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# Waiting for the top quark mass, $K^+ \to \pi^+\nu\bar{\nu}$, $B_s^0$-$\bar{B}_s^0$ mixing, and $CP$ asymmetries in $B$ decays

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Anticipating improved determinations of $m_t$, $|V_{ub}/V_{cb}|$, $B_K$, and $F_B\sqrt{B_B}$ in the next five years we will make an excursion into the future in order to find a possible picture of the unitarity triangle, the quark mixing, and $CP$ violation around the year 2000. We then analyze what impact on this picture the measurements of the four possibly cleanest quantities will have: $B(K^+ \to \pi^+\nu\bar{\nu})$, $x_d/x_s$, $\sin(2\alpha)$, and $\sin(2\beta)$. Our analysis shows very clearly that there is an exciting time ahead of us. In the course of our investigations we extend the analysis of the unitarity triangle beyond leading order in $\lambda$ and we derive several useful analytic formulas for quantities of interest.

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## I. INTRODUCTION

Among the quantities studied in the rich field of rare and $CP$-violating decays [1–7] the branching ratio $B(K^+ \to \pi^+\nu\bar{\nu})$, the ratio $x_d/x_s$ of $B_d^0$-$\bar{B}_d^0$ to $B_s^0$-$\bar{B}_s^0$ mixing, and a class of $CP$ asymmetries in neutral $B$ decays, all being essentially free from any hadronic uncertainties, stand out as ideally suited for the determination of the Cabibbo-Kobayashi-Maskawa (CKM) parameters. Simultaneously they appear to be in the reach of experimentalists in the next five to ten years. The decays $K_L \to \pi^0\nu\bar{\nu}$ and $B \to X_s\nu\bar{\nu}$ are also theoretically very clean but much harder to measure.

$B(K^+ \to \pi^+\nu\bar{\nu})$ and $x_d/x_s$ are probably the best quantities for the determination of the CKM element $V_{td}$ and consequently play important roles in constraining the shape of the unitarity triangle.

The decay $K^+ \to \pi^+\nu\bar{\nu}$ is dominated by short distance loop diagrams involving the heavy top quark and also receives sizable contributions from internal charm quark exchanges. The QCD corrections to this decay have been calculated in the leading logarithmic approximation a long time ago [8–10]. The recent calculation [11] of next-to-leading QCD corrections reduced considerably the theoretical uncertainty due to the choice of the renormalization scales present in the leading order expression. Since the relevant hadronic matrix element of the operator $\bar{s}\gamma_\mu(1-\gamma_5)d\,\bar{\nu}\gamma_\mu(1-\gamma_5)\nu$ can be measured in the leading decay $K^+ \to \pi^0 e^+\nu$, the resulting theoretical expression for $B(K^+ \to \pi^+\nu\bar{\nu})$ is only a function of the CKM parameters, the QCD scale $\Lambda_{\overline{\rm MS}}$, where $\overline{\rm MS}$ denotes the modified minimal subtraction scheme, and the quark masses $m_t$ and $m_c$. Moreover, because of the work of Ref. [11] the scales in $m_t$ and $m_c$ are under control so that the sensitivity of $B(K^+ \to \pi^+\nu\bar{\nu})$ to $m_c$ stressed in Refs. [12,13] is considerably reduced. The long distance contributions to $K^+ \to \pi^+\nu\bar{\nu}$ have been considered in Refs. [14–16] and found to be very small: two to three orders of magnitude smaller than the short distance contribution at the level of the branching ratio.

The top quark mass dependence and the QCD corrections to $B^0$-$\bar{B}^0$ mixing cancel in the ratio $x_d/x_s$, which depends only on the CKM parameters and SU(3)-flavor-breaking effects in the relevant hadronic matrix elements. These SU(3)-breaking effects contain much smaller theoretical uncertainties than the hadronic matrix elements present in $x_d$ and $x_s$ separately. The measurement of $x_d/x_s$ gives then a good determination of the ratio $|V_{td}/V_{ts}|$ and consequently of one side of the unitarity triangle.

The $CP$ asymmetry in the decay $B_d^0 \to \psi K_S$ allows in the standard model a direct measurement of the angle $\beta$ in the unitarity triangle without any theoretical uncertainties [5]. Similarly the decay $B_d^0 \to \pi^+\pi^-$ gives the angle $\alpha$, although in this case strategies involving other channels are necessary in order to remove hadronic uncertainties related to penguin contributions [17–21]. The determination of the angle $\gamma$ from $CP$ asymmetries in neutral $B$ decays is more difficult but not impossible [22].

At present $B(K^+ \to \pi^+\nu\bar{\nu})$, $x_d/x_s$, and the $CP$ asymmetries in neutral $B$ decays given by $\sin(2\phi_i)$ ($\phi_i = \alpha,\beta,\gamma$) can be predicted using the values of $|V_{ub}/V_{cb}|$ and $|V_{cb}|$ extracted from tree level $B$ decays, the analysis

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of the parameter $\epsilon_K$ describing the indirect $CP$ violation in $K \to \pi\pi$ decays, and the analysis of $x_d = (\Delta M)_B/\Gamma_B$ describing the size of $B_d^0$-$\bar{B}_d^0$ mixing.

All these ingredients are subject to theoretical uncertainties related to nonperturbative parameters entering the relevant formulas. Moreover, the last two require the value of $m_t$. Consequently the existing predictions for $B(K^+ \to \pi^+\nu\bar{\nu})$, $x_s$, and $CP$ asymmetries in $B$ decays are rather uncertain.

In this paper we would like to address the following questions.

What accuracy of theoretical predictions for $B(K^+ \to \pi^+\nu\bar{\nu})$, $x_s$, $\sin(2\phi_i)$, and the unitarity triangle could one expect around the year 2000 assuming reasonable improvements for the values of $| V_{cb} |$, $| V_{ub}/V_{cb} |$, $m_t$, and the nonperturbative parameters in question?

What would be the impact of a measurement of $B(K^+ \to \pi^+\nu\bar{\nu})$ on the CKM parameters and in particular on the value of $| V_{td} |$?

What would be the impact of a measurement of $x_s$ ?

What would be the impact of a measurement of $\sin(2\beta)$ and how important would be simultaneous measurements of $\sin(2\alpha)$ and $\sin(2\gamma)$?

How well should one measure $B(K^+ \to \pi^+\nu\bar{\nu})$, $\sin(2\beta)$, $V_{cb}$, $m_t$, and $x_d/x_s$ in order to obtain an acceptable determination of the CKM matrix on the basis of these five quantities alone?

As by-products of these studies, we will update the analysis of $B(K^+ \to \pi^+\nu\bar{\nu})$, $x_s$, $\sin(2\phi_i)$, and of the unitarity triangle in view of theoretical and experimental developments which took place in 1993, we will extend the analysis of the unitarity triangle beyond the leading order in the expansion parameter $\lambda =| V_{us} |$, and we will derive several approximate analytic formulas and bounds which should be useful in following the developments in this field in the 1990s.

Our paper is organized as follows. In Sec. II we extend the Wolfenstein parametrization and the analysis of the unitarity triangle beyond the leading order in $\lambda$ and we give improved formulas for $\sin(2\phi_i)$. In Sec. III we collect the formulas for $\varepsilon_K$, $B^0$-$\bar{B}^0$ mixing, and $B(K^+ \to \pi^+\nu\bar{\nu})$ beyond leading order in $\lambda$. In Sec. IV we list several analytic results which can be derived using Wolfenstein parametrization beyond leading $\lambda$, which to a very good accuracy represent exact numerical analysis. In Sec. V we systematically address the questions posed above. We end the paper with a brief summary and a number of conclusions.

## II. CABIBBO-KOBAYASHI-MASKAWA MATRIX

### A. Standard parametrization

We will dominantly use the standard parametrization [23]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{2.1}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ with $i$ and $j$ being generation labels $(i,j = 1,2,3)$. $c_{ij}$ and $s_{ij}$ can all be chosen to be positive. The measurements of the $CP$ violation in $K$ decays force $\delta$ to be in the range $0 < \delta < \pi$.

The extensive phenomenology of the last few years has shown that $s_{13}$ and $s_{23}$ are small numbers: $\sim 10^{-3}$ and $10^{-2}$, respectively. Consequently to an excellent accuracy $c_{13} = c_{23} = 1$ and the four independent parameters are given as

$$s_{12} =| V_{us} |, \quad s_{13} =| V_{ub} |, \quad s_{23} =| V_{cb} |, \quad \delta, \tag{2.2}$$

with the phase $\delta$ extracted from $CP$-violating transitions or loop processes sensitive to $| V_{td} |$. The latter fact is based on the observation that for $0 \leq \delta \leq \pi$, as required by the analysis of $CP$ violation, there is a one-to-one correspondence between $\delta$ and $|V_{td}|$ given by

$$| V_{td} |= \sqrt{a^2 + b^2 - 2ab\cos\delta}, \qquad a =| V_{cd}V_{cb} |, \qquad b =| V_{ud}V_{ub} | \tag{2.3}$$

### B. Wolfenstein parametrization beyond leading order

We will also use the Wolfenstein parametrization [24]. It is an approximate parametrization of the CKM matrix in which each element is expanded as a power series in the small parameter $\lambda =| V_{us} |= 0.22$:

$$V = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \varrho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \tag{2.4}$$

and the set (2.2) is replaced by

$$\lambda, \qquad A, \qquad \varrho, \qquad \eta\,. \tag{2.5}$$

The Wolfenstein parametrization has several nice features. In particular it offers in conjunction with the unitarity triangle a very transparent geometrical representation of the structure of the CKM matrix and allows one to derive several analytic results to be discussed below. This turns out to be very useful in the phenomenology of rare decays and of $CP$ violation.

