Radiative decay of vector quarkonium: Constraints on glueballs and light gluinos

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Given a resonance of known mass, width, and J^{PC}, we can determine its gluonic branching fraction $b_{R \to gg}$ from data on its production in radiative vector quarkonium decay $V \to \gamma R$. For most resonances $b_{R \to gg}$ is found to be $\sim 10\%$, consistent with being $q\bar{q}$ states, but we find that both pseudoscalars observed in the 1440 MeV region have $b_{R \to gg} \sim \frac{1}{2} - 1$ and $b(f_{0^{++}} \to gg) \sim \frac{1}{2}$. As data improve, $b_{R \to gg}$ should be a useful discriminator between $q\bar{q}$ and gluonic states and may permit quantitative determination of the extent to which a particular resonance is a mixture of glueball and $q\bar{q}$. We also examine the regime of validity of PQCD for predicting the rate of $V \to \gamma \eta_{\tilde{g}}$, the "extra" pseudoscalar bound state which would exist if there were light gluinos. From the CUSB limit on peaks in $\Upsilon \to \gamma X$, the mass range 3 GeV $\lesssim m(\eta_{\tilde{g}}) \sim 7$ GeV can be excluded. An experiment must be significantly more sensitive to exclude an $\eta_{\tilde{g}}$ lighter than this.

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I. INTRODUCTION

Radiative quarkonium decay has been experimentally studied both exclusively and inclusively. It is a particularly auspicious reaction for producing glueball resonances, as is evident from Fig. 1. In exclusive channels such as $J/\psi \to \gamma K \bar{K} \pi$, many resonances in the $K\bar{K}\pi$ invariant mass have been observed. In the inclusive process $\Upsilon \to \gamma X$, the absence of peaks in the photon energy spectrum provides an upper bound on the production of resonances[1][1, 2]:

$$\frac{\Gamma(\Upsilon \to \gamma R)}{\Gamma(\Upsilon \to \text{all})} \lesssim 8 \times 10^{-5}. \tag{1}$$

Qualitatively, a resonance that is prominent in $V \to \gamma R$ but not prominent in hadronic scattering can be a good candidate for a glueball. More quantitatively, "stickiness"[3], that is proportional to $\Gamma(V \to \gamma R)$ divided by the two-photon partial width $\Gamma(R \to \gamma\gamma)$, should be significantly larger for glueballs than for $q\bar{q}$ mesons. In the following we propose another quantitative measure of the gluonic content of a resonance: its branching ratio to gluons $b_{R \to gg}$. We describe how to extract it from experimental information on the width of the resonance and

the branching ratio for its production in radiative quarkonium decay $b(V \to \gamma R) \equiv \Gamma(\Upsilon \to \gamma R)/\Gamma(\Upsilon \to \text{all})$. We work in a naive parton approximation, but expect that the method can be shown to be more general. We will see that it seems to be a promising discriminator between glueballs and $q\bar{q}$ states, with glueball candidates having $b_{R \to gg} \sim \frac{1}{2} - 1$ and $q\bar{q}$ states having $b_{R \to gg} \sim \alpha_s^2 \sim 0.1$.

If the resonance R is a massive $Q\bar{Q}$ state such as the η_c, the branching ratio $b(V \to \gamma R)$ can be reliably calculated in perturbative QCD (PQCD) with a nonrelativistic potential for both V and R [4]. However in the interesting cases of R being a glueball or a bound state of
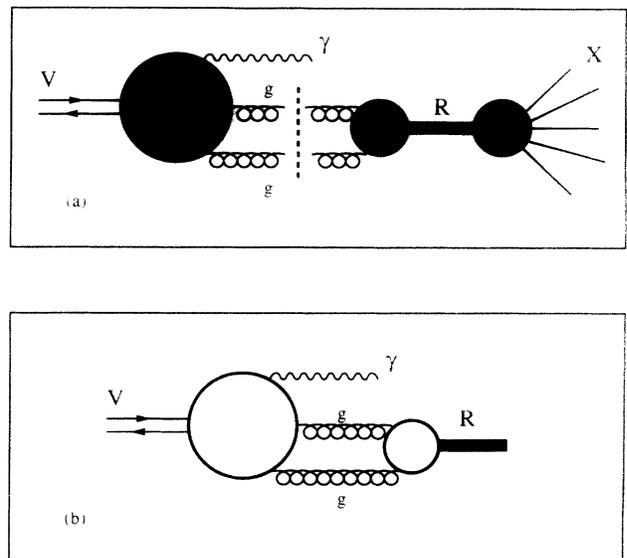
FIG. 1. The dominant contribution to $V \to \gamma R$.

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[1]The actual limit is slightly dependent on the mass of the resonance, but varies less than a factor of 2 as the mass varies between 1.5 GeV and 8 GeV. The exact value of the limit does not matter much when the resonance mass is large, as will be seen below, so we take the value in the range which is most relevant to analyzing data, ~ 2 GeV.

light gluinos,[2] no reliable means exists for making an absolute prediction for $b(V \to \gamma R)$. Therefore up to now we have been unable to make quantitative use of the exclusive data and the upper limit (1) to address the question of whether a given resonance is a glueball $q\bar{q}$ or possibly $\tilde{g}\tilde{g}$ state. This paper proposes a means to rectify this situation.

Let us begin by reviewing the case that the R is a bound state of massive quarks and V is the Υ. Since the mass of the b quark is large, PQCD is believed to be a reliable means of computing the decay $\Upsilon \to \gamma gg$. If the produced resonance R also contains heavy quarks, then PQCD can be reliably used to compute the branching ratio $b(\Upsilon \to \gamma R)$ in terms of $|R(0)|^2$, the resonance wave function at short distance,[3] α_s, and m_R. In practice, one takes a nonrelativistic model for the $Q\bar{Q}$ potential and fixes its parameters to give a correct prediction for $\Gamma(\eta_c \to e^+e^-)$, which also depends on $|R(0)|^2$. Having fixed the parameters of the potential, $|R(0)|^2$ for the other $Q\bar{Q}$ resonances is determined, assuming that they are described by the same nonrelativistic potential model. Following this procedure, the branching fractions $b(V \to \gamma R)$ have been predicted for the known $Q\bar{Q}$ mesons [4, 10] and are found to be small enough that they would not have been seen in the CUSB experiment. Kuhn [11] showed how to obtain $b(V \to \gamma R)$, when R is a light quark meson, in terms of its decay constant, e.g., f_π for R a pion.

If there were a gluino with mass $m_{\tilde{g}}$ large enough for the nonrelativistic potential description to be valid, we would have an extra O^{-+} resonance with[4] $m_R \approx 2m_{\tilde{g}}$. In this case, PQCD can be used to calculate $b(V \to \gamma\eta_{\tilde{g}})$[7–9]. The result is larger than that for the η_c by a factor ~ 21.5 due to the gluino being a color octet rather than triplet as we will detail below. For $m_{\eta_{\tilde{g}}} \gtrsim 3$ GeV, the PQCD calculation with nonrelativistic potential is probably a good enough approximation that the data can be used to exclude a gluino.

Of course the wave functions of glueballs and of mesons made of light quarks (or gluinos) cannot be treated with a nonrelativistic potential model. No one would try to calculate the width of a glueball of mass $\sim 1.5 - 2$, or even 3, GeV on the basis of knowing the η_c width. Nevertheless, we shall see that [Eq .(16) below] $b(\Upsilon \to \gamma R)$ for R a glueball, or a light $q\bar{q}$ or gluino resonance, can be estimated in terms of the observed width of the resonance Γ_R and its branching ratio to gluons $b_{R \to gg}$. Thus for a given resonance whose width has been measured,

by comparing this prediction with the CUSB limit we find an upper limit on $b_{R \to gg}$ for that resonance. For resonances observed in $J/\psi \to \gamma K\bar{K}\pi$, we can extract the product $b_{R \to gg} b(R \to K\bar{K}\pi)$. When $b(R \to K\bar{K}\pi)$ is known, $b_{R \to gg}$ can be determined. Otherwise the requirement $b(R \to K\bar{K}\pi) \lesssim 1$ provides a lower limit on $b_{R \to gg}$. In some cases these upper and lower bounds are quite close to one another, as we shall see below. If $b_{R \to gg} \sim 1$, it is a viable glueball candidate, while for a $q\bar{q}$ resonance we would expect $b_{R \to gg} \sim \alpha_s^2 \sim 1/10$. Mixing between $q\bar{q}$ and glueball resonances will give intermediate values of $b_{R \to gg}$. Although the data are still inadequate to draw firm conclusions, we will see possible examples of all three cases.

