

Tests of isospin symmetry breaking at $\phi(1020)$ meson factories

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In a model of isospin symmetry breaking we obtain the ($e^-e^+ \rightarrow \pi^-\pi^+$) amplitude Q and the isospin $I = 0$ and $I = 1$ relative phase ψ at the $\phi(1020)$ resonance in approximate agreement with experiment. The model predicts $\Gamma(\phi \rightarrow \omega\pi^0) \approx 4 \times 10^{-4}$ MeV. We have also obtained $\Gamma(\phi \rightarrow \eta'\gamma) = 5.2 \times 10^{-4}$ MeV. Measuring this partial width would strongly constrain η - η' mixing. The branching ratios \mathcal{B} of the isospin violating decays $\rho^+ \rightarrow \pi^+\eta$ and $\eta' \rightarrow \rho^\pm\pi^\mp$ are predicted to be $\mathcal{B}(\rho^+ \rightarrow \pi^+\eta) = 3 \times 10^{-5}$ and $\mathcal{B}(\eta' \rightarrow \rho^\pm\pi^\mp) = 4 \times 10^{-3}$, respectively, leading to $\mathcal{B}[\phi \rightarrow \rho^\pm\pi^\mp \rightarrow (\pi^\pm\eta)\pi^\mp \rightarrow (\pi^\pm\gamma\gamma)\pi^\mp] = 10^{-6}$ and $\mathcal{B}[\phi \rightarrow \eta'\gamma \rightarrow (\rho^\pm\pi^\mp)\gamma] = 2 \times 10^{-6}$.

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I. INTRODUCTION

It is generally believed that electromagnetic interactions and the mass differences of u and d quarks are the sources of the breaking of isospin symmetry [1]. Both lead, among other things, to the mixing of the isospin $I = 0$ and $I = 1$ members of the $SU(3)_f$ flavor nonets. The present paper deals with isospin-symmetry-violating meson decays that proceed via π^0 - η , π^0 - η' , ω - ρ^0 , and ϕ - ρ^0 mixings. Well-known examples are the isospin-forbidden decays $\eta \rightarrow 3\pi$ [1] (the main decay channel of the η) and $\psi' \rightarrow \psi\pi^0$ [2]. We concentrate on the decays $\phi \rightarrow \pi^+\pi^-$, $\phi \rightarrow \omega\pi^0$, $\rho^\pm \rightarrow \pi^\pm\eta$, and $\eta' \rightarrow \rho^\pm\pi^\mp$ that can thoroughly be investigated at ϕ meson factories. This is obvious for $\phi \rightarrow \pi^+\pi^-$ and $\phi \rightarrow \omega\pi^0$. Since the ϕ decays into $\rho\pi$ with a branching of 13%, ρ mesons will be produced at ϕ meson factories with a rate that is sufficient for detecting and investigating $\rho^\pm \rightarrow \pi^\pm\eta$. The ϕ is furthermore expected to decay into $\eta'\gamma$ with a branching of approximately 10^{-4} . This will presumably be sufficient for detecting and investigating $\eta' \rightarrow \rho^\pm\pi^\mp$.

II. INPUT PARAMETERS

We will use the matrix element

$$\langle \eta | H' | \pi^0 \rangle = -6000 \text{ MeV}^2. \quad (1)$$

We quote two determinations of this value. First, it follows from the recent determination [3]

$$\frac{m_d - m_u}{m_s} = 1/29 \quad (2)$$

of the quark mass ratios, together with the formula

$$\begin{aligned} \langle \eta | H' | \pi^0 \rangle^{\text{tadpole}} &= -\frac{m_d - m_u}{m_s} (m_K^2 - m_\pi^2) x_P \\ &= -6350 \text{ MeV}^2. \end{aligned} \quad (3)$$

This assumes the Zweig rule formula

$${}_{PS} \langle s\bar{s} | H' | \pi^0 \rangle^{\text{tadpole}} = 0. \quad (4)$$

