Partial and total inelasticities obtained from inclusive reaction data

J. Bellandi, R. J. M. Covolan, C. G. S. Costa, J. Montanha, and L. M. Mundim
Instituto de Física "Gleb Wataghin," Universidade Estadual de Campinas, Unicamp, 13083-970, Campinas, São Paulo, Brasil
(Received 17 December 1993)

We calculate the partial inelasticities referring to the production of pions, kaons, protons, and antiprotons from inclusive reaction data. From the partial inelasticities we also estimate the total inelasticity of a pp collision, which turns out to be rapidly increasing with energy.

PACS number(s): 13.85.Ni, 13.85.Tp

I. INTRODUCTION

A wide and diversified range of hadron interaction calculations in cosmic-ray and accelerator physics is strongly dependent on the inelasticity parameter, whose energy dependence is presently in question. The importance of this problem is enhanced nowadays when experiments for supercolliders are being outlined.

Inelasticity is usually defined as the fraction of the available energy released for multiparticle production in an inelastic hadron-hadron collision. Its value was evaluated a long time ago from cosmic-ray experiments [1] as being around 0.5, which was later confirmed at the CERN Intersecting Storage Rings (ISR) [2].

In the last few years the question concerning the energy dependence of the inelasticity (i.e., whether it is an increasing or decreasing function of the energy) was dealt with by a number of authors through different approaches, most of them in a model-dependent way. Generally speaking, the obtained results are not consistent [3,4] and the question remains unsolved [5].

We have already discussed this question [6,7] in connection with the behavior of the nucleonic cascade in the atmosphere. From the existing extensive air shower data, a definite conclusion cannot yet be reached.

In this paper we present an attempt at extracting partial inelasticities in a model-independent way from experimental data of inclusive reactions initiated by pp collisions (i.e., $pp \to cX$, $c=\pi^{\pm}$, $K^{\pm}$, $p^{\pm}$). From the results obtained for pion, kaon, and (anti)proton partial inelasticities we also estimate the behavior of the total inelasticity.

II. FITTING PROCEDURE

The average partial inelasticity $\langle k_c \rangle$ can be calculated in the following way:

$$\langle k_c \rangle = \int_0^1 x \frac{1}{\sigma_{\rm in}} \frac{d\sigma_c}{dx} dx \tag{1}$$

$$\simeq \frac{2\langle m_T \rangle}{\sqrt{s}} \int_0^{y_{\max}} \cosh y \frac{1}{\sigma_{\rm in}} \frac{d\sigma_c}{dy} dy \ , \tag{2}$$

where

$$x = \frac{E_c}{\sqrt{s}/2} = \frac{2m_T \cosh y}{\sqrt{s}} \ , \tag{3}$$

$d\sigma_c/dy$ is the inclusive cross section (integrated over p_T) and $\langle m_T \rangle = (m_c^2 + \langle p_T \rangle^2)^{1/2}$ is the average transversal mass (all variables here are given in the center-of-mass system).

Thus the energy dependence of the average partial inelasticity referring to the production of some particle c can be obtained from (2), since we provide $\langle p_T \rangle$ as a function of energy, and $1/\sigma_{\rm in} d\sigma_c/dy$ as a function of both s and y.

This last point is the essential difficulty in performing this kind of analysis, because we need data of $d\sigma_c/dy$ covering the whole rapidity range for a wide range of energy.

To the best of our knowledge there is just one set of inclusive production data that incorporates the whole rapidity range at ISR energy: that of Ref. [8]. We remind the reader that the cross sections of inclusive reactions measured at the ISR were reported in terms of invariant cross section $E_c d^3\sigma/d^3\mathbf{p}$, but not as $d\sigma_c/dy$, which we need here. Integration over p_T should take into account the nontrivial interplay between p_T and y dependence, which is related to the so-called seagull effect. Data at higher energies are unavailable.

