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Leptoproduction of heavy quarks. I. General formalism and kinematics of charged current and neutral current production processes

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Existing calculations of heavy quark production in charged current and neutral-current lepton-hadron scattering are formulated differently because of the artificial distinction of "light" and "heavy" quarks made in the traditional approach. A proper QCD formalism valid for a wide kinematic range from near threshold to energies much higher than the quark mass should treat these processes in a uniform way. We formulate a unified approach to both types of leptoproduction processes based on the conventional factorization theorem. In this paper, we present the general framework with complete kinematics appropriate for arbitrary masses, emphasizing the simplifications provided by the helicity formalism. We illustrate this approach with an explicit calculation of the leading-order contribution to the quark structure functions with general masses. This provides the basis for a complete QCD analysis of charged-current and neutral-current leptoproduction of charm and bottom quarks to be presented in subsequent papers.

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I. INTRODUCTION

Total inclusive lepton-hadron deep-inelastic scattering has been the keystone of the quark-parton picture and the QCD-based parton model. As the global QCD analysis of high energy interactions becomes more precise, other processes begin to play an increasingly important role in determining the parton distributions inside the nucleon [1–4]. For instance, semi-inclusive charm production in charged current and neutral current interactions in lepton-hadron scattering serves as a unique probe of the strange quark and charmed quark content of the nucleon [5–7]. In general, the production of heavy flavors in lepton-hadron and hadron-hadron colliders is a very important tool for quantitative QCD study and for searches for new physics [8–16].

Traditional analysis of massive quark production in deep-inelastic scattering (DIS) uses the simple light flavor parton model formulas (based on tree-level forward Compton scattering off the quark) with a "charm threshold" or "slow-rescaling" correction [17–19]. This prescription is still widely used in current literature, particularly for dimuon production in neutrino charged current scattering [5–7]; however, the applicable range of this approach is very limited—for the neutral current case by the mass of the initial state quark, and for both cases by the numerically important next-order gluon contribution [20]. In most neutral current charm production calculations and recent DESY HERA studies of heavy flavor production, a contrasting view has been prevalent: one forsakes the leading-order quark scattering mechanism and concentrates on the $O(\alpha_s)$ "gluon-fusion" processes [22,23]. Whereas this latter approach is appropriate when the hard scattering scale of the process, say Q, is of the same order of magnitude as the quark mass m [24], it is a poor approximation at high energies. In fact, when m/Q is small, these "gluon-fusion" diagrams contain large logarithms, i.e., factors of the form $\alpha_s^n \ln^n(m/Q)$, which vitiates the perturbation series as a good approximation. These large logarithms need to be resummed, which then yield quark-scattering contributions with properly evolved parton distribution for the not-so-heavy massive quark.

A consistent QCD analysis of this problem requires a renormalization scheme which contains the two conventional approaches as limiting cases—in their respective region of validity—and provides a smooth transition in the intermediate region where neither approximation is accurate. Such a scheme, motivated by the Collins-Wilczek-Zee [25] renormalization procedure, was proposed some time ago in the context of Higgs production, resulting in a satisfactory theory valid from threshold to asymptotic energies [26]. This approach also provides a natural framework for heavy quark production. It is particularly simple to implement in leptoproduction production processes, and has been applied to charm production in DIS in a previous short report [20].

The current paper is the first of a series which will give a detailed formulation of this problem. In systematically developing a consistent formulation of heavy-flavor production in DIS, one finds that conventional calculations, *even at the leading order level*, make implicit approximations inherited from the zero-mass parton model—such as the Callan-Gross relation and the choice of the scaling variable—which are not always valid in the presence of masses. In order to make a fresh start on a consistent theory including nonzero-mass partons, this first paper is devoted to a self-contained development of the general formalism of deeply inelastic scattering in the presence of masses which is valid for both charged and neutral current interactions. Much of this is kinematical in nature. In considering charm production in existing fixed target neutrino experiments, an important practical consideration is that the target nucleon mass is comparable to the charm quark mass, and *both* are non-negligible compared to the average energy scale Q of the process. Thus, for consistency, target mass effects should also be incorporated precisely [21]. To this end, we present a helicity formalism (along with the conventional tensor approach) to develop the general framework. It will become clear that whereas the conventional tensor method becomes quite complicated when both target mass and quark mass effects are properly incorporated, the helicity formalism retains the same simplicity throughout—due to its group-theory origin and to a key feature of the QCD parton model. To make the general formalism concrete, we shall apply this helicity approach to a complete leading order calculation of heavy flavor production in charged current DIS, and then compare with the conventional tensor calculations. Numerical studies will show that the complete calculation (with all masses retained) leads to significant differences in the calculated cross sections in certain regions of phase space. In the text of this paper, we shall emphasize the key elements of these developments. Most technical details are relegated to the appendices.

The second paper of this series [27] shall be focused on the consistent QCD formulation of heavy quark production in the context of order α_s calculation of this process, using the general kinematical formalism developed here. The emphasis will be on the formulation of a consistent renormalization and factorization scheme to reconcile the quark-scattering and the gluon-fusion mechanisms. The QCD framework developed there applies to all heavy quark processes, including hadroproduction. In subsequent paper, we shall study the phenomenological consequences of these calculations on the analysis of existing dimuon data from fixed target experiments, and on predictions of charm and bottom production at HERA.

II. SCATTERING AMPLITUDES

We consider a general lepton-hadron scattering process[1]

$$l_1(l_1) + N(P) \to l_2(l_2) + X(P_X) \qquad (1)$$

as depicted in Fig. 1 where the exchanged vector boson (γ, W, or Z) will be labeled by B and its momentum by q.

The lepton-boson and quark-boson couplings are specified by the following generic expression for the effective fermion-boson term in the electroweak Lagrangian:

$$\mathcal{L}_{\rm int}^{\rm EW} = -g_B[j_\mu^{(l)}(x) + J_\mu^{(h)}(x)]V_B^\mu(x), \qquad (2)$$

[1] In the production of a heavy quark Q, the final state is given by $X = Q + X'$ where X' is unobserved. For the purposes of the present discussion, we shall not single out Q from X.

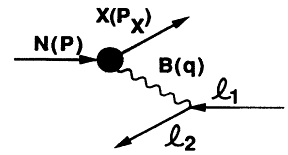

FIG. 1. The general lepton-hadron scattering process: $N(P) + l_1 \to X(P_X) + l_2$ via the exchange of a vector boson, $B(q)$. The lepton momenta are l_i while the initial and final hadronic momenta are P and P_X, respectively.

where a summation over B is implied. The gauge coupling constant g_B for the vector boson field V_B depends on B and their values as prescribed by the standard model are given in Table I.

Both the hadronic and fermionic current operators are defined by

$$\begin{aligned} J_\mu^{(f)}(x) &= \bar\psi_f(x)\gamma_\mu(g_V - g_A\gamma^5)\psi_f(x) \\ &= \bar\psi_f(x)\gamma_\mu[g_R(1+\gamma^5) + g_L(1-\gamma^5)]\psi_f(x) \ , \end{aligned} \qquad (3)$$

where ψ_f denotes a generic fermion field, and the vector and axial vector couplings $g_{V,A}$ are related to their chiral counterparts by $g_{L,R}$ by $g_{V,A} = g_L \pm g_R$. The values of those fermion coupling constants, according to the standard model, are given in Table II; however, we will keep them general in our considerations.

The scattering amplitude for the process of Eq. (1)—with particle momenta as shown in Fig. 1—is given by

$$\mathcal{M} = J_\mu^*(P,q)\frac{g_B^2 G^\mu{}_\nu}{Q^2 + M_B^2} j^\nu(q,l), \qquad (4)$$

where $q = l_1 - l_2$, $l = l_1 + l_2$, $Q^2 = -q^2 > 0$, and $G^\mu{}_\nu = g^\mu{}_\nu - q^\mu q_\nu/M_B^2$. The lepton current matrix element is given by

$$\begin{aligned} j^\mu(q,l) &= \langle l_2|j^\mu|l_1\rangle \\ &= \bar u(l_2)\gamma^\mu[g_R(1+\gamma^5) + g_L(1-\gamma^5)]u(l_1) \ . \end{aligned} \qquad (5)$$

The hadron current matrix element is kept in the general form: $J_\mu^*(P,q) = \langle P_X|J_\mu^\dagger|P\rangle$. For simplicity, we have suppressed the polarization indices for all external particles in Eq. (4). Furthermore, the term G_ν^μ can be replaced by g_ν^μ in actual applications since the term proportional to $q^\mu q_\nu$ (when contracted with the lepton current matrix element) yields terms proportional to m_l^2/Q^2 which are negligible at high energies.

An alternative expression to the above familiar formulation of the scattering amplitude which emphasizes the helicity of the exchanged vector boson is given by [28,29]:

$$\mathcal{M} = J_m^*(Q^2, P\cdot q)\frac{g_B^2 d^1(\psi)^m{}_n}{Q^2 + M_B^2} j^n(Q^2), \qquad (6)$$

where n and m are helicity indices for the vector boson,

TABLE I. The gauge couplings of the vector bosons according to the standard model.

B	γ	$W^\pm$	Z
g_B	$-e$	$\frac{g}{2\sqrt{2}}$	$\frac{g}{2\cos\theta_W}$

TABLE II. The gauge couplings of the vector bosons according to the standard model. V_{ij} represents the Cabibbo-Kobayashi-Maskawa (CKM) flavor mixing, if relevant, and Q_i is the fermion charge in units of $|e|$.

	γ	Z	$W^{\pm}$
g_V	Q_i	$T^i_{3L} - 2Q_i \sin^2\theta_W$	$1 \cdot V_{ij}$
g_A	0	T^i_{3L}	$1 \cdot V_{ij}$
g_R	$\frac{Q_i}{2}$	$-Q_i \sin^2\theta_W$	0
g_L	$\frac{Q_i}{2}$	$T^i_{3L} - Q_i \sin^2\theta_W$	$1 \cdot V_{ij}$

$j^n(Q^2)$ and $J^*_m(Q^2, P\cdot q)$ are the scalar helicity amplitudes for the two vertices shown in Fig. 1, and $d^1(\psi)$ is a spin-1 "rotation" matrix specifying the relative orientation of the two vertices. The derivation of this formula can be found in [28,29]; the precise definition of the rotation angle[2] ψ is given in Appendix A. (See also Appendix B for details.) We note that the structure of Eq. (6) is quite similar to Eq. (4) above. The advantages of using the helicity formulation in the QCD analysis of heavy quark production will be discussed in Section IV.

