String tension from monopoles in SU(2) lattice gauge theory

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We calculate the heavy quark potential from the magnetic current due to monopoles in four-dimensional SU(2) lattice gauge theory. The magnetic current is located in configurations generated in a conventional Wilson action simulation on a 16^4 lattice. The configurations are projected with high accuracy into the maximum Abelian gauge. The magnetic current is then extracted and the monopole contribution to the potential is calculated. The resulting string tension is in excellent agreement with the SU(2) string tension obtained by conventional means from the configurations. A comparison is made with the U(1) case, with emphasis on the differing periodicity properties of SU(2) and U(1) lattice gauge theories. The properties of the maximum Abelian gauge are discussed.

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I. INTRODUCTION

In this paper, we report on our calculations of the SU(2) string tension using monopoles. The monopoles were located in SU(2) lattice gauge theory configurations after the configurations were projected with high accuracy into the maximum Abelian gauge. These lattice calculations were motivated by an approach to continuum confinement outlined by 't Hooft some time ago [1]. In 't Hooft's framework, the first step is a partial gauge fixing, applied only to those gauge fields which are "charged," or have off-diagonal generators in the Lie algebra of the gauge group. The central idea is that the monopoles associated with the Abelian gauge invariance left after partial gauge fixing will control nonperturbative phenomena. This Abelian gauge invariance is associated with the fields whose generators are diagonal, called "photons." For an SU(2) gauge group with the generator T_3 diagonal, the gauge field A_μ^3 is the Abelian field or photon, and gauge fixing is done only on $A_\mu^a, a = 1, 2$, or equivalently the charged fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \pm iA_\mu^2).$$

For the particular choice of the gauge-fixing condition known as the maximum Abelian gauge, the continuum functional

$$G_c \equiv \frac{1}{2} \sum_\mu \int \left[(A_\mu^1)^2 + (A_\mu^2)^2\right] d^4x$$

$$= \sum_\mu \int (W_\mu^+ W_\mu^-) d^4x \qquad (1)$$

is minimized over all SU(2) gauge transformations, leading to the conditions

$$(\partial_\mu + igA_\mu^3)W_\mu^+ = (\partial_\mu - igA_\mu^3)W_\mu^- = 0,$$

where g is the SU(2) gauge coupling. In the remainder of this section, we discuss how this is turned into a specific calculational scheme on the lattice. Our results are described in Sec. II. Section III contains a discussion of the maximum Abelian gauge and our conclusions.

A. Lattice gauge fixing

The SU(2) lattice gauge theory is built out of link variables $U_\mu(x)$,

$$U_\mu(x) \equiv e^{igaA_\mu \cdot \tau},$$

where $\tau = \sigma/2$ are the generators of SU(2) in the fundamental representation, and a is the lattice spacing. On the lattice, the maximum Abelian gauge is obtained by maximizing the lattice functional

$$G_l = \sum_{x,\mu} \frac{\text{tr}}{2} \left[U_\mu^\dagger(x)\sigma_3 U_\mu(x)\sigma_3\right] \qquad (2)$$

over all SU(2) gauge transformations [2]. It is easy to show that, in the continuum limit, maximizing G_l is equivalent to minimizing G_c.

At the maximum, G_l will be stationary under gauge transformations. The demand that G_l be stationary with respect to a gauge transformation at an arbitrary site y leads to the requirement that

$$X(y) \equiv \sum_\mu \left[U_\mu(y)\sigma_3 U_\mu^\dagger(y) + U_\mu^\dagger(y - \hat{\mu})\sigma_3 U(y - \hat{\mu})\right] \qquad (3)$$

be diagonal. This can be accomplished by a gauge transformation $\Omega(y)$. However, the value of X at the nearest neighbors of y is affected by $\Omega(y)$, so the diagonalization of X over the whole lattice must be done iteratively.

