa_l^i+M^{il}\kappa_l^j+\lambda^{ijl}\rho_{2l}^*\ , \tag{2.8}$$

$$C^i=\hat C^i+L^j\kappa_j^i+M^{il}\rho_{2l}^*+\rho_2^l\kappa_l^i-\rho_1^i\ , \tag{2.9}$$

$$(m^2)_i^j=(\hat m^2)_i^j+\kappa_i^{*k}\kappa_k^j\ . \tag{2.10}$$

In (2.9), we have used the identity

$$\int d^4\theta\bar\eta\eta\Phi=\int d^2\theta\eta\Phi\ . \tag{2.11}$$

Obviously, other coupling constants in (2.5) are not affected by the redefinition (2.6).

The renormalization of the interactions (2.3) can now be done as follows. First, we renormalize the Lagrangian (2.5) under two conditions: that all the couplings in (2.5) are independent of each other and that the renormalized κ, ρ_1, and ρ_2 are set to 0. Then the bare couplings $(\hat A,\hat B,\hat C,\hat m^2,\kappa,\rho_1,\rho_2,\ldots)$ in (2.5) are expressed in terms of the renormalized couplings $(A,B,C,m^2,\ldots)$ in (2.3). Second, we obtain the bare couplings $(A,B,C,m^2,\ldots)$ in (2.3) by using the relations (2.7)–(2.10).

If the gauge group has a U(1) factor, we further need an additional term

$$\rho_3^A\int d^4\theta\bar\eta\eta D^\alpha W_\alpha^A=\frac{\rho_3^A}{4}\int d^4\theta(\bar D^2\bar\eta)(D^2\eta)V^A\ , \tag{2.12}$$

in the Lagrangian (2.5). By the field redefinition $V_A=V'_A-\bar\eta\eta\rho_3^A$, we can absorb (2.12) into m^2 as

$$\Delta(m^2)_i^j=2g_A(T^A)_i^j\rho_3^A\ . \tag{2.13}$$

[1]The usual definitions of A,B,C are our A^{ijk}/λ^{ijk}, B^{ij}/M^{ij}, C^i/L^i, respectively.

The appearance of the ρ_3 term complicates the following discussions, and we will discuss its consequences elsewhere.

III. EVALUATION OF DIVERGENT PARTS

In this section, we consider the calculation of the divergent supergraphs which are relevant to the renormalization of soft SUSY-breaking scalar interactions. We show that to the two-loop order the relevant divergences involving a spurion η are derived by simple algebra from those in the exact SUSY case. We follow the method given in Ref. [8].

By the nonrenormalization theorem [11,8] and discussions in the last section, it is sufficient for our study to calculate the divergent parts of vertex functions $\langle \bar{\Phi}\Phi \rangle$ and $\langle \Phi \rangle$ with and without η insertions.

The divergent parts of the vertex functions $\int d^4\theta \bar{\Phi}^i T_i^{(\eta)j} \Phi_j$ and $\int d^4\theta J^{(\eta)i}\Phi_i$ in the softly broken SUSY model are expanded in η as

$$T_i^{(\eta)j} = (T + T^{(1)}\eta + T^{(1)\dagger}\bar{\eta} + T^{(2)}\bar{\eta}\eta)_i^j , \tag{3.1}$$

and

$$J^{(\eta)i} = J^{(1)i}\bar{\eta} + J^{(2)i}\bar{\eta}\eta , \tag{3.2}$$

respectively. T_i^j in (3.1) is the two-point function in the exact SUSY case. By power counting, the factors $D\eta$ and $\bar{D}\,\bar{\eta}$ do not appear in these divergent parts. The renormalization of the coupling constants in the Lagrangian (2.3) are then expressed as

$$\begin{aligned} A^{ijk}(\text{bare})\mu^{-\epsilon} = {} & A^{ijk} + \tfrac{1}{2}A^{i'jk}T_{i'}^{i} + \tfrac{1}{2}A^{ij'k}T_{j'}^{j} + \tfrac{1}{2}A^{ijk'}T_{k'}^{k} \\ & - \lambda^{i'jk}T_{i'}^{(1)i} - \lambda^{ij'k}T_{j'}^{(1)j} \\ & - \lambda^{ijk'}T_{k'}^{(1)k} + O(1/\epsilon^2) , \end{aligned} \tag{3.3}$$

$$\begin{aligned} B^{ij}(\text{bare}) = {} & B^{ij} + \tfrac{1}{2}B^{i'j}T_{i'}^{i} + \tfrac{1}{2}B^{ij'}T_{j'}^{j} - M^{i'j}T_{i'}^{(1)i} \\ & - M^{ij'}T_{j'}^{(1)j} - \lambda^{ijk}J_k^{(1)*} + O(1/\epsilon^2) , \end{aligned} \tag{3.4}$$

$$\begin{aligned} C^{i}(\text{bare})\mu^{\epsilon} = {} & C^{i} + \tfrac{1}{2}C^{i'}T_{i'}^{i} - L^{i'}T_{i'}^{(1)i} - M^{ij}J_j^{(1)*} \\ & + J^{(2)i} + O(1/\epsilon^2) , \end{aligned} \tag{3.5}$$

$$\begin{aligned} (m^2)_i^j(\text{bare}) = {} & (m^2)_i^j + \tfrac{1}{2}(m^2)_{i'}^{j}T_i^{i'} + \tfrac{1}{2}(m^2)_i^{j'}T_{j'}^{j} \\ & + T_i^{(2)j} + O(1/\epsilon^2) , \end{aligned} \tag{3.6}$$

where μ is the renormalization scale and loop integrations are done in $D=4-2\epsilon$ dimension. The above relations are obtained from relations (2.7)–(2.10) by noting

$$\kappa_i^j(\text{bare}) = -T_i^{(1)j} + O(1/\epsilon^2) , \tag{3.7}$$

$$\rho_1^i(\text{bare}) = -J^{(2)i} + O(1/\epsilon^2) , \tag{3.8}$$

$$\rho_2^i(\text{bare}) = -J^{(1)i} + O(1/\epsilon^2) , \tag{3.9}$$

in the renormalization condition $(\kappa,\rho_1,\rho_2)(\text{renorm})=0$ in Sec. II. The equations (3.3)–(3.6), which are similar to the renormalization of the superpotential,

$$\begin{aligned} \lambda^{ijk}(\text{bare})\mu^{-\epsilon} = {} & \lambda^{ijk} + \tfrac{1}{2}\lambda^{i'jk}T_{i'}^{i} + \tfrac{1}{2}\lambda^{ij'k}T_{j'}^{j} \\ & + \tfrac{1}{2}\lambda^{ijk'}T_{k'}^{k}, \quad \text{etc.} , \end{aligned} \tag{3.10}$$

are remarkable consequences of the supergraph method.