We assume [4] $\theta_P = -20^\circ$ for the pseudoscalar meson mixing angle and have defined

$$|\eta\rangle = x_P \left| \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \right\rangle_{PS} + y_P |s\bar{s}\rangle_{PS}, \quad (5)$$

$$|\eta'\rangle = -y_P \left| \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \right\rangle_{PS} + x_P |s\bar{s}\rangle_{PS}, \quad (6)$$

such that $x_P = 1/\sqrt{3}(\cos\theta_P - \sqrt{2}\sin\theta_P) \approx 0.822$. We furthermore have

$$\langle \eta | H' | \pi^0 \rangle = \langle \eta | H' | \pi^0 \rangle^{\text{tadpole}} + \langle \eta | H' | \pi^0 \rangle^{\text{el}} \quad (7)$$

with [5] $\langle \eta | H' | \pi^0 \rangle^{\text{el}} = 520 \text{ MeV}^2$ leading to

$$\langle \eta | H' | \pi^0 \rangle = -5800 \text{ MeV}^2. \quad (8)$$

Second, Ref. [6] has found

$$\langle \eta | H' | \pi^0 \rangle = (-5900 \pm 600) \text{ MeV}^2. \quad (9)$$

In view of the large errors, we neglect the electromagnetic contribution and use the round number in Eq. (1), i.e.,

$$\langle \eta | H' | \pi^0 \rangle \approx \langle \eta | H' | \pi^0 \rangle^{\text{tadpole}} \approx -6000 \text{ MeV}^2. \quad (10)$$

In the same way we assume¹

¹We also note that [6] has found $\langle \eta' | H' | \pi^0 \rangle = (-5500 \pm 500) \text{ MeV}^2$ for $\theta_P = -13^\circ$. Using Eq. (9) together with these numbers one finds that ${}_{PS} \langle s\bar{s} | H' | \pi^0 \rangle \approx 0$ also in the approach of that reference.

$$\begin{aligned}
\langle \eta' | H' | \pi^0 \rangle &\approx -y_P \left\langle \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \middle| H' \middle| \pi_0 \right\rangle^{\text{tadpole}} \\
&= -\frac{y_P}{x_P} \langle \eta | H' | \pi^0 \rangle \\
&= -4200 \text{ MeV}^2.
\end{aligned} \tag{11}$$

We will also use [7]

$$\langle \omega | H' | \rho^0 \rangle = (-4520 \pm 600) \text{ MeV}^2. \tag{12}$$

For the ω - ϕ mixing angle the Gell-Mann-Okubo value [4] $\theta_V = 39^\circ$ will be used.

The couplings f_V of the vector mesons ρ , ω , ϕ to the photon are defined such that

$$\Gamma(V \rightarrow e^+ e^-) = \frac{\pi m_V \alpha^2}{3} f_V^{-2}. \tag{13}$$

where $\alpha = 1/137$ is the fine structure constant. Numerically $f_\rho^{-2} = 0.16$, $f_\omega^{-2} = 0.014$, $f_\phi^{-2} = 0.024$ in satisfactory agreement with the quark model relation 9:1:2 for these couplings. As also suggested by the quark model the f_V are assumed to be positive relative to each other.

One of the main ingredients of this work is the vector-vector-pseudoscalar meson coupling constant g defined by the effective Lagrangian

$$L_I = g/2 \epsilon_{\alpha\beta\gamma\delta} \sum_{a,b,c=0}^8 (\partial^\alpha V_a^\beta) V_b^\gamma (\partial^\delta P_c) d_{abc} \tag{14}$$

involving vector meson fields V_a^α with Lorentz index α , $SU(3)_f$ index a , and the pseudoscalar fields P_a . The d_{abc} are the well-known symmetric $SU(3)$ Clebsch-Gordan coefficients, $\epsilon_{\alpha\beta\gamma\delta}$ is the four-dimensional antisymmetric ϵ tensor, and ∂^α is a differentiation symbol. The experimental value of $\Gamma(\rho^0 \rightarrow \pi^0 \gamma)$ and the width formula

$$\Gamma(\rho^0 \rightarrow \pi^0 \gamma) = \frac{\alpha g^2}{96 f_\omega^2} \left(\frac{m_\rho^2 - m_{\pi^0}^2}{m_\rho} \right)^3 \tag{15}$$

imply $|g| = 0.0164 \text{ MeV}^{-1}$. This value will be used below.