Using the formalism developed below, the CUSB data can be turned into an upper limit on the gluonic width of any resonance produced in $\Upsilon \to \gamma R$. This is the best way to quote a limit on a possible $\eta_{\tilde{g}}$, since if it is not an already-observed resonance, its width is unknown. The question then becomes whether the allowed width is consistent with theoretical expectations for its width, for a given mass.

II. UNITARITY CALCULATION OF THE ABSORPTIVE CONTRIBUTION

Since we want a method of computing $b(V \to \gamma R)$ that is not limited to R being a massive $Q\bar{Q}$ state, let us see how far we can go with unitarity and analyticity. We begin by computing the "unitarity lower bound" on $\Gamma(V \to \gamma R)$ coming from the two gluon intermediate state, as illustrated in Fig. 1. This is an idea that cannot be precisely defined, since the gluon is not an asymptotic state. Our procedure can be made more rigorous by introducing a scale on which one sees two gluons in the intermediate state and does not resolve them further. This scale will be related to the ir cutoff and uv renormalization which must be introduced when working beyond tree approximation. However we begin with the most naive approach. We take the gluons to be massless. As a consistency check that this naive approach is reasonable, we verify below that giving the gluons masses $\sim \Lambda_{QCD}$ makes no significant difference to the conclusions. We will return later to the question of multigluon contributions and the resolution-size dependence.

In this approximation, the absorptive part of $\mathcal{M}^R_{V \to \gamma X}$ can be fixed in terms of the width $\Gamma(R \to gg)$, the inclusive radiative decay rate $\Gamma(V \to \gamma gg)$, and the rate $\Gamma(R \to X)$ as follows. Since we can safely ignore interactions between the photon and the final resonance R, unitarity tells us

$$\mathcal{M}_{V \to \gamma X}(P, k, \{p_i\})$$
$$= \sum_n \mathcal{M}_{V \to \gamma n}(P, k) \, \mathcal{M}_{n \to X}(s, \{p_i\}) \,, \quad (2)$$

where n labels the intermediate state and P, k, and $\{p_i\}$ are the four-momenta of V, γ, and the final state hadrons in X. s is the (invariant mass)^2 of the state X. Although J^{PC} of the resonance is fixed, in general more than one L, S state of two gluons can contribute for a given J^{PC}.

Now we want to rewrite (2) as an integral over s by inserting

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[2]While beam dump and collider experiments have ruled out light gluinos which decay in the apparatus, longer lived gluinos are not ruled out, except within certain ranges of mass and lifetime [5, 6]. Long-lived gluinos would form hadrons, with the ground state $\tilde{g}\tilde{g}$ being 0^{-+}. Demonstrating that the hadron spectrum cannot accomodate an additional flavor singlet pseudoscalar would thus exclude long-lived gluinos[7–9].

[3]For other than pseudoscalar resonances, the derivative of the short-distance wave function enters.

[4]Of course it can mix with nearby glueball or $q\bar{q}$ resonances, but we keep this discussion simple and we neglect mixing. It can be introduced with no conceptual difficulty.

$$1 = \int ds \, \delta^{(4)}\left(p - \sum_i p_i\right) d^4p \, \delta(s - p^2)$$

$$= \int \frac{ds}{2\pi} \frac{d^3\mathbf{p}}{(2\pi)^3 2E_p} (2\pi)^4 \, \delta^{(4)}\left(p - \sum_i p_i\right). \qquad (3)$$

where p is the four-momentum of the state X and $E_p \equiv \sqrt{\mathbf{p}^2 + s}$. Assuming that just a single two-gluon state dominates, or if more than one is important that their contributions add incoherently, we obtain

$$\Gamma^R_{V \to \gamma X} = \int \frac{ds}{2\pi} \left\{ \frac{1}{2M_V} \int \frac{d^3\mathbf{k}}{(2\pi)^3 2E_k} \frac{d^3\mathbf{p}}{(2\pi)^3 2E_p} (2\pi)^4 \delta^4(P - k - p)|\mathcal{M}_{V \to \gamma n}(P, k)|^2 \right\}$$

$$\times \left\{ \prod_i \left(\frac{d^3\mathbf{p_i}}{(2\pi)^3 E_{p_i}} \right) (2\pi)^4 \delta^4 \left(p - \sum_i p_i\right) |\mathcal{M}^R_{n \to X}(s, \{p_i\})|^2 \right\}$$

$$= \int \frac{ds}{2\pi} \frac{d\Gamma_{V \to \gamma n}}{ds} \left\{ \prod_i \left(\frac{d^3\mathbf{p_i}}{(2\pi)^3 E_{p_i}} \right) (2\pi)^4 \delta^4 \left(p - \sum_i p_i\right) |\mathcal{M}^R_{n \to X}(s, \{p_i\})|^2 \right\}, \qquad (4)$$

where

$$\frac{d\Gamma_{V \to \gamma n}}{ds} \equiv \frac{1}{2M_V} \int \frac{d^3\mathbf{k}}{(2\pi)^3 2E_k} \frac{d^3\mathbf{p}}{(2\pi)^3 2E_p} (2\pi)^4 \delta^4(P - k - p)|\mathcal{M}_{V \to \gamma n}(P, k)|^2. \qquad (5)$$

We define $\mathcal{P}^J_{S;L}$ to be the probability that the two gluons with spins s_1 and s_2 in a total spin state S and having orbital angular momentum L will combine to form the state with the J^{PC} of the resonance R. Then

$$\frac{d\Gamma_{V \to \gamma n}}{ds} = \frac{d\Gamma_{V \to \gamma gg}}{ds} \mathcal{P}^J_{S;L}. \qquad (6)$$

$\frac{d\Gamma_{V \to \gamma gg}}{ds}$ can in principle be taken directly from the measured inclusive radiative decay spectrum or, due to the heavy mass of the quark in V, should be reliably given by PQCD. If the latter route is taken, one would just project onto the relevant J^{PC} for the two gluons and automatically include the correct $\mathcal{P}^J_{S;L}$ [10]. Although within error bars the data on $\frac{d\Gamma_{V \to \gamma X}}{ds}$ agree with the PQCD predictions, we adopt here the PQCD approach since the data on $\frac{d\Gamma_{V \to \gamma X}}{ds}$ have large error bars for the s range of greatest interest, and projection onto the correct J^{PC} state of the two gluons is most reliably done using PQCD.[5]

Returning to (4), we require the matrix element $\mathcal{M}^R_{n \to X}$. For s near m^2_R it is given by the Breit-Wigner expression

$$\mathcal{M}^R_{n \to X} \equiv \frac{\mathcal{M}_{n \to R}\mathcal{M}_{R \to X}}{(s - m^2_R) + im_R\Gamma_R}, \qquad (7)$$

where Γ_R is the total decay width of the resonance. Now, neglecting possible variation of the matrix elements with s over the width of the resonance, the expression in large curly brackets in (4) is just

$$\{\ \} = \left\{ \frac{1}{\Phi_n(s)} \left(\frac{4 \, m^2_R \, \Gamma_{R \to n}\Gamma_{R \to X}}{(s - m^2_R)^2 + m^2_R \, \Gamma^2_R} \right) \right\}, \qquad (8)$$

where m_R is the mass of the resonance and $\Gamma_{R \to n}$, $\Gamma_{R \to X}$ and Γ_R are its partial and total decay widths in obvious notation. Equation (8) follows since

$$\int \prod_i \left[\frac{d^3\mathbf{p_i}}{(2\pi)^3 2E_i} \right] (2\pi)^4 \, \delta^{(4)}\left(p - \sum_i p_i\right) |\mathcal{M}_{R \to X}|^2$$

$$= 2 \, m_R \, \Gamma_{R \to X} \qquad (9)$$

and, in the approximation that $|\mathcal{M}_{n \to R}|^2$ is approximately constant over the resonance,

$$|\mathcal{M}_{n \to R}|^2 = \frac{2 \, m_R}{\Phi_n(s)} \, \Gamma_{R \to n}. \qquad (10)$$