III. CROSS-SECTION FORMULAS AND HADRON STRUCTURE FUNCTIONS

The cross-section formula for this process is (cf. Appendix A)

$$d\sigma = \frac{G_1 G_2}{2\Delta(s, m_{l_1}^2, M^2)} 4\pi Q^2 L^\mu_\nu W^\nu_\mu d\Gamma \ , \tag{7}$$

where $G_i = g_{B_i}^2/(Q^2 + M_{B_i}^2)$ is a shorthand for the boson coupling and propagator. The two indices B_1 and B_2 denoting the species of the exchanged vector bosons are implicitly summed over and kept distinct to accommodate the possibility of γ-Z interference, and $d\Gamma$ is the phase space of the final state lepton. The factor $4\pi Q^2$ is from the normalization of L and W. In the above expression we have introduced the dimensionless lepton and hadron tensors given by[3]

$$L^\mu{}_\nu = \frac{1}{Q^2} \overline{\sum_{\text{spin}}} \langle l_1 | j^\dagger_\nu | l_2 \rangle \langle l_2 | j^\mu | l_1 \rangle \ , \tag{8}$$

$$W^\mu{}_\nu = \frac{1}{4\pi} \overline{\sum_{\text{spin}}} (2\pi)^4 \delta^4(P + q - P_X) \times \langle P | J^\mu | P_X \rangle \langle P_X | J^\dagger_\nu | P \rangle \ . \tag{9}$$

The explicit expression for $L^\mu{}_\nu$ with general coupling constants is given in Appendix B. As is well known, the hadron tensor $W^\mu{}_\nu$ can be expanded in terms of a set of six independent basis tensors[4]

$$\begin{aligned} W^\mu{}_\nu = & -g^\mu{}_\nu W_1 + \frac{P^\mu P_\nu}{M^2} W_2 - i \frac{\epsilon^{Pq\mu}{}_\nu}{2M^2} W_3 \\ & + \frac{q^\mu q_\nu}{M^2} W_4 + \frac{P^\mu q_\nu + q^\mu P_\nu}{2M^2} W_5 \\ & + \frac{P^\mu q_\nu - q^\mu P_\nu}{2M^2} W_6 \ , \end{aligned} \tag{10}$$

where M is the target mass and $\epsilon^{Pq\mu\nu} = \epsilon^{\alpha\beta\mu\nu} P_\alpha q_\beta$. The scalar coefficient functions $\{W_i\}$ are the *invariant hadron structure functions* for this process.

By substituting the lepton and hadron tensors in Eq. (7) and partially integrating over the phase space of the final state lepton, one obtains, in the limit of negligible lepton masses, the well-known cross-section formula, generalized to arbitrary couplings,

$$\begin{aligned} \frac{d\sigma}{dE_2 d\cos\theta} = \frac{2E_2^2}{\pi M} \frac{G_1 G_2}{n_l} \Big\{ & g^2_{+l} \left[2W_1 \sin^2\frac{\theta}{2} + W_2 \cos^2\frac{\theta}{2} \right] \\ & \pm g^2_{-l} \left[\frac{E_1 + E_2}{M} W_3 \sin^2\frac{\theta}{2} \right] \Big\} \end{aligned} \tag{11}$$

where the $\pm$ sign for the W_3 term refers to the case of lepton and antilepton scattering, respectively. Here, E_1 and E_2 are the energies of the initial and final state leptons respectively in the laboratory frame, θ is the scattering angle of the lepton in the same frame, and n_l is the number of polarization states of the incoming lepton. To simplify the expression, we define $g^2_{\pm l} = g^2_{Ll} \pm g^2_{Rl}$, where g_{Ll} and g_{Rl} refer to the chiral couplings of the vector boson to the leptons.[5]

It is worth noting that the hadron structure functions $\{W_4, W_5, W_6\}$ do not appear on the right-hand side because they are multiplied by factors of lepton mass from the lepton vertex, *not because they are intrinsically small* compared to the familiar $\{W_1, W_2, W_3\}$. This will become relevant when we discuss the calculation of hard scattering cross sections involving heavy quarks.

It is by now customary to introduce the scaling structure functions F_i given by

$$\begin{aligned} F_1 &= W_1 \ , \\ F_2 &= \frac{\nu}{M} W_2 \ , \\ F_3 &= \frac{\nu}{M} W_3 \end{aligned} \tag{12}$$

in terms of which the expression for the differential cross section may be rewritten as

[2] For spacelike q, ψ is actually a hyperbolic angle specifying a Lorentz boost.

[3] Historically, the definition of $W^\mu{}_\nu$—and thus the definitions of W_i in Eq. (10)—contains an extra factor of M, the target mass. In view of scaling considerations, it is more natural to use the dimensionless definition. Also note that sums and integrals over all the unobserved hadronic final states X are implied in Eq. (9).

[4] In some papers, the tensor associated with W_1 is chosen to be the gauge invariant form $(-g^\mu_\nu + q^\mu q_\nu/q^2)$, and that associated with W_2 is obtained with the substitution $P^\mu \to P^\nu(g^\mu_\nu - q^\mu q_\nu/q^2)$; these changes (convenient for conserved currents) will modify the definitions of W_4, W_5, and W_6 only.

[5] The lepton chiral couplings appear explicitly because L^μ_ν has been evaluated. The corresponding hadron chiral couplings reside implicitly in the $\{W_i\}$ invariant structure functions.

$$\frac{d\sigma}{dxdy} = \frac{2ME_1}{\pi}\frac{G_1G_2}{n_l}\left\{g_{+l}^2\left[xF_1y^2 + F_2\left((1-y) - \left(\frac{Mxy}{2E_1}\right)\right)\right] \pm g_{-l}^2[xF_3y(1-y/2)]\right\}. \tag{13}$$

In the alternative helicity formalism, the expression for the cross section is given by

$$\frac{d\sigma}{dxdy} = \frac{yQ^2}{2\pi}\frac{G_1G_2}{n_l}\left\{g_{+l}^2\left[\frac{(F_+ + F_-)}{2}(1+\cosh^2\psi) + F_0\sinh^2\psi\right] \mp g_{-l}^2[(F_+ - F_-)\cosh\psi]\right\}, \tag{14}$$

where ψ is the hyperbolic rotation angle of Eq. (6), and we have introduced the *helicity structure functions* $\{F_\lambda, \lambda = +, 0, -\}$ which correspond to the physical forward Compton scattering helicity amplitudes

$$F_\lambda = \epsilon_\mu^{\lambda *}(P,q)W^\mu{}_\nu(P,q)\epsilon_\lambda^\nu(P,q) \quad \text{(no sum over } \lambda) \tag{15}$$

with *right-handed* (+), *longitudinal* (0), and *left-handed* (−) vector bosons respectively.[6] We note that the first term on the right hand side involves the *transverse* structure function $F_T = (F_+ + F_-)/2$ whereas the third term is the parity-violating term with F_+-F_- proportional to F_3 in Eq. (13). Eq. (14) should be familiar, as it is analogous to the corresponding well-known formulas for timelike vector boson production processes—Drell-Yan pairs and W, Z production—where the hyperbolic angle ψ is replaced by the center-of-mass angle θ for the final state lepton pair.

The helicity structure functions as defined above are naturally scaling functions. In addition, their direct physical interpretation leads to simple properties in the QCD parton model framework, as we shall see in the next section. Note that Eq. (14) does not show any explicit target mass dependence; all complications arising from the nonvanishing mass are contained in the definition of the rotation angle ψ through kinematics. This simplicity is a consequence of the underlying group-theoretical approach to the factorized structure of Fig. 1. The precise relations between the helicity structure functions and the invariant structure functions are found (cf. Appendix B) to be

$$\begin{aligned} F_+ &= F_1 - \frac{1}{2}\sqrt{1+\frac{Q^2}{\nu^2}}F_3\,, \\ F_- &= F_1 + \frac{1}{2}\sqrt{1+\frac{Q^2}{\nu^2}}F_3\,, \\ F_0 &= -F_1 + \left(1+\frac{Q^2}{\nu^2}\right)\left(\frac{1}{2x}\right)F_2\,. \end{aligned} \tag{16}$$

We see in the limit $M \to 0$ that $Q^2/\nu^2 \to 0$ and we obtain the approximation: $F_\pm \simeq F_1 \mp F_3/2$ and $F_0 \simeq -F_1 + F_2/2x$.

To leading order in the electroweak coupling, Eq. (11), Eq. (13), and Eq. (14) are completely general, assuming only Lorentz kinematics and small lepton masses. In particular, all results up to this point are independent of strong interaction dynamics. Aside from Eq. (14), they are well-established formulas explicitly generalized to include arbitrary couplings.

IV. THE QCD FACTORIZATION FORMULAS

Perturbative QCD allows one to relate the measurable hadron structure functions $\{F_i\}$ to the corresponding quantities involving elementary particles—the partons—which can be calculated in perturbation theory. This section states the basic QCD "factorization theorem" as it applies to deeply inelastic scattering processes and points out some important unfamiliar features in the presence of nonzero masses, especially when the initial state parton is a heavy quark.

A. Factorization of tensor amplitudes

The factorization theorem [30] states that, in the Bjorken limit, the dominant contributions to the hadronic tensor structure function has the factorized form of Fig. 2 with on-shell, collinear partons:

$$\begin{aligned} W_{\mu\nu}^{BN}(q,P,\ldots) &= \sum_a f_N^a \otimes \omega_{\mu\nu}^{Ba} \\ &= \sum_a \int \frac{d\xi}{\xi} f_N^a(\xi,\mu)\omega_{\mu\nu}^{Ba}(q,k_1,\ldots,\alpha_s(\mu)). \end{aligned} \tag{17}$$

In Eq. (17), the label a is summed over all parton species. The convolution integral variable ξ is the momentum fraction carried by the parton with respect to the hadron defined in terms of the ratio of light-cone momentum components $\xi = k_1^+/P^+$. The universal parton distribution functions f_N^a are scalars; scattering of

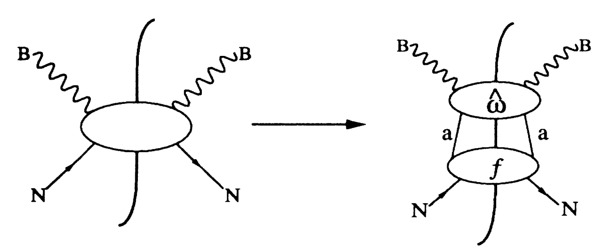

FIG. 2. Pictorial representation of the factorization theorem for the hadronic structure functions for inclusive deeply inelastic scattering. The process on the left is $N(P) + B(q) \to X(P_X)$, and the factorized process on the right is $N(P) \to a(k_1)$ (represented by the parton distribution function, f_N^a) with the successive hard scattering interaction $a(k_1) + B(q)$ (represented by $\omega_{\mu\nu}^a$). The vertical lines indicate an inclusive sum over the final states, $X(P_X)$.

[6]The choice of these labels—over the more obvious R, L, etc.—is constrained by the conflict between the left-handed and longitudinal designations. For $m_l = 0$, we can ignore $F_\lambda = \{F_{qq}, F_{q0}, F_{0q}\}$, cf. Appendix B.

the vector boson takes place with the partons via the hard-scattering factor $\omega^{Ba}_{\mu\nu}$ which can be aptly called the *parton structure function* tensor since it is entirely analogous to the hadron structure function tensor $W^{BN}_{\mu\nu}$ by substituting the hadron target N with the parton target a. Note that the tensor structure of $W^{BN}_{\mu\nu}$ is completely determined by that of $\omega_{\mu\nu}$. These features should be obvious by inspection of Fig. 2. Strictly speaking, the factorization theorem is established in this simple form only for certain specifically defined asymptotic regimes. We shall treat Eq. (17) as an ansatz and apply it in such a way that our results reduce to the known correct expressions in the limits $\Lambda \ll m_2 \simeq Q$ on the one hand, and $\Lambda < m_2 \ll Q$ on the other. (Here m_2 denotes a heavy quark mass.)