We first consider the calculation of the two-point function $T^{(\eta)}$. This function is obtained by inserting up to two η operators in (2.3) to propagators and vertices in the one particle irreducible (1PI) supergraphs that contribute to the renormalization of the $\int d^4\theta\bar{\Phi}\Phi$ operator.

We can commutate all η's and D operators at any step in our calculation since the commutator $[D,\eta]=D\eta$ does not contribute to the final results (3.1), apart from the subdivergences for vector self-interactions (see the last paragraph of this section). So the η insertions to vertices becomes almost trivial: the θ algebra and momentum integration are completely the same as those without the η insertions. The insertions can be done after all θ integrations in the supergraph have been reduced to a single integration $\int d^4\theta$.

The $m^2\bar{\eta}\eta$ operator insertions to chiral supermultiplet propagators can be treated in the same manner if we use the following propagator with a $m^2\bar{\eta}\eta$ insertion,

$$\langle \Phi_i(\theta_1)\bar{\Phi}^j(\theta_2)\rangle = \frac{i}{p^2}\delta^4(\theta_1-\theta_2)[\delta_i^j + (m^2)_i^j\bar{\eta}\eta] + (D\eta) , \tag{3.11}$$

instead of the exact and more involved one [10], since the relevant part of (3.11) is proportional to the exact SUSY propagator. Note that we need not specify whether $\eta=\theta_1^2$ or θ_2^2 in (3.11).

The gaugino mass insertions to vector supermultiplet propagators can also be done in the same manner after the following special care has been taken. In the exact SUSY case, we usually use the supersymmetric gauge-fixing term [14,8]

$$\mathcal{L}_{\text{GF}} = -\frac{1}{8\xi}\int d^4\theta (\bar{D}^2V^A)(D^2V^A) , \tag{3.12}$$

with the associated Faddeev-Popov ghost term

$$\mathcal{L}_{\text{FP}} = \int d^4\theta (b+\bar{b})\mathcal{L}_{gV}[(c+\bar{c}) + \coth(\mathcal{L}_{gV})(c-\bar{c})] , \tag{3.13}$$

where

$$\mathcal{L}_{gV}X \equiv [gV,X], \quad x\coth x = 1 + x^2/3 - x^4/45 + \cdots . \tag{3.14}$$

Here c and b are the Faddeev-Popov ghost and antighost, respectively, which are Grassmann-odd chiral superfields in the adjoint gauge representations. With the insertions of the gaugino mass term in (2.3), the propagator for vector supermultiplet is [10]

$$\begin{aligned} \langle V^A(\theta_1)V^B(\theta_2)\rangle = {} & -\frac{i}{2p^2}(1 + m_A\eta + m_A^*\bar{\eta} + 2|m_A|^2\bar{\eta}\eta)\frac{D^\alpha\bar{D}^2D_\alpha}{-8p^2}\delta^4(\theta_1-\theta_2)\delta^{AB} \\ & - \frac{i\xi}{2p^2}\frac{D^2\bar{D}^2 + \bar{D}^2D^2}{16p^2}\delta^4(\theta_1-\theta_2)\delta^{AB} + (D\eta) . \end{aligned} \tag{3.15}$$

Since the divergent vertex functions (3.1) and (3.2) are independent of the gauge fixing parameter ξ, we can choose two special values for ξ in (3.15): $\xi=0$ or $\xi=1+m_A\eta+m_A^*\overline{\eta}+2|m_A|^2\overline{\eta}\eta$. In both cases, the propagator becomes $(1+m_A\eta+m_A^*\overline{\eta}+2|m_A|^2\overline{\eta}\eta)$ times the one in the exact SUSY case. So the insertion of the gaugino mass term can be done as simply as the other η insertions.

Therefore, the relevant supergraphs for $T^{(\eta)}$ are obtained from those for T in the exact SUSY case with only dimensionless couplings, by the following rules A: (1) replace the Φ^3 interaction vertex λ^{pqr} by $\lambda^{pqr}-A^{pqr}\eta$; (2) replace the $\overline{\Phi}V^n\Phi(n=1,2,\dots)$ gauge interaction vertex $(T^n)_k^l$ by $(T^n)_k^q(\delta_q^l-(m^2)_q^l\overline{\eta}\eta)$; (3) replace the factor δ_k^l associated with an internal $\langle\Phi_k\overline{\Phi}^l\rangle$ line by $\delta_k^l+(m^2)_k^l\overline{\eta}\eta$; (4) multiply an internal $\langle V^AV^A\rangle$ line by $(1+m_A\eta+m_A^*\overline{\eta}+2|m_A|^2\overline{\eta}\eta)$; and (5) multiply the vector self-coupling $(V^A)^n(n=3,4,\dots)$ by $(1-m_A\eta-m_A^*\overline{\eta})$. Moreover, by a graph-theoretical argument, we can see that $T_i^{(\eta)j}$ is obtained from the final form of T_i^j by the following rules B: (1) replace λ^{pqr} by $\lambda^{pqr}-A^{pqr}\eta$; (2) replace gauge coupling g_A^2 by $g_A^2(1+m_A\eta+m_A^*\overline{\eta}+2|m_A|^2\overline{\eta}\eta)$; (3) insert $[\delta_p^{p'}+(m^2)_p^{p'}\overline{\eta}\eta]$ between contracted indices p and p' in λ and λ^*, respectively: $\lambda^{pqr}\lambda^*_{pq'r'}\to\lambda^{pqr}\lambda^*_{pq'r'}\;+\lambda^{pqr}(m^2)_p^{p'}\lambda^*_{p'q'r'}\overline{\eta}\eta$; and (4) for terms proportional to δ_i^j, multiply by $[1-(m^2)_i^j\overline{\eta}\eta]$.

Next we consider the tadpole function $J^{(\eta)}$ in (3.2). By power counting and chirality conservation, the relevant supergraphs should contain one and only one $\overline{\Phi}^2$ or $\overline{\Phi}^2\overline{\eta}$ vertex. Therefore, all the supergraphs for $J^{(\eta)i}$ correspond one by one to those for $T_j^{(\eta)i}$ by replacing $\overline{\Phi}^2\overline{\Phi}_j$ vertex in the latter by $\overline{\Phi}^2$. As a consequence, $J^{(\eta)i}$ is obtained from the $\overline{\eta}$-dependent part of $T_j^{(\eta)i}$ by replacing λ^*_{jkl} and A^*_{jkl} factors in $T_j^{(\eta)i}$ by M^*_{kl} and B^*_{kl}, respectively.