When using the Wolfenstein parametrization one should remember that it is an approximation and that in certain situations neglecting $O(\lambda^4)$ terms may give wrong results. The question then arises how to find $O(\lambda^4)$ and higher order terms? The point is that, as in any perturbative expansion, the $O(\lambda^4)$ and higher order terms are not unique. This is the reason why in different papers in the literature different $O(\lambda^4)$ terms can be found. The nonuniqueness of higher order terms in $\lambda$ is not troublesome however. As in any perturbation theory, different choices of expanding in $\lambda$ will result in different numerical values for the parameters in (2.5) being extracted from the data without changing the physics when all terms are summed up. Here it suffices to find an expansion in $\lambda$ which allows for simple relations between the parameters (2.2) and (2.5). This will also restore the unitarity of the CKM matrix which in the Wolfenstein parametrization as given in (2.4) is not satisfied exactly.

To this end we go back to (2.1) and we impose the relations

$$s_{12} = \lambda, \qquad s_{23} = A\lambda^2, \qquad s_{13}e^{-i\delta} = A\lambda^3(\varrho - i\eta)\,, \tag{2.6}$$

to *all orders* in $\lambda$. In view of the comments made above this can certainly be done. It follows that

$$\varrho = \frac{s_{13}}{s_{12}s_{23}}\cos\delta\,, \qquad \eta = \frac{s_{13}}{s_{12}s_{23}}\sin\delta\,. \tag{2.7}$$

We observe that (2.6) and (2.7) represent simply the change of variables from (2.2) to (2.5). Making this change of variables in the standard parametrization (2.1) we find the CKM matrix as a function of $(\lambda, A, \varrho, \eta)$, which satisfies unitarity exactly. We also note that in view of $c_{13} = 1 - O(\lambda^6)$ the relations between $s_{ij}$ and $|V_{ij}|$ in (2.2) are satisfied to high accuracy. The relations in (2.7) have been first used in Ref. [25]. However, our improved treatment of the unitarity triangle presented below goes beyond the analysis of Ref. [25].

The procedure outlined above gives automatically the corrections to the Wolfenstein parametrization in (2.4). Indeed expressing (2.1) in terms of Wolfenstein parameters using (2.6) and then expanding in powers of $\lambda$ we recover the matrix in (2.4) and in addition find explicit corrections of $O(\lambda^4)$ and higher order terms. $V_{ub}$ remains unchanged. The corrections to $V_{us}$ and $V_{cb}$ appear only at $O(\lambda^7)$ and $O(\lambda^8)$, respectively. For many practical purposes the corrections to the real parts can also be neglected. The essential corrections to the imaginary parts are

$$\Delta V_{cd} = -iA^2\lambda^5\eta, \qquad \Delta V_{ts} = -iA\lambda^4\eta\,. \tag{2.8}$$

These two corrections have to be taken into account in the discussion of $CP$ violation. However, the imaginary part of $V_{cs}$ which in our expansion in $\lambda$ appears only at $O(\lambda^6)$ can be fully neglected.

In order to improve the accuracy of the unitarity triangle discussed below we will also include the $O(\lambda^5)$ correction to $V_{td}$ which gives

$$V_{td} = A\lambda^3(1 - \bar{\varrho} - i\bar{\eta}) \tag{2.9}$$

with

$$\bar{\varrho} = \varrho\left(1 - \frac{\lambda^2}{2}\right), \qquad \bar{\eta} = \eta\left(1 - \frac{\lambda^2}{2}\right). \tag{2.10}$$

In order to derive analytic results we need accurate explicit expressions for $\lambda_i = V_{id}V_{is}^*$ where $i = c, t$. We have

$$\mathrm{Im}\lambda_t = -\mathrm{Im}\lambda_c = \eta A^2\lambda^5\,, \tag{2.11}$$

$$\mathrm{Re}\lambda_c = -\lambda\left(1 - \frac{\lambda^2}{2}\right), \tag{2.12}$$

$$\mathrm{Re}\lambda_t = -\left(1 - \frac{\lambda^2}{2}\right)A^2\lambda^5(1 - \bar{\varrho})\,. \tag{2.13}$$

Expressions (2.11) and (2.12) represent to an accuracy of 0.2% the exact formulas obtained using (2.1). The expression (2.13) deviates by at most 2% from the exact formula in the full range of parameters considered. In order to keep the analytic expressions in Secs. III and IV in a transparent form we have dropped a small $O(\lambda^7)$ term in deriving (2.13). After inserting expressions (2.11)–(2.13) in exact formulas for quantities of interest, further expansion in $\lambda$ should not be made.

### C. Unitarity triangle beyond leading order

The unitarity of the CKM matrix provides us with several relations of which

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \tag{2.14}$$

is the most useful one. In the complex plane relation (2.14) can be represented as a triangle, the so-called "unitarity–triangle" (UT). Phenomenologically this triangle is very interesting as it involves simultaneously the elements $V_{ub}$, $V_{cb}$, and $V_{td}$, which are under extensive discussion at present.

In the usual analyses of the unitarity triangle only terms $O(\lambda^3)$ are kept in (2.14) (Refs. [4,5,13,25–27]). It is, however, straightforwd to include the next-to-leading $O(\lambda^5)$ terms. We note first that

$$V_{cd}V_{cb}^* = -A\lambda^3 + O(\lambda^7). \tag{2.15}$$

Thus to an excellent accuracy $V_{cd}V_{cb}^*$ is real with $|V_{cd}V_{cb}^*| = A\lambda^3$. Keeping $O(\lambda^5)$ corrections and rescaling all terms in (2.14) by $A\lambda^3$ we find

$$\frac{1}{A\lambda^3}V_{ud}V_{ub}^* = \bar{\varrho} + i\bar{\eta}, \qquad \frac{1}{A\lambda^3}V_{td}V_{tb}^* = 1 - (\bar{\varrho} + i\bar{\eta}) \tag{2.16}$$

with $\bar{\varrho}$ and $\bar{\eta}$ defined in (2.10). Thus we can represent (2.14) as the unitarity triangle in the complex $(\bar{\varrho}, \bar{\eta})$ plane. This is shown in Fig. 1. The length of the side $CB$

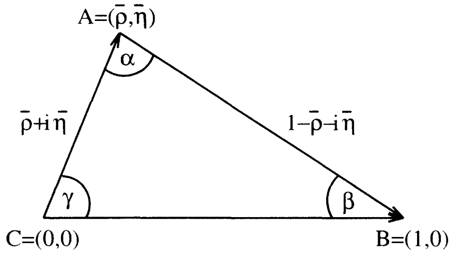


FIG. 1. Unitarity triangle in the complex $(\bar{\varrho},\bar{\eta})$ plane.

which lies on the real axis equals unity when Eq. (2.14) is rescaled by $V_{cd}V_{cb}^*$. We observe that beyond the leading order in $\lambda$ the point A *does not* correspond to $(\varrho,\eta)$ but to $(\bar{\varrho},\bar{\eta})$. Clearly within 3% accuracy $\bar{\varrho}=\varrho$ and $\bar{\eta}=\eta$. Yet in the distant future the accuracy of experimental results and theoretical calculations may improve considerably so that the more accurate formulation given here will be appropriate. For instance, the experiments at the CERN Large Hadron Collider (LHC) should measure $\sin(2\beta)$ to an accuracy of 2–3% (Ref. [28]).

Using simple trigonometry one can calculate $\sin(2\phi_i)$ in terms of $(\bar{\varrho},\bar{\eta})$ with the result

$$\sin(2\alpha)=\frac{2\bar{\eta}(\bar{\eta}^2+\bar{\varrho}^2-\bar{\varrho})}{(\bar{\varrho}^2+\bar{\eta}^2)[(1-\bar{\varrho})^2+\bar{\eta}^2]}, \tag{2.17}$$

$$\sin(2\beta)=\frac{2\bar{\eta}(1-\bar{\varrho})}{(1-\bar{\varrho})^2+\bar{\eta}^2}, \tag{2.18}$$

$$\sin(2\gamma)=\frac{2\bar{\varrho}\bar{\eta}}{\bar{\varrho}^2+\bar{\eta}^2}=\frac{2\varrho\eta}{\varrho^2+\eta^2}. \tag{2.19}$$

The lengths $CA$ and $BA$ in the rescaled triangle of Fig. 1 are denoted by $R_b$ and $R_t$, respectively, and are given by

$$R_b\equiv\frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|}=\sqrt{\bar{\varrho}^2+\bar{\eta}^2}=\left(1-\frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|, \tag{2.20}$$

$$R_t\equiv\frac{|V_{td}V_{tb}^*|}{|V_{cd}V_{cb}^*|}=\sqrt{(1-\bar{\varrho})^2+\bar{\eta}^2}=\frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|. \tag{2.21}$$

The expressions for $R_b$ and $R_t$ given here in terms of $(\bar{\varrho},\bar{\eta})$ are excellent approximations. Clearly $R_b$ and $R_t$ can also be determined by measuring two of the angles $\phi_i$:

$$R_b=\frac{\sin(\beta)}{\sin(\alpha)}=\frac{\sin(\alpha+\gamma)}{\sin(\alpha)}=\frac{\sin(\beta)}{\sin(\gamma+\beta)}, \tag{2.22}$$

$$R_t=\frac{\sin(\gamma)}{\sin(\alpha)}=\frac{\sin(\alpha+\beta)}{\sin(\alpha)}=\frac{\sin(\gamma)}{\sin(\gamma+\beta)}. \tag{2.23}$$

## III. BASIC FORMULAS

### A. Constraint from $\varepsilon_K$

The usual box diagram calculation together with the experimental value for $\varepsilon_K$ specifies a hyperbola in the $(\varrho,\eta)$ plane with $\eta>0$ (Refs. [13,26]). With our new coordinates $(\bar{\varrho},\bar{\eta})$ we get

$$\bar{\eta}\left[(1-\bar{\varrho})A^2\eta_2S(x_t)+P_0(\varepsilon)\right]A^2B_K=0.223. \tag{3.1}$$

Here

$$P_0(\varepsilon)=[\eta_3S(x_c,x_t)-\eta_1x_c]\frac{1}{\lambda^4}, \tag{3.2}$$

$$S(x_c,x_t)=x_c\left[\ln\frac{x_t}{x_c}-\frac{3x_t}{4(1-x_t)}\left(1+\frac{x_t}{1-x_t}\ln x_t\right)\right], \tag{3.3}$$

$$S(x_t)=x_t\left[\frac{1}{4}+\frac{9}{4}\frac{1}{(1-x_t)}-\frac{3}{2}\frac{1}{(1-x_t)^2}\right]+\frac{3}{2}\left[\frac{x_t}{x_t-1}\right]^3\ln x_t, \tag{3.4}$$

where $x_i=m_i^2/M_W^2$. $B_K$ is the renormalization group invariant nonperturbative parameter describing the size of $\langle\bar{K}^0\mid(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}\mid K^0\rangle$ and $\eta_i$ represent QCD corrections to the box diagrams.

