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Gluon fragmentation into P-wave heavy quarkonium

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The fragmentation functions for gluons to split into P-wave heavy quarkonium states are calculated to leading order in the QCD coupling constant. Long-distance effects are factored into two nonperturbative parameters: the derivative of the radial wave function at the origin and a second parameter related to the probability for a heavy-quark–antiquark pair that is produced in a color-octet S-wave state to form a color-singlet P-wave bound state. The fragmentation probabilities for a high transverse momentum gluon to split into the P-wave charmonium states χ_{c0}, χ_{c1}, and χ_{c2} are estimated to be 0.4×10^{-4}, 1.8×10^{-4}, and 2.4×10^{-4}, respectively. This fragmentation process may account for a significant fraction of the rate for the inclusive production of χ_{cJ} at large transverse momentum in $p\bar{p}$ colliders.

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Heavy quarkonium plays an important role in high-energy collider physics, because these states can probe physical processes at short distances of order $1/m_Q$, where m_Q is the heavy quark mass. Of particular importance experimentally are the 1^{--} S-wave states of charmonium and bottomonium, which have very clean signatures through their leptonic decay modes, and the J^{++} P-wave states with $J = 0,1,2$, which can also be observed through their radiative transitions into the 1^{--} states. In most previous studies of the direct production of heavy quarkonium [1], the dominant production mechanisms were assumed to be given by the Feynman diagrams that were lowest order in the QCD coupling constant α_s. It has recently been pointed out that the dominant mechanism for heavy quarkonium production at large transverse momentum p_T is *fragmentation*, the production of a high-energy parton with even larger transverse momentum which subsequently decays into the quarkonium state plus other partons [2]. While this mechanism is often of higher order in the QCD coupling constant α_s than conventional mechanisms, it is enhanced at large transverse momentum by powers of p_T/m_Q, and thus dominates at sufficiently large p_T.

The fragmentation of a parton i into any hadron H is described by a universal fragmentation function $D_{i\to H}(z,\mu)$, where z is the longitudinal momentum fraction of the hadron relative to the parton and μ is a factorization scale of order p_T [3]. If the fragmentation function is known at some initial momentum scale μ_0, then it can be determined at larger momentum scales μ by solving the Altarelli-Parisi evolution equations which sum up the leading logarithms of μ/μ_0. In Ref. [2], it was shown that in the case of heavy quarkonium, the fragmentation function $D(z,m_Q)$ at an initial scale of order m_Q can be calculated using perturbative QCD. The initial fragmentation functions for gluons to split into S-wave states of heavy quarkonium were calculated to leading order in α_s [2]. The fragmentation functions for heavy quarks to split into S-wave states have also been calculated to leading order [4–6], and these calculations have recently been extended to the P-wave states [7].

In this paper, we calculate the fragmentation functions for gluons to split into the P-wave states to leading order in α_s. For the sake of clarity, we describe the calculation in terms of the lowest P-wave states of the charmonium system: the $J^{PC} = J^{++}$ states χ_{cJ}, with $J = 0,1,2$, and the 1^{+-} state h_c. Our results apply equally well to the higher P-wave states of charmonium, as well as to the P-wave states of the bottomonium system. While a gluon can split into χ_{cJ} at order α_s^2 through the Feynman diagram in Fig. 1, gluon splitting into the h_c occurs first at one order higher in α_s. Since 1^{+-} states of heavy quarkonium like the h_c are difficult to observe experimentally, we concentrate in this paper on the J^{++} states.