To summarize, the divergent contributions to two-point and tadpole Φ functions involving η insertions, (3.1) and (3.2), are obtained from those of the two-point functions in the exact SUSY model by simple algebra B, apart from subdivergences for vector self-interactions.

The renormalization group equation for the coupling constant x_k is obtained from the relation between bare and renormalized coupling constants as

$$\beta(x_k)=\frac{dx_k}{d\ln\mu}+\epsilon r_k x_k=\left[r_l x_l\frac{\partial}{\partial x_l}-r_k\right]a_k^{(1)}(x)\,, \tag{3.16}$$

where

$$x_k(\text{bare})\mu^{-r_k\epsilon}=x_k+\sum_{n=1}^{\infty}\frac{a_k^{(n)}(x)}{\epsilon^n}\,, \tag{3.17}$$

and

$$r_k=\begin{cases}1 & \text{for } g,\ \lambda,\ A\,,\\ 0 & \text{for } M,\ B,\ m^2,\ m_A\,,\\ -1 & \text{for } L, C\,.\end{cases} \tag{3.18}$$

It is easily seen that the renormalization group equations for A, B, C, and m^2 are obtained by multiplying the coefficients of ϵ^{-1} on the right-hand sides of (3.3)–(3.6) by 2 for the one-loop contribution, or by 4 for the two-loop contribution.

The rules A for the η insertion do not apply to the vertices with only vector superfields, such as the gaugino mass term in (2.3) and the ρ_3 term in (2.12) since these vertices contain D operators in the divergent parts. There might appear problems in the calculation of $T^{(\eta)}$ if these vector vertices appear in the supergraphs as subdivergences. For example, in the two-loop $T^{(\eta)}$, a one-loop subdivergence of the two-point vector vertex appears. Nevertheless, we have checked that the rules A, when applied to the one-loop VV propagator, give a correct counterterm for the 1PI gaugino mass vertex, which is zero [1,8]. So the rules B are valid at least to the two-loop order. Further study on the vector vertex functions will be reported elsewhere.

IV. ONE-LOOP RENORMALIZATION GROUP EQUATIONS

In this section, we calculate the one-loop β functions for soft SUSY-breaking scalar couplings, using the supergraph method. Our results agree with the existing results [1] which have been calculated in the component field method, but the calculation is much simpler.

The one-loop contribution to the two-point function T_i^j in the exact SUSY case is [15]

$$T_i^j=\frac{1}{(4\pi)^2\epsilon}\left[\frac{1}{2}\lambda^*_{ikl}\lambda^{jkl}-2g_A^2C_A(\Phi_i)\delta_i^j\right]\,, \tag{4.1}$$

where we adopt the notation

$$C_A(\Phi_i)I=R_i{}^AR_i{}^A\,. \tag{4.2}$$

Following rules B, the divergent vertex functions with η are derived from (4.1) as follows:

$$T_i^{(1)j}=\frac{1}{(4\pi)^2\epsilon}\left[-\frac{1}{2}\lambda^*_{ikl}A^{jkl}-2g_A^2C_A(\Phi_i)m_A\delta_i^j\right]\,, \tag{4.3}$$

$$T_i^{(2)j}=\frac{1}{(4\pi)^2\epsilon}\left[\lambda^*_{ikl}(m^2)_{l'}^l\lambda^{jkl'}+\frac{1}{2}A^*_{ikl}A^{jkl}\right.$$
$$\left.-2g_A^2C_A(\Phi_i)[2|m_A|^2\delta_i^j-(m^2)_i^j]\right]\,, \tag{4.4}$$

$$J^{(1)i}=\frac{1}{(4\pi)^2\epsilon}\left[-\frac{1}{2}\lambda^{ikl}B^*_{kl}\right]\,, \tag{4.5}$$

$$J^{(2)i}=\frac{1}{(4\pi)^2\epsilon}\left[(m^2)_{l'}^l\lambda^{ikl'}M^*_{kl}+\frac{1}{2}A^{ikl}B^*_{kl}\right]\,. \tag{4.6}$$

By substituting these results into (3.3)–(3.6), we immediately obtain the β functions for soft SUSY-breaking scalar interactions:

$$(4\pi)^2\beta^{(1)}(A^{ijk})=\tfrac{1}{2}(\lambda^{iln}\lambda^*_{i'ln}A^{i'jk}+\lambda^{jln}\lambda^*_{j'ln}A^{ij'k}+\lambda^{kln}\lambda^*_{k'ln}A^{ijk'})+2g_A^2[C_A(\Phi_i)+C_A(\Phi_j)+C_A(\Phi_k)](2m_A\lambda^{ijk}-A^{ijk})$$
$$+\lambda^{i'jk}\lambda^*_{i'ln}A^{iln}+\lambda^{ij'k}\lambda^*_{j'ln}A^{jln}+\lambda^{ijk'}\lambda^*_{k'ln}A^{kln}\,, \tag{4.7}$$

$$(4\pi)^2\beta^{(1)}(B^{ij})=\tfrac{1}{2}(\lambda^{iln}\lambda^*_{i'ln}B^{i'j}+\lambda^{jln}\lambda^*_{j'ln}B^{ij'})+4g_A^2C_A(\Phi_i)(2m_AM^{ij}-B^{ij})+M^{i'j}\lambda^*_{i'ln}A^{iln}+M^{ij'}\lambda^*_{j'ln}A^{jln}+\lambda^{ijk}\lambda^*_{kln}B^{ln}\,, \tag{4.8}$$

$$(4\pi)^2\beta^{(1)}(C^i)=\tfrac{1}{2}\lambda^{iln}\lambda^*_{i'ln}C^{i'}+L^{i'}\lambda^*_{i'ln}A^{iln}+M^{ik}\lambda^*_{kln}B^{ln}+2\lambda^{ikl'}(m^2)^l_{l'}M^*_{kl}+A^{ikl}B^*_{kl}\,, \tag{4.9}$$

$$(4\pi)^2\beta^{(1)}[(m^2)^j_i]=\tfrac{1}{2}[\lambda^*_{ikl}\lambda^{i'kl}(m^2)^j_{i'}+\lambda^*_{j'kl}\lambda^{jkl}(m^2)^{j'}_i]+2\lambda^*_{ikl}(m^2)^l_{l'}\lambda^{jkl'}+A^*_{ikl}A^{jkl}-8g_A^2C_A(\Phi_i)|m_A|^2\delta^j_i\,. \tag{4.10}$$

For comparison, we also show the β function for λ^{ijk}:

$$(4\pi)^2\beta^{(1)}(\lambda^{ijk})=\tfrac{1}{2}(\lambda^{iln}\lambda^*_{i'ln}\lambda^{i'jk}+\lambda^{jln}\lambda^*_{j'ln}\lambda^{ij'k}+\lambda^{kln}\lambda^*_{k'ln}\lambda^{ijk'})-2g_A^2[C_A(\Phi_i)+C_A(\Phi_j)+C_A(\Phi_k)]\lambda^{ijk}\,. \tag{4.11}$$

The results (4.7)–(4.11) exactly agree with the existing results [1] if the gauge group has no U(1) factor. If the gauge group contains a U(1) factor, the additional term

$$\Delta\beta^{(1)}[(m^2)^j_i]=(4\pi)^{-2}2g_A^2(T^A)^j_i\mathrm{Tr}(T^Am^2) \tag{4.12}$$

appears. This is the ρ_3 contribution (2.13).