From the above, widths of the types $P \rightarrow \gamma\gamma$, $P \rightarrow V\gamma$, $V \rightarrow P\gamma$, and $V \rightarrow PV$ can be computed in overall agreement with experiment [8–10]. Details of the results depend on the assumed values of the meson mixing angles and $SU(3)_f$ -symmetry-breaking corrections and will not concern us here. As examples of straightforward checks of our mixing assumptions, we calculate

$$\begin{aligned}
\Gamma(\rho^0 \rightarrow \eta\gamma) &= \frac{\alpha g^2}{96 f_\rho^2} \left(\frac{m_\rho^2 - m_\eta^2}{m_\rho} \right)^3 (x_P)^2 \\
&= 0.174 (x_P)^2 \text{ MeV} = 0.119 \text{ MeV},
\end{aligned} \tag{16}$$

$$\begin{aligned}
\Gamma(\eta' \rightarrow \rho^0 \gamma) &= \frac{\alpha g^2}{32 f_\rho^2} \left(\frac{m_{\eta'}^2 - m_\rho^2}{m_{\eta'}} \right)^3 (y_P)^2 \\
&= 0.392 (y_P)^2 \text{ MeV} = 0.127 \text{ MeV},
\end{aligned} \tag{17}$$

$$\begin{aligned}
\Gamma(\phi \rightarrow \rho\pi) &= 3\Gamma(\phi \rightarrow \rho^0 \pi^0) = \frac{g^2}{4\pi} [q(\phi \rightarrow \rho^0 \pi^0)]^3 (x_V)^2 \\
&= 136 (x_V)^2 \text{ MeV} = 0.55 \text{ MeV},
\end{aligned} \tag{18}$$

and

$$\begin{aligned}
\Gamma(\phi \rightarrow \eta\gamma) &= \frac{\alpha g^2}{48 f_\phi^2} \left(\frac{m_\phi^2 - m_\eta^2}{m_\phi} \right)^3 (y_P)^2 \\
&= 0.38 (y_P)^2 \text{ MeV} = 0.122 \text{ MeV}.
\end{aligned} \tag{19}$$

A comparison with the experimental values $\Gamma(\rho^0 \rightarrow \eta\gamma) = 0.058 \text{ MeV}$, $\Gamma(\eta' \rightarrow \rho^0 \gamma) = 0.059 \text{ MeV}$, $\Gamma(\phi \rightarrow \rho^0 \pi^0) = 0.57 \text{ MeV}$, and $\Gamma(\phi \rightarrow \eta\gamma) = 0.057 \text{ MeV}$ indicates the quality of agreement to be expected *without corrections*. In particular, the poor agreement in the only case where $SU(3)_f$ is applied to $s\bar{s}$ states (i.e., $\phi \rightarrow \eta\gamma$) invites an $SU(3)_f$ -symmetry-breaking correction [9]. Overall fits [8–10] of course also improve the apparent agreement. Theoretical and experimental $\Gamma(\phi \rightarrow \rho\pi)$ compare surprisingly well, lending support to our choice of θ_V .

The width $\Gamma(\phi \rightarrow \eta'\gamma)$ can be written in terms of $\Gamma(\phi \rightarrow \eta\gamma)$ as

$$\begin{aligned}
\Gamma(\phi \rightarrow \eta'\gamma) &= \left(\frac{x_P}{y_P} \right)^2 \left(\frac{m_\phi^2 - m_{\eta'}^2}{m_\phi^2 - m_\eta^2} \right)^3 \Gamma(\phi \rightarrow \eta\gamma) \\
&= 5.2 \times 10^{-4} \text{ MeV}.
\end{aligned} \tag{20}$$

The numerical value follows from assuming that η and η' do not mix with mesons outside their $SU(3)_f$ nonet.