The presence of heavy quarks among the initial and final state partons in $\omega_{\mu\nu}$ has some important consequences. The most immediate one is that the range of integration in Eq. (17) will depend on the masses of the heavy quark as a simple consequence of the kinematics of the hard scattering. In leading order QCD, where the integration range reduces to a single point, this naturally gives rise to a generalized "slow-rescaling" variable which was originally proposed in the context of the simple parton model [17] (cf., Appendix A). In addition, the tensor structure of the perturbatively calculable $\omega^{\mu\nu}$ is clearly different from that of the naive parton model, even in leading order QCD. For example, the well-known Callan-Gross relation simply does not hold in the presence of heavy quark mass. A proper treatment of heavy quark production must use the correct hard-scattering amplitude $\omega^{Ba}_{\mu\nu}$ (calculated to the appropriate order, including quark masses) in conjunction with choosing the proper variable. A "slow-rescaling prescription" of a simple variable substitution is not sufficient, cf. Sec. VI.

In order to apply the factorization theorem to measurable quantities properly, we must re-express Eq. (17) in terms of the independent invariant structure functions $\{W_i\}$ or the helicity structure functions $\{F_\lambda\}$ in a precise way. Theoretical calculations of the parton-level hard amplitudes on the right-hand side of the equation usually yield the (parton) invariant or helicity amplitudes, not the tensor $\omega^{\mu\nu}$ itself. In the presence of target and heavy quark masses, we will find that the relations between the invariant structure functions at the hadron and the parton levels are far from being simple, as usually assumed in existing literature. In contrast, the connection between the corresponding helicity structure functions are completely transparent.

B. Invariant structure functions

The parton-level invariant amplitudes ω_i are defined in analogy to Eq. (10), as follows:[7]

$$\omega^\mu{}_\nu = -g^\mu{}_\nu\omega_1 + \frac{k_1^\mu k_{1\nu}}{Q^2}\omega_2 - i\frac{\epsilon^{k_1 q\mu}{}_\nu}{2Q^2}\omega_3 + \frac{q^\mu q_\nu}{Q^2}\omega_4 + \frac{k_1^\mu q_\nu + q^\mu k_{1\nu}}{2Q^2}\omega_5 + \frac{k_1^\mu q_\nu - q^\mu k_{1\nu}}{2Q^2}\omega_6\,, \tag{18}$$

where k_1 is the momentum of the incident parton. Substituting Eq. (18) in Eq. (17) and comparing $\omega^\mu{}_\nu$ with $W^\mu{}_\nu$ [Eq. (10)], we see that the relations between invariant structure functions at the hadron and the parton levels depend on the relation between k_1^μ and P^μ. Whereas the two momenta are proportional in the zero mass limit, this relation becomes nontrivial in the presence of *either* target mass or parton mass (cf. Appendix A). Since the vectors P, k_1, and q are collinear, we can parametrize k_1 as

$$k_1^\mu = \zeta_P P^\mu + \zeta_q q^\mu. \tag{19}$$

In the zero mass limit, $\zeta_P \to \xi$ and $\zeta_q \to 0$. In general, the coefficients (ζ_P, ζ_q) are rather complicated functions of the masses and the *convolution variable* ξ [cf. Eqs. (B19), (B20)]. Thus, the relations between the W_i and the ω_i are also rather complicated. Relevant formulas which relate W_i to ω_i are given in Appendix B.

C. Helicity structure functions

In sharp contrast to the above, the factorization theorem assumes a simple form when expressed in terms of the helicity basis. To see this, let us define the parton *helicity structure functions* ω_λ, in analogy to Eq. (15), by

$$\omega_\lambda = \epsilon_\mu^{\lambda *}(k_1,q)\omega^\mu{}_\nu\epsilon_\lambda^\nu(k_1,q) \quad (\text{no sum over } \lambda)\,. \tag{20}$$

In order to relate these to the hadron helicity structure functions F_λ, Eq. (15), it appears that one needs to re-express the vector-boson polarization vectors $\{\epsilon_\lambda^\nu(k_1,q)\}$ (defined using k_1 as the reference momentum) in terms of $\{\epsilon_\lambda^\nu(P,q)\}$ (defined using P as the reference momentum). The enormous simplification of the helicity approach follows from the fact that *the two sets of polarization vectors are in fact identical even in the presence of masses,* hence no transformation is needed! The reason for this is that for a given vector-boson momentum q, the reference momentum is used only to specify the direction of the polarization axis; the two seemingly different reference momenta k_1 and P actually specify the same set of polarization vectors because they are collinear in the QCD parton framework. Thus, we arrive at the straightforward formula:

$$F_\lambda^{BN}(q,P,\ldots) = \sum_a f_N^a \otimes \omega_\lambda^{B,a}\,. \tag{21}$$

This suggests that to explore the consequences of perturbative QCD on heavy quark production (as well as on all other processes), it is advantageous to perform the calculation in the helicity basis. The simple formula Eq. (21), together with Eq. (14), relate the calculation of hard scattering amplitudes directly to measurable cross

[7] In order to render the ω_i dimensionless, we use the natural variable Q rather than any parton mass in scaling the tensors so that the invariant structure functions have well-defined limits as $m/Q \to 0$. (Note that if the hadronic structure functions were originally defined this way, rather than using the target mass M as the scale factor, $\{W_i\}$ would be naturally "scaling.")

sections without any approximations or complications. Besides, since the parton-level helicity amplitudes have simple symmetry and structure, due to the basic chiral couplings of the theory, the results of this approach are often the most physical and compact to begin with.

V. LEADING ORDER QCD CALCULATION OF HEAVY FLAVOR PRODUCTION

To illustrate the use of the general formalism developed above, we apply it to the calculation of heavy quark production in leading order QCD. Existing applications of heavy quark production in DIS mostly concern charm production in charged current interactions at fixed-target energies. Since the charm mass is comparable to the target mass for existing neutrino experiments, and neither is negligible compared to the energy scale Q, it is reasonable to retain the target mass effects in order to be self consistent. Numerical comparisons of the complete calculation (with full target mass dependence) to the conventional one show that the difference can be significant in certain regions of the phase space.

The leading order diagram that contributes to ω_λ is shown in Fig. 3 and its contribution, including all masses and arbitrary couplings, is calculated explicitly in Appendix C. We consider charm production in charged current neutrino scattering. Since, the W-exchange process involves only left-handed chiral couplings (cf. Table II). The parton helicity structure functions for scattering from a strange quark are given by

$$\omega_\pm = g_{La}^2 \frac{Q^2+m_1^2+m_2^2\mp\Delta}{\Delta}\delta\left(\frac{\xi}{\chi}-1\right),$$
$$\omega_0 = g_{La}^2 \frac{(m_2^2-m_1^2)^2/Q^2+m_2^2+m_1^2}{\Delta}\delta\left(\frac{\xi}{\chi}-1\right), \quad (22)$$

where g_{La}^2 is the left-handed coupling of the W to the a-type parton, ξ is the convolution variable of Eq. (17), m_1 is the initial parton mass, m_2 is the heavy quark mass, and χ and Δ are given by

$$\chi = \eta\frac{(Q^2-m_1^2+m_2^2)+\Delta}{2Q^2}, \quad (23)$$

$$\Delta = \Delta[-Q^2, m_1^2, m_2^2], \quad (24)$$

where η [Eq. (A17)] is the target-mass corrected Bjorken x, and Δ is the triangle function [Eq. (A5)], both defined in Appendix A.

Substituting in Eq. (21), we obtain simple but nontrivial formulas for the hadron helicity structure functions. The δ function in Eq. (22) fixes the momentum fraction variable $\xi=\chi$. Since $\omega_0\neq 0$, we see explicitly that the longitudinal structure function cannot be neglected *even to leading order*. It is proportional to the quark masses when they are nonvanishing; thus, the Callan-Gross relation does not apply in its original form.

For charm-production, the initial parton is either a d or s quark; both can be treated as massless. In the limit $m_1\to 0$, one obtains

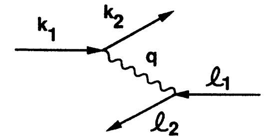

FIG. 3. Leading-order hard-scattering amplitude for heavy quark productions.

$$\omega_+ = 0, \quad (25)$$

$$\omega_- = g_{La}^2 2\delta(\xi/\chi-1), \quad (26)$$

$$\omega_0 = g_{La}^2\frac{m_2^2}{2Q^2}2\delta(\xi/\chi-1), \quad (27)$$

and $\chi=\eta(1+m_2^2/Q^2)$. Thus, the helicity structure functions assume the following simple form:

$$F_+ = 0, \quad (28)$$

$$F_- = g_{La}^2 2q_N^a(\chi), \quad (29)$$

$$F_0 = g_{La}^2\frac{m_2^2}{2Q^2}2q_N^a(\chi), \quad (30)$$

where an implicit sum over contributing parton species a is implied. By applying the general expression of Eq. (7), one obtains

$$\frac{d\sigma^\nu}{dxdy} = G_W^2 g_{Ll}^2 g_{La}^2 2q_N^a(\chi)\frac{yQ^2}{\pi} \times\left[\left(\frac{1+\cosh\psi}{2}\right)^2+\frac{m_2^2}{2Q^2}\frac{\sinh^2\psi}{2}\right], \quad (31)$$

where ψ is defined by Eq. (A23), $g_{Ll}=1$ and $g_{La}=\cos\theta_C(\sin\theta_C)$ for $a=s(d)$, respectively. Note, $G_W = g_{B_W}^2/(Q^2+M_W^2) = (G_F/\sqrt{2})/(1+Q^2/M_W^2)$.

The corresponding formula for antiquark production via lepton scattering, obtained from the interchange of g_{La} and g_{Ra} in the expressions for ω_λ, yields:

$$F_+ = g_{L\bar{a}}^2 2\bar{q}_N^{\bar{a}}(\chi), \quad (32)$$

$$F_- = 0, \quad (33)$$

$$F_0 = g_{L\bar{a}}^2\frac{m_2^2}{2Q^2}2\bar{q}_N^{\bar{a}}(\chi), \quad (34)$$

and

$$\frac{d\sigma^{\bar\nu}}{dxdy} = G_W^2 g_{Ll}^2 g_{L\bar{a}}^2 2\bar{q}_N^{\bar{a}}(\chi)\frac{yQ^2}{\pi} \times\left[\left(\frac{1-\cosh\psi}{2}\right)^2+\frac{m_2^2}{2Q^2}\frac{\sinh^2\psi}{2}\right]. \quad (35)$$

These results still retain the full kinematic target-mass dependence (cf. Appendix A). If one sets $M = 0$, the expressions for the cross section in Eqs. (31) and (35) stay unchanged; only the definitions of ψ and χ simplify. In particular

$$\chi \underset{m_1 \to 0}{\longrightarrow} \eta \left(1 + \frac{m_2^2}{Q^2}\right) \underset{M \to 0}{\longrightarrow} x \left(1 + \frac{m_2^2}{Q^2}\right) \qquad (36)$$

which is the "slow-rescaling" variable.