V. TWO-LOOP RENORMALIZATION GROUP EQUATIONS

In this section, we calculate the two-loop β functions for soft SUSY-breaking scalar couplings, using the supergraph method. We also discuss the difference between our results and the recent results in [6] which are obtained in the component field method.

We should first discuss the renormalization scheme. In general, the two-loop renormalization group equations depend on the renormalization scheme, except for those of the gauge couplings. In this paper, we use the $\overline{\rm DR}$ scheme (dimensional reduction [16] with modified minimal subtraction [17]) since it is most convenient in the supergraph calculation and respects SUSY, at least to the two-loop order [3,18].

The two-loop contribution to the two-point function T^j_i in the SUSY case is

$$T^j_i=\frac{-1+\epsilon}{2(4\pi)^4\epsilon^2}[4g_A^2g_B^2C_A(\Phi_i)C_B(\Phi_i)\delta^j_i+2g_A^4C_A(\Phi_i)(T_A(\Phi)-3C_A(V))\delta^j_i$$
$$+g_A^2(-C_A(\Phi_i)+2C_A(\Phi_l))\lambda^*_{ikl}\lambda^{jkl}-\tfrac{1}{2}\lambda^*_{ikl}\lambda^{lst}\lambda^*_{qst}\lambda^{jkq}]\,, \tag{5.1}$$

where we adopt the notation, in addition to (4.2),

$$C_A(V)=C_A(\text{adj.}),\quad T_A(\Phi_i)\delta^{AB}=\mathrm{Tr}R_i{}^AR_i^B,\quad T_A(\Phi)=\sum_i T_A(\Phi_i)\,. \tag{5.2}$$

The result (5.1) agrees with that in Ref. [19].

The divergent contribution to $T^{(1)}$, $T^{(2)}$, $J^{(1)}$, and $J^{(2)}$ are derived from (5.1) by using rules B as

$$T^{(1)j}_i=\frac{-1+\epsilon}{2(4\pi)^4\epsilon^2}[4g_A^2g_B^2C_A(\Phi_i)C_B(\Phi_i)(m_A+m_B)\delta^j_i+4g_A^4C_A(\Phi_i)(T_A(\Phi)-3C_A(V))m_A\delta^j_i$$
$$+g_A^2(-C_A(\Phi_i)+2C_A(\Phi_l))(\lambda^*_{ikl}\lambda^{jkl}m_A-\lambda^*_{ikl}A^{jkl})$$
$$+\tfrac{1}{2}(\lambda^*_{ikl}A^{lst}\lambda^*_{qst}\lambda^{jkq}+\lambda^*_{ikl}\lambda^{lst}\lambda^*_{qst}A^{jkq})]\,, \tag{5.3}$$

$$T^{(2)j}_i=\frac{-1+\epsilon}{2(4\pi)^4\epsilon^2}[4g_A^2g_B^2C_A(\Phi_i)C_B(\Phi_i)((2|m_A|^2+2|m_B|^2+m_Am_B^*+m_A^*m_B)\delta^j_i-(m^2)^j_i)$$
$$+2g_A^4C_A(\Phi_i)(T_A(\Phi)-3C_A(V))(6|m_A|^2\delta^j_i-(m^2)^j_i)$$
$$+g_A^2(-C_A(\Phi_i)+2C_A(\Phi_l))(2\lambda^*_{ikl}\lambda^{jkl}|m_A|^2-A^*_{ikl}\lambda^{jkl}m_A-\lambda^*_{ikl}A^{jkl}m_A^*$$
$$+A^*_{ikl}A^{jkl}+\lambda^*_{ikl}(m^2)^k_{k'}\lambda^{jk'l}+\lambda^*_{ikl}(m^2)^l_{l'}\lambda^{jkl'})$$
$$-\tfrac{1}{2}(A^*_{ikl}A^{lst}\lambda^*_{qst}\lambda^{jkq}+A^*_{ikl}\lambda^{lst}\lambda^*_{qst}A^{jkq}+\lambda^*_{ikl}A^{lst}A^*_{qst}\lambda^{jkq}$$
$$+\lambda^*_{ikl}\lambda^{lst}A^*_{qst}A^{jkq}+\lambda^*_{ikl}(m^2)^k_{k'}\lambda^{lst}\lambda^*_{qst}\lambda^{jk'q}+\lambda^*_{ikl}(m^2)^l_{l'}\lambda^{l'st}\lambda^*_{qst}\lambda^{jkq}$$
$$+\lambda^*_{ikl}\lambda^{lst}\lambda^*_{qst}(m^2)^q_{q'}\lambda^{jkq'}+2\lambda^*_{ikl}\lambda^{lst}(m^2)^{t'}_t\lambda^*_{qst'}\lambda^{jkq})]\,, \tag{5.4}$$

and

$$J^{(1)i}=\frac{-1+\epsilon}{2(4\pi)^4\epsilon^2}[2g_A^2C_A(\Phi_l)(\lambda^{ikl}M^*_{kl}m^*_A-\lambda^{ikl}B^*_{kl})+\tfrac{1}{2}(\lambda^{ikq}A^*_{qst}\lambda^{lst}M^*_{kl}+\lambda^{ikq}\lambda^*_{qst}\lambda^{lst}B^*_{kl})]\,, \tag{5.5}$$

$$J^{(2)i}=\frac{-1+\epsilon}{2(4\pi)^4\epsilon^2}[2g_A^2C_A(\Phi_l)(2\lambda^{ikl}M_{kl}^*|m_A|^2-\lambda^{ikl}B_{kl}^*m_A-A^{ikl}M_{kl}^*m_A^*$$
$$+A^{ikl}B_{kl}^*+\lambda^{ik'l}(m^2)_{k'}^kM_{kl}^*+\lambda^{ikl'}(m^2)_{l'}^lM_{kl}^*)$$
$$-\tfrac{1}{2}(\lambda^{ikq}\lambda_{qst}^*A^{lst}B_{kl}^*+A^{ikq}\lambda_{qst}^*\lambda^{lst}B_{kl}^*+\lambda^{ikq}A_{qst}^*A^{lst}M_{kl}^*+A^{ikq}A_{qst}^*\lambda^{lst}M_{kl}^*$$
$$+\lambda^{ik'q}(m^2)_{k'}^k\lambda_{qst}^*\lambda^{lst}M_{kl}^*+\lambda^{ikq}\lambda_{qst}^*\lambda^{l'st}(m^2)_{l'}^lM_{kl}^*$$
$$+\lambda^{ikq'}(m^2)_{q'}^q\lambda_{qst}^*\lambda^{lst}M_{kl}^*+2\lambda^{ikq}\lambda_{qst'}^*(m^2)_t^{t'}\lambda^{lst}M_{kl}^*)] \ . \tag{5.6}$$

