Shell model versus liquid drop model for strangelets

Jes Madsen
Institute of Physics and Astronomy, Aarhus University, DK-8000 Århus C, Denmark
(Received 17 January 1994)

An ansatz for the curvature contribution to the density of states for massive quarks in a bag is given and shown to reproduce exact mode-filling calculations. A mass formula for spherical lumps of three-flavor quark matter is derived self-consistently from an asymptotic expansion within the MIT bag model, taking into account bulk, surface, and curvature contributions. Shell model calculations are performed for a variety of strange quark masses and bag constants, and the results are shown to match nicely with the asymptotic expansion.

PACS number(s): 12.38.Mh, 12.39.Ba, 24.85.+p, 25.75.+r

Several experiments with relativistic heavy-ion colliders [1], as well as cosmic ray searches, have been performed or planned to test the possible (meta)stability of strange quark matter; a phase of three-flavor quark matter that could be the ground state of hadronic matter (thus more stable than iron) even at zero temperature and pressure. The first notion of this possibility seems to be due to Bodmer [2], and much work, also on the consequences for neutron stars and cosmology, has been performed since the idea was resurrected by Witten [3]. (See Ref. [4] for reviews and references on strange quark matter.)

It is therefore important to know the signatures to expect for small lumps of quark matter (strangelets) composed of up, down, and strange quarks. An important property is the mass (or energy per baryon). A calculation involving mode filling in a spherical MIT bag [5] was performed for *ud* systems by Vasak, Greiner, and Neise [6], and for two- and three-flavor systems for a few parameter sets by Farhi and Jaffe [7] and Greiner *et al.* [8]. Recently, Gilson and Jaffe [9] published a thorough investigation of low-mass strangelets for four different combinations of s-quark mass and bag constant. Such calculations, capable of showing shell effects, etc., are rather tedious, but important for an understanding of decay modes. For many applications, including generalization to finite temperature, a global mass formula analogous to the liquid drop model for nuclei is of great use.

An investigation of the strangelet mass formula within the MIT bag model was performed by Berger and Jaffe [10]. That investigation included Coulomb corrections and surface tension effects stemming from the depletion in the surface density of states due to the mass of the strange quark. Both effects were treated as perturbations added to a bulk solution with the surface contribution derived from a multiple reflection expansion.

Recently it was pointed out that another contribution to the energy, the curvature term, is dominant (and strongly destabilizing) at baryon numbers below a hundred [11]. A mass formula (liquid drop model) for systems of massless quarks including the curvature was derived in [12] and shown to fit well with shell model calculations.

However, the density of states correction due to the curvature was only known for massless quarks (and for infinite mass quarks, where the problem corresponds to the Dirichlet boundary conditions studied by Balian and Bloch [13]), whereas the surface tension is a function of the mass, vanishing for zero quark mass. Thus low-mass u and d quarks could be consistently included, but for realistic s-quark masses (expected to be in the range of 100–300 MeV), neither $m_s = 0$ nor $m_s \to \infty$ are terribly good approximations. It was argued in [11] that it seemed as if the shell model calculations of Ref. [8] required roughly two massless curvature contributions plus the surface energy. As will be shown below, the s-quark curvature contribution indeed goes through zero for intermediate s-quark mass.

In the ideal Fermi-gas approximation the energy of a system composed of quark flavors i is given by

$$E = \sum_i (\Omega_i + N_i \mu_i) + BV. \tag{1}$$

Here Ω_i, N_i, and μ_i denote thermodynamic potentials, total number of quarks, and chemical potentials, respectively. B is the bag constant and V is the bag volume.