In our numerical analysis we will use

$$\eta_1=1.1,\qquad\eta_2=0.57,\qquad \eta_3=0.36\ \text{(leading order)}, \tag{3.5}$$

from Refs. [29], [30], and [31–34], respectively. The values for $B_K$ are specified below.

### B. $B^0$-$\bar{B}^0$ mixing

The experimental knowledge of the $B_d^0$-$\bar{B}_d^0$ mixing described by the parameter $x_d=\Delta M/\Gamma_B$ determines $|V_{td}|$. Using the usual formulas for box diagrams with top quark exchanges one finds

$$x_d=|V_{td}|^2P(B_d^0-\bar{B}_d^0)S(x_t), \tag{3.6}$$

where

$$P(B_d^0-\bar{B}_d^0)=3.89\times10^3\left[\frac{\tau_{B_d}}{1.5\text{ ps}}\right]\left[\frac{F_{B_d}\sqrt{B_{B_d}}}{200\text{ MeV}}\right]^2\left[\frac{\eta_B}{0.55}\right] \tag{3.7}$$

and consequently

$$|V_{td}|=A\lambda^3R_t,\qquad R_t=1.63\frac{R_0}{\sqrt{S(x_t)}}. \tag{3.8}$$

Here

$$R_0 \equiv \sqrt{\frac{x_d}{0.72}}\left[\frac{200\ \mathrm{MeV}}{F_{B_d}\sqrt{B_{B_d}}}\right]\left[\frac{0.038}{\kappa}\right]\sqrt{\frac{0.55}{\eta_B}} \tag{3.9}$$

and

$$\kappa \equiv \mid V_{cb}\mid \left[\frac{\tau_B}{1.5\ \mathrm{ps}}\right]^{0.5}, \tag{3.10}$$

with $\tau_B$ being the $B$-meson lifetime. $\eta_B$ is the QCD factor analogous to $\eta_2$ and calculated to be $\eta_B = 0.55$ (Ref. [30]). $F_{B_d}$ is the $B$-meson decay constant and $B_{B_d}$ denotes a nonperturbative parameter analogous to $B_K$. The values of $x_d$, $F_{B_d}\sqrt{B_{B_d}}$, and $|V_{cb}|$ will be specified below.

It is well known (see for instance [27]) that the accuracy of the determination of $\mid V_{td}\mid$ and $R_t$ can be considerably improved by measuring simultaneously the $B_s^0$-$\bar B_s^0$ mixing described by $x_s$. Defining the ratio

$$R_{ds} = \frac{\tau_{B_d}}{\tau_{B_s}}\frac{m_{B_d}}{m_{B_s}}\left[\frac{F_{B_d}\sqrt{B_{B_d}}}{F_{B_s}\sqrt{B_{B_s}}}\right]^2, \tag{3.11}$$

we find

$$R_t = \frac{1}{\sqrt{R_{ds}}}\sqrt{\frac{x_d}{x_s}}\frac{1}{\lambda}\sqrt{1-\lambda^2(1-2\varrho)}, \tag{3.12}$$

and using (3.8) we find the matrix element $\mid V_{td}\mid$. The last factor in (3.12) describes a small departure of $\mid V_{ts}\mid$ from $\mid V_{cb}\mid$. The $\varrho$ dependence in (3.12) can safely be neglected. In this way $R_t$ does not depend either on $m_t$ or on $\mid V_{cb}\mid$. Since it is easier to calculate $R_{ds}$ than $R_0$, formula (3.12) gives a much more reliable determination of $R_t$ than (3.8), provided $x_s$ has been measured.

### C. The rare decay $K^+\to\pi^+\nu\bar\nu$

The $K^+\to\pi^+\nu\bar\nu$ branching ratio for one single neutrino flavor $l$ ($l = e,\mu,\tau$) is given by

$$B(K^+\to\pi^+\nu\bar\nu) = \frac{\alpha^2 B(K^+\to\pi^0e^+\nu)}{V_{us}^2 2\pi^2\sin^4\theta_W}\mid V_{cs}^*V_{cd}X_{NL}^l + V_{ts}^*V_{td}X(x_t)\mid^2 . \tag{3.13}$$

Summing over three neutrino flavors, using Eqs. (2.11)–(2.13) and setting

$$\alpha = \frac{1}{128},\qquad \sin^2\theta_W = 0.23,\quad B(K^+\to\pi^0e^+\nu) = 4.82\times10^{-2}, \tag{3.14}$$

we obtain

$$B(K^+\to\pi^+\nu\bar\nu) = 4.64\times10^{-11}A^4X^2(x_t)\frac{1}{\sigma}\left[(\sigma\bar\eta)^2 + \frac{2}{3}(\varrho_0^e-\bar\varrho)^2 + \frac{1}{3}(\varrho_0^\tau-\bar\varrho)^2\right] \tag{3.15}$$

with

$$\varrho_0^l = 1 + \frac{P_0^l}{A^2X(x_t)},\qquad P_0^l = \frac{X_{NL}^l}{\lambda^4},\qquad \sigma = \left(\frac{1}{1-\frac{\lambda^2}{2}}\right)^2 . \tag{3.16}$$

The function $X(x_t)$ is given as

$$X(x_t) = \eta_X X_0(x_t) \tag{3.17}$$

$$X_0(x_t) = \frac{x}{8}\left[-\frac{2+x}{1-x} + \frac{3x-6}{(1-x)^2}\ln x\right]\quad \text{with } \eta_X = 0.985 \tag{3.18}$$

where $\eta_X$ is the next-to-leading-order (NLO) correction calculated in Ref. [35] . For determining $P_0^l$ given in Table I we take the NLO results for $X_{NL}^l$ of Ref. [11]. Here $m_c \equiv \overline{m}_c(m_c)$.

The measured value of $B(K^+\to\pi^+\nu\bar\nu)$ determines an ellipse in the $(\bar\varrho,\bar\eta)$ plane centered at $(\varrho_0, 0)$ with

$$\varrho_0 = 1 + \frac{\bar P_0(K^+)}{A^2X(x_t)},\qquad \bar P_0(K^+) = \frac{2}{3}P_0^e + \frac{1}{3}P_0^\tau, \tag{3.19}$$

and having the axes squared

$$\bar\varrho_1^2 = r_0^2,\qquad \bar\eta_1^2 = \left(\frac{r_0}{\sigma}\right)^2, \tag{3.20}$$

where

$$r_0^2 = \frac{1}{A^4X^2(x_t)}\times\left[\frac{\sigma B(K^+\to\pi^+\nu\bar\nu)}{4.64\times10^{-11}} - \frac{2}{9}(P_0^e-P_0^\tau)^2\right]. \tag{3.21}$$

The last term in (3.21) is very small and can safely be

TABLE I. Values of $P_0^l$ for various $\Lambda_{\overline{MS}}$ (GeV) and $m_c$ (GeV).

| | $P_0^e$ | | | $P_0^\tau$ | | |
|---|---|---|---|---|---|---|
| $\Lambda_{\overline{MS}}\backslash m_c$ | 1.25 | 1.30 | 1.35 | 1.25 | 1.30 | 1.35 |
| 0.20 | 0.457 | 0.494 | 0.531 | 0.312 | 0.342 | 0.373 |
| 0.25 | 0.441 | 0.477 | 0.515 | 0.296 | 0.326 | 0.357 |
| 0.30 | 0.425 | 0.461 | 0.498 | 0.280 | 0.309 | 0.340 |
| 0.35 | 0.408 | 0.444 | 0.480 | 0.262 | 0.292 | 0.322 |

neglected.

The ellipse defined by $r_0$, $\varrho_0$, and $\sigma$ given above intersects for the allowed range of parameters with the circle (2.20) . This allows one to determine $\bar{\varrho}$ and $\bar{\eta}$ with

$$\bar{\varrho} = \frac{1}{1-\sigma^2}\left(\varrho_0 - \sqrt{\varrho_0^2 - (1-\sigma^2)(\varrho_0^2 - r_0^2 + \sigma R_b^2)}\right), \quad (3.22)$$

$$\bar{\eta} = \sqrt{R_b^2 - \bar{\varrho}^2}$$

and consequently

$$R_t^2 = 1 + R_b^2 - 2\bar{\varrho}, \quad (3.23)$$

where $\bar{\eta}$ is assumed to be positive.

In the leading order of the Wolfenstein parametrization

$$\sigma \to 1, \qquad \bar{\eta} \to \eta, \qquad \bar{\varrho} \to \varrho, \quad (3.24)$$

and $B(K^+ \to \pi^+\nu\bar{\nu})$ determines a circle in the $(\varrho, \eta)$ plane centered at $(\varrho_0, 0)$ and having the radius $r_0$ of (3.21) with $\sigma = 1$. Formulas (3.22) and (3.23) simplify then to

$$R_t^2 = 1 + R_b^2 + \frac{r_0^2 - R_b^2}{\varrho_0} - \varrho_0, \quad \varrho = \frac{1}{2}\left(\varrho_0 + \frac{R_b^2 - r_0^2}{\varrho_0}\right) \quad (3.25)$$

in accordance with Ref. [11].