By substituting these results into (3.3)–(3.6), we obtain the two-loop β functions for soft SUSY-breaking scalar interactions in the $\overline{\mathrm{DR}}$ scheme. The results are listed below:

$$(4\pi)^4\beta^{(2)}(A^{ijk})=4g_A^2g_B^2(C_A(\Phi_i)C_B(\Phi_i)+C_A(\Phi_j)C_B(\Phi_j)+C_A(\Phi_k)C_B(\Phi_k))(A^{ijk}-2(m_A+m_B)\lambda^{ijk})$$
$$+2g_A^4(C_A(\Phi_i)+C_A(\Phi_j)+C_A(\Phi_k))(T_A(\Phi)-3C_A(V))(A^{ijk}-4m_A\lambda^{ijk})$$
$$+g_A^2(-C_A(\Phi_i)+2C_A(\Phi_l))\lambda^{iln}\lambda_{i'ln}^*A^{i'jk}+g_A^2(-C_A(\Phi_j)+2C_A(\Phi_l))\lambda^{jln}\lambda_{j'ln}^*A^{ij'k}$$
$$+g_A^2(-C_A(\Phi_k)+2C_A(\Phi_l))\lambda^{kln}\lambda_{k'ln}^*A^{ijk'}$$
$$-\tfrac{1}{2}(\lambda^{inq}\lambda_{qst}^*\lambda^{lst}\lambda_{i'nl}^*A^{i'jk}+\lambda^{jnq}\lambda_{qst}^*\lambda^{lst}\lambda_{j'nl}^*A^{ij'k}+\lambda^{knq}\lambda_{qst}^*\lambda^{lst}\lambda_{k'nl}^*A^{ijk'})$$
$$+2g_A^2(\lambda^{inl}m_A-A^{inl})(C_A(\Phi_i)-2C_A(\Phi_l))\lambda_{i'nl}^*\lambda^{i'jk}$$
$$+2g_A^2(\lambda^{jnl}m_A-A^{jnl})(C_A(\Phi_j)-2C_A(\Phi_l))\lambda_{j'nl}^*\lambda^{ij'k}$$
$$+2g_A^2(\lambda^{knl}m_A-A^{knl})(C_A(\Phi_k)-2C_A(\Phi_l))\lambda_{k'nl}^*\lambda^{ijk'}$$
$$-(\lambda^{inq}\lambda_{qst}^*A^{lst}\lambda_{i'nl}^*+A^{inq}\lambda_{qst}^*\lambda^{lst}\lambda_{i'nl}^*)\lambda^{i'jk}$$
$$-(\lambda^{jnq}\lambda_{qst}^*A^{lst}\lambda_{j'nl}^*+A^{jnq}\lambda_{qst}^*\lambda^{lst}\lambda_{j'nl}^*)\lambda^{ij'k}$$
$$-(\lambda^{knq}\lambda_{qst}^*A^{lst}\lambda_{k'nl}^*+A^{knq}\lambda_{qst}^*\lambda^{lst}\lambda_{k'nl}^*)\lambda^{ijk'} \ , \tag{5.7}$$

$$(4\pi)^4\beta^{(2)}(B^{ij})=8g_A^2g_B^2C_A(\Phi_i)C_B(\Phi_i)(B^{ij}-2(m_A+m_B)M^{ij})+4g_A^4C_A(\Phi_i)(T_A(\Phi)-3C_A(V))(B^{ij}-4m_AM^{ij})$$
$$+g_A^2(-C_A(\Phi_i)+2C_A(\Phi_l))\lambda^{ikl}\lambda_{i'kl}^*B^{i'j}+g_A^2(-C_A(\Phi_j)+2C_A(\Phi_l))\lambda^{jkl}\lambda_{j'kl}^*B^{ij'}$$
$$-\tfrac{1}{2}(\lambda^{inq}\lambda_{qst}^*\lambda^{lst}\lambda_{i'nl}^*B^{i'j}+\lambda^{jnq}\lambda_{qst}^*\lambda^{lst}\lambda_{j'nl}^*B^{ij'})$$
$$+2g_A^2(\lambda^{inl}m_A-A^{inl})(C_A(\Phi_i)-2C_A(\Phi_l))\lambda_{i'nl}^*M^{i'j}$$
$$+2g_A^2(\lambda^{jnl}m_A-A^{jnl})(C_A(\Phi_j)-2C_A(\Phi_l))\lambda_{j'nl}^*M^{ij'}$$
$$-(\lambda^{inq}\lambda_{qst}^*A^{lst}\lambda_{i'nl}^*+A^{inq}\lambda_{qst}^*\lambda^{lst}\lambda_{i'nl}^*)M^{i'j}$$
$$-(\lambda^{jnq}\lambda_{qst}^*A^{lst}\lambda_{j'nl}^*+A^{jnq}\lambda_{qst}^*\lambda^{lst}\lambda_{j'nl}^*)M^{ij'}-4g_A^2C_A(\Phi_l)\lambda_{knl}^*(M^{nl}m_A-B^{nl})\lambda^{ijk}$$
$$-(\lambda_{knq}^*A^{qst}\lambda_{lst}^*M^{nl}+\lambda_{knq}^*\lambda^{qst}\lambda_{lst}^*B^{nl})\lambda^{ijk} \ , \tag{5.8}$$