In charmonium, the charmed quark and antiquark are nonrelativistic with typical velocity $v \ll 1$ and typical separation $1/(m_c v)$. Our calculation of the fragmentation function is based on separating short-distance effects involving the scale $1/m_c$ from long-distance effects involving scales of order $1/(m_c v)$ or larger. There are two distinct mechanisms that contribute to the fragmentation function at leading order in v [8], and we will refer to them as the *color-singlet mechanism* and the *color-octet mechanism.* The *color-singlet mechanism* is the production of a $c\bar{c}$ pair in a color-singlet ^3P_J state with separation of order $1/m_c$ in the quarkonium rest frame. The subsequent formation of the χ_{cJ} is a long-distance

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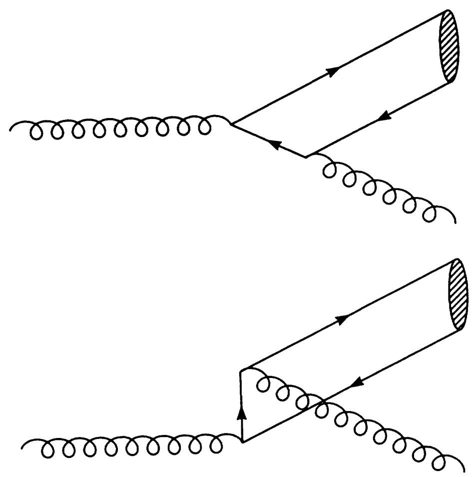

FIG. 1. Feynman diagrams for $g^* \to c\bar{c} + g$ which contribute to the color-singlet term in the fragmentation function for $g \to \chi_{cJ}$.

process with probability of order v^5. In addition to the volume factor $(m_c v/m_c)^3$, there is an extra suppression factor of v^2 from the wave function of the P state near the origin. The *color-octet mechanism* is the production of a $c\bar{c}$ pair in a color-octet $^3S_1$ state with separation of order $1/m_c$. The subsequent formation of the χ_{cJ} can proceed either through the dominant $|c\bar{c}\rangle$ component of the χ_{cJ} wave function or through the small $|c\bar{c}g\rangle$ component, which has a probability of order v^2. In the first case, the $c\bar{c}$ pair must radiate a soft gluon to make a transition to the color-singlet ^3P_J $|c\bar{c}\rangle$ state. In the second case, a soft gluon must combine with the $c\bar{c}$ pair to form a color-singlet $|c\bar{c}g\rangle$ state. In either case, the probability is of order v^5, with a volume factor of v^3 and an additional suppression of v^2 coming either from the probability of radiating a soft gluon or from the small probability of the $|c\bar{c}g\rangle$ component of the wave function. Since the color-singlet mechanism and the color-octet mechanism contribute to the fragmentation function at the same order in v, they must both be included for a consistent calculation.

Separating effects due to short distances of order $1/m_c$ from those of longer distance scales requires the introduction of an arbitrary factorization scale Λ in the range $m_c v \ll \Lambda \ll m_c$. The fragmentation functions for heavy quarkonium satisfy factorization formulas that involve this arbitrary scale. At leading order in v^2, the factorization formula for the fragmentation function for $g \to \chi_{cJ}$ has two terms:

$$D_{i\to\chi_{cJ}}(z,m_c) = \frac{H_1}{m_c} d_1^{(J)}(z,\Lambda) + (2J+1)\frac{H_8'(\Lambda)}{m_c} d_8(z) , \qquad (1)$$

where H_1 and $H_8'(\Lambda)$ are nonperturbative long-distance factors associated with the color-singlet and color-octet mechanisms, respectively. The short-distance factors $d_1^{(J)}(z,\Lambda)$ and $d_8(z)$ can be calculated using perturbation expansions in $\alpha_s(m_c)$. They are proportional to the fragmentation functions for a gluon to split into a $c\bar{c}$ pair with vanishing relative momentum and in the appropriate color-spin-orbital state: color-singlet ^3P_J state for $d_1^{(J)}$ and color-octet $^3S_1$ state for d_8. Note that in the factorization formula (1), the only dependence on Λ is in $d_1^{(J)}$ and H_8'. This simple form holds if the coefficients are calculated at most to next-to-leading order in α_s. Beyond that order, d_8 also acquires a weak dependence on Λ and J.