VI. COMPARISON WITH EXISTING CALCULATIONS

There are a variety of "slow-rescaling" prescriptions in the literature with varying degrees of accuracy [17]. Some analyses of charm production in DIS use a slow-rescaling corrected parton model prescription which consists of using the familiar zero-mass parton model cross-section with the substitution:

$$x \to \xi = x \left(1 + \frac{m_2^2}{Q^2}\right) . \qquad (37)$$

This prescription incorporates only the heavy quark mass effect for the on-mass shell kinematics—the delta function of Eq. (22)—but ignores corrections to the "body" of the partonic (hard) structure functions ω_λ in the same equation. It is therefore inherently inconsistent.

An improved treatment is obtained by using the exact expression for the Born diagram with $m_1 = 0$ and $M = 0$. The results are simple enough so that the final m_2 dependence can be rewritten to appear as a "slow-rescaling" corrected formula, as follows:

$$\frac{d\sigma}{dxdy} = G_W^2 g_{Li}^2 g_{La}^2 \frac{2Q^2}{\pi y} \left\{ \left[y + \frac{\xi}{x}(1-y) \right] q(\xi) + \left[y(y-1) + \frac{\xi}{x}(1-y) \right] \bar{q}(\xi) \right\} . \qquad (38)$$

By definition, this modified prescription ignores target mass effects in the parton kinematics that are not necessarily small compared with heavy quark effects. Equation (38) should be compared with Eq. (31) which has implicit M dependence in $\cosh\psi$, $\sinh\psi$, and χ.

Some papers include the target mass dependence of the cross section Eq. (13), i.e., the term $-Mxy/(2E_1)$, so that the cross section for neutrino production reads

$$\frac{d\sigma^\nu}{dxdy} = G_W^2 g_{Li}^2 g_{La}^2 \frac{2Q^2}{\pi y} \times \left\{ y + \frac{\xi}{x}(1-y) - \frac{\xi}{x}\left(\frac{Mxy}{2E_1}\right) \right\} q(\xi) . \qquad (39)$$

Numerically, this term has negligible effect; the $-Mxy/(2E_1)$ term does not approximate the true target mass dependence, and for all practical purposes, Eq. (38) and Eq. (39) are identical at the $\leq 2\%$ level.

We now present numerical results comparing cross sections calculated using the complete leading order formula Eq. (31) with that using the slow-rescaling prescription, Eq. (39). In Fig. 4 we compare the y and x dependence for $\nu + N \to \mu^- + c + X$ for neutrino energies ranging from 50 GeV to 300 GeV—a reasonable range for fixed target experiments. For simplicity, we only consider the dominant sub process: $W + s \to c$. As anticipated, for both the x and y distributions, the deviations decrease with increasing neutrino energy (hence, increasing Q^2) since the M^2/Q^2 and m_2/Q^2 terms are decreasing. The y distribution agrees well at large y, but deviates from the complete leading order result by more than 25% for small y where the effects of the charm mass threshold are significant. The deviation of the x distribution ranges from a few percent at small x to $\geq 25\%$ at large x. Thus the difference between the conventional slow-rescaling prescription and our approach, which is based on the factorization theorem, are not negligible. The main source of discrepancy arises from the charmed quark mass m_2 which is only slightly larger than the target mass M; the latter should not be neglected if effects due to the former are significant. In particular, the momentum fraction variable $\xi = \chi$ which enters the precise formula Eq. (31) is approximately

$$\xi = \chi \simeq x \left(1 + \frac{m_2^2}{Q^2}\right)\left(1 - \frac{x^2M^2}{Q^2}\right) \qquad (40)$$

when m_2^2/Q^2 and M^2/Q^2 are small, and $m_1 = 0$. In other words, the conventional "slow-rescaling" variable itself needs a target-mass correction.

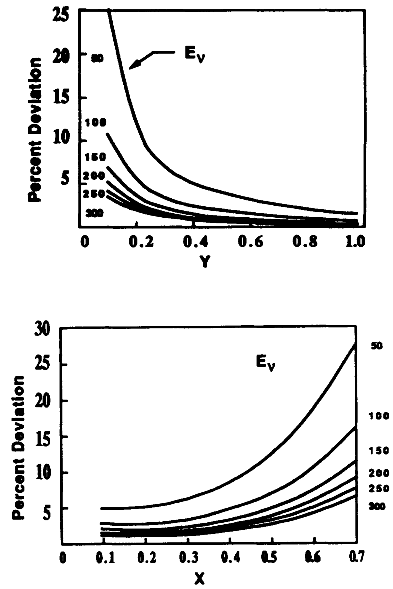

FIG. 4. Percent deviation of leading-order cross section between the "slow-rescaling," Eq. (39), and complete, Eq. (31), for $E_\nu = 80$ GeV, $m_c = 1.5$ GeV: (a) $d\sigma/dy(\nu + s \to c)$ integrated in x over the range $x = [0.1, 0.6]$; (b) $d\sigma/dx(\nu + s \to c)$ integrated in y over the range $y = [0.1, 0.8]$.

VII. CONCLUSIONS

The proper treatment of the effects of heavy quarks in the theoretical predictions of the differential cross section for deeply inelastic scattering processes is not completely solved in perturbative QCD. Strictly speaking, the familiar factorization theorem applies only to one scale problems, i.e., when either all quark masses are negligible compared to Q^2, or when the heavy quark mass m is of the same order of magnitude as Q^2.

The recent higher order calculations of heavy quark production which exclude massive partons and focus on the gluon-fusion diagrams apply only to the region in which $m^2 \sim Q^2$ and require a totally different treatment of charged and neutral current processes.

We formulate a unified approach to both types of processes that is based on the factorization theorem as an ansatz. We assume that the factorization theorem holds throughout the energy range of interest in the simple form $W = f \otimes \omega$. This ansatz produces the correct results in the regimes $Q^2 \sim m^2$ and $Q^2 \gg m^2$, and provides a *smooth* interpolation in the intermediate regions. We are able to treat both charged and neutral current processes by endowing the parton quarks with a mass and by not making *a priori* any assumptions about the relative importance of quark and gluon-initiated contributions. Instead, we take advantage of precisely the techniques that yield the proof of the factorization theorem to ensure that the final expressions conform to expectations in the $Q^2 \sim m^2$ and $Q^2 \gg m^2$ regions.

Working towards this goal, we have presented here the general framework. In order to illustrate the basics of our approach, we have presented an explicit calculation of the lowest order contribution to the quark structure functions. However, this contribution by itself is not sufficient for proper phenomenological analysis of DIS cross sections because of the importance of quark-gluon mixing in sea-quark initiated processes.

We have compared existing phenomenological analyses based on the lowest order process $W + q \to Q$, with the unified approach which retains all masses. For charged current charm production experiments ($W + s \to c$), the final state heavy quark mass m is comparable to the target mass M; hence, if the m dependence is retained, then the M dependence must also be retained for consistency. The m dependence results in the well-known "slow rescaling" adjustment of the scaling variable and the cross section. The target mass also adjusts the effective scaling variable, and can shift the cross section by up to 25% for fixed-target experiments.

For collider experiments such as the HERA *ep* facility, we would like to study charged and neutral current production of charm and bottom quarks. Such processes fall in the intermediate region where the heavy quarks are neither $Q^2 \sim m^2$ nor $Q^2 \gg m^2$; hence, we must carefully take the mass dependence into account.

In the second paper of this series we shall make use of the framework developed here to present a full next-to-leading order analysis of both charged and neutral current cross sections for deeply inelastic scattering.

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APPENDIX A: KINEMATICS

We summarize the details about the kinematics including target and heavy quark mass effects in this appendix. We begin with the lab frame kinematics for the overall process, and then examine the class of collinear frames including the brick wall (BW) frame. Finally, we consider the collinear frame for the partons, and relate the partonic quantities (including dot products) to the hadronic variables.[8]

1. Overall process

For the physical process

$$l_1(l_1) + N(P) \to l_2(l_2) + X(P_X) \tag{A1}$$

the following invariant variables are standard:

$$\begin{aligned} P^2 &= M^2 \ , \\ Q^2 &= -q^2 \ , \\ \nu &= \frac{P \cdot q}{\sqrt{P \cdot P}} = E_1 - E_2 \ , \\ x &= \frac{-q^2}{2P \cdot q} = \frac{Q^2}{2M\nu} \ , \\ y &= \frac{P \cdot q}{P \cdot l_1} = \frac{\nu}{E_1} \ , \end{aligned} \tag{A2}$$

where $q = l_1 - l_2$, and E_1 and E_2 are the laboratory energies of the incoming and outgoing leptons respectively.

The components of the relevant four-vectors in the lab frame are

$$\begin{aligned} P^\mu &= (M, \ 0, \ 0, \ 0) \ , \\ l_1^\mu &= (E_1, \ 0, \ 0, \ -E_1) \ , \\ l_2^\mu &= (E_2, \ -E_2 \sin\theta, \ 0, \ -E_2 \cos\theta) \ , \\ q^\mu &= (\nu, \ +E_2 \sin\theta, \ 0, \ -E_1 + E_2 \cos\theta) \ , \end{aligned} \tag{A3}$$

where, as throughout this paper, lepton masses are neglected.

The cross section for the deep inelastic scattering process is given by the standard form:

$$d\sigma = \frac{1}{2\Delta(s, m_{l_1}^2, M^2)} \overline{\sum_{\text{spin}}} |M^2| d\Gamma \tag{A4}$$

[8]We use the metric $g = \{+ - - -\}$ when necessary, but attempt to present the results in a metric independent fashion.

with M being the mass of the incident hadron, m_{l_1} the mass of the incident lepton, and the triangular function

$$\Delta(a,b,c) = \sqrt{a^2+b^2+c^2-2(ab+bc+ca)}\ . \tag{A5}$$

The sum and average over spins is given by

$$\overline{\sum_{\text{spin}}} = \frac{1}{n_l}\sum_{\text{spin}}$$

$$\text{with } n_l = \text{No. of initial spin states} = \begin{cases} 1 \text{ for } \nu,\bar{\nu} \\ 2 \text{ for } l^{\pm}\end{cases} . \tag{A6}$$

$d\Gamma$ represents the final state phase space, with all unobserved degrees of freedom to be integrated over,

$$d\Gamma = \widetilde{dl_2}(2\pi)^4\delta^4(P+l_1-P_X-l_2)d\Gamma_X \tag{A7}$$

with the notation (for invariant single-particle phase space)

$$\widetilde{dk} = \frac{d^4k}{(2\pi)^4}(2\pi)\delta_+(k^2-m_k^2) = \frac{d^3k}{(2\pi)^3 2k_0} \tag{A8}$$

and $d\Gamma_X$ representing the phase space factor for the hadronic final state. With the scattering amplitude given by Eq. (4), one can put the various pieces together to get:

$$d\sigma = \frac{G_1G_2}{2\Delta(s,m_{l_1}^2,M^2)}4\pi Q^2 L^{\mu}{}_{\nu}W^{\nu}{}_{\mu}\widetilde{dl_2}d\Gamma'\ , \tag{A9}$$

where $G_i = g_B^2/(Q^2+M_{B_i}^2)$, the subscripts on $g_{B_i}^2$ and $M_{B_i}^2$ indicate the type of exchanged vector boson, $d\Gamma'$ represents unintegrated hadron degrees of freedom (such as those associated with the production of a heavy quark), and the lepton (hadron) tensor $L^{\mu}{}_{\nu}(W^{\nu}{}_{\mu})$ is defined in Eq. (8) [Eq. (9)]. For convenience, W and L are defined to be dimensionless; these depart from some historical definitions by simple factors such as M. The factor of $4\pi Q^2$ comes from the normalization of W and L.