After gauge fixing, there is still manifest U(1) gauge invariance, and it is useful to factor a U(1) link vari-

able from $U_\mu(x)$, writing $U_\mu(x) = u_\mu(x)w_\mu(x)$, where $u_\mu(x) = \exp(i\phi_\mu^3\tau_3)$, and $w_\mu(x) = \exp(i\boldsymbol{\theta}_\mu \cdot \boldsymbol{\tau})$, with $\theta_\mu^3 \equiv 0$. The U(1) gauge transformation properties of u_μ and w_μ follow upon applying an Abelian transformation $\Omega_3 = \exp[i\alpha(x)\tau_3]$ to U_μ:

$$u_\mu(x) \to \Omega_3^\dagger(x + \hat\mu)u_\mu(x)\Omega_3(x),$$

$$w_\mu(x) \to \Omega_3^\dagger(x)w_\mu(x)\Omega_3(x),$$

so w_μ transforms as a charged chiral field at x. The angle ϕ_μ^3 can be extracted from the matrix elements of U_μ by expanding the gauge-fixed SU(2) link U_μ in Pauli matrices, writing

$$U_\mu = U_\mu^0 + i\sum_{k=1}^3 U_\mu^k \cdot \sigma_k \tag{4}$$

then $\phi_\mu^3 = 2\arctan(U_\mu^3/U_\mu^0)$.

The maximum Abelian gauge globally suppresses $|\boldsymbol{\theta}_\mu|$, or equivalently, tries to force w_μ to the identity matrix. Even so, it is nontrivial to expect that long-range effects associated with confinement are totally isolated in u_μ. The first concrete calculations to test this were performed by Suzuki and Yotsuyanagi [3]. In Wilson loops, they replaced each SU(2) link variable by the U(1) link variable u_μ, and found that full SU(2) results were obtained for Creutz ratios. This did not work for forms of partial gauge fixing other than the maximum Abelian gauge.

B. Monopoles in U(1)

In the U(1) lattice gauge theory, the usual form of a Wilson loop involves a line integral of the U(1) link variable $\phi_\mu(x)$ taken around a path specified by an integer-valued current $J_\mu(x)$:

$$\langle W_{U(1)}\rangle = \left\langle \exp\left(i\sum_x \phi_\mu^3 J_\mu\right)\right\rangle, \tag{5}$$

where angular brackets denote the expectation value over the ensemble of U(1) configurations. In addition, it is well established that confinement in U(1) is via monopoles [4, 5]. A Wilson loop originally expressed as in Eq. (5) can be factored into a perturbative term arising from one photon exchange, times a nonperturbative term arising from monopoles[1]:

$$\langle W_{U(1)}\rangle = \langle W_{phot}\rangle \langle W_{mon}\rangle. \tag{6}$$

An explicit formula for $\langle W_{mon}\rangle$ in U(1) is obtained by writing J_μ as the curl of a Dirac sheet variable [6]; $J_\mu = \partial_\nu D_{\mu\nu}$, where ∂_ν denotes a discrete derivative. Then $\langle W_{mon}\rangle$ is given by

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[1]Equation (6) can be derived as an exact formula only for the Villain form of the U(1) action. However, in Ref. [4], it was shown to work for other forms of the action, provided a coupling constant mapping was used to calculate $\langle W_{phot}\rangle$.

$$\langle W_{mon}\rangle = \left\langle \exp\left(\frac{i2\pi}{2}\sum_x D_{\mu\nu}(x)F_{\mu\nu}^*(x)\right)\right\rangle_m, \tag{7}$$

where $\langle\,\rangle_m$ denotes the sum over configurations of magnetic current. The sheet variable $D_{\mu\nu}$ is not unique. For the usual case of an $R \times T$ loop with $|J_\mu| = 1$, a useful choice is to set $D_{\mu\nu} = 1$ on the plaquettes of the flat rectangle with boundary J_μ, and $D_{\mu\nu} = 0$ on all other plaquettes. In Eq. (7), $F_{\mu\nu}^*$ is the dual of the field strength due to the magnetic current: $F_{\mu\nu}^*(x) = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}(x)$. The field strength itself is derived from a magnetic vector potential A_μ^m, $F_{\mu\nu} \equiv \partial_\mu A_\nu^m - \partial_\nu A_\mu^m$, where

$$A_\mu^m(x) = \sum_y v(x - y)m_\mu(y), \tag{8}$$

and m_μ is the integer-valued, conserved magnetic current.

In Ref. [3], results characteristic of confinement in SU(2) were obtained by replacing the full Wilson loop by a U(1) loop expressed in terms of the link variable ϕ_μ^3. Since confinement in U(1) lattice gauge theory itself is via monopoles, this raises the possibility of a monopole explanation of confinement in SU(2).