The phase space factor $\Phi_n(s)$ is $\frac{\lambda(s)}{8\pi}$, with

$$\lambda(s) \equiv \frac{1}{s} \sqrt{s^2 + m^4_1 + m^4_2 - 2m^2_1m^2_2 - 2s \, m^2_1 - 2s \, m^2_2} \qquad (11)$$

for a two particle intermediate state. For real gluons $m_1 = m_2 = 0$ and $\lambda(s) = 1$, while for $m_1 = m_2 = 200$ (500) MeV, $\lambda[(1.5\text{GeV})^2] = 0.96$ (0.75). Thus taking gluons to be massless or to have masses $O(\Lambda_{\text{QCD}})$ makes only a small difference in the conclusions,[6] so we simply set $\lambda(s) = 1$ hereafter.

Putting together Eqs. (4) and (8), we obtain the absorptive contribution to the rate

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[5]See [12] for a detailed treatment when data rather than PQCD are used.

[6]Except when R is a 1^{++}, in which case the leading absorptive amplitude vanishes for strictly massless gluons and the more rigorous treatment introducing a scale would be essential to obtaining a reliable result.

$$\Gamma^{R \ (abs)}_{V \to \gamma X} = 16\pi \ \mathcal{P}^J_{S;L} b_{R \to gg} b_{R \to X}$$

$$\times \int \frac{ds}{\pi \lambda(s)} \frac{d\Gamma_{V \to \gamma gg}}{ds} \frac{m^2_R \Gamma^2_R}{(s - m^2_R)^2 + m^2_R \Gamma^2_R} \ . \tag{12}$$

In the narrow width limit,

$$\lim_{\Gamma \to 0} \left[\frac{m \ \Gamma/\pi}{(s - m^2)^2 + m^2 \ \Gamma^2} \right] = \delta(s - m^2) \ . \tag{13}$$

As a result, the absorptive contribution to the branching ratio for $V \to \gamma R$ in the narrow resonance limit is

$$b^{(abs)}_{V \to \gamma R} = 16\pi \ m_R \Gamma_R \mathcal{P}^J_{S;L} \left(\frac{1}{\Gamma_V} \frac{d\Gamma_{V \to \gamma gg}}{ds} \Big|_{s = m^2_R} \right) b_{R \to gg}, \tag{14}$$

where Γ_V is the total width of the initial vector meson and $b_{R \to gg}$ is the gluonic branching fraction of[7] R.

Note that even when m_R is of order 1 GeV our unitarity calculation with a two-gluon intermediate state is a good approximation. This is because the quarks in V are quite massive so that three hard gluons are suppressed by $O(\alpha_s(m_b))$ in their contribution to $\Gamma(V \to \gamma X)$ compared to that of two gluons. Moreover, in the partonic spirit of this paper, by taking the resolution size of the "effective" gluons we are discussing to be large enough, we can arrange that[8] $\Gamma(R \to gg) \gg \Gamma(R \to ggg)$. This just means that where we encountered $\Gamma_R b_{R \to gg}$ in our our expression for the absorptive contribution to the width, we actually mean $\Gamma_R b_{R \to \text{tot gluonic}}$. With this understanding, that $b_{R \to gg}$ is to be identified with $b_{R \to \text{tot gluonic}}$, (14) holds even for light m_R.

III. DETERMINATION OF THE FULL AMPLITUDE

The number of gluons in the intermediate state, as well as the distinction between real and virtual gluons, depends on the scale size which has been chosen. This is analogous to how the identification of an event at the CERN e^+e^- collider LEP as a two-, three-, or four-jet event depends on the resolution size chosen for the jets. With a low-resolution definition of a gluon, most of the amplitude for $V \to \gamma X$ will be contained in the two-gluon state, while as the resolution is increased, states with more gluons will become more important. Furthermore, a state that appears under high resolution to contain three real gluons would appear to have one real and one virtual gluon as the resolution is lowered and two of

the gluons are merged. While real and absorptive contributions are separately resolution-size dependent, the predicted total rate of interest will not be. It is clearly important to compute real and imaginary parts consistently, however.

We might try to estimate the real part of the amplitude whose absorptive part we determined in the preceding section by using dispersion relations, avoiding the use of PQCD except to determine subtraction constants when needed. The feasibility of this idea can be assessed in the regime of applicability of PQCD by trying to recover the real part of the full PQCD amplitude for $\Upsilon \to \gamma \eta_c$ from its absorptive part [see Eq. (17)] in the physical region $m_R < m_V$. Doing so, one sees that the cut corresponding to $R \to \gamma V$ is essential to obtaining the correct result,[9] so that we cannot use dispersion relations over experimentally determined quantities to obtain the full amplitude from the absorptive contribution, and must find some alternative.

Even when m_R is small, PQCD gives a good approximation for the $V \to \gamma gg$ vertex (see Fig. 1), due to the heavy quark in V. The problem when m_R is small and the resolution scale of the gluons is large is that PQCD and the nonrelativistic potential do not give a correct description of the coupling of R to the two gluons. Perturbatively, the rate for $R \to gg$ is, for various[10] J^{PC}'s,

$$\Gamma_{0^{-+}} = \frac{8}{3} \frac{\alpha^2_s}{m^2_R} |R_S(0)|^2 \ , \tag{15a}$$

$$\Gamma_{0^{++}} = 96 \frac{\alpha^2_s}{m^4_R} |R'_P(0)|^2 \ , \tag{15b}$$

$$\Gamma_{1^{++}} \simeq 9.6 \frac{\alpha^2_s}{m^4_R} |R'_P(0)|^2 \ , \tag{15c}$$

$$\Gamma_{2^{++}} = \frac{128}{5} \frac{\alpha^2_s}{m^4_R} |R'_P(0)|^2 \ . \tag{15d}$$

$R(0)$ is the radial wave function of the bound state at $r = 0$ and $R'(0)$ is its derivative at $r = 0$.

Now let us take the PQCD prediction for $\Gamma(V \to \gamma R)$ of Refs. [10,4] and use these expressions (15) for the width of R to rewrite the PQCD formulas for $\Gamma(V \to \gamma R)$ in terms of the width Γ_R rather than $R(0)$ or $R'(0)$. This yields our central result which, for a 0^{+-}, is

$$b(V \to \gamma R) = b(V \to \gamma gg) \frac{m_R \Gamma_R b_{R \to gg}}{8\pi(\pi^2 - 9)m^2_V}$$

$$\times \left[1 - \left(\frac{m_R}{m_V} \right)^2 \right] |\hat{\mathcal{H}}^{PS}(x)|^2, \tag{16}$$

where $x = [1 - (\frac{m_R}{m_V})^2]$ and

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[7]Nominally, it is the two-gluon branching fraction, but when the scale of resolution of the gluons is taken large enough, as is implicit in our partonic discussion, this is just the total gluonic width of R.

[8]Because of the dominance of two-body phase space when m_R is not large, not because α^{eff}_{QCD} is small in the Rgg vertex.

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[9]G.R.F. thanks P. Landshoff for useful discussions on this issue.