Suppressing $d\Gamma'$, one obtains

$$\frac{d\sigma}{dxdy} = \frac{yQ^2}{8\pi}G_1G_2 L\cdot W\ . \tag{A10}$$

Note that the gauge couplings of the bosons g_{B_i} appear explicitly whereas the chiral couplings of the leptons $\{g_{Rl},g_{Ll}\}$ and hadrons $\{g_{Rh},g_{Lh}\}$ are kept with the currents, and hence reside in the respective tensors.

For completeness, we record the relations between various commonly used cross sections:

$$\begin{aligned}\frac{d\sigma}{dxdy} &= 2ME_1x\frac{d\sigma}{dxdQ^2} = 2ME_1^2y\frac{d\sigma}{dQ^2d\nu}\\ &= \frac{ME_1y}{E_2}\frac{d\sigma}{dE_2d\cos\theta}\ ,\end{aligned} \tag{A11}$$

which can be easily derived using the kinematic definitions in Eq. (A2).

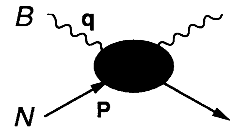

FIG. 5. Basic process for inclusive boson $B(q)$ nucleon $N(P)$ scattering: $N(P)+B(q)\to X(P_X)$, summed over the final state, $X(P_X)$.

2. The collinear frames

Since the underlying physical process is actually the scattering of a spacelike vector boson on a nucleon (cf. Fig. 5)

$$B(q)+N(P)\to X(P_X)\ , \tag{A12}$$

it is more natural to use frames in which the four-vectors (q,P) define the t-z plane. For parton-model considerations, it is convenient to specify these vectors in a general frame of this class by their light-cone coordinate components $(x^+,\vec{x},x^-)$, with $x^{\pm} = (x^0\pm x^3)/\sqrt{2}$, as:

$$\begin{aligned}P^{\mu} &= \left(P^+,\vec{0},\frac{M^2}{2P^+}\right)\ ,\\ q^{\mu} &= \left(-\eta P^+,\vec{0},\frac{Q^2}{2\eta P^+}\right)\ ,\end{aligned} \tag{A13}$$

where P^+ is arbitrary, and η is defined through the implicit equation:

$$2q\cdot P = \frac{Q^2}{\eta} - \eta M^2\ . \tag{A14}$$

η represents the generalization of the familiar Bjorken x in the presence of target mass, and it is related to the latter by

$$\frac{1}{x} = \frac{1}{\eta} - \eta\frac{M^2}{Q^2}\ . \tag{A15}$$

Clearly, η reduces to x in the zero target mass limit,

$$\eta \xrightarrow[M^2/Q^2\to 0]{} x\ , \tag{A16}$$

whereas, the general solution to Eq. (A15) is

$$\frac{1}{\eta} = \frac{1}{2x} + \sqrt{\frac{1}{4x^2}+\frac{M^2}{Q^2}}\ . \tag{A17}$$

We shall refer to this class of frames as the collinear frames. The laboratory frame (with the negative z axis aligned along $\vec{q}$) belongs to this class; it is obtained by setting $P^+ = M/\sqrt{2}$. The "infinite momentum frame," often used to derive the QCD asymptotic theorems, is obtained in the limit $P^+\to\infty$. Another useful frame in this class, used in the helicity formulation, is discussed in the following.

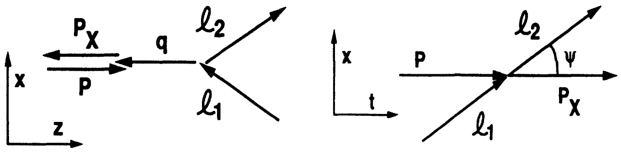

FIG. 6. (a) The standard hadron configuration in $\{x, z\}$ space. Note that the hadron momenta are collinear with the z axis, and the lepton momenta define the x-z plane. (b) This frame is related to the standard lepton configuration (Fig. 7 below) by a space-time rotation (i.e., boost) in the $\{x, t\}$ plane by the angle ψ.

3. The brick wall frame

The brick wall (BW) frame is the natural "rest frame" of the exchanged vector boson when its momentum q is spacelike, $q^2 = -Q^2 < 0$ (cf. Fig. 1). It is also one of the collinear frames, corresponding to setting $P^+ = Q/(\eta\sqrt{2})$ in Eq. (A13), and hence obtaining $q^0 = 0$ and $q^3 = -Q$. In the Cartesian coordinate system, (x^0, x^1, x^2, x^3), we have

$$\begin{aligned} q^\mu &= Q(0,\ 0,\ 0,\ -1)\ , \\ P^\mu &= \frac{1}{2Q}(\Delta_P,\ 0,\ 0,\ +\beta_1)\ , \\ P_X^\mu &= \frac{1}{2Q}(\Delta_P,\ 0,\ 0,\ -\beta_2)\ , \end{aligned} \tag{A18}$$

and we refer to this frame as the *standard hadron configuration*, Fig. 6, with

$$\begin{aligned} \Delta_P &= \Delta[-Q^2, P^2, P_X^2]\ , \\ \beta_1 &= Q^2 - P^2 + P_X^2\ , \\ \beta_2 &= Q^2 + P^2 - P_X^2\ . \end{aligned} \tag{A19}$$

In this frame, the lepton momenta are given by

$$\begin{aligned} l_1^\mu &= \frac{Q}{2}(\cosh\psi, \sinh\psi, 0, -1)\ , \\ l_2^\mu &= \frac{Q}{2}(\cosh\psi, \sinh\psi, 0, +1)\ , \end{aligned} \tag{A20}$$

which can be easily envisioned as being obtained from the *standard lepton configuration* [cf. the standard hadron configuration, Eq. (A18)], Fig. 7,

$$\begin{aligned} l_1^\mu &= \frac{Q}{2}(1,\ 0,\ 0,\ -1)\ , \\ l_2^\mu &= \frac{Q}{2}(1,\ 0,\ 0,\ +1)\ , \end{aligned} \tag{A21}$$

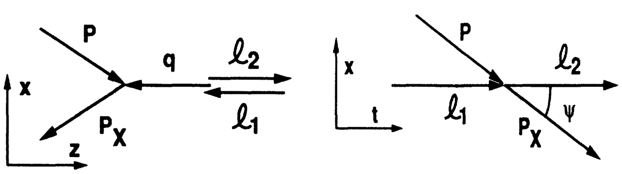

FIG. 7. (a) The standard lepton configuration in $\{x, z\}$ space. Note that the lepton momenta are collinear with the z axis, and the hadron momenta define the x-z plane; (b) The same frame seen in $\{x, t\}$ space.

by a "rotation" in the (t-x) plane (really a Lorentz boost) by the hyperbolic angle ψ. This is in analogy to the familiar c.m. rotation [in the (z-x) plane] between initial and final scattering states in a timelike situation. This is illustrated in Fig. 6 and Fig. 7.

The hyperbolic cosine can be obtained from the formula

$$\cosh\psi = \frac{2P\cdot(l_1 + l_2)}{\Delta[-Q^2, P^2, P_X^2]}\ . \tag{A22}$$

Evaluating the scalar productions in the laboratory frame, we relate $\cosh\psi$ to the more familiar variables

$$\begin{aligned} \cosh\psi &= \frac{E_1 + E_2}{\sqrt{Q^2+\nu^2}} = \frac{\eta^2 M^2 - Q^2 + 2\eta(s - M^2)}{\eta^2 M^2 + Q^2} \\ &\xrightarrow[M\to 0]{} \frac{(2-y)}{y}\,. \end{aligned} \tag{A23}$$

In developing the helicity formalism (Appendix B), we encounter the "spin-1 rotation matrix" for the vector boson polarization vectors under the above Lorentz boost from the configuration Eq. (A21) (Fig. 7) to Eq. (A20) (Fig. 6). The three-dimensional d matrix is

$$d^1(\psi) = \begin{bmatrix} \frac{1+\cosh\psi}{2} & \frac{-\sinh\psi}{\sqrt{2}} & \frac{1-\cosh\psi}{2} \\ \frac{-\sinh\psi}{\sqrt{2}} & \cosh\psi & \frac{+\sinh\psi}{\sqrt{2}} \\ \frac{1-\cosh\psi}{2} & \frac{+\sinh\psi}{\sqrt{2}} & \frac{1+\cosh\psi}{2} \end{bmatrix}\ . \tag{A24}$$

It is the SO(2,1) analogue of the familiar SO(3) rotation matrix.

4. Parton kinematics in the QCD parton model

In the QCD parton model (cf. Fig. 2), we have an initial state parton momentum k_1, whose light-cone components in a collinear frame are

$$k_1^\mu = \left(\xi P^+, \vec{0}, \frac{m_1^2}{2\xi P^+}\right)\ , \tag{A25}$$

where ξ is the fractional momentum carried by the parton. The momenta involved in the "hard scattering" consist of

$$q + k_1 \to k_x, \tag{A26}$$

where the final state, represented by the total momentum k_x, consists of either an on-mass-shell single parton [for the case of the leading-order (LO) calculation] or a continuum of multiparton configurations [for the next-to-leading-order (NLO) calculations and beyond].