C. Periodicity in SU(2)

Before applying Eqs. (7) and (8) to SU(2), account must be taken of the difference in periodicity for the standard U(1) and SU(2) actions. The above discussion was for a U(1) action periodic in link angles with period 2π. The SU(2) action is periodic in the U(1) link angle ϕ_μ^3 with period 4π. This follows from the formula $u_\mu(x) = \exp(i\phi_\mu^3\tau_3)$, where $\tau_3 = \sigma_3/2$. Alternatively, the process of temporarily setting w_μ to the identity on every link transforms the SU(2) action into a U(1) action with period 4π:

$$S \longrightarrow \frac{\beta}{2}\sum_{x,\mu>\nu}\left[1 - \cos\left(\frac{\phi_{\mu\nu}^3(x)}{2}\right)\right], \tag{9}$$

where $\phi_{\mu\nu}^3 = \partial_\mu\phi_\nu^3 - \partial_\nu\phi_\mu^3$.

While Eq. (9) is not intended as an approximation to the full SU(2) action, it does correctly reveal the monopole charges which will occur in SU(2). A Dirac string occurs when the plaquette angle $\phi_{\mu\nu}^3$ is an integer multiple of 4π, rather than 2π, so the magnetic current m_μ here is an even integer, or in other words the Abelian monopoles which occur in SU(2) are Schwinger monopoles [1, 7].

The replacement of w_μ by the identity in the action as in Eq. (9) is never actually done. The configurations are generated using full SU(2) dynamics. However, after projecting the configurations into the maximum Abelian gauge, the w_μ are set to the identity on every link in the calculation of Wilson loops. Only the U(1) link variable $u_\mu(x)$ is retained, so that the SU(2) Wilson loop becomes a U(1) loop:

$$W_{\rm SU(2)} \longrightarrow \frac{1}{2}\left\langle \left[\exp\left(i\sum_x \phi_\mu^3 J_\mu\right) \right.\right.$$
$$\left.\left. + \exp\left(-i\sum_x \phi_\mu^3 J_\mu\right)\right]\right\rangle, \quad (10)$$

where the conserved line current J_μ has $|J_\mu| = \frac{1}{2}$. Since in this approximation, the Wilson loop is built out of U(1) variables which involve only ϕ_μ^3, perturbative exchange of gluons coupled to τ_1 and τ_2 has clearly been suppressed. By analogy with the situation in U(1), perturbative exchange of the neutral gluon or "photon" coupled to τ_3 is still allowed, but is expected to reside in the SU(2) analogue of the $\langle W_{\rm phot}\rangle$ factor of Eq. (6), whereas the confining part of the potential is expected to reside in the factor $\langle W_{\rm mon}\rangle$. That the confining potential resides solely in $\langle W_{\rm mon}\rangle$ is a postulate which will be justified by our results.

Since $|J_\mu| = \frac{1}{2}$, the calculation of $\langle W_{\rm mon}\rangle$ for SU(2) involves a Dirac sheet variable with $|D_{\mu\nu}| = \frac{1}{2}$, along with a magnetic current m_μ which is an even integer. Both of these arose from the 4π periodicity of SU(2) in the link angle ϕ_μ^3. It is straightforward to transform back to the familiar case of 2π periodicity. Define $\bar\phi_\mu^3 = \phi_\mu^3/2 = \arctan(U_\mu^3/U_\mu^0)$, and require $\bar\phi_\mu^3 \in (-\pi, \pi]$. The location of the magnetic current starts with plaquette angles $\bar\phi_{\mu\nu}^3$ constructed from $\bar\phi_\mu^3$:

$$\bar\phi_{\mu\nu}^3(x) = \partial_\mu\bar\phi_\nu^3 - \partial_\nu\bar\phi_\mu^3$$
$$= \bar\phi_\mu^3(x) + \bar\phi_\nu^3(x+\hat\mu) - \bar\phi_\mu^3(x+\hat\nu) - \bar\phi_\nu^3(x). \quad (11)$$