The nonperturbative parameters H_1 and H_8' can be rigorously defined as matrix elements of four-quark operators in nonrelativistic QCD [9]. Their dependence on Λ is given by renormalization group equations whose coefficients can be calculated as perturbation series in $\alpha_s(\Lambda)$ [8]. To order α_s, H_1 is scale invariant and H_8' satisfies

$$\Lambda\frac{d}{d\Lambda}H_8'(\Lambda) = \frac{16}{27\pi}\alpha_s(\Lambda)H_1 . \qquad (2)$$

If the factorization scale Λ is chosen to be much less than m_c, this equation can be used to sum up large logarithms of m_c/Λ:

$$H_8'(m_c) = H_8'(\Lambda) + \frac{16}{27\beta_0}\ln\left(\frac{\alpha_s(\Lambda)}{\alpha_s(m_c)}\right) H_1 , \qquad (3)$$

where $\beta_0 = (33-2n_f)/6$ is the first coefficient in the β function for QCD with n_f flavors of light quarks. The parameter H_1 can be related to the derivative of the nonrelativistic radial wave function at the origin for the P-wave states:

$$H_1 \approx \frac{9}{2\pi}\frac{|R_P'(0)|^2}{m_c^4}\left[1+O(v^2)\right] . \qquad (4)$$

This parameter can be determined phenomenologically from the annihilation rates of the χ_{cJ} states. Using recent high precision measurements of the light hadronic decay rates of χ_{c1} and χ_{c2}, H_1 has been determined to be approximately 15 MeV [10]. The parameter H_8' was introduced in Ref. [11] in a calculation of the rate for the decay $b \to \chi_{cJ} + X$, which also receives contributions from both the color-singlet and color-octet mechanisms for χ_{cJ} production. The prime on H_8' is a reminder that this parameter is not related in any rigorous way to the corresponding parameter H_8 that appears in decays of the χ_{cJ} states into light hadrons. Using data on the inclusive decays of B mesons into charmonium, its value for $\Lambda = m_c$ has been estimated to be $H_8'(m_c) \approx 3$ MeV [11]. This parameter also enters into the inclusive decay rate of the Υ into P-wave charmonium states [12].

We now turn to the calculation of the coefficient $d_1^{(J)}(z,\Lambda)$ in the color-singlet contribution to the fragmentation function. We follow the method and notation of Ref. [2]. Let $\mathcal{A}_\alpha$ denote the amplitude for $g^* \to c\bar{c}(^3P_J,\underline{1}) + g$ corresponding to the Feynman diagram in Fig. 1. The $c\bar{c}$ pair have almost equal momenta, and are in a color-singlet ^3P_J state. The amplitude $\mathcal{A}_\alpha$ can be written down in terms of $R_P'(0)$ using standard Feynman rules for quarkonium processes [13]. Multiplying $\mathcal{A}_\alpha$ by its complex conjugate and summing over final colors and spins, we obtain the generic form

$$\sum \mathcal{A}_\alpha \mathcal{A}^*_\beta = \frac{H_1}{m_c} [A_J(s)(-g_{\alpha\beta}) + B_J(s) p_\alpha p_\beta + C_J(s)(p_\alpha q_\beta + q_\alpha p_\beta) + D_J(s) q_\alpha q_\beta] , \quad (5)$$

where p and q are the four-momenta of the $c\bar{c}$ pair and the fragmenting gluon g^*, respectively, and $s = q^2$. The strategy is to reduce this expression in the limit $q_0 \gg m_c$ to the polarization sum $(-g_{\alpha\beta} + \cdots)$ for an on-shell gluon multiplied by a function of s and z, where z is the longitudinal momentum fraction of the $c\bar{c}$ pair relative to the fragmenting gluon. Terms in (5) that are proportional to q_α or q_β can be dropped, because in the axial gauge, q_α and q_β are of order m_c^2/q_0 when contracted with the numerator of the propagator of the virtual gluon. In the $p_\alpha p_\beta$ term, we can set $p = zq + p_\perp$, where $p_\perp$ is the transverse part of the four-vector p. After averaging over the directions of $p_\perp$, $p_\alpha p_\beta$ can be replaced by $-g_{\alpha\beta}\mathbf{p}_\perp^2/2$, up to terms that are suppressed in the axial gauge. The terms in (5) that contribute to fragmentation then reduce to

$$\sum \mathcal{A}_\alpha \mathcal{A}^*_\beta \approx \frac{H_1}{m_c} \left(A_J(s) + \frac{\mathbf{p}_\perp^2}{2} B_J(s) \right) (-g_{\alpha\beta}) . \quad (6)$$