For the LO calculation presented in Sec. V, with $k_x = k_2 = k_1 + q$, we can evaluate the argument of the δ function which enforces the on-shell condition for the final state heavy quark:

$$k_2^2 - m_2^2 = \frac{Q^2(\xi - \chi_+)(\xi - \chi_-)}{\eta\xi}\ , \tag{A27}$$

where

$$\chi_{\pm} = \eta \frac{(Q^2 - m_1^2 + m_2^2) \pm \Delta[-Q^2, m_1^2, m_2^2]}{2Q^2} \quad \text{(A28)}$$

and η is defined in Eq. (A17). The limits on ξ (see below) dictate that the only physical root is

$$\xi = \chi \equiv \chi_+ \,. \quad \text{(A29)}$$

This variable reduces to the "slow-rescaling" variable $x(1 + m_2^2/Q^2)$ in the limit $m_1 \to 0$ *and* $M \to 0$. Substituting Eq. (A29) in the second factor in Eq. (A27), we obtain

$$\delta_+(k_2^2 - m_2^2) = \frac{\delta\left(\frac{\xi}{\chi} - 1\right)}{\Delta[-Q^2, m_1^2, m_2^2]} \,. \quad \text{(A30)}$$

When the final state consists of multipartons (for NLO and beyond), the c.m. energy of the subprocess $\hat{s}$ must be greater than a threshold $\hat{s}_{\text{th}}$, which is equal to either m_2^2 or $4m_2^2$, depending on whether a single heavy quark (charged current case) or a heavy quark-antiquark pair (neutral current case) is produced. Since

$$\begin{aligned} \hat{s} &= (k_1 + q)^2 = m_1^2 - Q^2 + 2k_1 \cdot q \\ &= \left(Q^2 + \frac{\eta}{\xi} m_1^2\right)\left(\frac{\xi}{\eta} - 1\right) \geq \hat{s}_{\text{th}}, \end{aligned} \quad \text{(A31)}$$

it is easy to see that the threshold condition imposes the constraint $\xi \geq \xi_{\text{th}}$ on the parton momentum fraction variable where

$$\xi_{\text{th}} = \eta \frac{(Q^2 - m_1^2 + \hat{s}_{\text{th}}) + \Delta[-Q^2, m_1^2, \hat{s}_{\text{th}}]}{2Q^2} \,. \quad \text{(A32)}$$

(Note that for $\hat{s}_{\text{th}} = m_2^2$, $\xi_{\text{th}} = \chi_+ \equiv \chi$.) On the other hand, the condition that $P_X^+ = P^+(1 - \xi) \geq 0$ requires $\xi \leq 1$. Hence, ξ, which is also the integration variable for the convolution in the fundamental factorization theorm [Eq. (17)], has the following range:

$$1 \geq \xi \geq \xi_{\text{th}} = \eta \frac{(Q^2 - m_1^2 + \hat{s}_{\text{th}}) + \Delta[-Q^2, m_1^2, \hat{s}_{\text{th}}]}{2Q^2}. \quad \text{(A33)}$$

We recall that η is the generalization of Bjorken x incorporating the target mass effect. Thus the lower limit for ξ is modified by both target mass *and* heavy quark mass. This aspect of mass-dependence has been overlooked in existing literature.

5. Dot productions of lepton and parton momenta

In the explicit calculation of cross sections using the contraction of lepton and hadron tensors (cf. Sec. V and Appendix C), one needs the scalar products of the lepton and hadron four-vectors. This calculation is subtle because the variable $\xi = k_1^+/P_+$ is invariant for boosts along the z axis, but not for other boosts or rotations.

In the BW frame, the light-cone components of the two parton momenta are

$$\begin{aligned} k_1^\mu &: \frac{Q}{\sqrt{2}}\left(\frac{\xi}{\eta},\ \vec{0},\ \frac{\eta}{\xi}\frac{m_1^2}{Q^2}\right), \\ k_2^\mu &: \frac{Q}{\sqrt{2}}\left(\frac{\xi - \eta}{\eta},\ \vec{0},\ 1 + \frac{\eta}{\xi}\frac{m_1^2}{Q^2},\right). \end{aligned} \quad \text{(A34)}$$

Using the explicit components of the lepton momenta given in Eq. (A20), it is then straightforward to show

$$(k_1 \cdot l_1) = \frac{1}{2}\frac{\xi}{\eta}Q^2\left(\frac{\cosh\psi + 1}{2}\right) + \frac{1}{2}\frac{\eta}{\xi}m_1^2\left(\frac{\cosh\psi - 1}{2}\right), \quad \text{(A35)}$$

$$(k_1 \cdot l_2) = \frac{1}{2}\frac{\xi}{\eta}Q^2\left(\frac{\cosh\psi - 1}{2}\right) + \frac{1}{2}\frac{\eta}{\xi}m_1^2\left(\frac{\cosh\psi + 1}{2}\right). \quad \text{(A36)}$$

To contrast the simplicity and symmetry of this group theoretic approach with a more traditional "brute force" calculation *in the collinear frame*, we compare:

$$(k_1 \cdot l_1) = \frac{Q^2\xi^2(s - M^2 + M^2\eta) + m_1^2(s\eta^2 - M^2\eta^2 - Q^2\eta)}{2\xi(Q^2 + M^2\eta^2)}, \quad \text{(A37)}$$

$$(k_1 \cdot l_2) = \frac{Q^2\xi^2(s - M^2 - Q^2/\eta) + m_1^2\eta^2(s + M^2\eta - M^2)}{2\xi(Q^2 + M^2\eta^2)}. \quad \text{(A38)}$$

Although it is not obvious, Eqs. (A35) and (A36) are identical to Eqs. (A37) and (A38); however, the symmetries of the problem are more apparent in Eqs. (A35) and (A36).

In the limit of zero masses, we have the usual relations where $(k_1 \cdot l_1) \to \hat{s}/2$ and $(k_1 \cdot l_2) \to \hat{u}/2$ with no ξ dependence. However, if we wish to obtain the correct mass dependence, we must include the proper ξ dependence in our calculation.

Once we have $(k_1 \cdot l_1)$ and $(k_1 \cdot l_2)$, we can use $k_1 + l_1 = k_2 + l_2$ to easily compute the other necessary combinations via

$$\begin{aligned} (k_2 \cdot l_2) &= (k_1 \cdot l_1) - \left(\frac{m_2^2 - m_1^2}{2}\right), \\ (k_2 \cdot l_1) &= (k_1 \cdot l_2) + \left(\frac{m_2^2 - m_1^2}{2}\right). \end{aligned} \quad \text{(A39)}$$

APPENDIX B: STRUCTURE FUNCTIONS AND CROSS-SECTIONS

Since the precise treatment of the mass effects is emphasized in this paper, we include here some details on the derivation of structure function and cross-section formulas used in the text, especially for the less familiar helicity vertices and structure functions.

1. Tensor amplitudes and invariant structure functions

We begin by recording the expression for the lepton tensor, Eq. (8). In the limit of zero lepton mass, it is

$$L^{\mu\nu} = \frac{1}{Q^2}\overline{\sum_{\text{spin}}}\bar{u}(l_1)\Gamma^\mu u(l_2)\cdot\bar{u}(l_2)\Gamma^{\nu\dagger}u(l_1) = \frac{1}{Q^2}\frac{1}{n_l}\mathrm{Tr}[l_1\Gamma^\mu l_2\Gamma^{\nu\dagger}], \tag{B1}$$

where n_l counts the number of incoming helicity states. Using a general $V-A$ coupling of the form, Eq. (5),

$$\Gamma^\mu = \gamma^\mu[g_{Rl}(1+\gamma_5)+g_{Ll}(1-\gamma_5)] \tag{B2}$$

the result is

$$L^{\mu\nu} = \frac{8}{Q^2}\frac{1}{n_l}\left\{g_{+l}^2\left[l_1^\mu l_2^\nu + l_2^\mu l_1^\nu - g^{\mu\nu}\frac{Q^2}{2}\right] - g_{-l}^2[i\epsilon^{\mu\nu\rho\sigma}l_{1\rho}l_{2\sigma}]\right\}. \tag{B3}$$

The independent components of the hadron tensor $W_{\mu\nu}$ are expressed in terms of invariant (i.e., Lorentz scalar) structure functions defind as [Eq. (10)]

$$W^\mu{}_\nu = -g^\mu{}_\nu W_1 + \frac{P^\mu P_\nu}{M^2}W_2 - i\frac{\epsilon^{Pq\mu}{}_\nu}{2M^2}W_3 + \frac{q^\mu q_\nu}{M^2}W_4 + \frac{P^\mu q_\nu + q^\mu P_\nu}{2M^2}W_5 + \frac{P^\mu q_\nu - q^\mu P_\nu}{2M^2}W_6 . \tag{B4}$$

Contracting the lepton and hadron tensors and evaluating the scalar productions of the four-vectors in the laboratory frame [cf. Eq. (A3)], one obtains

$$W\cdot L = \frac{16E_1E_2}{n_lQ^2}\left\{g_{+l}^2\left[2\sin^2\frac{\theta}{2}W_1 + \cos^2\frac{\theta}{2}W_2\right] + g_{-l}^2\left[\frac{E_1+E_2}{M}\sin^2\frac{\theta}{2}W_3\right]\right\} . \tag{B5}$$

The structure functions $\{W_4, W_5, W_6\}$ do not appear on the right-hand side of this equation because the dot product of q^μ with the lepton tensor $L^{\mu\nu}$ gives rise to a factor proportional to some combinations of the lepton masses which is neglected here. Equation (B5), in conjunction with Eqs. (A10)–(A11), form the bases for the derivation of the cross-section formula (11) in Sec. III.

2. Helicity vertices and structure functions

We now turn to the calculation of helicity amplitudes, vertices, and structure functions. We use the helicity labels $\lambda_{1,2}$ for the leptons; $\sigma_{1,2}$ for the hadrons, and $\{m,n\}$ for the bosons. Lower indices are for incoming particles; and upper indices are for outgoing particles. The scattering amplitude for the basic process, Eq. (1), can be written in the factorized form in the helicity basis [28,29]:

$$\mathcal{M}_{\lambda_1\sigma_1}^{\lambda_2\sigma_2} = J_{\sigma_1 m}^{*\sigma_2}(Q^2, q\cdot P)\frac{g_B^2 d^1(\psi)^m{}_n}{Q^2+M_B^2}j_{\lambda_1}^{\lambda_2 n}(Q^2) , \tag{B6}$$

where $d^1(\psi)^m{}_n$ is a spin-1 SO (2,1) "rotation matrix" in the brick wall frame of the process corresponding to q^μ: $(0,0,0,-Q)$ [cf. Eq. (A24)]. The scalar lepton helicity vertex function is

$$j_{\lambda_1}^{\lambda_2 n}(Q) = \epsilon_\mu^{n*}\langle l_2,\lambda_2|j^\mu|l_1,\lambda_1\rangle = \bar{u}_{\lambda_2}(l_2)\epsilon^{n*}\cdot\Gamma u_{\lambda_1}(l_1) \tag{B7}$$

and the corresponding hadron vertex function is

$$J_{\sigma_1 m}^{*\sigma_2}(Q^2, q\cdot P) = \langle P_X,\sigma_2|J_\mu^\dagger|P,\sigma_1\rangle\epsilon_m^\mu . \tag{B8}$$

Much of the simplicity of the helicity approach results from the fact that the lepton vertex function is extremely simple in the limit of zero lepton masses. *For left-handed (right-handed) coupling, there is only one non-vanishing vertex function for which all three particles are left-handed (right-handed);* it is simply given by

$$j_L^{LL}(Q) = j_{-1/2}^{-1/2-1}(Q) = \sqrt{8Q^2} . \tag{B9}$$

(Likewise, $j_R^{RR}(Q) = -\sqrt{8Q^2}$ in the case of right-handed coupling.) Thus, upon squaring the scattering amplitude, Eq. (B6), one obtains

$$\overline{\sum_{\text{spin}}}|M^2| \propto d^1(\psi)^{-1}{}_m d^1(-\psi)^n{}_{-1}W^m n , \tag{B10}$$

where $W^m{}_n$ is the helicity forward Compton scattering amplitude for initial state vector boson polarization n and final state polarization m:

$$W^m{}_n = \epsilon_\mu^{m*}(P,q)W^\mu{}_\nu(P,q)\epsilon_n^\nu(P,q) . \tag{B11}$$

For totally inclusive process, this amplitude must be diagonal in (m,n) due to angular momentum conservation;[9] hence, the right-hand side becomes $d^1(\psi)^{-1}{}_m d^1(\psi)^m{}_{-1}F_m$ where the diagonal helicity amplitude $W^m{}_m$ is identified with the *helicity structure function* F_m, cf. Eq. (15).