The plaquette angle $\bar\phi_{\mu\nu}^3$ is resolved into a Dirac string contribution, plus a fluctuating part:

$$\bar\phi_{\mu\nu}^3 = 2\pi\bar n_{\mu\nu} + \tilde\phi_{\mu\nu}^3, \quad (12)$$

where $\tilde\phi_{\mu\nu}^3 \in (-\pi, \pi]$ and $\bar n_{\mu\nu}$ is an integer [8]. The integer-valued magnetic current $\bar m_\mu = m_\mu/2$ is determined by the net flux of Dirac strings into an elementary cube:

$$\bar m_\mu = -\tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial_\nu\bar n_{\alpha\beta}. \quad (13)$$

We now simply use $\bar m_\mu$ in Eq. (8) to obtain A_μ, and from it, $F_{\mu\nu}^*$. Finally $\langle W_{\rm mon}\rangle$ for SU(2) is obtained from Eq. (7) where the Dirac sheet variable now has $|D_{\mu\nu}| = 1$. We have in effect used the Dirac condition [9] on the product of electric and magnetic charge to transform from 4π to 2π periodicity. In the interaction of a distribution of magnetic current with a single electric charge, if the magnetic current is halved and the electric current is doubled, the same result is obtained, since the interaction only depends on the quantized product of electric and magnetic charge.

D. Monopole location in one-cubes

To summarize, we use Eq. (7) to calculate $\langle W_{\rm mon}\rangle$ for SU(2), after extracting the magnetic current $\bar m_\mu$ from SU(2) configurations projected into the maximum Abelian gauge. In the identification of $\bar m_\mu$, elementary

cubes were used, so a magnetic charge is located at the center of a spacial one-cube, likewise for other components of the magnetic current. This is done mainly for practical reasons; if (say) two-cubes were used instead, the effective lattice size would become 8^4 instead of 16^4. Although this practical reason was dominant in our calculations, it is worth pointing out that there is no conflict between monopoles being extended objects, and using one-cubes to locate them. The monopole location procedure finds a monopole by locating the end of a Dirac string. Therefore, we must be in a gauge where there are Dirac strings attached to monopoles. Consider for the moment the case of a 't Hooft–Polyakov monopole in the continuum [10]. The gauge where a Dirac string is present is usually described as having the Higgs field along the isotopic three-axis. A completely equivalent description is that the gauge fields are in the maximum Abelian gauge. In this gauge, the photon field A_μ^3 takes the form appropriate for an elementary point monopole. The extended structure of the monopole involves only the charged gauge fields, $W_\mu^\pm$ [11]. In a gauge with these properties, the center of the monopole may be located by finding the end of its Dirac string, using the finest spacial scale available.

In calculating the *effects* of monopoles on heavy quarks, the assumption is made that despite their finite size, only the long-range Coulombic fields produced by monopoles contribute to the confining potential. This produces a clear calculational procedure for computing $\langle W_{\rm mon}\rangle$, which as will be seen in the next section, works very well. However, further theoretical understanding of this assumption is certainly needed.

II. MONOPOLE CALCULATIONS

A. Simulation and gauge fixing

Our simulations were done on a 16^4 lattice, using the standard Wilson form of the SU(2) action. Three β values were used; $\beta = 2.40, 2.45$, and 2.50. At each β, after equilibration, 500 configurations were saved. Saved configurations were separated by 20 updates of the lattice, where a lattice update consisted of one heat bath sweep [12], plus one or two overrelaxation sweeps [13]. Each of these configurations was then projected into the maximum Abelian gauge using the overrelaxation method of Mandula and Ogilvie, with their parameter $\omega = 1.70$ [14]. The overrelaxation process was stopped after the off-diagonal elements of $X(x)$ of Eq. (3) were sufficiently small. Expanding $X(x)$ in Pauli matrices,

$$X = X^0(x) + i\sum_{k=1}^{3} X^k(x)\sigma_k, \quad (14)$$

we used

$$\langle |X^{\rm ch}|^2\rangle \equiv \frac{1}{L^4}\sum_x \left[|X^1(x)|^2 + |X^2(x)|^2\right] \quad (15)$$

as a measure of the average size of the off-diagonal matrix elements of X over the lattice, and required $\langle |X^{\rm ch}|^2\rangle \leq$

10^{-10}. This condition was reached in approximately 1000 overrelaxation sweeps.