Energy-momentum conservation in the form $s = (\mathbf{p}_\perp^2 + 4m_c^2)/z + \mathbf{p}_\perp^2/(1-z)$ can be used to eliminate $\mathbf{p}_\perp^2$ in (6) in favor of s and z. The fragmentation probability is obtained by dividing the coefficient of $(-g_{\alpha\beta})$ by s^2 for the propagator of the virtual gluon, and then integrating over the phase space of the $c\bar{c}$ pair and the gluon in the final state. The phase space integral can be expressed compactly in terms of integrals over s and z [2]. The resulting expression for the integral over z of $d_1^{(J)}(z,\Lambda)$ is

$$\int_0^1 dz\, d_1^{(J)}(z,\Lambda) = \frac{1}{16\pi^2} \int_{s_{\min}(\Lambda)}^\infty ds \int_{4m_c^2/s}^1 dz\, \frac{1}{s^2} \left(A_J(s) + \frac{(1-z)(zs-4m_c^2)}{2} B_J(s) \right) . \quad (7)$$

We have anticipated the presence of an infrared divergence associated with a soft gluon in the final state by imposing a lower cutoff Λ on the energy of the gluon in the quarkonium rest frame. This translates into a lower limit on s: $s_{\min}(\Lambda) = 4m_c^2(1+\Lambda/m_c)$. The calculations of the functions $A_J(s)$ and $B_J(s)$ in (7) involve some rather complicated algebra, but the final results are relatively simple. Interchanging orders of integration in (7), we can read off the functions $d_1^{(J)}(z,\Lambda)$:

$$d_1^{(J)}(z,\Lambda) = \frac{\alpha_s^2}{27} \int_{4m_c^2/z}^\infty ds\, \frac{m_c^2}{s^2(s-4m_c^2)^4} f_J(s,z) , \quad z < \left(1 + \frac{\Lambda}{m_c}\right)^{-1} \quad (8)$$

$$= \frac{\alpha_s^2}{27} \int_{s_{\min}(\Lambda)}^\infty ds\, \frac{m_c^2}{s^2(s-4m_c^2)^4} f_J(s,z) , \quad z > \left(1 + \frac{\Lambda}{m_c}\right)^{-1} , \quad (9)$$

where

$$f_0(s,z) = (s-12m_c^2)^2 \left[(s-4m_c^2)^2 - 2(1-z)(zs-4m_c^2)s\right] , \quad (10)$$

$$f_1(s,z) = 6s^2 \left[(s-4m_c^2)^2 - 2(1-z)(zs-4m_c^2)(s-8m_c^2)\right] , \quad (11)$$

$$f_2(s,z) = 2\left[(s-4m_c^2)^2(s^2+96m_c^4) - 2(1-z)(zs-4m_c^2)s(s^2-24sm_c^2+96m_c^4)\right] . \quad (12)$$

For $z < (1+\Lambda/m_c)^{-1}$, the integral over s in (8) can be calculated straightforwardly. The cutoff Λ can be set to zero everywhere except in terms proportional to $1/(1-z)$, which diverge upon integrating over z. In the $1/(1-z)$ terms, the limit $\Lambda \ll m_c$ must be taken more carefully, and it gives rise to a plus distribution:

$$\frac{1}{1-z}\,\theta\left(1 - z - \frac{\Lambda}{m_c}\right) \longrightarrow \frac{1}{(1-z)_+} - \ln\frac{\Lambda}{m_c}\,\delta(1-z) . \quad (13)$$