[10]The decay rates of the S-wave quarkonia are given in Ref. [8] and the decay rates of the P-wave quarkonia are given in Ref. [13].

$$\hat{\mathcal{H}}^{PS}(x) = \frac{4}{x}\left\{ L_2(1-2x) - L_2(1) - \frac{x}{1-2x}\ln\ 2x - \frac{1-x}{2-x}\left(2L_2(1-x) - 2L_2(1) + \frac{1}{2}\ln^2(1-x)\right)\right\}$$
$$+ i\pi\frac{4}{x}\frac{1-x}{2-x}\ln\ (1-x) , \tag{17}$$

$$L_2(x) = -\int_0^x dt\,\frac{nln(1-t)}{t} . \tag{18}$$

For 0^{++}, 1^{++}, and 2^{++} the $\frac{1}{8\pi}$ factor in (16) becomes $\frac{4}{3\pi}$, $\frac{\cong 40}{3\pi}$ and $\frac{5}{\pi}$, respectively, and $\hat{\mathcal{H}}^{PS}$ is replaced by the function for the appropriate J^{PC} given in [10]. Explicit forms for $\hat{\mathcal{H}}^{S,V,T}$ can also be found in [12].

The form of the result (16) agrees with that of the unitarity calculation (14) in being proportional to $\Gamma_R b_{R\to gg}$. Its absorptive part correctly reproduces the absorptive part obtained by unitarity, using PQCD to obtain $V \to \gamma gg$, as we have argued is reliable on account of the large quark mass in V. By construction it agrees with the full PQCD nonrelativistic potential result, when that is applicable to computing $\Gamma(R \to gg)$, i.e., when R contains massive quarks. Finally, for light mesons it agrees with the light-cone QCD result of Kuhn [11] when the f_R and α_s dependence is removed in favor of $\Gamma(R \to gg)$ computed from (15a) using the relation (19) between f_R and $R(0)$.

We therefore propose that (16) is a good approximate expression for Okubo-Zweig-Iizuka- suppressed radiative production of *any* resonance. Further work is needed to assign an error to it, because it involves not only the $O(\alpha_s)$ error due to dropping the hard three-gluon states, but also the assumption that the relative size of real and imaginary parts is correctly given by PQCD even when the nonrelativistic potential cannot be used to evaluate the $R - gg$ vertex. A productive line of reasoning to put this on a more rigorous footing and allow estimation of the error would be to follow Kuhn's discussion for light $q\bar{q}$ mesons [11], since the dynamics of his light $q\bar{q}$ system should be similar to a glueball and an $\eta_{\tilde{g}}$, if $m(\tilde{g})$ is close enough to the mass of a strange quark. Note that our discussion of the resolution-size dependence of the *description* of the intermediate state (number of gluons, their virtuality, etc.) suggests that corrections to (16) come only from the hard three-gluon intermediate state and thus are $O(\alpha_s(M_V/2))$. As long as data rather than a model are used for Γ_R and one does not attempt to distinguish between real and imaginary part contributions to the total rate, soft gluon corrections may be completely included. A careful treatment with scale size introduced is required to verify this conjecture.

It is interesting that the contribution of the imaginary part of (16) is very tiny compared to that of the real part, but the total prediction of (16) is similar to the unitarity prediction (14), using data for $\frac{d\Gamma_{V\to\gamma X}}{ds}$ and a crude model for $\mathcal{P}^J_{S;L}$ [12]. This is consistent with the intuition that typical hadrons are composed of somewhat virtual quarks and gluons, so that the relative importance of virtual gluons in the PQCD calculation is much greater than that of virtual hadrons when a hadronic basis for the calculation is used.

IV. SOME APPLICATIONS TO DATA

Figures 2–12 show the upper and lower limits on $b_{R\to gg}$ which are obtained by comparing the prediction of Eq. (16) with the CUSB and exclusive data. We take $b(\Upsilon \to \gamma X) = 0.03$[14] and $b(J/\psi \to \gamma X) = 0.06$[15]. In general, the branching fraction of the resonance into the specific mode in which it is observed in the exclusive radiative decay experiments is not known, although of course it is no greater than 1. Except when the branching fraction is known, the exclusive data therefore only give a lower limit on $b_{R\to gg}$. In some instances it is seen that the CUSB upper limit and the exclusive lower limits are quite near, resulting in an estimate of $b_{R\to gg}$ and the predictions that (a) with a modest increase in sensitivity this resonance should show up in $\Upsilon \to \gamma X$ and (b) the exclusive branching fraction must be near 1.