$$(4\pi)^4\beta^{(2)}(C^i)=2g_A^2C_A(\Phi_l)\lambda^{ikl}\lambda_{i'kl}^*C^{i'}-\tfrac{1}{2}\lambda^{ikq}\lambda_{qst}^*\lambda^{lst}\lambda_{i'kl}^*C^{i'}-4g_A^2C_A(\Phi_l)(\lambda^{ikl}m_A-A^{ikl})\lambda_{i'kl}^*L^{i'}$$
$$-(\lambda^{ikq}\lambda_{qst}^*A^{lst}\lambda_{i'kl}^*+A^{ikq}\lambda_{qst}^*\lambda^{lst}\lambda_{i'kl}^*)L^{i'}-4g_A^2C_A(\Phi_l)\lambda_{jnl}^*(M^{nl}m_A-B^{nl})M^{ij}$$
$$-(\lambda_{jnq}^*A^{qst}\lambda_{lst}^*M^{nl}+\lambda_{jnq}^*\lambda^{qst}\lambda_{lst}^*B^{nl})M^{ij}$$
$$+4g_A^2C_A(\Phi_l)(2\lambda^{ikl}M_{kl}^*|m_A|^2-\lambda^{ikl}B_{kl}^*m_A-A^{ikl}M_{kl}^*m_A^*+A^{ikl}B_{kl}^*+\lambda^{ik'l}(m^2)_{k'}^kM_{kl}^*+\lambda^{ikl'}(m^2)_{l'}^lM_{kl}^*)$$
$$-(\lambda^{ikq}\lambda_{qst}^*A^{lst}B_{kl}^*+A^{ikq}\lambda_{qst}^*\lambda^{lst}B_{kl}^*+\lambda^{ikq}A_{qst}^*A^{lst}M_{kl}^*$$
$$+A^{ikq}A_{qst}^*\lambda^{lst}M_{kl}^*+\lambda^{ik'q}(m^2)_{k'}^k\lambda_{qst}^*\lambda^{lst}M_{kl}^*+\lambda^{ikq}\lambda_{qst}^*\lambda^{l'st}(m^2)_{l'}^lM_{kl}^*$$
$$+\lambda^{ikq'}(m^2)_{q'}^q\lambda_{qst}^*\lambda^{lst}M_{kl}^*+2\lambda^{ikq}\lambda_{qst'}^*(m^2)_t^{t'}\lambda^{lst}M_{kl}^*) \ , \tag{5.9}$$

$$
\begin{aligned}
(4\pi)^4\beta^{(2)}((m^2)^j_i) ={}& 8g_A^2 g_B^2 C_A(\Phi_i) C_B(\Phi_i)(2|m_A|^2+2|m_B|^2+m_A m_B^*+m_A^* m_B)\delta^j_i \\
&+24 g_A^4 C_A(\Phi_i)(T_A(\Phi)-3C_A(V))|m_A|^2\delta^j_i \\
&+(-g_A^2 C_A(\Phi_i)+2g_A^2 C_A(\Phi_l))(\lambda^*_{ikl}\lambda^{i'kl}(m^2)^j_{i'}+\lambda^*_{j'kl}\lambda^{jkl}(m^2)^{j'}_i) \\
&-\tfrac{1}{2}(\lambda^*_{ikl}\lambda^{lst}\lambda^*_{qst}\lambda^{i'kq}(m^2)^j_{i'}+\lambda^*_{j'kl}\lambda^{lst}\lambda^*_{qst}\lambda^{jkq}(m^2)^{j'}_i) \\
&+2g_A^2(-C_A(\Phi_i)+2C_A(\Phi_l))(2\lambda^*_{ikl}\lambda^{jkl}|m_A|^2-A^*_{ikl}\lambda^{jkl}m_A-\lambda^*_{ikl}A^{jkl}m_A^* \\
&\qquad\qquad +A^*_{ikl}A^{jkl}+\lambda^*_{ikl}(m^2)^k_{k'}\lambda^{jk'l}+\lambda^*_{ikl}(m^2)^l_{l'}\lambda^{jkl'}) \\
&-A^*_{ikl}A^{lst}\lambda^*_{qst}\lambda^{jkq}-A^*_{ikl}\lambda^{lst}\lambda^*_{qst}A^{jkq}-\lambda^*_{ikl}A^{lst}A^*_{qst}\lambda^{jkq} \\
&-\lambda^*_{ikl}\lambda^{lst}A^*_{qst}A^{jkq}-\lambda^*_{ikl}(m^2)^k_{k'}\lambda^{lst}\lambda^*_{qst}\lambda^{jk'q}-\lambda^*_{ikl}(m^2)^l_{l'}\lambda^{l'st}\lambda^*_{qst}\lambda^{jkq} \\
&-\lambda^*_{ikl}\lambda^{lst}\lambda^*_{qst}(m^2)^q_{q'}\lambda^{jkq'}-2\lambda^*_{ikl}\lambda^{lst}(m^2)^{t'}_t\lambda^*_{qst'}\lambda^{ikq} \,.
\end{aligned}
\tag{5.10}
$$

For comparison, we show the β function for λ^{ijk}:

$$
\begin{aligned}
(4\pi)^4\beta^{(2)}(\lambda^{ijk}) ={}& 4g_A^2 g_B^2(C_A(\Phi_i)C_B(\Phi_i)+C_A(\Phi_j)C_B(\Phi_j)+C_A(\Phi_k)C_B(\Phi_k))\lambda^{ijk} \\
&+2g_A^4(C_A(\Phi_i)+C_A(\Phi_j)+C_A(\Phi_k))(T_A(\Phi)-3C_A(V))\lambda^{ijk} \\
&+g_A^2(-C_A(\Phi_i)+C_A(\Phi_l))\lambda^{inl}\lambda^*_{i'nl}\lambda^{i'jk}+g_A^2(-C_A(\Phi_j)+2C_A(\Phi_l))\lambda^{jnl}\lambda^*_{j'nl}\lambda^{ij'k} \\
&+g_A^2(-C_A(\Phi_k)+2C_A(\Phi_l))\lambda^{knl}\lambda^*_{k'nl}\lambda^{ijk'}-\tfrac{1}{2}(\lambda^{inq}\lambda^*_{qst}\lambda^{lst}\lambda^*_{i'nl}\lambda^{i'jk}+\lambda^{jnq}\lambda^*_{qst}\lambda^{lst}\lambda^*_{j'nl}\lambda^{ij'k} \\
&\qquad\qquad +\lambda^{knq}\lambda^*_{qst}\lambda^{lst}\lambda^*_{k'nl}\lambda^{ijk'}) \,.
\end{aligned}
\tag{5.11}
$$

As in the last section, the contribution of ρ_3 (2.13) is not included in $\beta(m^2)$.