III. THE DECAY $\phi \rightarrow \pi^+ \pi^-$

The observed decays of the $\phi(1020)$ into purely hadronic final states are $\phi \rightarrow K\bar{K}$ (fraction: 0.83), $\rho\pi$ (0.13), $\pi^+ \pi^- \pi^0$ (0.024), and $\pi^+ \pi^-$ (8×10^{-5}). This pattern can be understood if the physical ϕ is an almost ideally mixed $s\bar{s}$ vector meson (i.e., $I^G = 0^-$) that decays into $\pi^+ \pi^-$ electromagnetically [Fig. 1(c)] as well as hadronically via an isospin-symmetry-violating mixing with ρ^0 [Fig. 1(b)]. More specifically, we assume the resonant part A_ϕ of the invariant $\langle \gamma | \pi^+ \pi^- \rangle$ ampli-

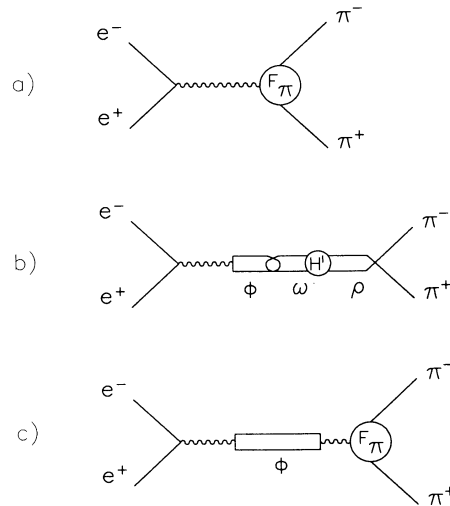


FIG. 1. Diagrams used to compute ψ and Q of $e^- e^+ \rightarrow \pi^- \pi^+$ near the $\phi(1020)$ resonance.

tude [Fig. 1(a)] around $\sqrt{s} = 1020$ MeV to be given by Figs. 1(b) and 1(c). The total amplitude

$$A(s) = F_\pi(s) + A_\phi(s) \quad (21)$$

also contains a nonresonant smooth interpolation $F_\pi(s)$ of the total form factor A of the π over the ϕ resonance region. As an illustrative example we will saturate $F_\pi(s)$ by the Breit-Wigner propagator of the ρ in the normalization $F_\pi(m_\rho^2) = 1/i\Gamma_\rho m_\rho$ such that

$$F_\pi(s) = \frac{1}{s - m_\rho^2 + i\Gamma_\rho m_\rho}. \quad (22)$$

Since $A_\phi(m_\rho^2) \approx 0$, we also have $A(m_\rho^2) \approx F_\pi(m_\rho^2) = 1/i\Gamma_\rho m_\rho$. From Figs. 1(b) and 1(c) we read

$$A(s) = F_\pi(m_\phi^2) + \frac{f_\rho m_\phi^2}{f_\phi m_\rho^2} \frac{1}{s - m_\phi^2 + i\Gamma_\phi m_\phi} \times \left(\frac{x_V \langle \omega | H' | \rho^0 \rangle}{s - m_\rho^2 + i\Gamma_\rho m_\rho} + \frac{\alpha\pi}{f_\phi f_\rho} F_\pi(m_\phi^2) \right) \quad (23)$$

for $s \approx m_\phi^2$. In analogy with the pseudoscalar case we have defined $x_V = 1/\sqrt{3}(\cos\theta_V - \sqrt{2}\sin\theta_V)$.