### D. $B^0$ decays and superweak models

Although the $CP$ asymmetries in $B^0$ decays in which the final state is a $CP$ eigenstate offer a way to measure the angles of the unitarity triangle, they may in principle fail to distinguish the standard model from superweak models. As discussed by Gérard and Nakada [36] and by Liu and Wolfenstein [37], nonvanishing asymmetries are also expected in superweak scenarios. In order to rule out superweak models one has to measure the asymmetries in two distinct channels and find that they differ from each other. As an example, consider $B^0 \to \psi K_S$ $(CP = -1)$ and $B^0 \to \pi^+\pi^-$ $(CP = 1)$ for which the time integrated asymmetries are

$$A_{CP}(\psi K_S) = -\sin(2\beta)\frac{x_d}{1+x_d^2}, \qquad A_{CP}(\pi^+\pi^-) = -\sin(2\alpha)\frac{x_d}{1+x_d^2}. \quad (3.26)$$

Generally these two asymmetries could differ in the standard model both in sign and magnitude. In a superweak model however these asymmetries differ only by the sign of the $CP$ parity of the final state. Yet as emphasized by Winstein [38] if $\sin 2\beta = -\sin 2\alpha$, it will be impossible to distinguish the standard model result from superweak models. This will happen for any $\bar{\varrho} > 0$ and $\bar{\eta}$ given by [38]

$$\bar{\eta} = (1-\bar{\varrho})\sqrt{\frac{\bar{\varrho}}{2-\bar{\varrho}}} \quad (3.27)$$

as can be easily verified using (2.17) and (2.18). Consequently $(\bar{\varrho}, \bar{\eta})$ must lie sufficiently away from the curve of Eq. (3.27) in order to rule out the superweak scenario on the basis of $B^0$ decays to $CP$ eigenstates. We will investigate in Sec. V whether this is likely to happen in the future experiments.

## IV. ANALYTIC RESULTS

Now, we want to give a list of results following from the formulas above which can be presented in an analytic form. Some of these results appeared previously in the literature.

### A. Lower bounds on $m_t$ and $B_K$ from $\varepsilon_K$

The hyperbola (3.1) intersects the circle given by (2.20) in two points. It is usually stated in the literature that one of these points corresponds to $\bar{\varrho} < 0$ and the other one to $\bar{\varrho} > 0$. For most values of $A$, $B_K$, and $m_t$ this is in fact true. However, with decreasing $A$, $B_K$, and $m_t$, the hyperbola (3.1) moves away from the origin of the $(\bar{\varrho}, \bar{\eta})$ plane and both solutions can appear for $\bar{\varrho} < 0$. For sufficiently low values of these parameters the hyperbola and the circle only touch each other at a small negative value of $\bar{\varrho}$. In this way a lower bound for $m_t$ as a function of $B_K$, $V_{cb}$ and $\mid V_{ub}/V_{cb} \mid$ can be found.

With an accurate approximation for $S(x_t)$,

$$S(x_t) = 0.784 x_t^{0.76}, \quad (4.1)$$

one can derive an analytic lower bound on $m_t$ (Ref. [39]), which to an accuracy of 2% reproduces the exact numerical result. It is given by

$$(m_t)_{\min} = M_W \left[\frac{1}{2A^2}\left(\frac{1}{A^2 B_K R_b} - 1.2\right)\right]^{0.658}. \quad (4.2)$$

A detailed analysis of (4.2) can be found in Ref. [39]. Here we want to stress that once $m_t$ has been deter-

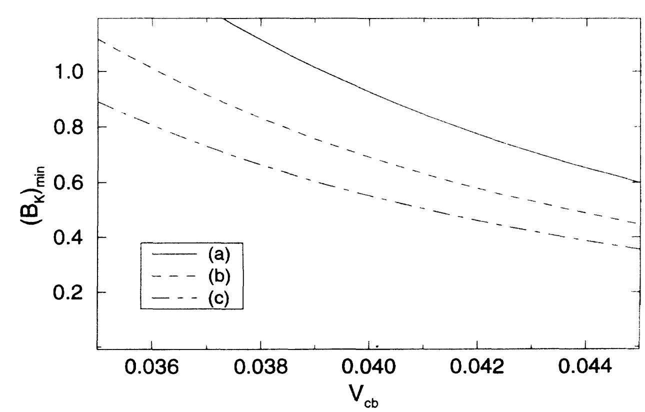


FIG. 2. Lower bound on $B_K$ for $\mid V_{ub}/V_{cb} \mid = 0.06$ (a), $\mid V_{ub}/V_{cb} \mid = 0.08$ (b), $\mid V_{ub}/V_{cb} \mid = 0.10$ (c) from $\varepsilon_K$ and $m_t < 180\,\mathrm{GeV}$.

mined, the same analysis gives the minimal value of $B_K$ consistent with measured $\varepsilon_K$ as a function of $| V_{cb} |$ and $| V_{ub}/V_{cb} |$. We find

$$(B_K)_{\min} = \left[A^2 R_b \left(2x_t^{0.76} A^2 + 1.2\right)\right]^{-1} . \tag{4.3}$$

Choosing $m_t = 180$ GeV we show $(B_K)_{\min}$ as a function of $| V_{cb} |$ for different values of $| V_{ub}/V_{cb} |$ in Fig. 2. For lower values of $m_t$ the bound is stronger. We observe that for $m_t \leq 180$ GeV, $| V_{ub}/V_{cb} | \leq 0.10$, and $| V_{cb} | \leq 0.040$ only values $B_K > 0.55$ are consistent with $\varepsilon_K$ in the framework of the standard model.

### B. Upper bound on sin(2β)

For the present range of $R_b$ the angle $\beta$ is smaller than 45°. This allows one to derive an upper bound on $\sin(2\beta)$, which depends only on $R_b$. As shown in Fig. 3 it is found to be

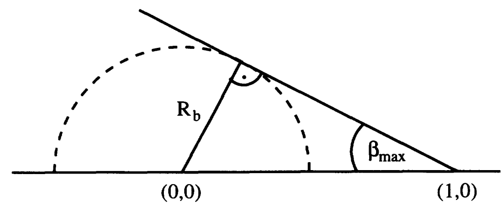


FIG. 3. Determination of $(\sin(2\beta))_{\max}$.

$$(\sin(2\beta))_{\max} = 2R_b\sqrt{1 - R_b^2} . \tag{4.4}$$

This implies

$$(\sin(2\beta))_{\max} = \begin{cases} 0.795 & (\beta_{\max} = 26.3^\circ) , \quad | V_{ub}/V_{cb} | = 0.10 , \\ 0.663 & (\beta_{\max} = 20.8^\circ) , \quad | V_{ub}/V_{cb} | = 0.08 , \\ 0.513 & (\beta_{\max} = 15.4^\circ) , \quad | V_{ub}/V_{cb} | = 0.06 . \end{cases} \tag{4.5}$$

A lower bound on $\sin(2\beta)$ can only be found numerically as it depends on $\bar{\eta}$. The result can be inferred from our numerical analysis in Sec. V.

### C. sin(2β) from $\varepsilon_K$ and $B^0$-$\bar{B}^0$ mixing

Combining (3.1) and (3.8) one can derive an analytic formula for $\sin(2\beta)$. We find

$$\sin(2\beta) = \frac{1}{1.33 A^2 \eta_2 R_0^2} \left[\frac{0.223}{A^2 B_K} - \bar{\eta} P_0(\varepsilon)\right] . \tag{4.6}$$

$P_0(\varepsilon)$ is weakly dependent on $m_t$ and for $150 \leq m_t \leq 180$ GeV one has $P_0(\varepsilon) \approx 0.26 \pm 0.02$. As $\bar{\eta} \leq 0.45$ for $| V_{ub}/V_{cb} | \leq 0.1$ the first term in parentheses is generally by a factor of 2–3 larger than the second term. Since this dominant term is independent of $m_t$, the values for $\sin(2\beta)$ extracted from $\varepsilon_K$ and $B^0$-$\bar{B}^0$ mixing show only a weak dependence on $m_t$, as stressed in particular in Ref. [6].

### D. Ambiguity in $\bar{\varrho}$

It is well known that in the analysis of $\varepsilon_K$ with fixed $| V_{ub}/V_{cb} |$ and $|V_{cb}|$ one gets two solutions for $(\bar{\varrho}, \bar{\eta})$ with $\bar{\eta}$ being larger for the solution with larger $\bar{\varrho}$ . The solution of this ambiguity in $\bar{\varrho}$ is very important for $CP$-violating decays $K_L^0 \to \pi^0 e^+ e^-$, $K_L^0 \to \pi^0 \nu\bar{\nu}$ and the $CP$ asymmetries in $B$ decays governed by $\sin(2\beta)$, because $B(K_L^0 \to \pi^0 e^+ e^-)$, $B(K_L^0 \to \pi^0 \nu\bar{\nu})$, and $\sin(2\beta)$ are larger for the solution with larger $\bar{\varrho}$. The preferred solution in searches of $CP$ violation corresponds in most cases to $\bar{\varrho} \geq 0$.

This should be contrasted with any $CP$-conserving transition sensitive to $| V_{td} |$, such as $B^0$-$\bar{B}^0$ mixing , $K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \mu\bar{\mu}$, $B \to \mu\bar{\mu}$, which for given values of $m_t$, $F_B\sqrt{B_B}$ , $V_{cb}$, $x_d$ determine uniquely the value of $\bar{\varrho}$.

Although several analyses of this determination have been presented in the literature (see in particular Ref. [13]) , we think it is useful to have simple analytic expressions to help us to answer immediately, whether or not the favored solution $\bar{\varrho} \geq 0$ is chosen.