In the multiple reflection expansion framework of Balian and Bloch [13], the thermodynamical quantities can be derived from a density of states of the form

$$\frac{dN_i}{dk} = 6\left\{\frac{k^2 V}{2\pi^2} + f_S\left(\frac{m_i}{k}\right) kS + f_C\left(\frac{m_i}{k}\right) C + \cdots\right\}, \tag{2}$$

where area $S = \oint dS$ ($= 4\pi R^2$ for a sphere) and curvature $C = \oint \left(\frac{1}{R_1} + \frac{1}{R_2}\right) dS$ ($= 8\pi R$ for a sphere). Curvature radii are denoted R_1 and R_2. For a spherical system $R_1 = R_2 = R$. The functions f_S and f_C will be discussed below.

In terms of volume, surface, and curvature densities, $n_{i,V}$, $n_{i,S}$, and $n_{i,C}$, the number of quarks of flavor i is

$$N_i = \int_0^{k_{Fi}} \frac{dN_i}{dk} dk = n_{i,V} V + n_{i,S} S + n_{i,C} C, \tag{3}$$

with Fermi momentum $k_{Fi} = (\mu_i^2 - m_i^2)^{1/2} = \mu_i(1 - \lambda_i^2)^{1/2}$; $\lambda_i \equiv m_i/\mu_i$.

The corresponding thermodynamical potentials are related by

$$\Omega_i = \Omega_{i,V}V + \Omega_{i,S}S + \Omega_{i,C}C, \tag{4}$$

where $\partial\Omega_i/\partial\mu_i = -N_i$, and $\partial\Omega_{i,j}/\partial\mu_i = -n_{i,j}$. The volume terms are given by

$$\Omega_{i,V} = -\frac{\mu_i^4}{4\pi^2}\left((1-\lambda_i^2)^{1/2}(1-\frac{5}{2}\lambda_i^2) + \frac{3}{2}\lambda_i^4 \ln\frac{1+(1-\lambda_i^2)^{1/2}}{\lambda_i}\right), \tag{5}$$

$$n_{i,V} = \frac{\mu_i^3}{\pi^2}(1-\lambda_i^2)^{3/2}. \tag{6}$$

The surface contribution from massive quarks is derived from $f_S(m/k) = -\{1 - (2/\pi)\arctan(k/m)\}/8\pi$ as [10]

$$\Omega_{i,S} = \frac{3}{4\pi}\mu_i^3\left[\frac{(1-\lambda_i^2)}{6} - \frac{\lambda_i^2(1-\lambda_i)}{3} - \frac{1}{3\pi}\left(\arctan\left[\frac{(1-\lambda_i^2)^{1/2}}{\lambda_i}\right] - 2\lambda_i(1-\lambda_i^2)^{1/2} + \lambda_i^3\ln\left[\frac{1+(1-\lambda_i^2)^{1/2}}{\lambda_i}\right]\right)\right], \tag{7}$$

$$n_{i,S} = -\frac{3}{4\pi}\mu_i^2\left[\frac{(1-\lambda_i^2)}{2} - \frac{1}{\pi}\left(\arctan\left[\frac{(1-\lambda_i^2)^{1/2}}{\lambda_i}\right] - \lambda_i(1-\lambda_i^2)^{1/2}\right)\right]. \tag{8}$$

For massless quarks $\Omega_{i,S} = n_{i,S} = 0$, whereas $f_C(0) = -1/24\pi^2$ gives [11,12] $\Omega_{i,C} = \mu_i^2/8\pi^2$; $n_{i,C} = -\mu_i/4\pi^2$.