Using these results for the squared amplitude, $|M^2|$, keeping all factors, and making use of the explicit form of the d matrix, Eq. (A24), we obtain $L\cdot W$, which appears in the cross-section formula Eq. (A10):

[9] In principle, there can be mixing among $\{W^q{}_q, W^q{}_0, W^0{}_q\}$. Since the coefficients of these terms are proportional to m_l^2/Q^2, we only concern ourselves with $\{W^+{}_+, W^0{}_0, W^-{}_-\}$.

$$W \cdot L = \frac{8}{n_l} \left\{ g_{Rl}^2 \left[F_+ \left(\frac{1+\cosh\psi}{2} \right)^2 + F_0 \left(\frac{-\sinh\psi}{\sqrt{2}} \right)^2 + F_- \left(\frac{1-\cosh\psi}{2} \right)^2 \right] \right.$$
$$\left. + g_{Ll}^2 \left[F_+ \left(\frac{1-\cosh\psi}{2} \right)^2 + F_0 \left(\frac{+\sinh\psi}{\sqrt{2}} \right)^2 + F_- \left(\frac{1+\cosh\psi}{2} \right)^2 \right] \right\} . \quad \text{(B12)}$$

This leads to the general formula, Eq. (14), for the cross section given in Sec. III.

3. Relations between invariant and helicity structure functions

To derive the relations between the invariant and helicity structure functions, we first examine the polarization vectors for a vector boson with momentum q in the helicity basis. With respect to an arbitrary reference momentum p, the "longitudinal" polarization vector is

$$\epsilon_0^\mu(p,q) = \frac{(-q^2)p^\mu + (p\cdot q)q^\mu}{\sqrt{(-q^2)[(p\cdot q)^2 - q^2p^2]}} \quad \text{(B13)}$$

with $-q^2 = Q^2 > 0$ for the spacelike q^μ. It is also useful to define the "scalar" polarization:

$$\epsilon_q^\mu(p,q) = \frac{q^\mu}{\sqrt{-q^2}} \,. \quad \text{(B14)}$$

In a collinear frame where the z component of q^μ is positive, the transverse polarization vectors are given by

$$\epsilon_\pm^\mu(p,q) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0) \,. \quad \text{(B15)}$$

For the z component of q^μ negative, we rotate the above about the y axis by π. These polarization vectors depend on the reference vector p^μ only to the extent that it defines the t-z plane in conjunction with q^μ. For the transverse polarization vectors, this is obvious. For the longitudinal vector, $\epsilon_0^\mu(p,q)$, this follows from the fact that it is merely the unit vector in the t-z plane orthogonal to q^μ. The reference vector p^μ is used only to define this plane and to provide the nonvanishing perpendicular component for projecting onto ϵ_0^μ. The two distinct reference vectors in the plane, such as P^μ (the target momentum) and k_1^μ (the initial state parton momentum) used in the text, define the same set of polarization vectors for the vector boson. As discussed in Sec. IV, this is the key point which leads to the simple factorization formula for the helicity structure functions in the QCD parton framework.

To project out the transverse helicity amplitudes, the following representations are useful:

$$\epsilon_+^\mu(p,q)\epsilon_+^{\nu *}(p,q) - \epsilon_-^\mu(p,q)\epsilon_-^{\nu *}(p,q) = \frac{i\epsilon^{\mu\nu pq}}{\sqrt{(p\cdot q)^2 - q^2p^2}} \,,$$

$$\epsilon_+^\mu(p,q)\epsilon_+^{\nu *}(p,q) + \epsilon_-^\mu(p,q)\epsilon_-^{\nu *}(p,q)$$
$$= -g^{\mu\nu} + \epsilon_0^\mu(p,q)\epsilon_0^{\nu *}(p,q) - \epsilon_q^\mu(p,q)\epsilon_q^{\nu *}(p,q) \,. \quad \text{(B16)}$$

The second relation is simply completeness.

Applying the above polarization vectors to the definition of the helicity structure functions, Eq. (15),

$$F_\lambda = \epsilon_\mu^{\lambda *}(P,q) W^\mu{}_\nu(P,q) \epsilon_\lambda^\nu(P,q) \quad \text{(no sum over } \lambda) \,, \quad \text{(B17)}$$

and using the representation of $W^\mu{}_\nu(P,q)$ in terms of the invariant structure functions, Eq. (B4), we obtain:

$$F_+ = W_1 - \frac{\nu}{2M}\sqrt{1+\frac{Q^2}{\nu^2}}\,W_3 \,,$$
$$F_- = W_1 + \frac{\nu}{2M}\sqrt{1+\frac{Q^2}{\nu^2}}\,W_3 \,,$$
$$F_0 = -W_1 + \left(1+\frac{\nu^2}{Q^2}\right) W_2 \,. \quad \text{(B18)}$$

The complete transformation matrix to convert hadron helicity amplitudes to invariant amplitudes ($W_\lambda = f \otimes \omega_\lambda = t_\lambda^i W_i$) is given in Table III. The coefficients for the inverse transformation $(t^{-1})_i^\lambda$ are given in Table IV.

4. Relations Between Hadron and Parton Tensors

As discussed in Sec. IV, the k_1 four-vector is not simply proportional to P, but in general contains a mixture

TABLE III. Transformation matrix to convert hadron helicity amplitudes to invariant amplitudes: $W_{\lambda\lambda} = f \otimes \omega_\lambda = t_\lambda^i W_i$. Note, we use the short hand notation $F_\lambda \equiv W_{\lambda\lambda}$. We have defined $\rho^2 = 1 + Q^2/\nu^2$ (note $\rho \to 1$ in the DIS limit).

t_λ^i	$F_1 \equiv W_1$	$F_2 \equiv (\nu/M)W_2$	$F_3 \equiv (\nu/M)W_3$	W_4	W_5	W_6
$F_+ \equiv W_{++}$	1	0	$\frac{-\rho}{2}$	0	0	0
$F_- \equiv W_{--}$	1	0	$\frac{+\rho}{2}$	0	0	0
$F_0 \equiv W_{00}$	-1	$\frac{\rho^2}{2x}$	0	0	0	0
W_{qq}	1	$\frac{1}{2x}$	0	$\frac{2Q^2}{M^2}$	$\frac{-\nu}{M}$	0
$W_{0q} + W_{q0}$	0	$\frac{\rho}{x}$	0	0	$\frac{-\rho\nu}{M}$	0
$W_{0q} - W_{q0}$	0	0	0	0	0	$\frac{-\rho\nu}{M}$

TABLE IV. Transformation matrix to convert hadron invariant amplitudes to helicity amplitudes: $W_i = (t^{-1})_i^\lambda W_\lambda$. Note that we use the shorthand notation $F_\lambda \equiv W_{\lambda\lambda}$. We have also used $F_1 = W_1$, $F_2 = (\nu/M)W_2$, and $F_3 = (\nu/M)W_3$. We have defined $\rho^2 = 1 + Q^2/\nu^2$ (note $\rho \to 1$ in the DIS limit). Note that as $M \to 0$, $\{W_4, W_5, W_6\}$ decouple from $\{F_+, F_0, F_-\}$.

$(t^{-1})_i^\lambda$	$F_+ \equiv W_{++}$	$F_- \equiv W_{--}$	$F_0 \equiv W_{00}$	W_{qq}	$(W_{0q} + W_{q0})$	$(W_{0q} - W_{q0})$
$F_1 \equiv W_1$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
$F_2 \equiv (\nu/M)W_2$	$\frac{x}{\rho^2}$	$\frac{x}{\rho^2}$	$\frac{2x}{\rho^2}$	0	0	0
$F_3 \equiv (\nu/M)W_3$	$\frac{-1}{\rho}$	$\frac{+1}{\rho}$	0	0	0	0
W_4	$\frac{-M^2}{4\nu^2\rho^2}$	$\frac{-M^2}{4\nu^2\rho^2}$	$\frac{M^2}{2Q^2\rho^2}$	$\frac{M^2}{2Q^2}$	$\frac{-M^2}{2Q^2\rho}$	0
W_5	$\frac{M}{\nu\rho^2}$	$\frac{M}{\nu\rho^2}$	$\frac{2M}{\nu\rho^2}$	0	$\frac{-M}{\nu\rho}$	0
W_6	0	0	0	0	0	$\frac{-M}{\nu\rho}$

of P and q given by

$$k_1^\mu = \zeta_P P^\mu + \zeta_q q^\mu ,$$

$$\zeta_P = \frac{Q^2\xi^2 + m_1^2\eta^2}{\xi(Q^2 + M^2\eta^2)} , \tag{B19}$$

$$\zeta_q = \frac{\eta(m_1^2 - M^2\xi^2)}{\xi(Q^2 + M^2\eta^2)} . \tag{B20}$$

Note that this mixing depends on both M and m_1. The result is that the hadron tensors and the parton tensors are mixed. Specifically,

$$W_i = c_i^j f \otimes \omega_j , \tag{B21}$$

where the c_i^j coefficients are given in Table V. The coefficients for the inverse transformation $(c^{-1})_j^i$ are given in Table VI.

This is in contrast to the corresponding result for the hadron helicity amplitudes where there is no mixing:

$$F_\lambda = W_{\lambda\lambda} = f \otimes \omega_\lambda . \tag{B22}$$

APPENDIX C: LEADING ORDER CALCULATION WITH MASSES

We present the details of the leading order calculation with the full mass dependence both as an illustration of general points made in the text of the paper, and as a concrete example to check the self-consistency of the tensor and helicity formalisms developed in the text. Although the calculation is straightforward, the results with the full mass dependence do not exist in the literature, and have not been used in the analysis of experimental data—as emphasized in this paper.