#### 1. $B^0$-$\bar{B}^0$ mixing

We require that $R_t \leq \sqrt{1 + R_b^2}$. Then for a given value of $R_b$ one gets a positive $\bar{\varrho}$. Using the analytic formula (4.1) and introducing the "scaling" variable [4]

$$z(B_d^0) = m_t \left[\frac{\kappa}{0.038}\right]^{1.32} , \tag{4.7}$$

we find using (3.8) and (3.9) the condition

$$F_{B_d}\sqrt{B_{B_d}} \geq \sqrt{\frac{0.55}{\eta_B}} \sqrt{\frac{x_d}{0.72}} \left[\frac{179 \text{ GeV}}{z(B_d^0)}\right]^{0.76} \frac{200 \text{ MeV}}{\sqrt{1 + R_b^2}} . \tag{4.8}$$

When this inequality is satisfied the favored solution with $\bar{\varrho} \geq 0$ is bound to be chosen. Setting $\eta_B = 0.55$ we plot in Fig. 4 the smallest value of $F_B\sqrt{B_B}$ consistent with (4.8) as a function of $z(B_d^0)$ for different values of $|V_{ub}/V_{cb}|$ and $x_d = 0.72$. We observe that for $z(B_d^0) \leq 180$ GeV one needs $F_{B_d}\sqrt{B_{B_d}} \geq 180$ MeV in order to

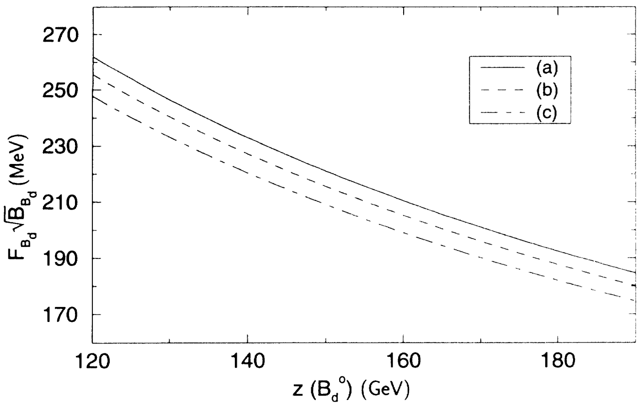


FIG. 4. Lower bound on $F_{B_d}\sqrt{B_{B_d}}$ for $|V_{ub}/V_{cb}| = 0.06$ (a), $|V_{ub}/V_{cb}| = 0.08$ (b), $|V_{ub}/V_{cb}| = 0.10$ (c) necessary for $\bar{\varrho} \geq 0$.

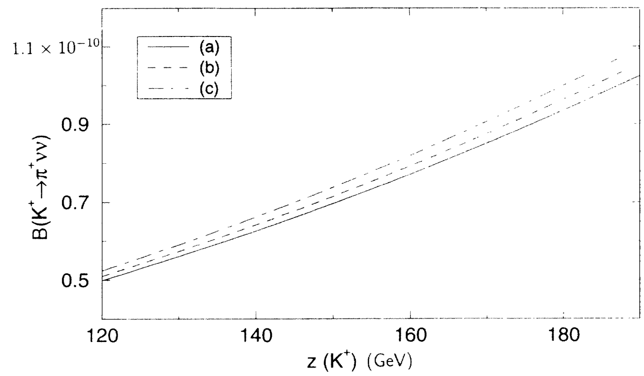


FIG. 5. Upper bound on $B(K^+ \to \pi^+\nu\bar{\nu})$ for $|V_{ub}/V_{cb}| = 0.06$ (a), $|V_{ub}/V_{cb}| = 0.08$ (b), $|V_{ub}/V_{cb}| = 0.10$ (c) necessary for $\bar{\varrho} \geq 0$.

have $\bar{\varrho} \geq 0$.

Using (3.12) we can also find a minimal value for $x_s$ consistent with $R_t \leq \sqrt{1+R_b^2}$. One gets, to a very good approximation,

$$(x_s)_{\min} = \frac{x_d}{R_{ds}\lambda^2}\frac{1}{1+R_b^2}\,. \tag{4.9}$$

For $R_{ds} = 1$ and $R_b = 1/3$ we have $(x_s)_{\min} \simeq 18.6 x_d$.

### 2. $K^+ \to \pi^+\nu\bar{\nu}$

An analogous condition can be derived from the decay $K^+ \to \pi^+\nu\bar{\nu}$ by requiring $\sqrt{{\varrho_0}^2 + (R_b\sigma)^2} \geq r_0$ with $\varrho_0$ and $r_0$ defined in (3.19) and (3.21), respectively. Neglecting the tiny contribution of the second term in (3.21), using the formula

$$X(x_t) = 0.65 x_t^{0.575}\,, \tag{4.10}$$

which reproduces the function $X(x_t)$ to an accuracy of 0.5% for the range of $m_t$ considered in this paper, and introducing the variable [4]

$$z(K^+) = m_t \left[\frac{V_{cb}}{0.038}\right]^{1.74}\,, \tag{4.11}$$

we find the condition

$$B(K^+ \to \pi^+\nu\bar{\nu}) \leq 4.64 \times 10^{-11}\frac{1}{\sigma}\left\{\left[0.40\left(\frac{z(K^+)}{M_W}\right)^{1.15} + \bar{P}_0(K^+)\right]^2 + 0.16\left(\frac{z(K^+)}{M_W}\right)^{2.30}(R_b\,\sigma)^2\right\}\,. \tag{4.12}$$

This bound is shown in Fig. 5 as a function of the variable $z(K^+)$. Although this solution is welcome in searches for $CP$ violation, the experimental bound on $B(K^+ \to \pi^+\nu\bar{\nu})$, which could be reached in the coming years [40], will be most probably above it.

## V. PHENOMENOLOGICAL ANALYSIS

### A. First look

In order to describe the situation of 1994 after a possible top quark discovery we first make the following choices for the relevant parameters.

Range I:

$$|V_{cb}| = 0.038 \pm 0.004\,, \quad |V_{ub}/V_{cb}| = 0.08 \pm 0.02\,,$$
$$B_K = 0.7 \pm 0.2\,, \quad \sqrt{B_{B_d}}F_{B_d} = (200 \pm 30)\ \text{MeV}\,,$$
$$x_d = 0.72 \pm 0.08\,, \quad m_t = (165 \pm 15)\ \text{GeV}\,. \tag{5.1}$$

The values of $|V_{cb}|$ and $|V_{ub}/V_{cb}|$ given here are consistent with the recent summary in [41]. The values of $B_K$ cover comfortably the range of most recent lattice ($B_K = 0.825 \pm 0.027$) (Ref. [42]) and 1/N ($B_K = 0.7 \pm 0.1$) (Ref. [43]) results. They also touch the range of values obtained in the hadron duality approach ($B_K = 0.4 \pm 0.1$) (Ref. [44]). $\sqrt{B_{B_d}}F_{B_d}$ given here is in the range of various lattice and QCD sum rule estimates [45]. $x_d$ is in accordance with the most recent average of CLEO and ARGUS data [7] and it is compatible with the data from the CERN $e^+e^-$ collider LEP. We set $\tau_B = 1.5$ ps (Ref. [46]) in the whole analysis because the existing small error on $\tau_B$ ($\Delta\tau_B = \pm 0.04$ ps) has only a very small impact on our numerical results.

The choice for $m_t$ requires certainly an explanation. The high precision electroweak studies give in the standard model typically $m_t \simeq 165 \pm 30$ GeV, where the central value corresponds to $m_H = 300$ GeV (Ref. [47]). Since we work in the standard model we expect that $m_t$ will be found in this range. A top quark discovery at Fermilab Tevatron will certainly narrow this range by at least a factor of 2. It is of interest to see what impact this would have for the phenomenology considered here. At this level of accuracy one has to state how $m_t$ is defined. The QCD corrections to $\varepsilon_K$, $B^0$-$\bar{B}^0$ mixing, and $K^+ \to \pi^+\nu\bar{\nu}$ used here correspond to the running top quark mass in the $\overline{\rm MS}$ scheme evaluated at $m_t$, i.e., $m_t$ in (5.1) and in all formulas of this paper represents $\overline{m_t}(m_t)$. The physical top quark mass as the pole of the renormalized propagator is then given by

$$m_t^{\rm phys}(m_t) = m_t \left[1 + \frac{4\alpha_s(m_t)}{3\pi}\right] . \qquad (5.2)$$

For the range of $m_t$ considered here $m_t^{\rm phys}$ is higher than $m_t$ by $7 \pm 1$ GeV.

For $\Lambda_{\overline{\rm MS}}$ and $m_c$ affecting $B(K^+ \to \pi^+\nu\bar{\nu})$ we use

$$\Lambda_{\overline{\rm MS}} = (0.275 \pm 0.075) \text{ GeV}$$

$$m_c \equiv \overline{m}_c(m_c) = (1.3 \pm 0.05) \text{ GeV} . \qquad (5.3)$$

In Fig. 6 (graph I) we show the resulting unitarity triangle . To this end the analysis of $\varepsilon_K$ and of $B_d^0$-$\bar{B}_d^0$ mixing have been used. In Table II we show the resulting ranges for $\delta$, $\sin(2\phi_i)$, $B(K^+ \to \pi^+\nu\bar{\nu})$, $|V_{td}|$, and $x_s$ corresponding to the choice of the parameters in (5.1). In calculating $x_s$ we have set $R_{ds} = 1$.

We observe the following.

The uncertainty in the value of $\sin(2\beta)$ is moderate. We find $\sin(2\beta) \simeq 0.59 \pm 0.21$. Consequently a large asymmetry $A_{CP}(\psi K_s)$ is expected. In particular $\sin(2\beta) \geq 0.38$.

The uncertainties in $\sin(2\alpha)$ and in $\sin(2\gamma)$ are huge.

Similarly the uncertainties in the predicted values of $B(K^+ \to \pi^+\nu\bar{\nu})$, $|V_{td}|$, and $x_s$ are large.