In order to overcome this difficulty and obtain $d\sigma_c/dy$ as a function of s and y we have carried out a simultaneous fit involving the available data in terms of y [8,9], data of $d\sigma_c/dy|_{y=0}$ obtained from Refs. [10,11], and data of mean multiplicity [12,13] of the produced particle c, since

$$\int_0^{y_{\max}} \frac{1}{\sigma_{\rm in}} \frac{d\sigma_c}{dy} dy = \frac{\langle n_c \rangle}{2} \ . \tag{4}$$

The expression used in the fitting procedure was borrowed from Ref. [14]:

$$\frac{1}{\sigma_{\rm in}} \frac{d\sigma_c}{dy} = A \{ \exp[-(y+y_0)^2/2\alpha^2] + \exp[-(y-y_0)^2/2\alpha^2] \} \ . \tag{5}$$

where

0556-2821/94/50(1)/297(5)/$06.00

TABLE I. Parameters of Eqs. (5) and (6) obtained by fitting experimental data of mean multiplicity and inclusive cross section.

	a_1	a_2	a_3	a_4	a_5	a_6
π^-	0.1044	−0.0393	0.1524	0.1193	0.1416	0.0652
π^+	0.0845	0.1775	0.1642	0.1949	0.1451	0.1264
K^-	0.0137	−0.0303	0.1599	−0.1765	0.1473	−0.2226
K^+	0.0177	−0.0360	0.0776	0.4227	0.0818	0.2601
$\bar{p}$	0.0069	−0.0208	0.1742	−0.4616	0.2590	−1.0387

$$A = a_1 \ln s + a_2 ,$$
$$y_0 = a_3 \ln s + a_4 , \qquad (6)$$
$$\alpha = a_5 \ln s + a_6 ,$$

and the fit was carried out by using the CERN's MINUIT routine.

The fit parameters obtained are shown in Table I. Figures 1 and 2 present some of the data set used in the fitting procedure and the respective fitted curves.

In our procedure we actually have fitted only data referring to the inclusive production of $\pi^{\pm}$, $K^{\pm}$, and $\bar{p}$. For the proton-inclusive production, some considerations are required. Firstly, data of proton-inclusive production involve both diffractive and nondiffractive production mechanisms and there is no unambiguous way of separating them in these two contributions. As we are interested here in the latter contribution, we estimate nondiffractive (ND) proton-inclusive production based on the fit for antiproton-inclusive production.

In addition, in pp collisions positive particles are preferentially produced on the whole rapidity range at ISR energies [see, e.g., Figs. 1(a) and 1(b)]. At those energies, ND proton production is usually estimated to be double the antiproton production, but, as energy becomes higher, positive and negative particle distributions tend to equalize. This is reflected in the mean multiplicity as can be seen in Fig. 2 for pions and kaons.

In order to take into account these effects, we write

$$\frac{d\sigma_p}{dy} = \lambda(s) \frac{d\sigma_{\bar{p}}}{dy} , \qquad (7)$$

where $\lambda(s)$ is parametrized as

$$\lambda(s) = 1 + 34.2 s^{-0.52} \qquad (8)$$

from $d\sigma_{p,\bar{p}}/dy|_{y=0}$ data [10].

The assumption inserted in the parameter $\lambda(s)$ by which it has no dependence on rapidity is quite reasonable. For instance, the ratios $(d\sigma_{\pi^+}/dy)/(d\sigma_{\pi^-}/dy)$ and $(d\sigma_{K^+}/dy)/(d\sigma_{K^-}/dy)$ for the data of Figs. 1(a) and 1(b) only differ significantly from a constant very near to y_{max}.

Thus, summarizing, we calculate the inclusive cross section $(1/\sigma_{in})(d\sigma/dy)$ to produce $\pi^{\pm}$, $K^{\pm}$, and $\bar{p}$ by the parametrization (5) and (6) with the parameters given in Table I. Nondiffractive proton production is evaluated from (7) and (8).