A few comments are in order. Since the hadronic isospin-symmetry-breaking Hamiltonian is proportional to $(u\bar{u} - d\bar{d})$, the ϕ couples hadronically to the ρ via ϕ - ω mixing only. This yields the factor x_V . The contribution of $\omega \rightarrow \gamma \rightarrow \rho^0$ is contained in $\langle \omega | H' | \rho^0 \rangle$. The contribution of the ρ^0 at $\sqrt{s} \approx m_\phi$ is then parametrized by the Breit-Wigner propagator. This is entirely correct for the hadronic ω - ρ^0 transition. For the photonic transition $\omega \rightarrow \gamma \rightarrow \rho^0$ it may be argued that the coupling is to $F_\pi(m_\phi^2)$ rather than to the Breit-Wigner propagator of the ρ^0 . It is however easy to see that the photonic contribution to $\langle \omega | H' | \rho^0 \rangle$ is small. Namely, the contribution of $\omega \rightarrow \gamma \rightarrow \pi^-\pi^+$ to $\omega \rightarrow \pi^-\pi^+$ can be obtained from an obvious modification of Eq. (23). If there were no other contribution, $\Gamma(\omega \rightarrow \pi^-\pi^+) = 0.005$ MeV (whereas the experimental width is 0.19 MeV). We follow [11] in factorizing the $\langle \gamma | \pi^+\pi^- \rangle$ amplitude in the neighborhood of the $\phi(1020)$ such that the s -dependent factor containing the ϕ resonance reads

$$1 + Q \frac{e^{i\psi} m_\phi \Gamma_\phi}{s - m_\phi^2 + im_\phi \Gamma_\phi} = 1 + \frac{f_\rho m_\phi^2}{f_\phi m_\rho^2} \frac{Y}{s - m_\phi^2 + im_\phi \Gamma_\phi}, \quad (24)$$

where Y is given by

$$Y = \frac{\alpha\pi}{f_\rho f_\phi} m_\rho^2 + \left| \frac{F_\pi(m_\rho^2)}{F_\pi(m_\phi^2)} \right| \frac{m_\rho \Gamma_\rho x_V \langle \omega | H' | \rho^0 \rangle}{m_\phi^2 - m_\rho^2 + i\Gamma_\rho m_\rho} e^{iR}, \quad (25)$$

with R the phase of $F_\pi(m_\phi^2)$ at the ϕ resonance:

$$F_\pi(m_\phi^2) = |F_\pi(m_\phi^2)| e^{-iR}. \quad (26)$$

Assuming for m_ϕ and Γ_ϕ the values of (Ref. [4]) $m_\phi = 1019.4$ MeV and $\Gamma_\phi = 4.43$ MeV, the observables to be determined experimentally are ψ and

$$Q = \left(\frac{36B(\phi \rightarrow \pi^-\pi^+)B(\phi \rightarrow e^-e^+)}{\alpha^2(1 - 4m_\pi^2/m_\phi^2)^{3/2}|F_\pi|^2} \right)^{1/2}. \quad (27)$$

Reference [11] finds

$$\psi = (20 \pm 13)^\circ \quad (28)$$

together with

$$Q = 0.07 \pm 0.02. \quad (29)$$

Using $|F_\pi|^2 = 2.9 \pm 0.2$ and $B(\phi \rightarrow e^-e^+) = 3 \times 10^{-4}$ (as has also been obtained in [11]), this yields

$$\Gamma(\phi \rightarrow \pi^+\pi^-) = 2.8 \times 10^{-4} \text{ MeV}. \quad (30)$$

The value $\Gamma(\phi \rightarrow \pi^+\pi^-) = (3.5_{-1.8}^{+2.2}) \times 10^{-4}$ MeV [4] combines this result with the much higher $\Gamma(\phi \rightarrow \pi^+\pi^-) = 8 \times 10^{-4}$ MeV of Ref. [12].

Using R as a parameter, we present our results in Table I. Approximate agreement with experiment is obtained for R between -170° and 15° . A more meaningful test will hopefully be provided by DAΦNE.