For $z > (1+\Lambda/m_c)^{-1}$, the limit $\Lambda \ll m_c$ can be taken only after evaluating the integral over s in (9). This gives rise to additional end-point contributions proportional to $\delta(1-z)$. Our final result for the short-distance factor multiplying H_1/m_c in the fragmentation function is

$$d_1^{(J)}(z,\Lambda) = \frac{2\alpha_s^2}{81} \left[(2J+1)\frac{z}{(1-z)_+} + \left(Q_J - (2J+1)\ln\frac{\Lambda}{m_c}\right)\delta(1-z) + P_J(z) \right] , \quad (14)$$

where the coefficients Q_J are

$$Q_0 = \frac{13}{12} , \quad Q_1 = \frac{23}{8} , \quad Q_2 = \frac{121}{24} , \quad (15)$$

and the functions $P_J(z)$ are

$$P_0(z) = \frac{z(85-26z)}{8} + \frac{9(5-3z)}{4} \ln(1-z) , \quad (16)$$

$$P_1(z) = -\frac{3z(1+4z)}{4} , \quad (17)$$

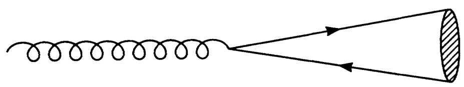

FIG. 2. Feynman diagram for $g^* \to c\bar{c}$ which contributes to the color-octet term in the fragmentation function for $g \to \chi_{cJ}$.

$$P_2(z) = \frac{5z(11-4z)}{4} + 9(2-z)\,\ln(1-z)\,. \qquad (18)$$

We next consider the color-octet coefficient $d_8(z)$ in the fragmentation formula (1). At leading order in α_s, this contribution to the fragmentation function comes from the subprocess $g^* \to c\bar{c}({}^3S_1, \underline{8})$ given by the Feynman diagram in Fig. 2. The c and $\bar{c}$ have equal momenta $q/2$, and are in a color-octet ${}^3S_1$ state. The projection onto this state can be reduced to a simple Feynman rule:

$$v(q/2)\bar{u}(q/2) \;\to\; \frac{R_8(0)}{\sqrt{16\pi m_c}} T^a_{ij}\; \not\epsilon(q)(\not q + 2m_c)\,, \qquad (19)$$

where $\epsilon^\mu(q)$ is the polarization four-vector of the ${}^3S_1$ state and i, j, and a are the color indices of the quark, antiquark, and color-octet state, respectively. The parameter $R_8(0)$ is a fictitious "color-octet radial wave function at the origin" related to the nonperturbative matrix element $H_8'(\Lambda)$ by $H_8' = (2/3\pi)|R_8(0)|^2/m_c^2$. The square of the amplitude $\mathcal{A}_\alpha$ for the subprocess $g^* \to c\bar{c}$, summed over final-state colors and spins, is

$$\sum \mathcal{A}_\alpha \mathcal{A}^*_\beta \;=\; 6\pi\alpha_s m_c^3 H_8'(\Lambda)\,\left(-g_{\alpha\beta} + \frac{q_\alpha q_\beta}{4m_c^2}\right)\,. \qquad (20)$$

The $q_\alpha q_\beta$ term can be dropped because q_α is of order m_c^2/q_0 when contracted with the numerator of the virtual gluon propagator in axial gauge. The expression therefore reduces to the polarization sum $(-g_{\alpha\beta} + \ldots)$ for an on-shell gluon multiplied by $6\pi\alpha_s m_c^3 H_8'$. Dividing by $(4m_c^2)^2$ for the virtual gluon propagator, we obtain the fragmentation probability $(3\pi\alpha_s/8)H_8'/m_c$. This probability can be identified with the second term in (1), integrated over z and summed over $J = 0,1,2$. This term in the fragmentation function contributes only at the end point $z = 1$. We can therefore identify the function $d_8(z)$ in (1) to be