Since $\pi^{\pm}$, $K^{\pm}$, and $p^{\pm}$ constitute the bulk of the charged particles produced in a (anti)proton-proton collision, we can check our parametrizations at collider energies by estimating the inclusive charged-particle pseudorapidity distribution $(1/\sigma_{in})(d\sigma/d\eta)$ from them. To do that we transform y into η by an approximated

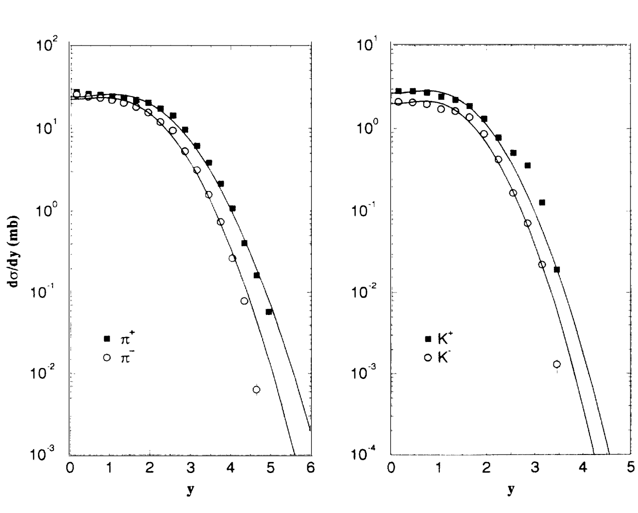

FIG. 1. Comparison between Eqs. (5) and (6) (parameters in Table I) and inclusive cross section data of the reaction $pp \to cX$, where c is (a) $\pi^{\pm}$ and (b) $K^{\pm}$.

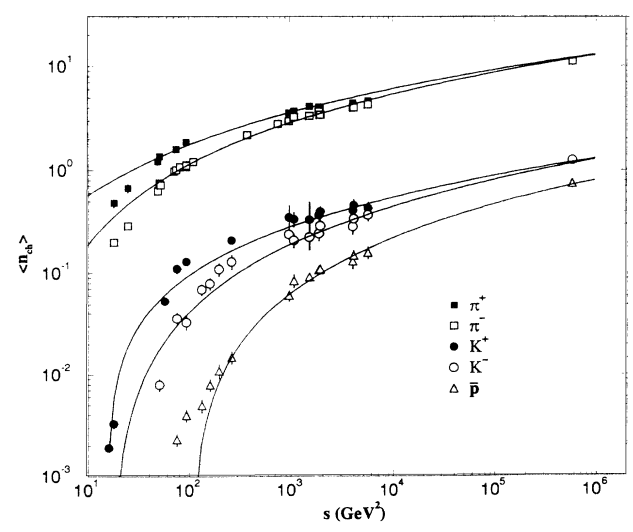

FIG. 2. Fit of mean multiplicity of charged particles in the reaction $pp \to cX$, where c is shown in the figure, by using Eqs. (4), (5), and (6) (the parameters are in Table I).

TABLE II. Parameters obtained by fitting data of $\langle p_T \rangle$ from Refs. [11,13] by the expression $\langle p_T \rangle = a + b \ln s$.

	π^-	π^+	K^-	K^+	$\bar{p}$
a	0.2610	0.2858	0.3282	0.3631	0.3473
b	0.0123	0.0089	0.0153	0.0129	0.0216

relation, that is, $\langle p_T \rangle \sinh\eta \simeq \langle m_T \rangle \sinh y$, where $\langle p_T \rangle = a + b \ln s$ with the parameters given in Table II. As can be seen in Fig. 3, the agreement obtained with experimental data is quite fair (we emphasize that these data were not used in the fitting procedure). This tells us that we can extrapolate our parametrizations at least until energies reached at the CERN Super Proton Synchrotron (SPS) with a good confidence level.

III. PARTIAL AND TOTAL INELASTICITIES

By admitting that the parametrizations extrapolate to high energies in an acceptable way, we calculate the partial inelasticities for $\pi^\pm$, $K^\pm$, and $p^\pm$ by Eq. (2) with the respective fitted expressions. The results are shown in Fig. 4.