The curvature terms have never been derived for massive quarks, but as will be shown in this paper, the following ansatz (found from analogies with the surface term and other known cases) works:

$$f_C\left(\frac{m}{k}\right) = \frac{1}{12\pi^2}\left\{1 - \frac{3}{2}\frac{k}{m}\left(\frac{\pi}{2} - \arctan\frac{k}{m}\right)\right\}. \tag{9}$$

This expression has the right limit for massless quarks ($f_C = -1/24\pi^2$) and for infinite mass, which corresponds to the Dirichlet boundary conditions studied by Balian and Bloch [13] ($f_C = 1/12\pi^2$). Furthermore, the expression gives perfect fits to mode-filling calculations (see the figures and the discussion below). From this ansatz one derives the following thermodynamical potential and density:

$$\Omega_{i,C} = \frac{\mu_i^2}{8\pi^2}\left[\lambda_i^2\ln\frac{1+(1-\lambda_i^2)^{1/2}}{\lambda_i} + \frac{\pi}{2\lambda_i} - \frac{3\pi\lambda_i}{2} + \pi\lambda_i^2 - \frac{1}{\lambda_i}\arctan\frac{(1-\lambda_i^2)^{1/2}}{\lambda_i}\right]; \tag{10}$$

$$n_{i,C} = \frac{\mu_i}{8\pi^2}\left[(1-\lambda_i^2)^{1/2} - \frac{3\pi}{2}\frac{(1-\lambda_i^2)}{\lambda_i} + \frac{3}{\lambda_i}\arctan\frac{(1-\lambda_i^2)^{1/2}}{\lambda_i}\right]. \tag{11}$$

With these prescriptions the differential of $E(V, S, C, N_i)$ is given by

$$dE = \sum_i (\Omega_{i,V}dV + \Omega_{i,S}dS + \Omega_{i,C}dC + \mu_i dN_i) + BdV. \tag{12}$$

Minimizing the total energy at fixed N_i by taking $dE = 0$ for a sphere gives the pressure equilibrium constraint

$$B = -\sum_i \Omega_{i,V} - \frac{2}{R}\sum_i \Omega_{i,S} - \frac{2}{R^2}\sum_i \Omega_{i,C}. \tag{13}$$

Eliminating B from Eq. (1) then gives the energy for a spherical quark lump as

$$E = \sum_i\left(N_i\mu_i + \frac{1}{3}\Omega_{i,S}S + \frac{2}{3}\Omega_{i,C}C\right). \tag{14}$$

The importance of the curvature contribution is demonstrated in Fig. 1, which shows the energy per baryon for fixed bag constant of a bag filled with one flavor of quarks, quark mass ranging from 1 to 450 MeV. The shell model results are derived in a similar manner as described by Gilson and Jaffe [9], except that no zero-point correction has been added. One sees that a significant positive curvature energy arises for low quark mass (where the surface term approaches zero). For a quark mass around 150 MeV (for the given choice of B) the curvature term is zero, and the surface term alone reproduces the shell model calculations, whereas for higher m_i the surface term overshoots, but the shell model results are reproduced when including the now *negative* curvature contribution. For high masses one approaches the Dirichlet boundary conditions, but since quark masses are always lower than the chemical potential rather than infinite, the Dirichlet limit is only a rough approximation, and the full surface and curvature terms should be used.

Equally nice fits are obtained for other choices of B and m_s. Small changes in the ansatz for the curvature density of states lead to clearly visible errors in the fits to the mode-filling calculations, so even though the curvature term has not been derived from the multiple reflection expansion (and attempts to do it have been unsuccessful), it seems fair to conclude that this is the proper term, and that surface plus curvature corrections are sufficient (and equally necessary) to reproduce the overall behavior of mode-filling calculations.

Figures 2 and 3 compare shell model and liquid drop model calculations for three-flavor strangelets for massless u and d quarks, and massive s quarks for a variety of parameters. Again, the overall behavior of the energy per baryon is fitted perfectly by the liquid drop results. For low m_s, the energy is roughly that of bulk plus three massless curvature contributions. For intermediate m_s, two massless curvature terms plus the s-quark surface term are important, and for high s-quark mass, where no s quarks are present, the energy per baryon is given by E/A in bulk plus the curvature terms from two massless quarks. (The approach of Ref. [10] gives *constant* E/A in the low as well as high m_s limit since only surface tension is included, and for intermediate values of m_s