TABLE II. Ranges for scan of basic parameters for range I, shown in Eq. (5.1). Split according to the two different solutions for the CKM phase $\delta$ in the first and second quadrant.

| | 1. Quadrant | | 2. Quadrant | |
|---|---|---|---|---|
| | Min | Max | Min | Max |
| $\delta$ | 44.5 | 90.0 | 90.0 | 135.9 |
| $\sin(2\alpha)$ | −0.67 | 0.74 | 0.50 | 1.00 |
| $\sin(2\beta)$ | 0.50 | 0.80 | 0.38 | 0.74 |
| $\sin(2\gamma)$ | 0 | 1.00 | −1.00 | 0 |
| $\mid V_{td} \mid \times 10^3$ | 6.9 | 10.0 | 8.6 | 11.8 |
| $x_s$ | 10.8 | 24.2 | 7.7 | 14.4 |
| $B(K^+ \to \pi^+\nu\bar{\nu}) \times 10^{10}$ | 0.62 | 1.39 | 0.67 | 1.46 |

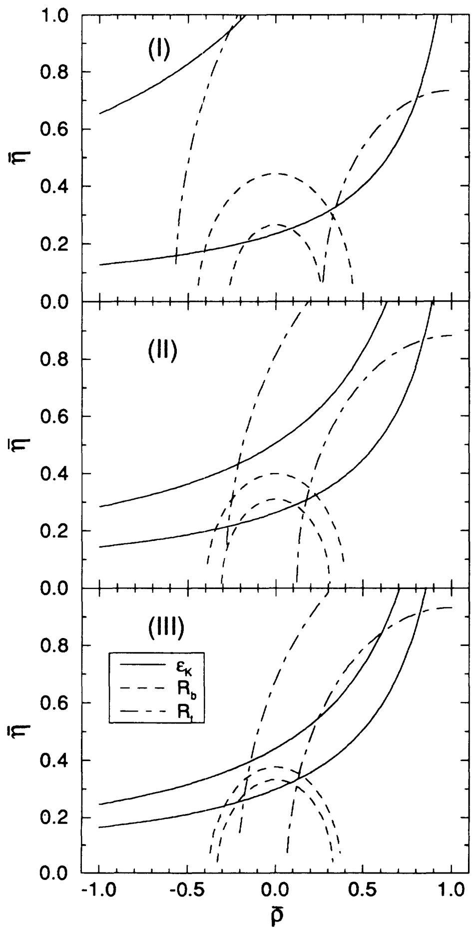


FIG. 6. Unitarity triangle in the $(\bar{\varrho}, \bar{\eta})$ plane determined by $\varepsilon_K$, $|V_{ub}/V_{cb}|$, and $x_d$ using ranges I, II, and III as in Eqs. (5.1), (5.4), and (5.5), respectively.

### B. A look in the future

It is to be expected that the uncertainties in (5.1) will be reduced in the next five years through the improved determinations of $|V_{cb}|$ and $|V_{ub}/V_{cb}|$ at CLEO II [7], the improved measurements of $x_d$, and the discovery of the top quark giving an improved $m_t$ range. We also anticipate that the extensive efforts of theorists, in particular using the lattice methods, will considerably reduce the errors on $B_K$ and $\sqrt{B_B}F_B$.

We consider the following ranges of parameters.

Range II:

$$|V_{cb}| = 0.040 \pm 0.002\,, \quad |V_{ub}/V_{cb}| = 0.08 \pm 0.01\,,$$
$$B_K = 0.75 \pm 0.07\,, \quad \sqrt{B_{B_d}}F_{B_d} = (185 \pm 15)\ \text{MeV}\,,$$
$$x_d = 0.72 \pm 0.04\,, \quad m_t = (170 \pm 7)\ \text{GeV}\,. \tag{5.4}$$

Range III:

$$|V_{cb}| = 0.040 \pm 0.001\,, \quad |V_{ub}/V_{cb}| = 0.08 \pm 0.005\,,$$
$$B_K = 0.75 \pm 0.05\,, \quad \sqrt{B_{B_d}}F_{B_d} = (185 \pm 10)\ \text{MeV}\,,$$
$$x_d = 0.72 \pm 0.04\,, \quad m_t = (170 \pm 5)\ \text{GeV}\,. \tag{5.5}$$

For $\Lambda_{\overline{\text{MS}}}$ and $m_c$ we use

$$\Lambda_{\overline{\text{MS}}} = 0.3\,\text{GeV}\,, \qquad m_c = 1.3\,\text{GeV}\,. \tag{5.6}$$

For each range we repeat the analysis of Sec. V A. The results are given in Fig. 6 (graphs II and III) and Tables III and IV.

We observe the following.

The uncertainty in the value of $\sin(2\beta)$ has been considerably reduced. We find

$$\sin(2\beta) = \begin{cases} 0.60 \pm 0.14 & \text{(range II)}\,, \\ 0.61 \pm 0.09 & \text{(range III)}\,. \end{cases} \tag{5.7}$$

The uncertainties in $\sin(2\alpha)$ and $\sin(2\gamma)$ although somewhat reduced remain very large.

For $|V_{td}|$, $x_s$, and $B(K^+ \to \pi^+\nu\bar{\nu})$ we find

$$|V_{td}| = \begin{cases} (9.5 \pm 1.4) \times 10^{-3} & \text{(range II)}\,, \\ (9.4 \pm 1.0) \times 10^{-3} & \text{(range III)}\,, \end{cases} \tag{5.8}$$

$$x_s = \begin{cases} 13.3 \pm 4.3 & \text{(range II)}\,, \\ 12.9 \pm 2.8 & \text{(range III)}\,, \end{cases} \tag{5.9}$$

$$B(K^+ \to \pi^+\nu\bar{\nu}) = \begin{cases} (1.07 \pm 0.24) \times 10^{-10} & \text{(range II)}\,, \\ (1.03 \pm 0.15) \times 10^{-10} & \text{(range III)}\,. \end{cases} \tag{5.10}$$

TABLE III. Same as in Table II but for range II, shown in Eq. (5.4).

| | 1. Quadrant | | 2. Quadrant | |
|---|---|---|---|---|
| | Min | Max | Min | Max |
| $\delta$ | 60.9 | 90.0 | 90.0 | 122.5 |
| $\sin(2\alpha)$ | −0.30 | 0.69 | 0.57 | 1.00 |
| $\sin(2\beta)$ | 0.57 | 0.73 | 0.46 | 0.69 |
| $\sin(2\gamma)$ | 0 | 0.85 | −0.91 | 0 |
| $\mid V_{td} \mid \times 10^3$ | 8.1 | 9.8 | 9.0 | 10.8 |
| $x_s$ | 11.2 | 17.6 | 9.1 | 13.0 |
| $B(K^+ \to \pi^+\nu\bar{\nu}) \times 10^{10}$ | 0.83 | 1.22 | 0.86 | 1.3 |

TABLE IV. Same as in Table II but for range III, shown in Eq. (5.5).

| | 1. Quadrant | | 2. Quadrant | |
|---|---|---|---|---|
| | Min | Max | Min | Max |
| $\delta$ | 69.0 | 90.0 | 90.0 | 113.7 |
| $\sin(2\alpha)$ | 0.01 | 0.66 | 0.60 | 0.99 |
| $\sin(2\beta)$ | 0.60 | 0.70 | 0.52 | 0.66 |
| $\sin(2\gamma)$ | 0 | 0.67 | −0.69 | 0 |
| $\mid V_{td} \mid \times 10^3$ | 8.4 | 9.6 | 9.1 | 10.4 |
| $x_s$ | 11.9 | 15.6 | 10.1 | 13.3 |
| $B(K^+ \to \pi^+\nu\bar{\nu}) \times 10^{10}$ | 0.88 | 1.12 | 0.92 | 1.18 |

This exercise implies that if the accuracy of various parameters given in (5.4) and (5.5) is achieved the determination of $|V_{td}|$ and the predictions for $\sin(2\beta)$ and $BR(K^+ \to \pi^+\nu\bar{\nu})$ are quite accurate. A sizable uncertainty in $x_s$ remains, however.

Another important message from this analysis is the inability of a precise determination of $\sin(2\alpha)$ and $\sin(2\gamma)$ on the basis of $\varepsilon_K$, $B^0 - \bar{B}^0$, $|V_{cb}|$, and $|V_{ub}/V_{cb}|$ alone. Although the great sensitivity of $\sin(2\alpha)$ and $\sin(2\gamma)$ to various parameters has been already stressed by several authors, in particular in Refs. [27,26,48,49], our analysis shows that even with the improved values of the parameters in question, as given in (5.4) and (5.5), a precise determination of $\sin(2\alpha)$ and $\sin(2\gamma)$ should not be expected in this millennium.

The fact that $\sin(2\beta)$ can be much easier determined than $\sin(2\alpha)$ and $\sin(2\gamma)$ is easy to understand. Since $R_t$ is generally by at least a factor of 2 larger than $R_b$, the angle $\beta$ is much less sensitive to the changes in the position of the point $A = (\bar{\varrho}, \bar{\eta})$ in the unitarity triangle than the remaining two angles.

### C. The impact of $B(K^+ \to \pi^+\nu\bar{\nu})$ and $x_d/x_s$

$B(K^+ \to \pi^+\nu\bar{\nu})$ and $x_d/x_s$ determine $|V_{td}|$ and $R_t$. If our expectations for the ranges discussed above are correct we should be able to have a rather accurate prediction for $B(K^+ \to \pi^+\nu\bar{\nu})$ using the analysis of $\varepsilon_K$ and of $B_d^0$-$\bar{B}_d^0$ mixing. Measuring $B(K^+ \to \pi^+\nu\bar{\nu})$ to similar accuracy would either confirm the standard model predictions or indicate some physics beyond the standard model.

We infer from Tables III and IV that measurements of $B(K^+ \to \pi^+\nu\bar{\nu})$ with the accuracy of $\pm 10\%$ would be very useful in this respect.