As we see, pion inelasticities increases very slowly with energy and the kaon ones remain practically constant, while proton and antiproton inelasticities grow very fast. Figure 4 (and the following) also displays, for each kind of particle, two dotted curves that establish the upper and lower limits for the respective inelasticity. These limits were estimated from the error matrix furnished by the MINUIT routine. As we can see, the errors present a growing spread with energy as we go beyond the fit's region, giving an idea of the resultant uncertainty in extrapolating the parametrizations at supercollider energies. Figure 5 displays the sum of the six calculated partial inelasticities $\langle k_{\rm tot}^{\rm ch} \rangle$, which practically corresponds to the total charged-particle inelasticity. The curve labeled $\langle k_{\rm total} \rangle$ is an estimation of the total inelasticity, which includes neutral particle production. This estimation was made based on the following assumptions: (1) $k_{\pi^0} = (k_{\pi^+} + k_{\pi^-})/2$; (2) $k_{K^0} + k_{\bar{K}^0} = k_{K^+} + k_{K^-}$; (3) $k_\eta = k_{K^+} + k_{K^-}$; (4) $k_n + k_{\bar{n}} = k_p + k_{\bar{p}}$; (5) $k_{\rm hyperons} = 2/3(k_p + k_{\bar{p}})$.

Assumptions (1), (2), and (4) are quite natural; on the contrary, (3) and (5) require some comments. The shape of the η rapidity distribution is very similar to that of kaons at $\sqrt{s} = 27.5$ GeV [8], but $\langle n \rangle_\eta$ surpasses $\langle n \rangle_{K^\pm}$ at collider energies [16]. Thus assumption (3) can possibly underestimate k_η. Assumption (5) is exclusively based on mean multiplicity data measured at collider energies [16], so it is very crude.

In spite of the roughness of assumptions (3) and (5), the estimate of $\langle k_{\rm tot} \rangle$ (as well as $\langle k_{\rm tot}^{\rm ch} \rangle$), shown in Fig. 5, presents an unequivocal increase with energy.

In Fig. 6 we show the same partial inelasticities, but grouped in a different form. For comparison, also shown are results obtained from the formula

$$\langle k_{\rm tot} \rangle = 4 \frac{\sigma_{\rm el}}{\sigma_{\rm tot}} \left[1 - \frac{\sigma_{\rm el}}{\sigma_{\rm tot}} \right] = 4 \left[1 - \frac{\sigma_{\rm in}}{\sigma_{\rm tot}} \right] \frac{\sigma_{\rm in}}{\sigma_{\rm tot}} , \quad (9)$$

derived in [6].

This equation was derived by establishing a simple ansatz identifying inelasticity and the inelastic overlap function, $\langle k \rangle = \langle G \rangle$, in connection with the grey disk model [6], so it should be taken just as a qualitative tendency. For this reason the agreement seen in Fig. 6 between Eq.

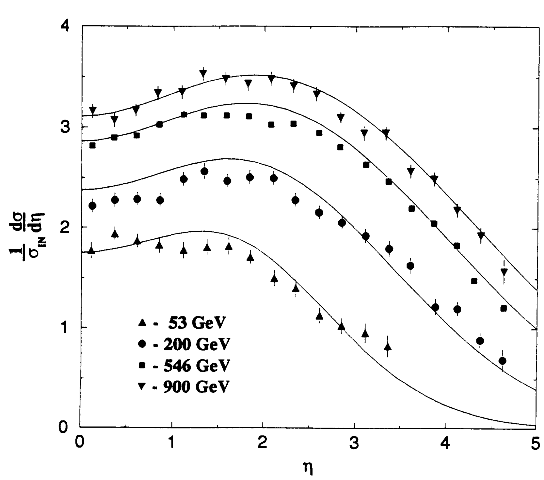

FIG. 3. Inclusive charged-particle pseudorapidity distribution obtained by summing the contribution of $\pi^\pm$, $K^\pm$, and $p^\pm$ in comparison with experimental data. Data are from Ref. [15].