From each gauge-fixed SU(2) link, the U(1) link angle $\bar{\phi}_\mu^3$ was extracted using the formula $\bar{\phi}_\mu^3 = \arctan(U_\mu^3/U_\mu^0)$, as described in Sec. I. Then, making use of the plaquette angles $\bar{\phi}_{\mu\nu}^3$, the magnetic current $\bar{m}_\mu(x)$ was found. Although the values $\bar{m}_\mu = 0 \pm 1 \pm 2$ are allowed by Eqs. (12) and (13), the overwhelming fraction of links carrying current had $|\bar{m}_\mu| = 1$; $|\bar{m}_\mu| = 2$ rarely occurred. Only a few percent of the links actually carried current. We define f_m as the number of links with nonzero current, divided by the total number of links, $4L^4$. Our results for f_m are recorded in Table I, and agree well with those in the literature [15, 16].

B. The quark potential from monopoles

With the magnetic current $\bar{m}_\mu$ in hand, the monopole Wilson loops for SU(2) are completely determined. Applying Eqs. (7) and (8), for $\beta = 2.40$ and 2.50 all $R \times T$ loops up to 7×10 were measured and averaged over configurations. For $\beta = 2.45$, the maximum size was increased to 8×12. From the monopole Wilson loops, monopole contributions to the potential, denoted by $V_{mon}(R)$, were extracted by performing straight line fits to $\ln(\langle W(R,T)_{mon}\rangle)$ vs T. The fits for $R \geq 2$ were over the interval $T = R + 1$ to T_{max}, except for R_{max}, where $T = R$ to T_{max} was used. Finally, the monopole contribution to the string tension σ was extracted by fitting $V_{mon}(R)$ to the form $V_{mon}(R) = \alpha/R + \sigma R + V_0$, over the interval $R = 2$ to $R = R_{max}$. The results are shown in Fig. 1, and tabulated in Table II.

As a glance at either Table II or Fig. 1 shows, the monopole potentials are essentially linear at all values of R, with negligible Coulomb terms. This is expected on the basis of the discussion of Sec. I. The steps of first setting $w_\mu = 1$ on each link of the Wilson loop, and then calculating only the monopole factor $\langle W_{mon}\rangle$ of the resulting U(1) loop, effectively suppresses the Coulomb terms arising from single gluon exchange.

The crucial question for the present paper is whether or not the monopole string tensions agree with those for full SU(2). Of the various ways of determining the SU(2) string tension, the "torelon" method of Michael and Teper is perhaps the best comparison. The torelon method involves the temporal correlation between loops which are wrapped around the entire lattice in the spacial direction. The method determines the string tension directly, without the need to separate linear from Coulomb terms. For $\sqrt{\sigma}$, Michael and Teper give 0.258(2) at $\beta = 2.4$ on a 16^4 lattice and 0.185(2) at $\beta = 2.5$ on
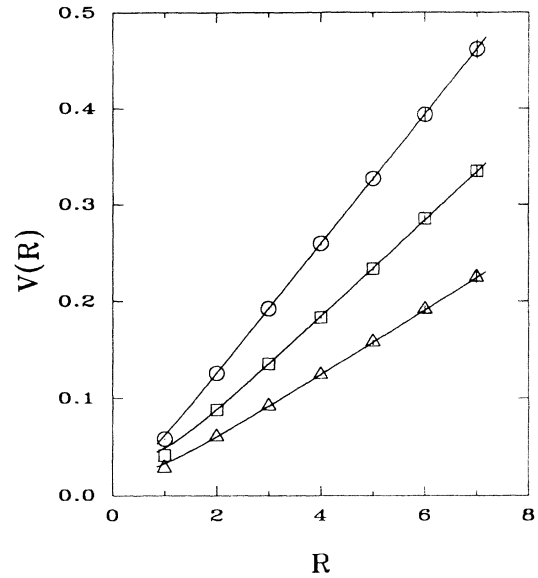

FIG. 1. The potentials extracted from monopole Wilson loops at $\beta = 2.40$ (circles), 2.45 (squares), and 2.50 (triangles). The solid lines are the linear-plus-Coulomb fits to each potential.