The *parton structure function* tensor $\omega_{\mu\nu}^{Ba}$, representing the vector boson (B) and parton (a) forward Compton scattering amplitude, is entirely analogous to $W_{\mu\nu}^{BN}$—replacing the hadron target N by the parton target a. The leading order diagram, Fig. 3, gives rise to

$$\omega_\nu^\mu = \frac{1}{4\pi}(2\pi)\delta_+(k_2^2 - m_2^2) \times \overline{\sum_{\text{spin}}} \langle k_1, \sigma_1 | j^\mu | k_2, \sigma_2 \rangle \langle k_2, \sigma_2 | j_\nu^* | k_1, \sigma_1 \rangle . \tag{C1}$$

For quarks, the spin sum and average on the right-hand side is

$$\begin{aligned}\frac{1}{2}\mathrm{Tr}[(\not k_1 + m_1)\Gamma^\mu(\not k_2 + m_2)\Gamma^{\nu *}] = {} & 4g_{R_a}^2\{-g^{\mu\nu}(k_1 \cdot k_2) + k_1^\mu k_2^\nu + k_2^\mu k_1^\nu + i\epsilon^{\mu\nu\rho\sigma}k_{1\rho}k_{2\sigma}\} \\ & +4g_{L_a}^2\{-g^{\mu\nu}(k_1 \cdot k_2) + k_1^\mu k_2^\nu + k_2^\mu k_1^\nu - i\epsilon^{\mu\nu\rho\sigma}k_{1\rho}k_{2\sigma}\} \\ & +4(g_{R_a}g_{L_a} + g_{L_a}g_{R_a})\{+g^{\mu\nu}(m_1 m_2)\},\end{aligned} \tag{C2}$$

TABLE V. Transformation matrix to convert parton invariant amplitudes to hadron invariant amplitudes: $W_i = c_i^j f \otimes \omega_j$. Note that as $M \to 0$, $\{W_4, W_5, W_6\}$ decouple from $\{\omega_i\}$.

c_i^j	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
$F_1 \equiv W_1$	1	0	0	0	0	0
$F_2 \equiv (\nu/M)W_2$	0	$\frac{\zeta_P^2}{2x}$	0	0	0	0
$F_3 \equiv (\nu/M)W_3$	0	0	$\frac{\zeta_P}{2x}$	0	0	0
W_4	0	$\frac{\zeta_q^2 M^2}{Q^2}$	0	$\frac{M^2}{Q^2}$	$\frac{\zeta_q M^2}{Q^2}$	0
W_5	0	$\frac{2\zeta_P\zeta_q M^2}{Q^2}$	0	0	$\frac{\zeta_P M^2}{Q^2}$	0
W_6	0	0	0	0	0	$\frac{\zeta_P M^2}{Q^2}$

TABLE VI. Transformation matrix to convert hadron invariant amplitudes to parton invariant amplitudes: $\omega_j = (c^{-1})^i_j f \otimes W_i$.

$(c^{-1})^i_j$	$F_1 \equiv W_1$	$F_2 \equiv (\nu/M)W_2$	$F_3 \equiv (\nu/M)W_3$	W_4	W_5	W_6
ω_1	1	0	0	0	0	0
ω_2	0	$\frac{2x}{\zeta_P^2}$	0	0	0	0
ω_3	0	0	$\frac{2x}{\zeta_P}$	0	0	0
ω_4	0	$\frac{2x\zeta_q^2}{\zeta_P^2}$	0	$\frac{Q^2}{M^2}$	$\frac{-\zeta_q Q^2}{\zeta_P M^2}$	0
ω_5	0	$\frac{-4x\zeta_q}{\zeta_P^2}$	0	0	$\frac{Q^2}{\zeta_P M^2}$	0
ω_6	0	0	0	0	0	$\frac{Q^2}{\zeta_P M^2}$

where $\{g_{R_a}, g_{L_a}\}$ are the couplings of the a-type parton to the boson, and the on-mass-shell δ function is given by Eq. (A27).

This expression for ω^μ_ν can be used in two ways: (i) it can be substituted into the general factorization theorem formula, Eq. (17), and then contracted with L^ν_μ to yield leading order cross sections directly, cf. Eq. (A10); or (ii) it can be used to calculate the helicity structure functions through Eq. (20) and Eq. (21) before substituting into the general cross-section formula Eq. (14). We shall do both, and demonstrate the consistency of the two approaches. Although at leading order these two methods are comparable in the ease of use, the helicity approach provides a more efficient way of calculating higher orders. It also provides additional insight on the structure of the physical amplitudes, as we will discuss.

We begin with the helicity approach using

$$\omega_\lambda = \frac{1}{4\pi} 2\pi\delta_+(k_2^2 - m_2^2) \overline{\sum_{\sigma_1,\sigma_2} J^{*\sigma_1\sigma_2}_\lambda J^\lambda_{\sigma_1\sigma_2}}$$
$$= \epsilon^{\mu *}_\lambda(k,q)\omega_{\mu\nu}(k,q)\epsilon^\nu_\lambda(k,q) \quad \text{no sum on } \lambda \,, \tag{C3}$$

and Eqs. (C1) and (C2) above for $\omega_{\mu\nu}(k,q)$; the helicity structure functions at the parton level can be evaluated. We obtain,[10]

$$\omega_\lambda = \delta\left(\frac{\xi}{\chi} - 1\right)\left(g^2_{R_a}\Omega^{RR}_\lambda + 2g_{R_a}g_{L_a}\Omega^{RL}_\lambda + g^2_{L_a}\Omega^{LL}_\lambda\right) \,, \tag{C4}$$

where the superscripts (R, L) refer to right-handed and left-handed chiral couplings at the hadron vertices, and the Ω's are given in Table VII.

The partonic helicity structure functions $\{\omega_\lambda\}$ exhibit many physically interesting features which are obscured in the conventional Dirac trace method. For example, there are obvious symmetries under $g_{R_a} \leftrightarrow g_{L_a}$ when the vector boson helicity is flipped. Additionally, there is a clear order of magnitude separation of the amplitudes when $m^2_{1,2}/Q^2$ become small (high energy limit): all the longitudinal structure functions, as well as the mixed chirality ones, become of $O(m^2_{1,2}/Q^2)$.

Because of the direct relationship between the hadronic helicity structure functions $\{F_\lambda\}$ to the partonic helicity structure functions $\{\omega_\lambda\}$, the $\{F_\lambda\}$ functions are essentially given by the expressions above multiplied by the relevant parton distribution functions evaluated at $\xi = \chi$ [due to the delta function in Eq. (C4)]. Substituting these expressions in the general formula for $L \cdot W$, Eq. (B12), we obtain

$$L \cdot W = q(\xi) \otimes \frac{8}{n_l}\left\{ g^2_{Rl}\left[\omega_+\left(\frac{1+\cosh\psi}{2}\right)^2 + \omega_0\left(\frac{-\sinh\psi}{\sqrt{2}}\right)^2 + \omega_-\left(\frac{1-\cosh\psi}{2}\right)^2\right] + g^2_{Ll}\left[\omega_+\left(\frac{1-\cosh\psi}{2}\right)^2 + \omega_0\left(\frac{+\sinh\psi}{\sqrt{2}}\right)^2 + \omega_-\left(\frac{1+\cosh\psi}{2}\right)^2\right]\right\} \tag{C5}$$

with $\{\omega_+, \omega_0, \omega_-\}$ given by Eq. (C4). The corresponding results for the antiquark process is obtained by the substitution $g_{R_a} \leftrightarrow g_{L_a}$.

TABLE VII. The helicity amplitudes for the leading-order process $l_1 + k_1(m_1) \to l_2 + k_2(m_2)$, with $\Delta = \Delta[-Q^2, m_1^2, m_2^2]$.

$\Omega^{\chi\chi'}_\lambda$	$\chi\chi' = RR$ $g^2_{R_a}$	$\chi\chi' = RL = LR$ $2g_{R_a}g_{L_a}$	$\chi\chi' = LL$ $g^2_{L_a}$
$\lambda = +$	$\frac{Q^2+m_1^2+m_2^2+\Delta}{\Delta}$	$\frac{-2m_1m_2}{\Delta}$	$\frac{Q^2+m_1^2+m_2^2-\Delta}{\Delta}$
$\lambda = 0$	$\frac{(m_1^2+m_2^2)+(m_1^2-m_2^2)^2/Q^2}{\Delta}$	$\frac{+2m_1m_2}{\Delta}$	$\frac{(m_1^2+m_2^2)+(m_1^2-m_2^2)^2/Q^2}{\Delta}$
$\lambda = -$	$\frac{Q^2+m_1^2+m_2^2-\Delta}{\Delta}$	$\frac{-2m_1m_2}{\Delta}$	$\frac{Q^2+m_1^2+m_2^2+\Delta}{\Delta}$

[10]Note that we have used $\Omega^{RL} = \Omega^{LR}$ to simplify Eq. (C4), and ω is symmetric under $\Omega^{RL} \to \Omega^{LR}$.

Alternately, we can compute this in the tensor representation by contracting ω^{μ}_{ν} with L^{ν}_{μ}, Eq. (B3), to obtain

$$L\cdot\omega = \frac{1}{n_l}\frac{2^6}{Q^2}\frac{\delta\left(\frac{\xi}{\chi}-1\right)}{\Delta[-Q^2,m_1^2,m_2^2]}\left\{\begin{array}{lll} & (g_{R_a}^2 g_{Rl}^2 + g_{L_a}^2 g_{Ll}^2) & (k_1\cdot l_1)(k_2\cdot l_2) \\ + & (g_{R_a}^2 g_{Ll}^2 + g_{L_a}^2 g_{Rl}^2) & (k_1\cdot l_2)(k_2\cdot l_1) \\ - & g_{R_a} g_{L_a}(g_{Rl}^2 + g_{Ll}^2) & (m_1 m_2)(l_1\cdot l_2) \end{array}\right\}. \tag{C6}$$

Applying the convolution integral and inserting the scalar products between lepton and quark momenta derived in A5 into Eq. (C6) leads to:

$$L\cdot W = \frac{1}{n_l}\frac{2^6}{Q^2}\frac{q(\chi)}{\Delta[-Q^2,m_1^2,m_2^2]} \times\left\{\begin{array}{lll} & (g_{R_a}^2 g_{Rl}^2 + g_{L_a}^2 g_{Ll}^2) & (Q^2\chi^2 d_- + m_1^2\eta^2 d_+ + \chi\eta Q^2)(Q^2\chi^2 d_+ + m_1^2\eta^2 d_-)/(2^2\eta^2\chi^2) \\ + & (g_{R_a}^2 g_{Ll}^2 + g_{L_a}^2 g_{Rl}^2) & (Q^2\chi^2 d_+ + m_1^2\eta^2 d_- - \chi\eta Q^2)(Q^2\chi^2 d_- + m_1^2\eta^2 d_+)/(2^2\eta^2\chi^2) \\ - & g_{R_a} g_{L_a}(g_{Rl}^2 + g_{Ll}^2) & (m_1 m_2)Q^2/2 \end{array}\right\}, \tag{C7}$$

where $d_\pm = (\cosh\psi \pm 1)/2$ are elements of the $d^1(\psi)$ matrix. A special case of these results—charm production in neutrino scattering—is discussed in Sec. V.

Although it is far from obvious, Eqs. (C5) and (C7) are in fact identical (as some tedious algebra will prove). The difference in appearance is simply that the helicity approach exploits the symmetries of the problem; hence, these symmetries are manifest in the final representation of the cross section, Eq. (C5).

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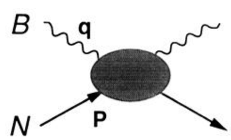

FIG. 5. Basic process for inclusive boson $B(q)$ nucleon $N(P)$ scattering: $N(P) + B(q) \rightarrow X(P_X)$, summed over the final state, $X(P_X)$.