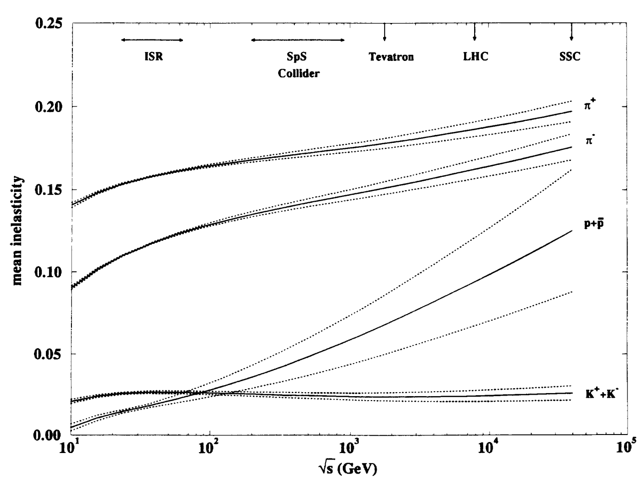

FIG. 4. Partial charged-particles inelasticities calculated by Eq. (2) with parametrization (5). Dotted curves show the upper and lower limits for each case.

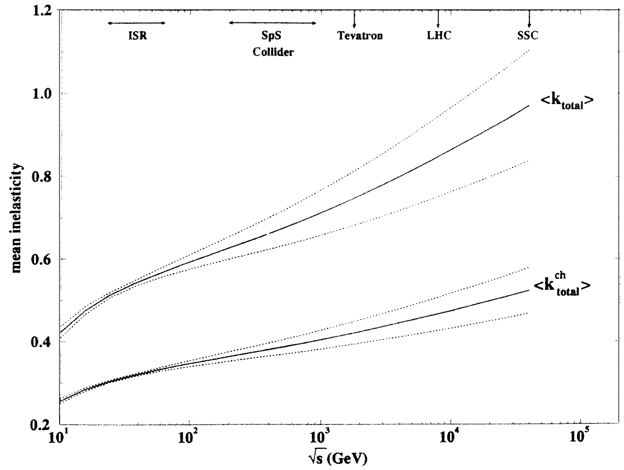

FIG. 5. Sum of partial charged-particle inelasticities (labeled $\langle k_{\rm tot}^{\rm ch} \rangle$) compared to the total inelasticity ($\langle k_{\rm tot} \rangle$), which includes neutral particle contributions.

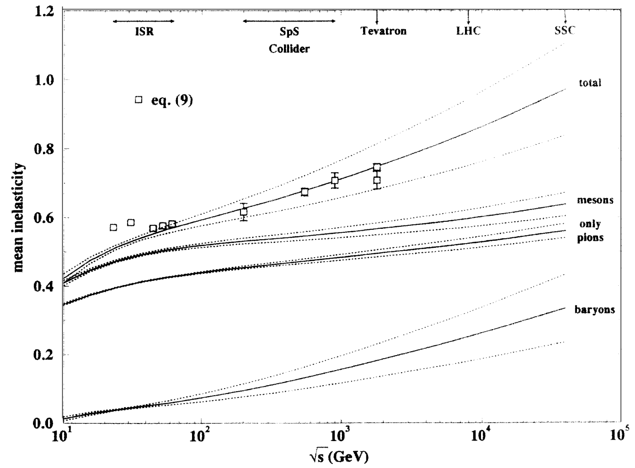

FIG. 6. Total inelasticity in comparison with the one given by Eq. (9). The figure also shows the contributions of pions (only), mesons (pions, kaons, and eta), and baryons [(anti)protons, neutrons, and hyperons].

(9) and the result of the present analysis is very surprising.

Another analysis about inelasticity with a basis in inclusive reactions was recently made by Dias de Deus and Pádua [17]. In their analysis, pion inelasticity results are constant, while in the present one $\langle k_{\pi^\pm} \rangle$ increases very slowly with energy (Fig. 4), even though the results of $\langle k_{\pi^\pm} \rangle$ of both analyses are consistent at ISR energies. Charged-particle inelasticities, on the contrary, show opposite trends.