The accuracy of predictions for $x_s$ is poorer as seen in (5.9). A measurement of $x_s$ at a $\pm 10\%$ level will have therefore a considerable impact on the determination of the CKM parameters and in particular $R_t$ [see (3.12)], provided $R_{ds}$ is known within 10% accuracy. A numerical exercise is presented in Sec. V E.

### D. The impact of $CP$ asymmetries in $B$ decays

Measuring the $CP$ asymmetries in neutral $B$ decays will give the definitive answer whether the CKM description of $CP$ violation is correct. Assuming that this is in fact the case, we want to investigate the impact of the measurements of $\sin(2\phi_i)$ on the determination of the unitarity triangle.

Since in the rescaled triangle of Fig. 1 one side is known, it suffices to measure two angles to determine the triangle completely.

It is well known that the measurement of the $CP$ asymmetry in the decay $B^0 \to \psi K_s$ should give a measurement of $\sin(2\beta)$ without any theoretical uncertainties. One expects that prior to LHC experiments the error on $\sin(2\beta)$ should amount roughly to $\Delta\sin(2\beta) = \pm 0.06$ (Refs. [7,50,51]). The measurement of $\sin(2\alpha)$ is more difficult. It requires in addition the measurement of several channels in order to eliminate the penguin contributions. An error $\Delta\sin(2\alpha) = \pm 0.10$ prior to LHC could however be achieved at a SLAC $B$ factory [50].

In Fig. 7 we show the impact of such measurements and also plot the curve (3.27) which represents superweak models. Specifically we take

$$\sin(2\beta) = \begin{cases} 0.60 \pm 0.18\,, & \text{(5.11a)} \\ 0.60 \pm 0.06\,, & \text{(5.11b)} \end{cases}$$

as an illustration of two measurements of $\sin(2\beta)$ with two different accuracies. Next we take the following three choices for $\sin(2\alpha)$:

$$\sin(2\alpha) = \begin{cases} -0.20 \pm 0.10 & \text{(I)}\,, \\ 0.10 \pm 0.10 & \text{(II)}\,, \\ 0.70 \pm 0.10 & \text{(III)}\,. \end{cases} \tag{5.12}$$

In Fig. 8 we replace the impact of $\sin(2\alpha)$ by the impact of a measurement of $\sin(2\gamma)$ keeping $\sin(2\beta)$ unchanged. We choose the values

$$\sin(2\gamma) = \begin{cases} -0.50 \pm 0.10 & \text{(I)}\,, \\ 0 \pm 0.10 & \text{(II)}\,, \\ 0.50 \pm 0.10 & \text{(III)}\,. \end{cases} \tag{5.13}$$

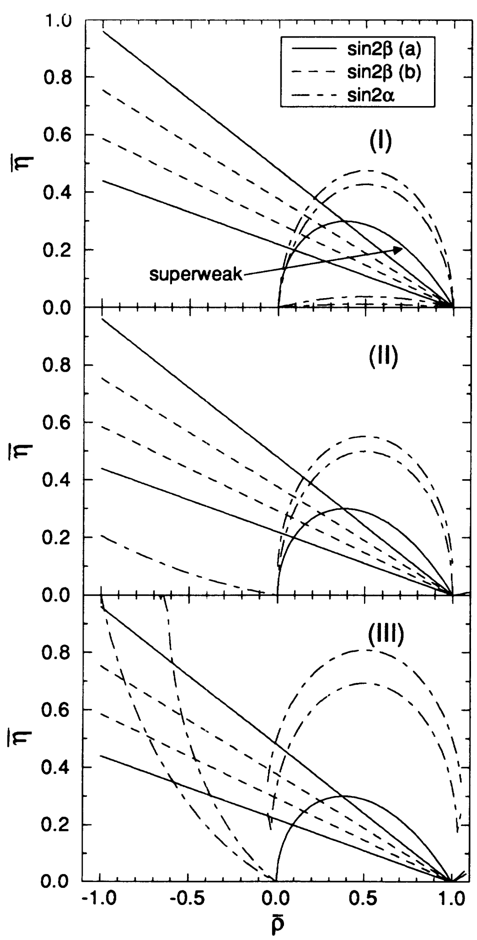


FIG. 7. Determination of the unitarity triangle in the $(\bar{\varrho}, \bar{\eta})$ plane by measuring $\sin(2\beta)$ and $\sin(2\alpha)$ as in Eqs. (5.11) and (5.12), respectively. For $\sin(2\alpha)$ we always find two solutions in $(\bar{\varrho}, \bar{\eta})$ and for $\sin(2\beta)$ we only use the solution consistent with $|V_{ub}/V_{cb}| \le 0.1$.

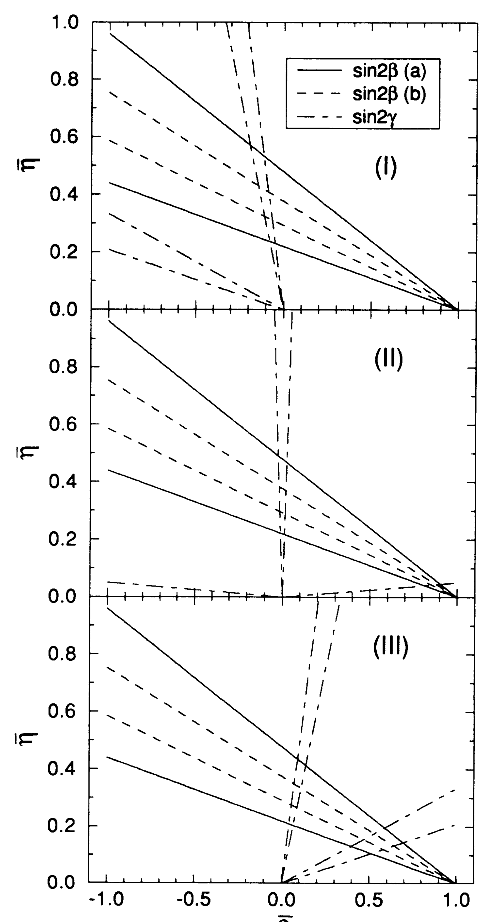


FIG. 8. Determination of the unitarity triangle in the $(\bar{\varrho}, \bar{\eta})$ plane by measuring $\sin(2\beta)$ and $\sin(2\gamma)$ as in Eqs. (5.11) and (5.13), respectively. For $\sin(2\gamma)$ we always find two solutions in $(\bar{\varrho}, \bar{\eta})$ and for $\sin(2\beta)$ we only use the solution consistent with $|V_{ub}/V_{cb}| \le 0.1$.

TABLE V. Predicted ranges for various quantities calculated by restricting $\sin(2\alpha)$ and $\sin(2\beta)$ to the ranges of (5.14) and using $|V_{cb}|$, $x_d$, and $m_t$ of (5.4). There is no allowed solution for the second quadrant.

| | Min | Max |
|---|---|---|
| $\delta$ | 69.5 | 77.8 |
| $\sin(2\gamma)$ | 0.42 | 0.66 |
| $\mid V_{td} \mid \times 10^3$ | 8.4 | 9.1 |
| $x_s$ | 15.0 | 17.5 |
| $B(K^+ \to \pi^+ \nu\bar{\nu}) \times 10^{10}$ | 0.90 | 1.12 |

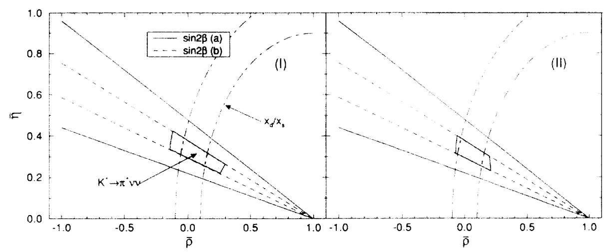


FIG. 9. Allowed ranges in the $(\bar{\varrho}, \bar{\eta})$ plane with constraints from Eqs. (5.15) and (5.16) for $B(K^+ \to \pi^+ \nu\bar{\nu})$ and $R_t = 1.0 \pm 0.1$.

We observe that the measurement of $\sin(2\alpha)$ or $\sin(2\gamma)$ in conjunction with $\sin(2\beta)$ at the expected precision will have a large impact on the accuracy of the determination of the unitarity triangle and of the CKM parameters. In order to show this more explicitly we take, as an example,

$$\sin(2\beta) = 0.60 \pm 0.06\,, \qquad \sin(2\alpha) = 0.10 \pm 0.10\,, \tag{5.14}$$

and give in Table V the predicted ranges for $\delta$, $\sin(2\gamma)$, $B(K^+ \to \pi^+ \nu\bar{\nu})$, $|V_{td}|$, and $x_s$ corresponding to the values of $\sin(2\beta)$ and $\sin(2\alpha)$ given in (5.14) and $|V_{cb}|$, $x_d$, and $m_t$ of (5.4). We use only the solution of $\sin(2\beta)$ consistent with $|V_{ub}/V_{cb}| \leq 0.1$.

It should be stressed that this impressive accuracy can only be achieved by measuring $\sin(2\alpha)$ or $\sin(2\gamma)$ in addition to $\sin(2\beta)$. This is easy to understand in view of the fact that the expected accuracy of the measurements of $\sin(2\alpha)$ and $\sin(2\gamma)$ is considerably higher than the corresponding accuracy of the predictions on basis of $\varepsilon_K$, $B^0$-$\bar{B}^0$ mixing, $|V_{ub}/V_{cb}|$, and $|V_{cb}|$ alone.

### E. $K^+ \to \pi^+ \nu\bar{\nu}$, $\sin(2\beta)$, $|V_{cb}|$, $m_t$, and $x_d/x_s$

We would like to address now our last question posed in the Introduction.