$$d_8(z) \;=\; \frac{\pi\alpha_s}{24}\,\delta(1-z)\,. \qquad (21)$$

The total fragmentation function at leading order in α_s is given by the factorization formula (1), with the color-singlet coefficient given in (14) and the color-octet coefficient given in (21). To avoid large logarithms of m_c/Λ in the color-singlet coefficient, we can choose $\Lambda = m_c$. We thus arrive at the final expressions for the fragmentation functions of gluon splitting into χ_{cJ} to leading order in α_s:

$$\begin{aligned} &D_{g\to\chi_{cJ}}(z, 2m_c) \\ &\quad\approx \frac{2\alpha_s^2(2m_c)}{81}\frac{H_1}{m_c}\left[(2J+1)\frac{z}{(1-z)_+} + Q_J\,\delta(1-z) \,+\, P_J(z)\right] \\ &\qquad + (2J+1)\,\frac{\pi\alpha_s(2m_c)}{24}\frac{H_8'(m_c)}{m_c}\,\delta(1-z)\,, \end{aligned} \qquad (22)$$

where Q_J and $P_J(z)$ given by (15)–(18). The choice of the scale μ in the running coupling constant is independent of the choice of factorization scale Λ. We have followed Ref. [2] in choosing $\mu = 2m_c$, which is the minimum value of the invariant mass of the virtual gluon. If we wish to use a value for the factorization scale Λ in (22) which is significantly smaller than m_c, we should use the solution (3) to the renormalization group equation for $H_8'(\Lambda)$ to sum up the leading logarithms of m_c/Λ.

Rough estimates of the gluon fragmentation contribution to the production of the χ_{cJ} states at large transverse momentum in any high-energy process can be obtained by multiplying the cross sections for producing gluons with transverse momentum larger than $2m_c$ by appropriate fragmentation probabilities. Integrating the initial fragmentation functions (22) over z, we obtain the probabilities

$$\begin{aligned} P_{g\to\chi_{cJ}} \;&\approx\; -\,R_J\,\frac{\alpha_s^2(2m_c)H_1}{108m_c} \\ &\quad + (2J+1)\,\frac{\pi\alpha_s(2m_c)H_8'(m_c)}{24m_c}\,, \end{aligned} \qquad (23)$$

where $R_0 = 5$, $R_1 = 4$, and $R_2 = 16$. Notice that with the choice $\Lambda = m_c$ for the factorization scale, the color-singlet pieces give rise to negative contributions to the initial fragmentation probabilities. Requiring that all the probabilities (23) be positive, we obtain an interesting lower bound on $H_8'(m_c)$:

$$H_8'(m_c) \;>\; \frac{10\alpha_s(2m_c)}{9\pi}\,H_1\,. \qquad (24)$$

Using $H_1 \approx 15$ MeV, $m_c = 1.5$ GeV, and $\alpha_s(2m_c) = 0.26$, we find $H_8'(m_c) > 1.4$ MeV. The estimate $H_8'(m_c) \approx 3$ MeV obtained in Ref. [11] is consistent with this lower bound. Using the value $H_8'(m_c) \approx 3$ MeV, our estimates for the initial fragmentation probabilities in (23) are 0.4×10^{-4}, 1.8×10^{-4}, and 2.4×10^{-4} for χ_{c0}, χ_{c1}, and χ_{c2}, respectively. The production of χ_{cJ} states contributes to the inclusive rate for production of the 1^{--} charmonium state J/ψ through the radiative decay $\chi_{cJ} \to J/\psi + \gamma$. Multiplying the fragmentation probabilities given above by the appropriate radiative branching fractions of 0.7%, 27%, and 14%, we find that the probability of a J/ψ in a gluon jet is approximately 8×10^{-5}. This is more than an order of magnitude larger than the probability 3×10^{-6} for the direct fragmentation of a gluon into J/ψ that was obtained in Ref. [2].