IV. THE DECAY $\phi \rightarrow \omega\pi^0$

The amplitude $\langle \phi | \omega\pi^0 \rangle$ is determined by the contributions of Fig. 2. We emphasize that, in contrast to Fig. 1(c), we have in Fig. 2(c) *only to take the contribution of the $\rho^0(770)$ into account, i.e., not the full $F_\pi(M_\phi^2)$* . Namely interpreting $F_\pi(s)$ in the neighborhood of the $\phi(1020)$ as the *sum* of the contributions of the ($I = 1$) vector mesons $\rho(770)$, $\rho(1450)$, and $\rho(1700)$, the meson full listings of Ref. [4] suggest that *only the $\rho(770)$ considerably couples to $\omega\pi^0$* . Thus we may write, for the amplitude (taking the $\rho^0\omega\pi^0$ coupling from Sec. II rather than directly from the Gell-Mann-Sharp-Wagner calculation [10,13] of $\omega \rightarrow 3\pi$),

$$A = \sum_{j=a}^e A^{(j)} \quad (31)$$

TABLE I. With the use of R of Eq. (26) as a parameter, the predicted values of ψ and Q [Eq. (27)] are listed and compared to experiment [11].

Condition	R (degree)	ψ (degree)	Q
Experiment		-20 ± 13	0.07 ± 0.02
Destructive interference	-165.7	0	0.089
	-150	-6.4	0.091
	-120	-15	0.10
Phase ψ minimal	-93	-17	0.12
Constructive interference	14.3	0	0.16
Phase ψ maximal	122	17	0.12

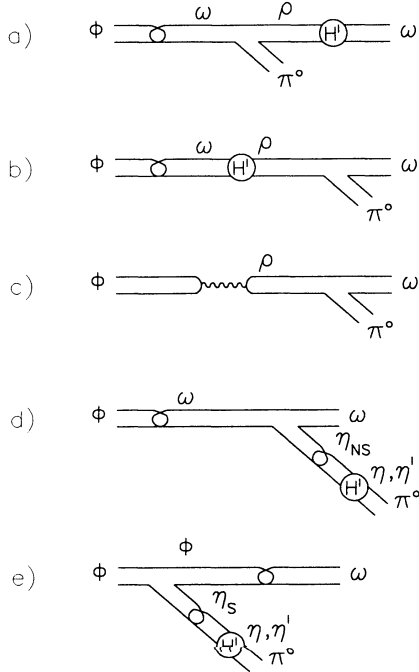


FIG. 2. Diagrams used to compute the width $\Gamma(\phi \rightarrow \omega\pi^0)$.

with

$$A^{(a)} = \frac{gx_V \langle \omega | H' | \rho^0 \rangle}{m_\omega^2 - m_\rho^2 + im_\rho \Gamma_\rho}, \quad (32)$$

$$A^{(b)} = \frac{gx_V \langle \omega | H' | \rho^0 \rangle}{m_\phi^2 - m_\rho^2 + i\Gamma_\rho m_\rho}, \quad (33)$$

$$A^{(c)} = \frac{\alpha g \pi m_\rho^2}{f_\phi f_\rho} \frac{1}{m_\phi^2 - m_\rho^2 + i\Gamma_\rho m_\rho}, \quad (34)$$

$$A^{(d)} = x_V g \left(x_P \frac{\langle \eta | H' | \pi^0 \rangle}{m_{\pi^0}^2 - m_\eta^2} - y_P \frac{\langle \eta' | H' | \pi^0 \rangle}{m_{\pi^0}^2 - m_{\eta'}^2} \right), \quad (35)$$

and

$$A^{(e)} = \sqrt{2} x_V g \left(y_P \frac{\langle \eta | H' | \pi^0 \rangle}{m_{\pi^0}^2 - m_\eta^2} + x_P \frac{\langle \eta' | H' | \pi^0 \rangle}{m_{\pi^0}^2 - m_{\eta'}^2} \right). \quad (36)$$

The width is

$$\Gamma(\phi \rightarrow \omega\pi^0) = |A|^2 \frac{[q(\phi \rightarrow \omega\pi^0)]^3}{12\pi}. \quad (37)$$

The input parameters we use yield $\Gamma(\phi \rightarrow \omega\pi^0) = 4 \times 10^{-4}$ MeV.