The predictions in the figures have some imprecision because the widths are poorly known in many cases. Moreover, there is some intrinsic error in our method that we have not estimated, but it is at least $O(\alpha_s)$ from neglect of the three-hard gluon state. Nonetheless, the results are interesting and generally plausible. Even though DM2 and Mark III disagree on the order and exact widths of the three resonances they see in the 1440 MeV region, both pseudoscalars seen in both experiments are likely to be glueballs or the result of mixing a glueball with a $q\bar{q}$ state. Details of these states are given in Table I. Figures 13–16 refer to particles whose radiative production in J/ψ decay is given in the Particle Data Group tables (1990 edition, generally). The $\eta(1490)$ and $f_{0^{++}}(1720)$ are difficult to classify as pure $q\bar{q}$, since their $b_{R\to gg}$'s are $\sim 50\%$. The $\eta(1760)$ could be either a $q\bar{q}$ or gluonic state, given the spread in the lower and upper limits. The exclusive production for $f_1(1285)$ is above the upper limit, given a 12% branching fraction to $K\bar{K}\pi$, so that there is some internal inconsistency. Perhaps the problem is in the CUSB upper limit: they have a poorly understood contribution to their data from nonresonant processes in the region $m_R \lesssim 1.5$ GeV [1]. It could be wise to mentally attribute some additional systematic error to the CUSB limits for the low mass region. Instead, this discrepancy between upper and lower limits for the $f_1(1285)$ may reflect the intrinsic error of our method. If the latter is the case, our method will not be very useful unless the problem is isolated to 1^{++} production. The ques-
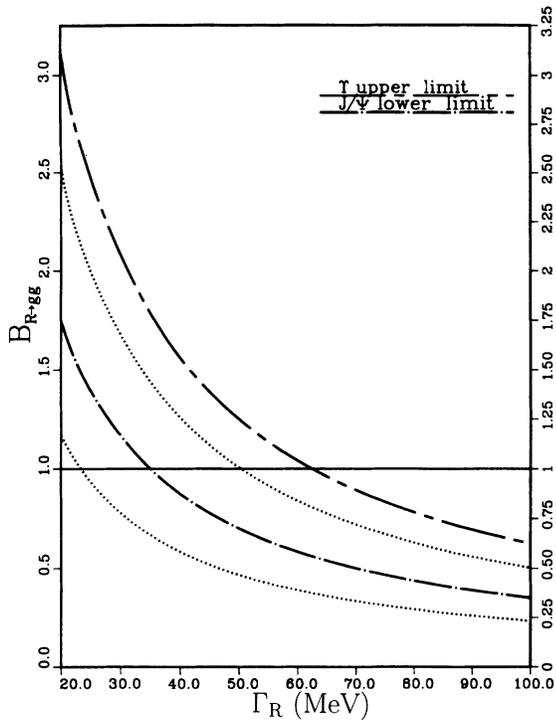
FIG. 2.   The lower limit obtained on the gluonic branching fractions of the resonance $0^{-+}(1416)$ reported by Mark III (dot-dashed lines, with dotted lines at ± 1 s.d.) and upper limits from CUSB (long-dashed lines).
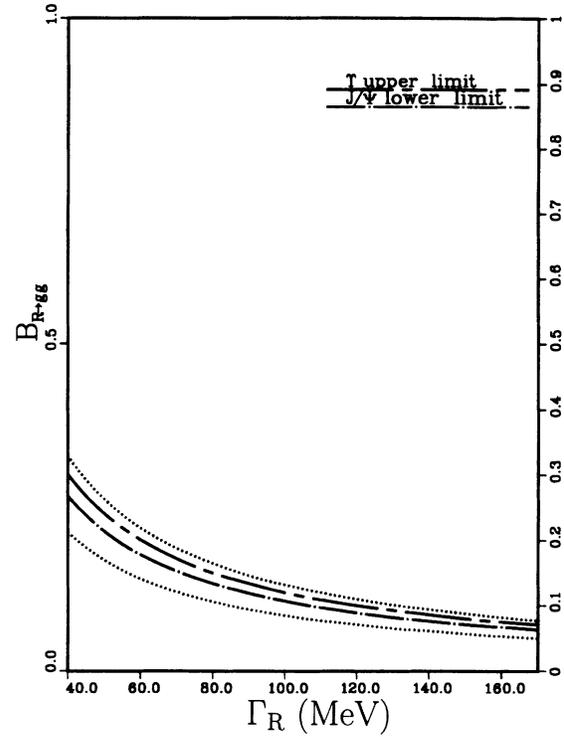
FIG. 4.   The lower limit obtained on the gluonic branching fractions of the resonance $1^{++}(1443)$ reported by Mark III (dot-dashed lines, with dotted lines at ± 1 s.d.) and upper limits from CUSB (long-dashed lines).
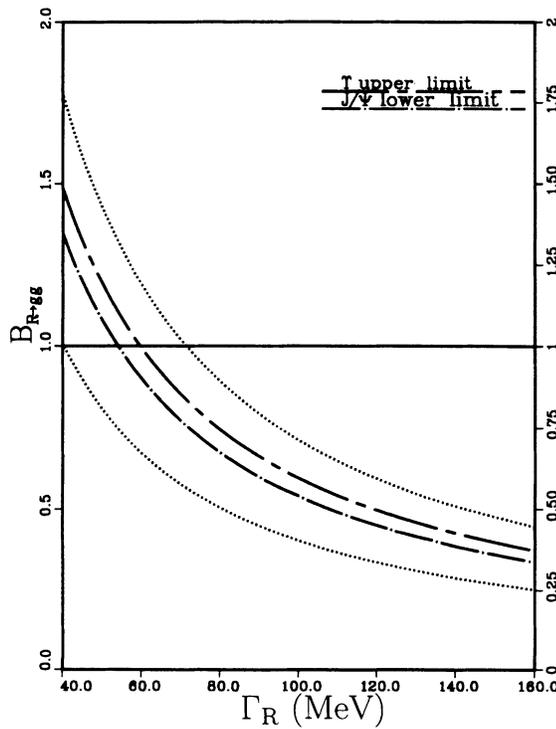
FIG. 3.   The lower limit obtained on the gluonic branching fractions of the resonance $0^{-+}(1490)$ reported by Mark III (dot-dashed lines, with dotted lines at ± 1 s.d.) and upper limits from CUSB (long-dashed lines).