How well should one measure $B(K^+ \to \pi^+ \nu\bar{\nu})$, $\sin(2\beta)$, $|V_{cb}|$, $m_t$, and $x_d/x_s$ in order to obtain an acceptable determination of the CKM matrix on the basis of these five quantities alone? As we stated at the beginning of this paper, $K^+ \to \pi^+ \nu\bar{\nu}$ and $\sin(2\beta)$ are essentially free of any theoretical uncertainties. $|V_{cb}|$, however, is easier to determine than $|V_{ub}/V_{cb}|$ and once the top quark is discovered, $m_t$ should be known relatively well. Finally $x_d/x_s$ determines directly $R_t$ by means of Eq. (3.12).

In Fig. 9 we show the result of this exercise taking (5.6) and

$$\sin(2\beta) = 0.60 \pm 0.06\,, \quad |V_{cb}| = 0.040 \pm 0.001\,, \quad m_t = (170 \pm 5)\ \mathrm{GeV}\,, \tag{5.15}$$

$$B(K^+ \to \pi^+ \nu\bar{\nu}) = \begin{cases} (1.0 \pm 0.2) \times 10^{-10} & (\mathrm{I})\,, \\ (1.0 \pm 0.1) \times 10^{-10} & (\mathrm{II})\,. \end{cases} \tag{5.16}$$

In Table VI we give the predicted ranges of various quantities for the two cases considered.

In addition we show in Fig. 9 the result of a possible measurement of $x_d/x_s$ corresponding to $R_t = 1.0 \pm 0.1$. We observe that provided the expected accuracy of measurements is achieved we should have a respectable determination of $|V_{td}|$ this way. Figure 9 indicates that for the $\Delta V_{cb}$ and $\Delta m_t$ assumed here, $B(K^+ \to \pi^+ \nu\bar{\nu})$ must be measured with a precision of $\pm 10\%$ to be competitive with $\Delta R_t = \pm 10\%$ extracted hopefully in the future from $x_d/x_s$. The uncertainty in the predictions for $\sin(2\alpha)$ and $\sin(2\gamma)$ is very large as in the analysis of Sec. V B.

### F. $\varepsilon_K$, $B_d^0$-$\bar{B}_d^0$ mixing, $\sin(2\beta)$, and $\sin(2\alpha)$

It is useful to combine the results of Secs. V A, V B, and V D by making the customary $\sin(2\beta)$ versus $\sin(2\alpha)$ plot [5]. This plot demonstrates very clearly the correlation between $\sin(2\alpha)$ and $\sin(2\beta)$. The allowed ranges for $\sin(2\alpha)$ and $\sin(2\beta)$ corresponding to the choices of the parameters in (5.1), (5.4), and (5.5) are shown in Fig. 10 together with the results of the independent measurements of $\sin(2\beta) = 0.60 \pm 0.06$ and $\sin(2\alpha)$ given by (5.12). The latter are represented by dark shaded rectangles. The black rectangles illustrate the accuracy of future LHC measurements [$\Delta \sin(2\alpha) = \pm 0.04$, $\Delta \sin(2\beta) = \pm 0.02$] (Ref. [28]).

We also show the results of an analysis in which the

TABLE VI. Ranges of various quantities calculated with constraints from Eqs. (5.15) and (5.16).

| | (I) | | (II) | |
|---|---|---|---|---|
| | Min | Max | Min | Max |
| $\sin(2\alpha)$ | −0.917 | 0.978 | −0.691 | 0.973 |
| $\sin(2\gamma)$ | −0.704 | 1.000 | −0.418 | 0.976 |
| $\mid V_{td} \mid \times 10^3$ | 6.9 | 10.3 | 7.6 | 9.7 |

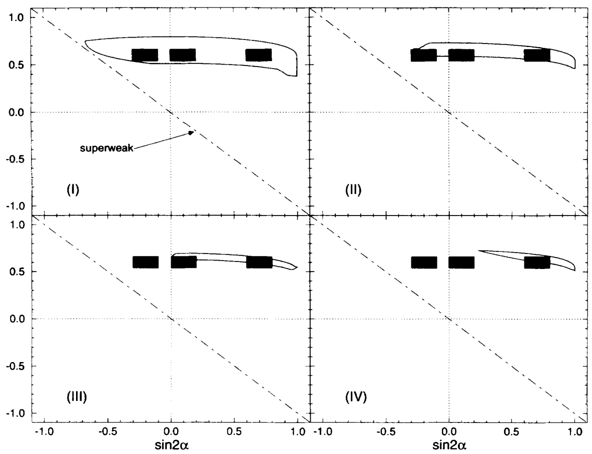


FIG. 10. $\sin(2\alpha)$ versus $\sin(2\beta)$ plot corresponding to the parameter ranges I–IV as in (5.1), (5.4), (5.5), and (5.17) and the dark shaded rectangles given by (5.12) and (5.11b). The black rectangles illustrate the accuracy of future LHC measurements.

accuracy of various parameters is as in (5.4) but with the central values modified.

Range IV:

$$\begin{aligned} |V_{cb}| &= 0.038 \pm 0.002\,, & |\,V_{ub}/V_{cb}\,| &= 0.08 \pm 0.01\,, \\ B_K &= 0.70 \pm 0.07\,, & \sqrt{B_{B_d}}F_{B_d} &= (185 \pm 15)\ \mathrm{MeV}\,, \\ x_d &= 0.72 \pm 0.04\,, & m_t &= (165 \pm 7)\ \mathrm{GeV}\,. \end{aligned} \tag{5.17}$$

In addition we show the prediction of superweak theories which in this plot is represented by a straight line.

There are several interesting features on this plot.

The impact of the direct measurements of $\sin(2\beta)$ and $\sin(2\alpha)$ is clearly visible in this plot.

In cases III and IV we have examples where the measurements of $\sin(2\alpha)$ are incompatible with the predictions coming from $\varepsilon_K$ and $B^0$-$\bar{B}^0$ mixing. This would be a signal for physics beyond the standard model. The measurement of $\sin(2\alpha)$ is essential for this.

Case IV shows that for a special choice of parameters the predictions for the asymmetries coming from $\varepsilon_K$, $B^0$-$\bar{B}^0$ mixing, $|\,V_{cb}\,|$, and $|\,V_{ub}/V_{cb}\,|$ can be quite accurate when these four constraints can only be satisfied simultaneously in a small area of the $(\bar{\varrho},\bar{\eta})$ space. Decreasing $|\,V_{cb}\,|$, $|\,V_{ub}/V_{cb}\,|$, and $m_t$ and increasing $F_B$ would make the allowed region in case IV even smaller.

We also observe that the future measurements of asymmetries and the improved ranges for the parameters relevant for $\varepsilon_K$ and $B^0$-$\bar{B}^0$ mixing will probably allow one to rule out the superweak models.

## VI. SUMMARY AND CONCLUSIONS

The top quark discovery and the measurements of $B(K^+ \rightarrow \pi^+\nu\bar{\nu})$, $x_s$ and of $CP$-violating asymmetries in $B$ decays will play crucial roles in the determination of the CKM parameters and in the tests of the standard model. Similarly the improvements in the determination of the CKM elements $V_{ub}$ and $V_{cb}$ in tree level $B$ decays and the improved calculations of the nonperturbative parameters like $B_K$ and $\sqrt{B_B}F_B$ will advance our understanding of weak decay phenomenology. In this paper we have made an excursion into the future trying to see what one could expect in this field in the coming five to ten years prior to LHC experiments.

In the first part of the numerical analysis we have investigated how the top quark discovery together with the improved determinations of $|V_{ub}/V_{cb}|$, $|V_{cb}|$, $B_K$, and $\sqrt{B_B}F_B$ would allow for the determination of the unitarity triangle and more accurate predictions for $K^+ \rightarrow \pi^+\nu\bar{\nu}$, $B_s^0$-$\bar{B}_s^0$ mixing, and $\sin(2\phi_i)$. Our main findings in this part can be summarized as follows: We expect that around the year 2000 satisfactory predictions for $|\,V_{td}\,|$, $\sin(2\beta)$, and $B(K^+ \rightarrow \pi^+\nu\bar{\nu})$ should be possible; a sizable uncertainty in $x_s$ and huge uncertainties in $\sin(2\alpha)$ and in $\sin(2\gamma)$ will remain, however.

In the second part of our analysis we have investigated the impact of future measurements of $B(K^+ \rightarrow \pi^+\nu\bar{\nu})$, $x_s$, and $\sin(2\phi_i)$. Our main findings in this second part can be summarized as follows: The measurements of $\sin(2\alpha)$, $\sin(2\beta)$, and $\sin(2\gamma)$ will have an impressive impact on the determination of the CKM parameters and the tests of the standard model; this impact is further strengthened by combining the constraints considered in the two parts of our analysis as seen most clearly in Fig. 10; future LHC $B$ physics experiments around the year 2005 will refine these studies as evident from Fig. 10 and Ref. [28].

In our analysis we have concentrated on quantities which have either been already measured $(\varepsilon_K, x_d)$ or quantities which are practically free from theoretical uncertainties such as $x_d/x_s$, $K^+ \rightarrow \pi^+\nu\bar{\nu}$, and certain asymmetries in $B$ decays. We stress at this point, however, that the measurements of $\varepsilon'/\varepsilon$, $B \rightarrow s\gamma$, $K_L \rightarrow \mu^+\mu^-$, $K_L \rightarrow \pi^0 e^+e^-$, $K_L \rightarrow \pi^0\nu\bar{\nu}$, and other rare decays discussed in the literature are also very important for our understanding of weak decays. In particular a measurement of a nonzero $\mathrm{Re}(\varepsilon'/\varepsilon)$, to be expected in few years from now, will most probably give the first signal of direct $CP$ violation. Unfortunately, all these decays are either theoretically less clean than the decays considered here or they are more difficult to measure. Clearly some dramatic improvements in the experimental techniques and in nonperturbative methods could change this picture in the future.

We hope that our investigations and the analytic formulas derived in this paper will facilitate the waiting for $m_t$, $K^+ \rightarrow \pi^+\nu\bar{\nu}$, $B_s^0$-$\bar{B}_s^0$, mixing and $CP$ asymmetries in $B$ decays. There is clearly a very exciting time ahead of us.

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