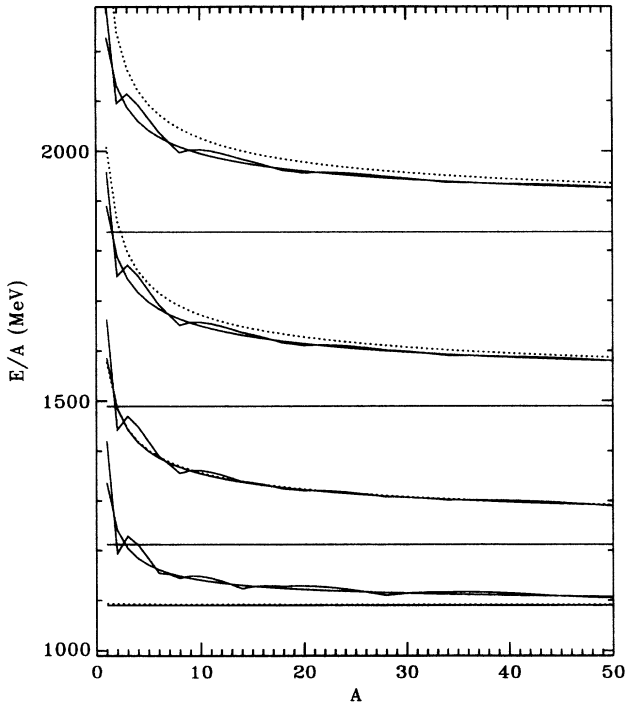

FIG. 1. Energy per baryon of one-flavor quark lumps at fixed bag constant ($B^{1/4}$ = 145 MeV), for quark masses of 1, 150, 300, and 450 MeV (bottom to top). For each mass four curves are shown: Horizontal lines are the bulk values; dotted curves bulk plus surface terms; spiky curves are the shell model calculations, and smooth curves on top of the shell model results are the liquid drop model results including surface *and* curvature. Notice that the dotted curve is almost indistinguishable from the bulk curve for m_s = 1 MeV, and overlaps with the full calculation for m_s = 150 MeV.

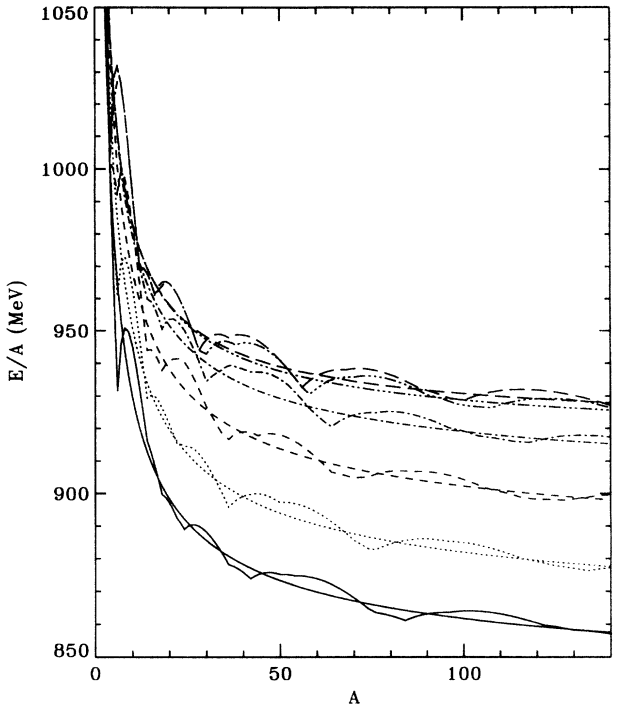

FIG. 2. Energy per baryon for three-flavor strangelets for $B^{1/4}$ = 145 MeV (the lowest possible value for *ud* quark matter to remain unstable). Pairs of curves with the same signature show shell model and liquid drop model calculations for the same m_s ranging from 50 to 300 MeV in steps of 50 MeV (bottom to top). For even higher m_s no s-quarks are present, and results look like those for m_s = 300 MeV.