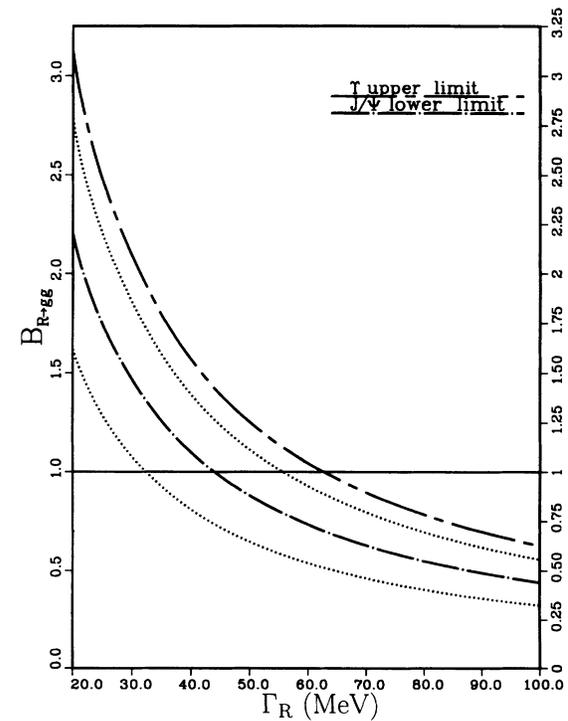
FIG. 5.   The lower limit obtained on the gluonic branching fractions of the resonance $0^{-+}(1421)$ reported by DM2 (dot-dashed lines, with dotted lines at ± 1 s.d.) and upper limits from CUSB (long-dashed lines).
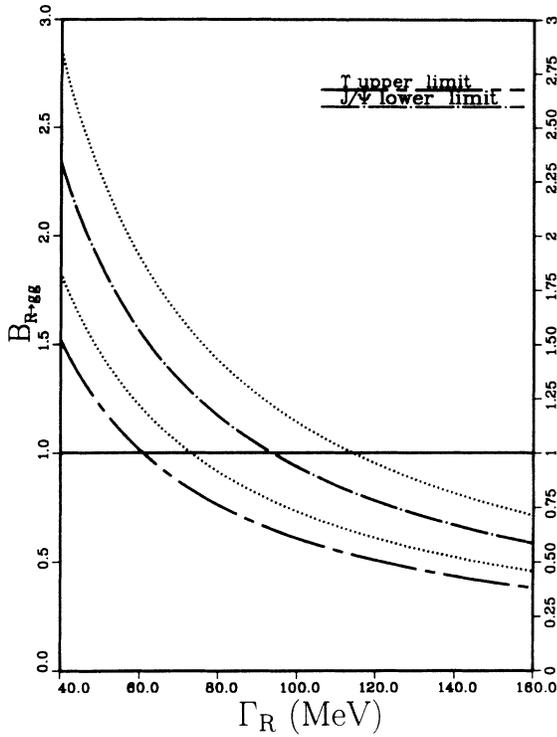
FIG. 6.   The lower limit obtained on the gluonic branching fractions of the resonance $0^{-+}(1459)$ reported by DM2 (dot-dashed lines, with dotted lines at ± 1 s.d.) and upper limits from CUSB (long-dashed lines).
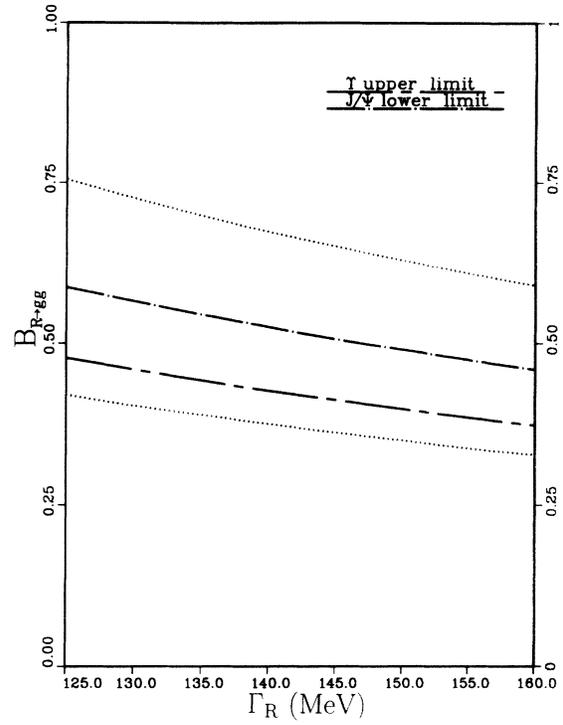
FIG. 8.   CUSB upper limit (long-dashed lines) on $b[\eta(1490) \to gg]$, and lower limits from $\Gamma[J/\psi \to \gamma\eta(1490) \to \gamma X] = 1.0 \pm 0.2 \times 10^{-3}$ (dot-dashed lines, with dotted lines at ± 1 s.d.).
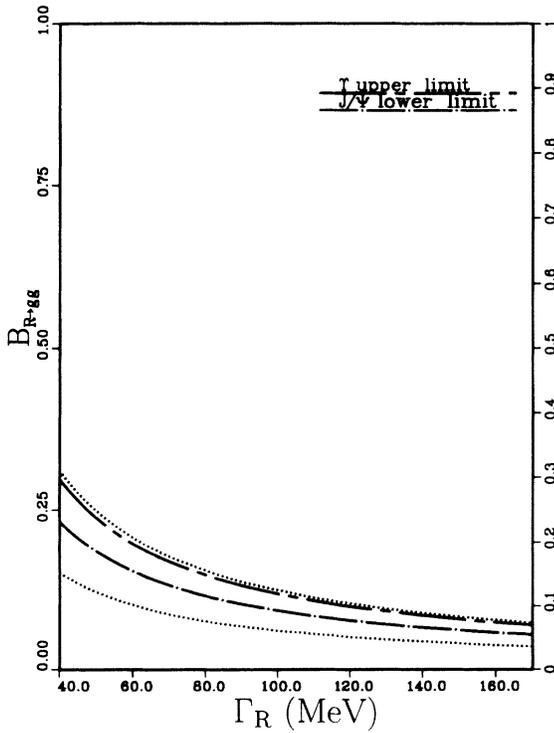
FIG. 7.   The lower limit obtained on the gluonic branching fractions of the resonance $1^{++}(1462)$ reported by DM2 (dot-dashed lines, with dotted lines at ± 1 s.d.) and upper limits from CUSB (long-dashed lines).
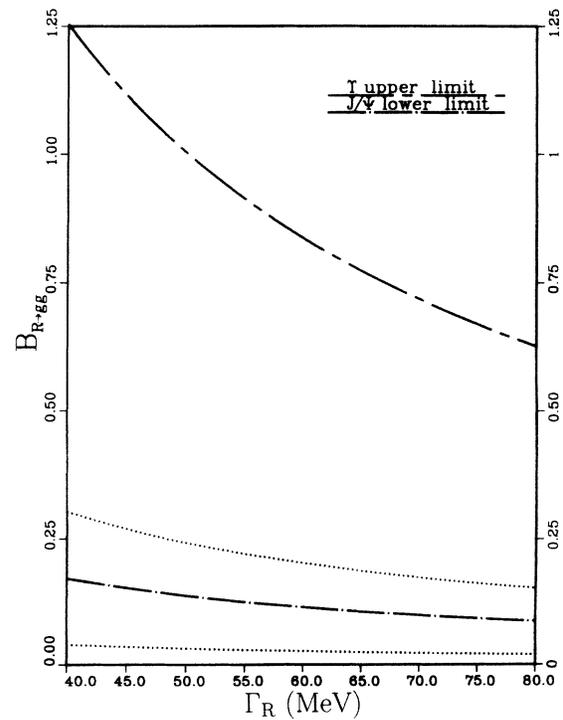
FIG. 9.   CUSB upper limit (long-dashed lines) on $b[\eta(1760) \to gg]$ and lower limits from $\Gamma[J/\psi \to \gamma\eta(1760) \to \gamma X] = 1.3 \pm 0.9 \times 10^{-4}$ (dot-dashed lines, with dotted lines at ± 1 s.d.).
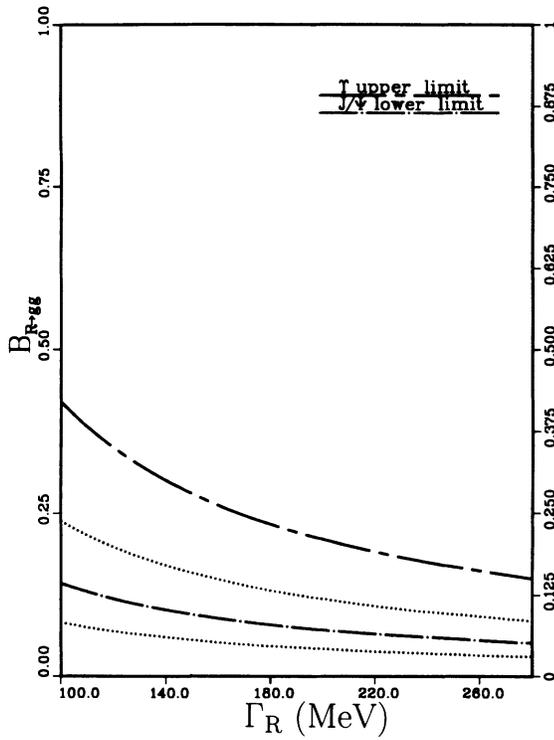
FIG. 10. CUSB upper limit (long-dashed lines) on $b[\eta(2100) \to gg]$ and lower limits from $\Gamma[J/\psi \to \gamma\eta(2100) \to \gamma X] = 2.9 \pm 0.6 \times 10^{-4}$ (dot-dashed lines, with dotted lines at ± 1 s.d.).
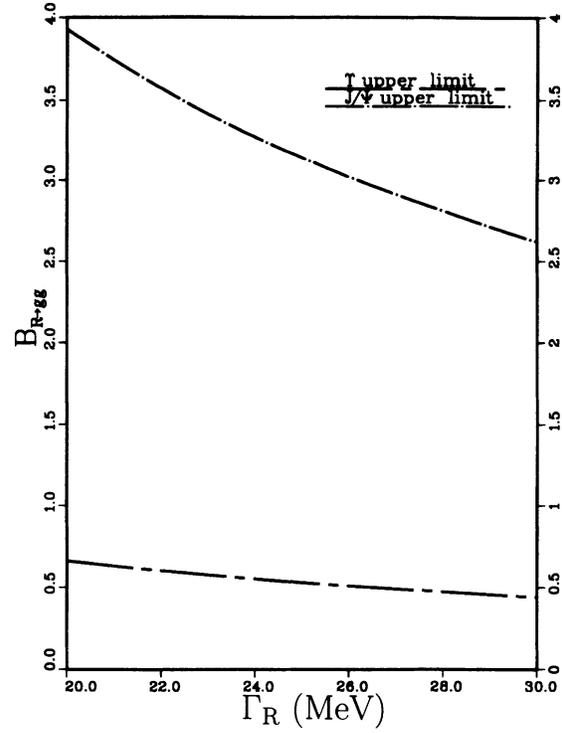
FIG. 12. CUSB upper limit (long-dashed lines) on $b[f_{1++}(1285) \to gg]$ and lower limits from $\Gamma[J/\Psi \to \gamma f_{1++}(1285) \to \gamma X] = 7.0 \pm 2.0 \times 10^{-4}$ (dot-dashed lines, with dotted lines at ± 1 s.d.).
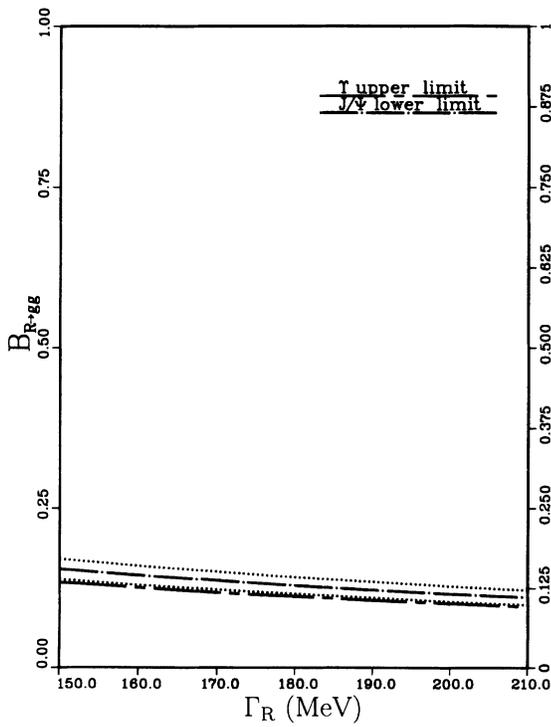
FIG. 11. CUSB upper limit (long-dashed lines) on $b[f_{2++}(1270) \to gg]$ and lower limits from $\Gamma[J/\psi \to \gamma f_{2++}(1270) \to \gamma X] = 1.4 \pm 0.2 \times 10^{-3}$ (dot-dashed lines, with dotted lines at ± 1 s.d.).
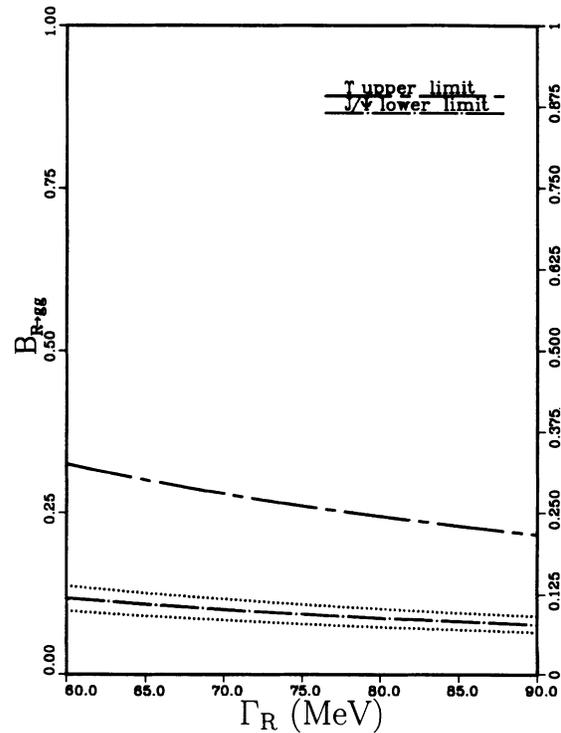
FIG. 13. CUSB upper limit (long-dashed lines) on $b[f_{2++}(1525) \to gg]$ and lower limits from $\Gamma[J/\psi \to \gamma f_{2++}(1525) \to \gamma X] = 6.3 \pm 1.0 \times 10^{-4}$ (dot-dashed lines, with dotted lines at ± 1 s.d.).