The methods used above to calculate the fragmentation functions $D_{g\to\chi_{cJ}}(z)$ can also be used to calculate the distribution of the transverse momentum $p_\perp$ of the χ_{cJ} relative to the gluon jet. This transverse momentum is related to the invariant mass s of the gluon jet by $s = (\mathbf{p}_\perp^2 + 4m_c^2)/z + \mathbf{p}_\perp^2/(1-z)$. For the color-singlet contribution, the s distribution is obtained by integrating over z in (7). For the color-octet contribution, the s distribution is a δ function at $s = 4m_c^2$. Adding these two contributions we obtain

$$\frac{dP_{g\to\chi_{c0}}}{ds} = \frac{2\alpha_s^2 m_c H_1}{81}\,\frac{(s-12m_c^2)^2}{s^3(s-4m_c^2)}\,\theta(s - s_{\min}(\Lambda)) + \frac{\pi\alpha_s H_8'(\Lambda)}{24m_c}\,\delta(s-4m_c^2)\,, \tag{25}$$

$$\frac{dP_{g\to\chi_{c1}}}{ds} = \frac{4\alpha_s^2 m_c H_1}{27}\,\frac{s+4m_c^2}{s^2(s-4m_c^2)}\,\theta(s - s_{\min}(\Lambda)) + 3\frac{\pi\alpha_s H_8'(\Lambda)}{24m_c}\,\delta(s-4m_c^2)\,, \tag{26}$$

$$\frac{dP_{g\to\chi_{c2}}}{ds} = \frac{4\alpha_s^2 m_c H_1}{81}\,\frac{s^2+12sm_c^2+96m_c^4}{s^3(s-4m_c^2)} \times\theta(s - s_{\min}(\Lambda)) + 5\frac{\pi\alpha_s H_8'(\Lambda)}{24m_c}\,\delta(s-4m_c^2)\,, \tag{27}$$

where $s_{\min}(\Lambda) = 4m_c^2(1+\Lambda/m_c)$. Integrating over s, we recover the fragmentation probabilities given in (23). The cutoff dependence of the color-singlet terms in (25)–(27) is canceled by the Λ dependence of the parameter $H_8'(\Lambda)$ in the color-octet terms. The color-singlet terms in these invariant mass distributions were obtained previously by Hagiwara, Martin, and Stirling [14], up to an error of 4π in the overall coefficient. They did not include the color-octet contributions, so their answers were sensitive to the value of the infrared cutoff Λ. In the region near the lower end point $s = 4m_c^2$, the distributions (25)–(27) must of course be smeared over an appropriate range in $p_\perp$ before they can be compared with experimental data.

We have calculated the fragmentation functions for gluons to split into P-wave quarkonium states to leading order in α_s. The fragmentation functions satisfy a factorization formula with two nonperturbative parameters H_1 and H_8' which can be determined from other processes involving the annihilation and production of P-wave states. These fragmentation functions are universal and can be used to calculate the rates for the direct production of P-wave states at large transverse momentum in any high-energy process. They are also needed to calculate the total production rate of the 1^{--} states from the fragmentation mechanism, since the P-wave states have significant rates for transitions to the 1^{--} states. The fragmentation probabilities for $g\to\chi_{c1}$ and $g\to\chi_{c2}$ were estimated to be on the order of 10^{-4}. This is large enough that gluon fragmentation into χ_{cJ} should account for a significant fraction of the χ_{cJ}'s that are observed at large transverse momentum in hadron colliders. Fragmentation into χ_{cJ} followed by its radiative decay may also account for a significant fraction of the J/ψ's that are produced at large p_T.

While this paper was being written, we received a paper by Ma [15], in which the color-singlet term in the fragmentation function for $g\to\chi_{c1}$ is calculated for longitudinally and transversely polarized χ_{c1} separately. After summing over polarizations, his result agrees with ours except near the end point $z=1$. In the end-point region, Ma's fragmentation function is sensitive to an infrared cutoff.

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