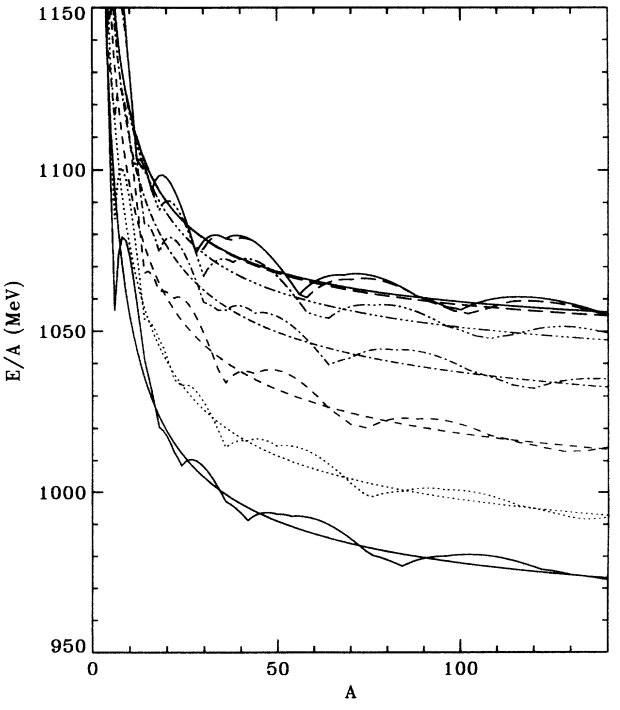

FIG. 3. As Fig. 2, but for $B^{1/4}$ = 165 MeV (the highest possible value for stable *uds*-quark matter with m_s = 0). Now the highest m_s that can be distinguished is 350 MeV.

the significant contribution of u and d quark curvature is lacking).

The calculations neglect Coulomb energies, the phenomenological zero-point energy, and the gluon exchange described by the strong coupling, α_s. Coulomb energies can be self-consistently included (see [12]), but are at most a few MeV. The zero-point energy was included in [9] as $-1.84/R$, and reduces the energies for very small A somewhat. In E/A this term contributes (for *fixed* chemical potentials, which is not quite correct for $A \to 1$ [14]) like $A^{-4/3}$, compared to curvature ($A^{-2/3}$) and surface ($A^{-1/3}$). Significant changes occur only for $A < 10$, and can also be accounted for by an extension of the liquid drop model. There is no easy way to include the strong coupling.

A detailed study of strangelet decay modes (an important issue for experimental strangelet searches) must ultimately rely on shell model calculations. The main scope of this paper has been to show, that the general structure of shell model results for multiquark systems can be explained by a liquid drop model mass-formula including volume, surface, and curvature contributions. Since such calculations are much easier to perform than the shell model calculations, this allows the overall structure of strangelets to be studied for a range of parameters, and facilitates generalizations to finite temperature, studies of phase transitions and mixed phase structures in heavy-ion physics, cosmology, and neutron stars [15]. For a consistent liquid drop treatment, the full curvature term for a massive quark, introduced here for the first time, turns out to be crucial.

[1] Some of these experiments are reviewed by J. Sandweiss, in *Strange Quark Matter in Physics and Astrophysics*, Proceedings of the International Workshop, Aarhus, Denmark, 1991, edited by J. Madsen and P. Haensel [Nucl. Phys B (Proc. Suppl.) **24B**, 234 (1991)]; see also H. J. Crawford *et al.*, *ibid.*, p. 251; F. S. Rotondo, *ibid.*, p. 265; M. Aoki *et al.*, Phys. Rev. Lett. **69**, 2345 (1992); H. J. Crawford, M. S. Desai, and G. L. Shaw, Phys. Rev. D **48**, 4474 (1993); K. Borer *et al.*, Phys. Rev. Lett. **72**, 1415 (1994).