TABLE I. Predicted branching ratios for $J/\psi \rightarrow \gamma R, R \rightarrow K\bar{K}\pi$, without the factor $b_R \equiv b_{R \rightarrow gg} \times b(R \rightarrow K\bar{K}\pi)$, for the three resonances in the $\iota(1430)$ region found by Mark III and DM2, respectively. Gluonic states would have $b_{R \rightarrow gg} \sim 1$, so that dividing the experimental result by the last colum would produce $b(R \rightarrow K\bar{K}\pi)$

R	m_R(MeV)	Γ_R(MeV)	$B^{\mathrm{expt}}(J/\Psi \rightarrow \gamma R, R \rightarrow K\bar{K}\pi)$	B^{pred}/b_R
$0^{-+}_{(1426)}$	$1426^{+10.6}_{-9.4}$	$54^{+39.2}_{-31.9}$	$0.66^{+0.17+0.24}_{-0.16-0.15} \times 10^{-3}$	$1.02^{+0.74}_{-0.60} \times 10^{-3}$
$0^{-+}_{(1490)}$	$1490^{+16.3}_{-17.9}$	$91^{+68.7}_{-49.0}$	$1.03^{+0.21+0.26}_{-0.18-0.19} \times 10^{-3}$	$1.74^{+1.31}_{-0.94} \times 10^{-3}$
$1^{++}_{(1443)}$	$1443^{+7.6}_{-6.3}$	$68^{+30.1}_{-20.1}$	$0.87^{+0.14+0.14}_{-0.14-0.11} \times 10^{-3}$	$5.53^{+3.89}_{-1.64} \times 10^{-3}$
$0^{-+}_{(1421)}$	1421 ± 14	63 ± 18	$0.83 \pm 0.13 \pm 0.18 \times 10^{-3}$	$1.19 \pm 0.34 \times 10^{-3}$
$0^{-+}_{(1459)}$	1459 ± 5	75 ± 9	$1.78 \pm 0.21 \pm 0.33 \times 10^{-3}$	$1.43 \pm 0.18 \times 10^{-3}$
$1^{++}_{(1462)}$	1462 ± 20	129 ± 41	$0.76 \pm 0.15 \pm 0.21 \times 10^{-3}$	$10.6 \pm 3.43 \times 10^{-3}$

tion can be decided experimentally when the branching ratios $B(R \rightarrow K\bar{K}\pi)$ are known and the resonances are actually seen in $\Upsilon \rightarrow \gamma R$.

Our conclusions above regarding which resonances may be glueballs are generally consistent with other indicators. One interesting point which we do not pursue here is the possibility that there is actually an "extra" flavor singlet pseudoscalar around 1.4 GeV [3, 16], compared to expectations from filling known $q\bar{q}$ nonets and the predicted glueball spectrum. This will have to await further experimental elucidation of this mass region. Other applications of this method, including more figures and tables, can be found in [12].

FIG. 14. CUSB upper limit (long-dashed lines) on $b[f_{0^{++}}(1720) \rightarrow gg]$, and lower limits from $\Gamma[J/\psi \rightarrow \gamma f_{0^{++}}(1720) \rightarrow \gamma X] = 9.7 \pm 1.2 \times 10^{-4}$ (dot-dashed lines, with dotted lines at ± 1 s.d.).

V. LIMITS ON GLUINO MASS

CUSB has claimed [1] to exclude a gluino with mass in the range $0.21 < m(\tilde{g}) < 3.6$ GeV, without, however, seeing any other resonances in their spectrum. It is on account of the strong gluonic coupling of the gluino compared to the quark that it was noted [7–9] that, when PQCD and the nonrelativistic potential model are applicable, the $\eta_{\tilde{g}}$ will be produced at a significantly larger level than a $Q\bar{Q}$ resonance and in particular at a level which should be seen at CUSB. Let us now address the question as to the range of validity of these calculations, which rely on PQCD and the nonrelativistic potential model,[11] and find out what can be said when we cannot use them.

From Eq. (16) and the bound (1) on $b(\Upsilon \rightarrow \gamma X)$, knowing the radiative branching fraction of the Υ to be 0.03, we can extract an upper bound on the width of the $\eta_{\tilde{g}}$ as a function of its mass. This is shown as the solid curve in

FIG. 15. Experimental limit on $b_{R \rightarrow gg}\Gamma_R$ for any resonance produced in $\Upsilon \rightarrow \gamma X$ from CUSB (solid curve). Nonrelativistic potential + PQCD prediction for width of an $\eta_{\tilde{g}}$ for $\Lambda_{\mathrm{QCD}} = 100$ and 200 MeV (lower and upper dashed lines). "Mesonic wave function model" (see text) for the width of an $\eta_{\tilde{g}}$ (dot-dashed lines).

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[11]References [7, 9] themselves remark that their analysis applies only for $m(\eta_{\tilde{g}}) \gtrsim 3$ GeV.

Fig. 15. Evidently, if the $\eta_{\tilde{g}}$ has a width less than ~ 40 MeV, it cannot be excluded for any mass. Figure 15 also shows the nonrelativistic potential model (NRPM) and/or PQCD prediction for the width, for $\Lambda_{QCD} = 100$ and 200 MeV (the lower and upper dashed curves, respectively). It is obtained [7–9] by replacing the $\frac{8}{3}$ in Eq. (15a) by 18 and using $|R(0)|^2 = (\frac{9m_R}{4m(\eta_c)})^{\frac{3}{2}}|R(\eta_c)(0)|^2$.

Based on the fact that the experimental limit lies above the theoretical prediction for $m(\eta_{\tilde{g}}) \lesssim 7$ GeV, CUSB concluded that they could exclude gluino masses below[12] 3.6 GeV. However for $m(\eta_{\tilde{g}}) \lesssim 3$ GeV the predicted width becomes very large and may signal a failure of this model for the width. Physically, the way that the width of an $\eta_{\tilde{g}}$ can be large is for the constituents to be so massive that the bound state is very small, leading to a large wave function at the origin, while due to the large color charge compared to a quark, the intrinsic gluino-gluon coupling strength is large compared to the quark-gluon coupling. Since the color charge is important both in making the system tightly bound, and thus concentrated at the origin, and in increasing the coupling to the final state gluons, the effect of a larger color charge enters twice as we saw above, leading to the observation of Refs. [7–9] that an $\eta_{\tilde{g}}$ would be very prominent in the radiativge decay spectrum if the nonrelativistic potential model and PQCD were relevant.