a 20^4 lattice [17]. Both numbers are in excellent agreement with the monopole results of Table II. As an additional check that full SU(2) results were being obtained, we performed linear-plus-Coulomb fits to our own SU(2) Wilson loops, which were obtained from the same set of configurations using an analytic form of the multihit method [18], along with the smearing method [19], as noise reduction techniques. The resulting string tension is presented in Table III, and again there is agreement with the monopole string tension to within statistical errors. Statistical errors were estimated using standard jack-knife methods. Similar results for the SU(2) string tension using monopoles have recently been obtained by Shiba and Suzuki [20]. The Coulomb terms coming from the monopoles and full SU(2) naturally differ, as explained above. No attempt to calculate the SU(2) analog of the factor $\langle W_{phot}\rangle$ is made in this paper.

C. The string tension and magnetic current loop size

To further investigate confinement via monopoles, the magnetic current was resolved into individual loops, each of which separately conserves current. Not all sizes of loops are expected to contribute to the confining potential. For example in the U(1) lattice gauge theory, it is possible to show analytically that a random distribution

TABLE I. The fraction of links carrying magnetic current at each value of β.

β	f_m
2.40	$2.75(6) \times 10^{-2}$
2.45	$1.95(1) \times 10^{-2}$
2.50	$1.36(1) \times 10^{-2}$

TABLE II. The string tension and Coulomb coefficient obtained from fits to the potentials calculated using the monopole Wilson loops at each β.

β	σ	α
2.40	0.068(2)	0.01(1)
2.45	0.051(1)	0.02(1)
2.50	0.034(1)	0.01(1)

TABLE III. The string tension and Coulomb coefficient for the full SU(2) potential at each β.

β	σ	α
2.40	0.072(3)	-0.28(2)
2.45	0.049(1)	-0.29(1)
2.50	0.033(2)	-0.29(1)

TABLE V. The monopole string tension from all current loops with $\geq$ 50, 100, and 200 links.

β	$\sigma(\geq 50)$	$\sigma(\geq 100)$	$\sigma(\geq 200)$
2.40	0.068(3)	0.066(3)	0.063(3)
2.45	0.050(3)	0.049(3)	0.044(3)
2.50	0.035(2)	0.034(2)	0.029(2)

of loops much smaller in size than the Wilson loop under consideration does not affect the long-range potential, but only renormalizes the Coulomb term. At the opposite extreme, again in U(1), it is known that the string tension can be calculated accurately using only the contribution coming from very large loops of magnetic current [21].

For the present case of SU(2), we first obtained a rough measure of how the current is distributed over loops of various size. This was done by counting the number of current-carrying links residing in loops of size up to and including 10, 20, 50, and 100 links. In Table IV, the results are shown as a fraction of the total current. As can be seen from the last column of the table, including all loops of magnetic current with up to 100 links accounts for approximately half of the current-carrying links at each coupling. The remainder of the current consists of a small number ($\sim$ 3) of loops, each containing typically several hundred links.

To study how the string tension depends on magnetic current loop size, we computed $\langle W_{mon} \rangle$ with a cut on loop size. Since experience with U(1) shows that large loops are what is important in confinement, in computing $\langle W_{mon} \rangle$ from Eq. (7), only those loops of magnetic current with more than n_{cut} links were included. The results for the string tension with $n_{cut} = 50, 100,$ and 200 are shown in Table V. For $n_{cut} = 50$ the answers are within statistical errors of the full string tension for all couplings. However, as n_{cut} is increased the string tension decreases steadily so that by $n_{cut} = 200$, there is a statistically significant deviation from the full string tension. This shows that in SU(2) lattice gauge theory, the string tension cannot be explained by retaining only the very largest loops of current, unlike the situation in U(1). To make this point quantitatively, we compare the SU(2) results with our work in U(1) lattice gauge theory on a 24^4 lattice. In Ref. [21], we considered a U(1) coupling corresponding to a string tension of $\sigma = 0.058(2)$, intermediate between the SU(2) string tension at $\beta = 2.40$ and $\beta = 2.45$. For this case and other nearby couplings, the U(1) string tension is stable under an increase of n_{cut} to at least $n_{cut} = 1000$, whereas deviations are already significant in SU(2) for $n_{cut} = 200$. Admittedly, two different lattice sizes are being compared here, but since our

TABLE IV. The fraction of total magnetic current contained in loops with $\leq$ 10, 20, 50, and 100 links.