V. THE DECAYS $\rho^\pm \rightarrow \pi^\pm \eta$ AND $\eta' \rightarrow \rho^\pm \pi^\mp$

Finally we consider the decays $\rho^\pm \rightarrow \pi^\pm \eta$ and $\eta' \rightarrow \rho^\pm \pi^\mp$. They obviously violate G parity and thus isospin symmetry (since charge conjugation symmetry holds for these strong and/or electromagnetic decays). The strong

isospin-symmetry-conserving matrix element leading to the proposed decays via $\pi^0 \eta$ and $\pi^0 \eta'$ mixing, is in both cases $\langle \rho^\pm | \pi^\mp \pi^0 \rangle$.

With q the decay momenta of the P -wave decays we thus obtain

$$\begin{aligned} \Gamma(\rho^+ \rightarrow \pi^+ \eta) &= \left(\frac{q(\rho \rightarrow \pi \eta)}{q(\rho \rightarrow \pi \pi)} \right)^3 \left(\frac{\langle \eta | H' | \pi^0 \rangle}{m_\pi^2 - m_\eta^2} \right)^2 \\ &\quad \times \Gamma(\rho^+ \rightarrow \pi^+ \pi^0) \\ &= 4 \times 10^{-3} \text{ MeV} \end{aligned} \quad (38)$$

and, taking both final charge states together,

$$\begin{aligned} \Gamma(\eta' \rightarrow \rho^\pm \pi^\mp) &= 2 \times 3 \left(\frac{q(\eta' \rightarrow \rho \pi)}{q(\rho \rightarrow \pi \pi)} \right)^3 \left(\frac{\langle \eta' | H' | \pi^0 \rangle}{m_\pi^2 - m_{\eta'}^2} \right)^2 \\ &\quad \times \Gamma(\rho^+ \rightarrow \pi^+ \pi^0) \\ &= 7 \times 10^{-4} \text{ MeV}. \end{aligned} \quad (39)$$

The factor of 3 in the second formula is due to the summation (rather than averaging) over the spin orientations of the ρ . The results for the branching ratios \mathcal{B} are $\mathcal{B}(\rho^+ \rightarrow \pi^+ \eta) = 3 \times 10^{-5}$ and $\mathcal{B}(\eta' \rightarrow \rho^\pm \pi^\mp) = 4 \times 10^{-3}$, leading to branchings $\mathcal{B}[\phi \rightarrow \rho^\pm \pi^\mp \rightarrow (\pi^\pm \eta) \pi^\mp \rightarrow (\pi^\pm \gamma \gamma) \pi^\mp] = 10^{-6}$ and $\mathcal{B}[\phi \rightarrow \eta' \gamma \rightarrow (\rho^\pm \pi^\mp) \gamma] = 2 \times 10^{-6}$. Thus meaningful conclusions on these decays can be obtained at ϕ meson factories [14].

VI. CONCLUSIONS

In conclusion, we note that our understanding of the processes considered here and those connected to them by $SU(3)_f$ symmetry, vector meson dominance, and isospin symmetry breaking will be strongly enhanced by accurate e^-e^+ annihilation experiments in the $\phi(1020)$ resonance region. As to orders of magnitude, mainstream low-energy phenomenology can accommodate the observed $\psi = (-20 \pm 13)^\circ$ and $Q = 0.07 \pm 0.02$ within the large experimental errors. Our input data favor a ψ that is negative and nearer to 0° and/or a larger Q [implying a larger $\Gamma(\phi \rightarrow \pi^- \pi^+)$]. The width $\Gamma(\phi \rightarrow \omega\pi^0)$ is predicted to be approximately 4×10^{-4} MeV. Our predictions concerning $\rho^\pm \rightarrow \pi^\pm \eta$ and $\eta' \rightarrow \rho^\pm \pi^\mp$ are stated at the end of the preceding section.

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