However, if the gluino is lighter than perhaps $\frac{1}{2} - 1$ GeV, the bound state properties should resemble those of gluons and strange quarks. We do not know much about the former, but we do know that the latter form hadrons whose decay constants are remarkably similar. (See Fig. 16, which shows the measured pseudoscalar decay constants.) These which are related to the wave function at the origin by[13]

$$f_R = \sqrt{\frac{3R_R^2(0)}{\pi m_R}}. \tag{19}$$

The similarity in value of the various nonet pseudoscalar decay constants means that for bound states of light constituents $|R(0)|^2 \sim m_R$ (hadronic volume)$^{-1}$. The volume of the hadron is mainly governed by the confinement scale and has little to do with the mass of the light constituent. Furthermore, with light constituents, the individuality of the constituent is lost amidst the sea of gluons and $q\bar{q}$ pairs, so that whether the constituents are quarks or gluinos should not matter much, if the gluino is light. If this is a correct interpretation, one would expect $f_R \sim f_\pi$ for R a pseudoscalar bound state of light gluinos. Now taking $R(0)$ for the $\eta_{\tilde{g}}$ from Eq. (19) with $f_R = 120$ MeV and using Eq. (15a), one obtains the $\eta_{\tilde{g}}$ width shown in the dot-dashed curves ("mesonic wave function model") in Fig. 15. The upper curve corre-

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[12]Aside from being insensitive if the $\eta_{\tilde{g}}$ is too light to decay to pions, which they say occurs for a gluino mass less than 0.21 GeV.

[13]This formula can be derived or found in Refs. [17, 18] and also is implicit in the connection between refs. [11] and [10].
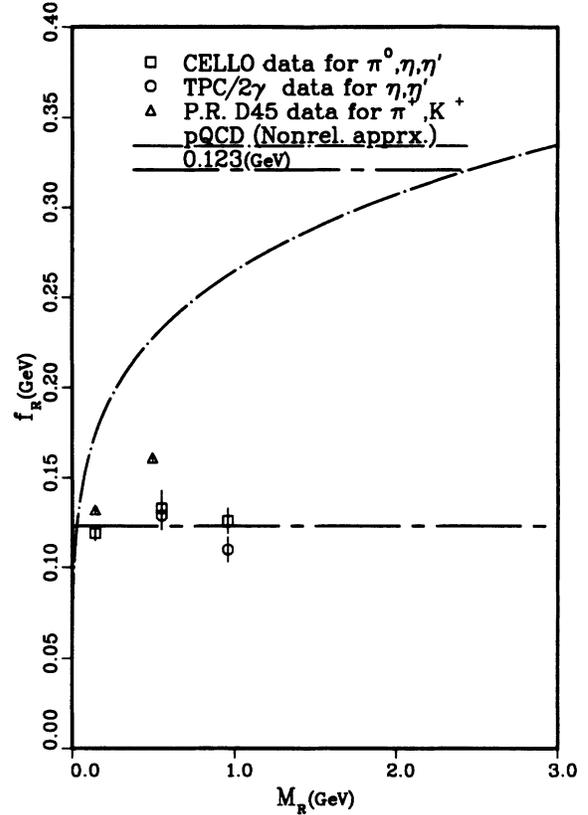
FIG. 16. f_R for nonet pseudoscalars and prediction from nonrelativistic potential model.

sponds to the NRPM-PQCD expression (15a), replacing $\frac{8}{3}$ in Eq. (15a) by 18, with $\alpha_s = \frac{12\pi}{25}\ln(\frac{m_R}{\Lambda_{QCD}})^2$ for $\Lambda_{QCD} = 100$ MeV, and the lower one is obtained by replacing the factor $18\alpha_s^2$ with 1, suitable if the interaction strength saturates at low energy scale. Either way, one obtains a much smaller prediction for the $\eta_{\tilde{g}}$ width, ~ 10 MeV, than from the NRPM-PQCD, basically because the NRPM formula for $|R(0)|$ grossly overestimates it for relativistic constituents. Note, however, that when the constituents are relativistic, the decay $R \to gg$ presumably probes a larger spatial portion of the wave function than merely the point at the origin, so that the NRPM-PQCD formula itself [Eq. (15)] may not be applicable. Hence using it with even a perfect estimate for $R(0)$ does not necessarily give a reliable estimate of the actual width and the curves' mesonic wave function model only serves to give an indication of the large uncertainty in modeling the width.

Predicting the width of an $\eta_{\tilde{g}}$ is a good problem for lattice gauge theory. Until such predictions are available, the most conservative approach is to take the $\eta_{\tilde{g}}$ width to be $\frac{1}{10} - 1$ times the typical width of glueball candidates. The motivation for this is that in order to communicate with quarks, the gluino pair must first convert to gluons, requiring at least two powers of α_s more than are present in a glueball decay rate. At the same time, since the system is strongly interacting these factors of α_s need not be small, leading to the above estimate.

Thus we argue that since CUSB does not see glueballs,

its data cannot be used to exclude an $\eta_{\tilde{g}}$ of a similar mass. For a resonance of ~ 1.5 GeV, the width limit from CUSB is ~ 70 MeV, roughly the width of the pseudoscalar glueball candidates in the iota region (see Table I), and we cannot exclude an $\eta_{\tilde{g}}$ in this region. The mass range between this and ~ 3 GeV is ambiguous. At its upper end, even though we cannot have complete confidence in the NRPM-PQCD calculation on theoretical grounds,[14] the CUSB limit of ~ 50 MeV is well below the NRPM-PQCD prediction of $150 - 250$ MeV, so there is a comfortable margin of error. At the lower end of the range, glueballs exist and they certainly must be visible in the experiment before drawing conclusions about the absence of an $\eta_{\tilde{g}}$. When the width of an $\eta_{\tilde{g}}$ has been well determined from lattice QCD, one can learn from Eq. (16) how sensitive a search is required to observe them in $V \rightarrow \gamma R$. Until that time, to be conservative we conclude that the CUSB experiment can only be used to rule out the range $\sim 3 \lesssim m(\eta_{\tilde{g}}) \lesssim 7$ GeV.

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[14]We cannot be completely confident due to the large color charge of gluinos: the factor $\frac{8}{3}\alpha_s^2$ for $q\bar{q}$ becomes $18\alpha_s^2$ for gluinos, so that perturbation theory is less reliable than for an η_c of the same mass.

VI. SUMMARY

We have proposed a method of predicting the branching ratio for production of any resonance in $V \rightarrow \gamma R$, which only requires knowing the mass, total width, and gluonic branching fraction $b_{R \rightarrow gg}$ of the resonance. We applied it to determining or obtaining limits on $b_{R \rightarrow gg}$ for a number of known flavor singlet resonances, identifying the best glueball candidates as the ones for which $b_{R \rightarrow gg}$ can be near 1. Of the states we examined, these are the two pseudoscalars in the 1440 MeV region, the $f_{0^{++}}$ and possibly the $\eta(1760)$.

We also found limits on the total width of a possible gluino-gluino bound state, since for such a state one would have $b_{R \rightarrow gg} \sim 1$. These limits are in conflict with the nonrelativistic potential model plus PQCD prediction of Refs. [7–9], so that in its region of validity the existence of an $\eta_{\tilde{g}}$ can be excluded, i.e., for the range $3 \lesssim m(\eta_{\tilde{g}}) \lesssim 7$ GeV. We argued that for lower masses the NRPM-PQCD calculation is not applicable, and instead one can only say that the width of the $\eta_{\tilde{g}}$ is less than the bound shown in Fig. 15, i.e.,~ 70 MeV for a mass of about 1.5 GeV or ~ 50 MeV for a mass greater than 2 GeV.

ACKNOWLEDGMENT

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FIG. 1.   The dominant contribution to $V \to \gamma R$.