	Max loop size			
β	10	20	50	100
2.40	0.33	0.41	0.46	0.48
2.45	0.38	0.46	0.51	0.54
2.50	0.43	0.51	0.56	0.59

16^4 SU(2) string tension is within statistical errors of the 24^4 SU(2) numbers, it is reasonable to assume that the distribution of loops we found on 16^4 would be similar to that on a 24^4 lattice.

Small loops of magnetic current play no role in confinement for either SU(2) or U(1). In the present work, we performed additional calculations of $\langle W_{mon} \rangle$ with a cut on loop size, but this time including only loops of current with less than n_{cut} links. The string tension was statistically zero for $n_{cut} = 50$ and 100 for all three values of β. The Coulomb terms were small, of magnitude $\approx$ 10% of the Coulomb term for full SU(2), and attractive in sign. For $n_{cut} = 200$, a very small contribution to the string tension was seen for $\beta = 2.5$.

We may summarize our investigation of current loops by saying that for SU(2), somewhat more than half the magnetic current plays a role in the confining potential. The string tension can be explained by retaining all magnetic current loops of intermediate size ($\approx$ 50 links) and larger. A reasonable guess for how large a loop must be to play a role in the confining potential would be a physical extent $O(1/\sqrt{\sigma})$ or greater. For U(1), there is a class of huge magnetic current loops present only in the confined phase, and the string tension can be explained with a restriction only to these very large loops.

D. Abelian–Wilson loops

Another possible route to the string tension is Eq. (10), which expresses the assumption that after partial gauge fixing, the physics of confinement is contained in the U(1) link variable ϕ_μ^3. So far, we have concentrated on the extraction of the string tension from $\langle W_{mon} \rangle$, which is based on the further assumption that the U(1) approximation to the full SU(2) Wilson loop, Eq. (10), can be factored into a photon part $\langle W_{phot} \rangle$, times a monopole part $\langle W_{mon} \rangle$. The physical motivation for this was to investigate the monopole confinement mechanism, but the method also has clear computational advantages. The noise level in the monopole Wilson loops is much lower than in the full SU(2) loops, and the string tension is easier to identify, due to the very small Coulomb terms in the monopole contribution to the potential. For completeness, we have returned to Eq. (10) to calculate the U(1) approximation to the SU(2) Wilson loops directly in terms of the link variable ϕ_μ^3, and extracted the string tension and Coulomb term. The statistical accuracy of these results is poor compared to those obtained from the full SU(2) Wilson loops, due to the absence of the multihit technique. Partial gauge fixing produces a sequence of U(1) configurations, but the U(1) action that would produce these configurations is unknown. Lacking the usual noise reduction techniques, we perform a simple
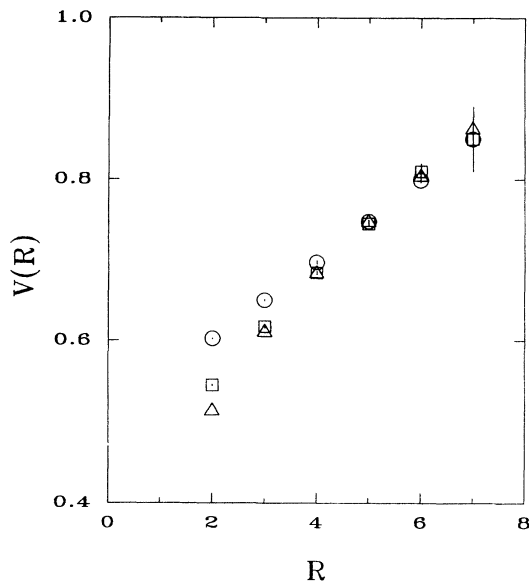
FIG. 2. Comparison of the monopole potentials (circles), the U(1) approximation to full SU(2) potentials (squares) and the full SU(2) potentials (triangles) at $\beta = 2.45$.

average over these U(1) configurations to obtain Wilson loops, and extract the string tension and Coulomb term from the resulting potential. The Coulomb term is now significant, but still less than that for full SU(2). This is expected since exchange of neutral gluons or photons is allowed in Eq. (10). The string tension is consistent with the full SU(2) string tension as well as that extracted from $\langle W_{\mathrm{mon}} \rangle$ [22]. In Fig. 2, we show the potentials derived from the full SU(2) loops, from the monopole loops, and from Eq. (10), for $\beta = 2.45$. A constant has been added to the monopole and U(1) potentials for the purpose of comparing the R dependence of the three potentials in Fig. 2. The shift of the potentials by a constant is not physically relevant in that it does not affect the functional dependence on R of the potentials. The plots for $\beta = 2.40$ and 2.50 are similar.