IV. CONCLUSIONS

Concluding, we notice that the increased rate with energy of the total inelasticity obtained in the present analysis implies a fast vanishing of the leading-particle effect originating from diffractive production. However, such an effect has usually been observed in cosmic-ray physics at superghigh energies [18]. This apparent contradiction disappears by noticing that what we have obtained here is the average behavior for the inelasticity in a very large number of collisions (since in the present analysis we have just used accelerator data), while fluctuations surely have a very important role in a single cosmic-ray event. A last remark is that the increase rate of $\langle k_{\rm tot} \rangle$ was not predicted by any model and/or analysis until now [3], except for those of Ref. [6] and Ref. [19]. One hopes that the measurements of leading-particle distribution carried out by the UA8 Collaboration [20] can provide some clues to clarify this question very soon.

ACKNOWLEDGMENTS

The authors would like to thank Brazilian governmental agencies CAPES, CNPq, and FAPESP for financial support. They also thank Y. V. Fisyak who graciously provided the data of Ref. [8], and F. S. Navarra for useful discussions.

[1] See, for instance, E. L. Feinberg, Phys. Rep. **5**, 237 (1972).
[2] M. Basile *et al.*, Nuovo Cimento A **73**, 329 (1983).
[3] C. E. Navia *et al.*, Prog. Theor. Phys. **88**, 53 (1992); Yu. M. Shabelski *et al.*, J. Phys. G **18**, 281 (1992); T. K. Gaisser *et al.*, in *Proceedings of the XXIst International Cosmic-Ray Conference,* Adelaide, Australia, 1990, edited by R. J. Protheroe (Graphic Services, Northfield, South Australia, 1990), Vol. 8, p. 55.
[4] J. Dias de Deus and A. B. Pádua, Phys. Lett. B **315**, 188 (1993); F. O. Durães and F. S. Navarra, Brazilian J. Phys. **23**, 209 (1993); M. T. Nazirov and P. A. Usik, J. Phys. G **18**, L7 (1992); D. Linkai and Z. Qingpi, Phys. Lett. B **297**, 201 (1992).
[5] The papers quoted in [3] compared mean inelasticity obtained from different approaches. Reference [4] is a compilation of recent papers about the same subject, but not included in the analyses referred to in [3].
[6] J. Bellandi *et al.*, Phys. Lett. B **262**, 102 (1991).
[7] J. Bellandi *et al.*, J. Phys. G **18**, 579 (1992); Phys. Lett. B **279**, 149 (1992).
[8] LEBC-EHS Collaboration, M. Aguilar-Benitez *et al.*, Z. Phys. C **50**, 405 (1991).
[9] A. E. Brenner *et al.*, Phys. Rev. D **26**, 1497 (1992).
[10] K. Guettler *et al.*, Nucl. Phys. **B116**, 77 (1976); B. Alpher *et al.*, *ibid.* **B100**, 237 (1975).
[11] G. J. Alner *et al.*, Nucl. Phys. **B258**, 505 (1985).
[12] M. Antinucci *et al.*, Lett. Nuovo Cimento **6**, 121 (1973); UA5 Collaboration, K. Aplgård *et al.*, Phys. Lett. **121B**, 209 (1983).
[13] A. M. Rossi *et al.*, Nucl. Phys. **B84**, 269 (1975).
[14] J. N. Capdevielle, J. Phys. G **15**, 909 (1989).
[15] UA5 Collaboration, G. J. Alner *et al.*, Z. Phys. C **23**, 1

(1986).
[16] See, for instance, data compilation in C. Geich-Gimbel, Int. J. Mod. Phys. A **4**, 1527 (1989).
[17] See the first paper quoted in [4].
[18] See, for instance, S. Yamashita, J. Phys. Soc. Jpn. **54**, 529 (1985).
[19] F. O. Durães and F. S. Navarra, Brazilian J. Phys. **23**, 209 (1993).
[20] UA8 Collaboration, P. Schlein, in "Proceedings of the 13th European Cosmic Ray Symposium," Geneva, Switzerland, 27–31 July 1992, edited by P. K. F. Grieder [Nucl. Phys. B **33A,B**, 41 (1993)].