Finally, we compare our results to the other calculation. Recently, the two-loop β functions for A, B, and m^2 have been obtained in Ref. [6] by using the component field method. They have used the following method: apply the general formulas given in [2] to the softly broken SUSY model in the Wess-Zumino gauge and convert them to the $\overline{\text{DR}}$ scheme by the one-loop finite transformation [5]. Our results agree with theirs for A and B, but do not for m^2. The difference is

$$
\begin{aligned}
(4\pi)^4\beta^{(2)}((m^2)^j_i)[6]-\text{Eq. }(5.10) ={}& -2g_A^2(T^A)^j_i(T^A)^q_p(m^2)^r_q\lambda^*_{rst}\lambda^{pst}+8g_A^2 g_B^2(T^A)^j_i\mathrm{Tr}(T^A C_B(\Phi_r)m^2) \\
&+8g_A^4 C_A(\Phi_i)\mathrm{Tr}(T_A(\Phi_r)m^2)\delta^j_i-8g_A^4 C_A(\Phi_i)C_A(V)|m_A|^2\delta^j_i \,.
\end{aligned}
\tag{5.12}
$$

The first two terms are the contributions from ρ_3 (2.13) that we neglected in the present analysis and they cause no problem. The remaining two terms show, however, a serious discrepancy between our results and those of Ref. [6]. The discrepancy can be clearly shown for the following simple case: G is simple, $\lambda=M=L=0$, $A=B=C=m_A=0$, and $(m^2)^j_i=m_i^2\delta^j_i$. The renormalization group equation for m^2 is then

$$
(4\pi)^4\frac{dm_i^2}{d\ln\mu}=\begin{cases}0 \text{ [Eq. (5.10)]}, \\ 8g_A^4 C_A(\Phi_i)\sum_r T_A(\Phi_r)m_r^2 \text{ (Ref. [6])}\,. \end{cases}
\tag{5.13}
$$

We have found by a direct calculation in the $\overline{\text{DR}}$ scheme that, for this simple case, the component field method does reproduce our result. However, we have so far been unable to trace the origin of the discrepancy, and the definite conclusion is left to further studies.

VI. CONCLUSION

In this paper, we have obtained the renormalization group equations for soft SUSY-breaking scalar couplings to the two-loop order, by using the supergraph method. We have shown that, because of the nonrenormalization theorem, the renormalization group equations for these couplings are obtained by evaluating one- and two-point functions of chiral superfields involving the spurion field η, if the gauge group has no U(1) factor. Under the same condition, we have also shown that the one- and two-loop relevant divergent terms involving η can be obtained by simple algebra from those in the exact SUSY case. Our results basically agree with the existing results which are obtained by the component field method, but the calculation is much simpler once the SUSY two-point function T^j_i in (3.1) has been given. A discrepancy in the two-loop β function for the scalar mass term is found between our results and those of Ref. [6] that are obtained by finite transformation from the non-SUSY β functions of the component field. Although we confirmed in a simple case the validity of our result by an explicit calculation in the component field method, we have not been able to trace the origin of the discrepancy.

Our method of obtaining the β functions for the soft SUSY-breaking terms from the exact SUSY results might be extended to higher-loop orders, if the following problems are solved: the potential inconsistency [20] of the $\overline{\text{DR}}$ scheme and the renormalization of the vector and ghost superfields in softly broken SUSY models.

If the gauge group has a U(1) factor, such as the standard model, we should add the ρ_3 contribution (2.13) to $\beta(m^2)$. The analysis including this contribution will be reported elsewhere.

Note added in proof. After this paper was accepted for publication we received a paper [21] which found a different result for $\beta^{(2)}(m^2)$. Reference [21] points out that for the proper renormalization in the $\overline{\mathrm{DR}}$ scheme, the mass terms $\tilde{m}_A$ for the ϵ scalars (the last 2ϵ components of the vector fields V_μ^A) and their counterterms should be included into $\mathcal{L}_{\text{soft}}$. As a result, Eq. (5.10) is modified as

$$\begin{aligned}(4\pi)^4\beta^{(2)}_{\mathrm{DR}}[(m^2)^j_i]=&(5.10)+(\rho_3-\text{contribution})\\ &+8\delta^j_i g_A^4 C_A(\Phi_i)\{2\,\mathrm{Tr}[T_A(\Phi_r)m^2]-2C_A(V)|m_A|^2+[T_A(\Phi)-3C_A(V)]\tilde{m}_A^2\}\\ &-2g_A^2[C_A(\Phi_i)+2C_A(\Phi_l)]\lambda^*_{ikl}\lambda^{jkl}\tilde{m}_A^2\ .\end{aligned}$$

We have checked that this result is obtained in the superfield formalism after inclusion of the ϵ-scalar masses,

$$\frac{\tilde{m}_A^2}{2}V_\mu^A V_\nu^A\hat{g}^{\mu\nu}=\frac{\tilde{m}_A^2}{2}\int d^4\theta\,\bar{\eta}\eta\frac{1}{16g^2}\bar{\sigma}_\mu^{\dot{\alpha}\alpha}\bar{D}_{\dot{\alpha}}(e^{-2gV}D_\alpha e^{2gV})\bar{\sigma}_\nu^{\dot{\beta}\beta}\bar{D}_{\dot{\beta}}(e^{-2gV}D_\beta e^{2gV})\hat{g}^{\mu\nu}\ .$$

Here $\hat{g}^{\mu\nu}$ is the 2ϵ-dimensional metric tensor. In the superfield formalism the extra contribution to $\beta^{(2)}_{\mathrm{DR}}(m^2)$ above comes from the additional terms to $T^{(2)}$ in (3.1), which are produced by the $\tilde{m}^2$ insertion to supergraphs and by the subdivergence $\bar{\eta}\eta(\bar{D}\bar{\sigma}\,DV)^2$ which is absent in naive calculation. We have found that the corresponding additional terms to $J^{(2)}$ in (3.2) modifies $\beta^{(2)}(C)$ of (5.9) as

$$(4\pi)^4\beta^{(2)}_{\mathrm{DR}}(C^i)=(5.9)-4g_A^2C_A(l)\lambda^{ikl}M^*_{kl}\tilde{m}_A^2\ .$$

Details of the discussion will be presented elsewhere. We thank I. Jack, D.R.T. Jones, S.P. Martin, and M.T. Vaughn for clarifying discussions on this problem.

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APPENDIX

Here we list our notation and conventions for the superfield formalism.