III. CONCLUSIONS AND SUMMARY

The present work was done entirely in the maximum Abelian gauge, stimulated by the pioneering work of Suzuki and Yotsuyanagi [3]. In an average way over the lattice, the maximum Abelian gauge forces the fluctuations in the charged sector to be as small as possible. This gauge also has a variational formulation [Eqs. (1) and (2)], and is renormalizable in the continuum limit [23]. While these are desirable features, it should be possible to capture the physics of confinement with other forms of partial gauge fixing. That this has not been possible so far in lattice calculations may have something to do with the monopole location procedure. As mentioned in Sec. I D, the location procedure finds a monopole by locating the end of its Dirac string, so if monopoles are to be correctly located using the Abelian flux over one-cubes, the gauge which is used must attach a string to

a monopole at distance scales right down to the lattice spacing. For a 't Hooft–Polyakov monopole in the continuum, the requirement that the string go all the way to the center of the monopole singles out the maximum Abelian gauge. It is easy to check that another gauge discussed by 't Hooft [1], where the charged field strength $G_{12}^{\pm}$ is set to zero, does not have this property. Starting from the "hedgehog" form of the monopole solution, going to the gauge where $G_{12}^{\pm} = 0$ will be accomplished by the same gauge transformation as going to the maximum Abelian gauge in the region far from the monopole, therefore, here there is no distinction between the two gauges. However, inside the extended structure of the monopole, the two begin to deviate and only the maximum Abelian gauge has a Dirac string extending to the origin of the monopole.[2] Assuming a similar phenomenon occurs on the lattice, a calculation which uses one-cubes to locate the monopoles in the lattice version of the gauge $G_{12}^{\pm} = 0$ may then locate the monopoles incorrectly. This interpretation of the difference between the maximum Abelian gauge and other forms of partial gauge fixing is supported by work on three-dimensional SU(2) lattice gauge theory by Trottier et al. [24]. They show that the density of monopoles as found in the maximum Abelian gauge has the correct scaling law as a function of coupling, using one-cubes to locate the monopoles. However, for other forms of partial gauge fixing such as $G_{12}^{\pm} = 0$, the correct scaling law is obtained only after the size of the cube used to locate the monopoles is increased considerably. This would say that while other forms of partial gauge fixing are in principle equivalent, for practical reasons having to do with lattice size the maximum Abelian gauge is likely to continue to be favored in calculations with monopoles.

We have presented evidence in favor of 't Hooft's picture of confinement. Although our lattice size and number of configurations is moderate, the numbers obtained from monopoles for the string tension are in excellent agreement with full SU(2) numbers on larger lattices with higher statistics. It would be of interest to repeat the present work on a larger lattice with several thousand configurations, in order to search for any possible systematic difference between the full SU(2) string tension and the string tension deduced from monopoles. A larger lattice would also allow a move toward weaker coupling and a smaller string tension. As the correlation length defined by the string tension grows, so presumably must the physical size of monopoles which has been ignored in the present work. An interesting issue is whether the string tension will continue to be dominated by the long range Coulombic fields of the monopoles. Study of the gluon propagator offers a different line of attack on this question. We are presently calculating both charged and neutral gluon propagators from our 16^4 configurations, after a final gauge fixing which puts the photon link variable ϕ_{μ}^{3} in the lattice Landau gauge [25]. A detailed report will be presented elsewhere, but our preliminary

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[2] At the center of the monopole, the hedgehog solution already satisfies $G_{12}^{\pm} = 0$.

calculations show that the neutral gluon or photon propagator is intrinsically larger and of longer range than the charged gluon propagator.

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