[2] A. R. Bodmer, Phys. Rev. D **4**, 1601 (1971).

[3] E. Witten, Phys. Rev. D **30**, 272 (1984).

[4] *Strange Quark Matter in Physics and Astrophysics* [1].

[5] T. A. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D **12**, 2060 (1975).

[6] D. Vasak, W. Greiner, and L. Neise, Phys. Rev. C **34**, 1307 (1986).

[7] E. Farhi and R. L. Jaffe, Phys. Rev. D **30**, 2379 (1984).

[8] C. Greiner, D.-H. Rischke, H. Stöcker, and P. Koch, Phys. Rev. D **38**, 2797 (1988).

[9] E. P. Gilson and R. L. Jaffe, Phys. Rev. Lett. **71**, 332 (1993).

[10] M. S. Berger and R. L. Jaffe, Phys. Rev. C **35**, 213 (1987); **44**, 566(E) (1991).

[11] J. Madsen, Phys. Rev. Lett. **70**, 391 (1993).

[12] J. Madsen, Phys. Rev. D **47**, 5156 (1993). An error in the curvature term in Eq. (11) of that paper propagates through, but fortunately causes only insignificant errors in the numerical results. Equations (12), (13), and (14) in the present paper are the correct versions of Eqs. (11), (14), and (17) in that paper (apart from Coulomb terms, which are unchanged). In addition, Eqs. (35) and (37) in the paper should just be $E = 4BV \propto B^{1/4}$, and the factor of 5 should be 4 in Eq. (38). The rest of the equations and conclusions remain unchanged.

[13] R. Balian and C. Bloch, Ann. Phys. (N.Y.) **60**, 401 (1970).

[14] Crawford, Desai, and Shaw [1] employ a mass formula with surface and curvature contributions to E/A of $cb_6A^{-1/3} + cb_7A^{-2/3}$, where b_6, b_7, and c are constants. This form is not appropriate for small A in view of the present investigation. The relative weight of the two terms depends on B and m_s, and the A dependence is not as simple due to changes in μ_i. For constant μ_i calculated in bulk the mass formulas for $B^{1/4} = 145$ MeV (as in Fig. 2) are (with m_s running from 0 to 300 in steps of 50; all energies in MeV): $E/A = 829 + 351A^{-2/3}$; $835 + 61A^{-1/3} + 277A^{-2/3}$; $852 + 83A^{-1/3} + 241A^{-2/3}$; $874 + 77A^{-1/3} + 232A^{-2/3}$; $896 + 53A^{-1/3} + 242A^{-2/3}$; $911 + 22A^{-1/3} + 266A^{-2/3}$; $917 + 296A^{-2/3}$. These "bulk approximations" are generally good to better than 2 MeV for $A > 100$, 5 MeV for $A \approx 50$, 10 MeV for $A \approx 10$, and 20 MeV for $A \approx 5$, when compared to the complete liquid drop model results shown in the figures. They always undershoot because the actual chemical potentials increase when A decreases. For massless quarks the results scale with $B^{1/4}$. For finite m_s no simple scaling applies. For $B^{1/4} = 165$ MeV (Fig. 3) one finds $985 + 93A^{-1/3} + 265A^{-2/3}$ for $m_s = 150$, and $1027 + 46A^{-1/3} + 284A^{-2/3}$ for $m_s = 250$.

[15] For recent discussions of finite-size effects in these contexts, see I. Mardor and B. Svetitsky, Phys. Rev. D **44**, 878 (1991); N. K. Glendenning, *ibid.* **46**, 1274 (1992); H. Heiselberg, C. J. Pethick, and E. F. Staubo, Phys. Rev. Lett. **70**, 1355 (1993); M. L. Olesen and J. Madsen, Phys. Rev. D **47**, 2313 (1993); **49**, 2698 (1994).