The conventions for 2-component spinors are

$$\begin{aligned}&\epsilon^{\alpha\beta}=\pm1=-\epsilon_{\alpha\beta},\quad \epsilon^{\dot{\alpha}\dot{\beta}}=(\epsilon^{\alpha\beta})^*\ ,\\ &\psi^\alpha=\epsilon^{\alpha\beta}\psi_\beta,\quad \psi_\alpha=\epsilon_{\alpha\beta}\psi^\beta\ ,\\ &\psi\xi=\psi^\alpha\xi_\alpha,\quad \bar{\psi}\bar{\xi}=\bar{\psi}_\alpha\bar{\xi}^\alpha\ ,\\ &\sigma^\mu_{\alpha\dot{\alpha}}=(1,\sigma_{1-3}),\quad \bar{\sigma}^{\mu\dot{\alpha}\alpha}=\epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}}\sigma^\mu_{\beta\dot{\beta}}\ ,\\ &g_{\mu\nu}=(+,-,-,-)\ .\end{aligned}\tag{A1}$$

The notation for superfields is

$$\begin{aligned}&\int d^2\theta\theta^2=1,\quad \int d^4\theta\bar{\theta}^2\theta^2=1\ ,\\ &D_\alpha=\frac{\partial}{\partial\theta^\alpha}-i(\sigma^\mu\bar{\theta})_\alpha\partial_\mu,\quad \bar{D}_{\dot{\alpha}}=-\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}+i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu\ ,\\ &\{D_\alpha,\bar{D}_{\dot{\alpha}}\}=2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\ .\end{aligned}\tag{A2}$$

By using the following decompositions of the superfields in the Wess-Zumino gauge,

$$\begin{aligned}&\Phi(\theta)=\phi+\sqrt{2}\theta\psi+\theta^2F\ ,\\ &V(\theta)=-\theta\sigma^\mu\bar{\theta}V_\mu+\bar{\theta}^2\theta\chi+\theta^2\bar{\theta}\bar{\chi}+\tfrac{1}{2}\theta^2\bar{\theta}^2D'\ ,\end{aligned}\tag{A3}$$

we can show that Eq. (2.1) gives the properly normalized Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{SUSY}}=&-\tfrac{1}{4}(\partial_\mu V_\nu-\partial_\nu V_\mu-ig[V_\mu,V_\nu])^2+i\chi\sigma^\mu(\partial_\mu\bar{\chi}-ig[V_\mu,\bar{\chi}])+\tfrac{1}{2}D'^2+|(\partial_\mu-igV_\mu)\phi|^2\\ &+i\bar{\psi}\bar{\sigma}^\mu(\partial_\mu-igV_\mu)\psi+|F|^2-\sqrt{2}g(\phi^\dagger\chi\psi+\bar{\psi}\bar{\chi}\phi)+g\phi^\dagger D'\phi+\left[\frac{\partial W(\phi)}{\partial\phi_i}F_i-\frac{1}{2}\frac{\partial^2W(\phi)}{\partial\phi_i\partial\phi_j}\psi_i\psi_j\right]+\text{H.c.}\ ,\end{aligned}\tag{A4}$$

where

$$W(\phi)=\tfrac{1}{6}\lambda^{ijk}\phi_i\phi_j\phi_k+\tfrac{1}{2}M^{ij}\phi_i\phi_j+L^i\phi_i\ ,\tag{A5}$$

and

$$\mathcal{L}_{\text{soft}}=-\left[\frac{1}{6}A^{ijk}\phi_i\phi_j\phi_k+\frac{1}{2}B^{ij}\phi_i\phi_j+C^i\phi_i+\frac{m_A}{2}\chi^A\chi^A\right]+\text{H.c.}-\phi^{*i}(m^2)^j_i\phi_j\ .\tag{A6}$$

[1] K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita, Prog. Theor. Phys. **68**, 927 (1992); **70**, 330(E) (1983); **71**, 413 (1984); J. P. Derendinger and C. A. Savoy, Nucl. Phys. **B237**, 307 (1984); B. Gato, J. León, J. Pérez-Mercader, and M. Quirós, *ibid.* **B253**, 285 (1985); N. K. Falck, Z. Phys. C **30**, 247 (1986).

[2] M. E. Machacek and M. T. Vaughn, Nucl. Phys. **B222**, 83 (1983); **B236**, 221 (1984); **B249**, 70 (1985).

[3] I. Jack, Phys. Lett. **147B**, 405 (1984).

[4] Y. Yamada, Phys. Lett. B **316**, 109 (1993); Phys. Rev. Lett. **72**, 25 (1994).

[5] S. P. Martin and M. T. Vaughn, Phys. Lett. B **318**, 331 (1993).

[6] S. P. Martin and M. T. Vaughn, Phys. Rev. D **50**, 2282 (1994).

[7] R. Delbourgo, Nuovo Cimento **25A**, 646 (1975); A. Salam and J. Strathdee, Nucl. Phys. **B86**, 142 (1975); K. Fujikawa and W. Lang, *ibid.* **B88**, 61 (1975).

[8] M. T. Grisaru, M. Roček, and W. Siegel, Nucl. Phys. **B159**, 429 (1979).

[9] L. Girardello and M. T. Grisaru, Nucl. Phys. **B194**, 65 (1982).

[10] J. A. Helayël-Neto, Phys. Lett. **135B**, 78 (1984); F. Feruglio, J. A. Helayël-Neto, and F. Legovini, Nucl. Phys. **B249**, 533 (1985); M. Scholl, Z. Phys. C **28**, 545 (1985).

[11] J. Wess and B. Zumino, Phys. Lett. **49B**, 52 (1974); I. Iliopoulos and B. Zumino, Nucl. Phys. **B76**, 310 (1974).

[12] D. Chang and A. Gangopadhyaya, Phys. Rev. D **33**, 1771 (1986).

[13] S. Ferrara and B. Zumino, Nucl. Phys. **B79**, 413 (1974); A. Salam and J. Strathdee, Phys. Lett. **51B**, 353 (1974); Phys. Rev. D **11**, 1521 (1974).

[14] S. Ferrara and O. Piguet, Nucl. Phys. **B93**, 261 (1975).

[15] R. Barbieri, S. Ferrara, L. Maiani, F. Palambo, and C. A. Savoy, Phys. Lett. **115B**, 212 (1982).

[16] W. Siegel, Phys. Lett. **84B**, 193 (1979); D. M. Capper, D. R. T. Jones, and P. van Nieuwenhuizen, Nucl. Phys. **B167**, 479 (1980).

[17] W. A. Bardeen, A. J. Buras, D. W. Duke, and T. Muta, Phys. Rev. D **18**, 3998 (1978).

[18] P. Howe, A. Parkes, and P. West, Phys. Lett. **147B**, 409 (1984).

[19] P. West, Phys. Lett. **137B**, 371 (1984); D. R. T. Jones and L. Mezincescu, *ibid.* **138B**, 293 (1984).

[20] W. Siegel, Phys. Lett. **94B**, 37 (1980); L. V. Avdeev and A. A. Vladimirov, Nucl. Phys. **B219**, 262 (1983).

[21] I. Jack and D. R. T. Jones, Liverpool Report No. LTH 334, 1994 (unpublished).