PHYSICAL REVIEW D VOLUME 50, NUMBER 4 15 AUGUST 1994

Growth of density inhomogeneities in Newtonian cosmological models with variable Λ

R. C. Arcuri and I. Waga
Universidade Federal do Rio de Janeiro, Instituto de Física, C.P. 68528, 21945, 970, Rio de Janeiro, RJ, Brazil
(Received 28 April 1993)

We analyze the growth of density perturbations in Newtonian cosmological models with continuous matter creation and obtain the time evolution equation for the mass density contrast. The special case where matter is created by the decay of a cosmological term (Λ) varying as $\Lambda = 3\beta H^2$ is examined. Constraints on the value of β from the observed high isotropy of the cosmic microwave background radiation are also obtained.

PACS number(s): 98.80.Hw

I. INTRODUCTION

Recently, much effort has been made to investigate the possibility that the Universe has a nonvanishing cosmological term (Λ). In the past, the cosmological term was quite often introduced when there was some apparent conflict between theory and observation. Later on, when a better understanding of the observational data was achieved, the hypothesis of a $\Lambda \neq 0$ universe was discarded for the sake of simplicity.

The interest in a cosmological term now reappears motivated by two main reasons. First, with a Λ term cosmologists can reconcile the theoretical appeal of the inflationary models with observations. A cosmological term helps to solve the "low-energy density problem," i.e., it could be responsible for the missing mass necessary to "close" the Universe [1]. Second, with this term we can get a theoretical age for a flat universe in the observed range, even for a present high value of the Hubble parameter (H_p) [2,3].

There is also an additional reason that makes physicists believe that the contribution of the cosmological term could be appreciable. According to quantum field theory, the vacuum state has zero-point fluctuations to whose energy the gravitational field is sensitive. Lorentz invariance implies that the vacuum contribution to the energy-momentum tensor has the same form as that of a cosmological term ($\rho_v g_{\mu\nu}$). This vacuum contribution to Λ added to the bare cosmological term gives rise to an effective cosmological term. However, the estimated value given by quantum field theory is 50–120 orders of magnitude above the maximum value that observations indicate for the effective cosmological term. The miraculous cancellation between the bare and the vacuum contributions to the effective cosmological term, or more specifically, the smallness of this term, constitutes the so-called "cosmological constant problem" [3,4].

One way to solve the Λ problem is to assume that as the Universe evolves the effective cosmological term decreases to its present value. From this point of view, Λ is small today because the Universe is old. Several cosmological models having this common property have recently been proposed in the literature [5–9]. Some of them also present the characteristic that matter is continuously created as a consequence of Λ decay.

Cosmological models with matter creation have mainly been associated with the steady-state universe [10]. Of special interest for us is the McCrea interpretation of the steady-state universe. Differently from the usual theory, where in general a C field is introduced, McCrea has shown that matter creation can be treated by the standard methods of general relativity provided that the possibility of a "zero-point" stress is allowed [11]. In fact, we can say that, more than 40 years ago, McCrea introduced what we call today "variable Λ cosmologies with matter creation." The only difference is that in the present models the perfect cosmological principle is not assumed, and so the energy density is not necessarily constant during the universe expansion.

The problem of galaxies formation in a steady-state universe was examined by several authors [12–14]. The calculations (usually made in the Newtonian context) assume, however, that the Universe mass density is constant, as required by the perfect cosmological principle. In the present paper, we shall relax this hypothesis and will reconsider, still in the Newtonian framework, the problem of the growth of small perturbations in a universe with continuous matter creation. Since we are mainly interested in variable-Λ cosmologies, we shall also introduce a time-dependent cosmological term and relate its time derivative to the rate of mass creation.

The evolution of density perturbations in variable-Λ cosmologies in a scenario in which only radiation is created has been examined by Abdalla and Abdel-Rahman [15]. This scenario could appear, for instance, when the vacuum decay is a consequence of the oscillations of some scalar field. In this case, since the Hubble parameter is relatively low at the matter-dominated phase, it can be argued that the mass of the created particles would also be very low. However, the precise mechanism of vacuum decay is not yet known and in general we would expect that if vacuum decays it will decay to radiation and/or matter. Usually, it is assumed that the vacuum couples only to the dominant component. So, differently from Ref. [15], we shall here explore the case in which only nonrelativistic matter is created.

This paper is organized as follows. We obtain in Sec. II, in the linear approximation, the time evolution equation for the mass density contrast for a Newtonian universe with matter creation. In Sec. III we relate the rate of matter creation to the time derivative of the cosmological term and obtain the time dependence of the density contrast in the special case in which the cosmological term decays as $\Lambda=3\beta H^2$. Finally, the isotropy of the cosmic background radiation is used to estimate an upper limit to the parameter β.

II. DIFFERENTIAL EQUATION FOR δ

The fundamental hydrodynamical equations that describe the motion of the cosmic fluid here considered are

$$\left[\frac{\partial \mathbf{u}}{\partial t}\right]_r+(\mathbf{u}\cdot\nabla_r)\mathbf{u}=-\nabla_r\Phi\ , \tag{1}$$

$$\left[\frac{\partial \rho}{\partial t}\right]_r+\nabla_r\cdot(\rho\mathbf{u})=\Psi\ , \tag{2}$$

and

$$\nabla_r^2\Phi=4\pi G\rho-\Lambda\ . \tag{3}$$

Equations (1)–(3) are, respectively, the momentum conservation equation (the Euler equation), the continuity equation, and Poisson's equation. Here $\mathbf{u}$ is the velocity of a fluid element of volume, ρ is the fluid mass density, Φ is the gravitational potential, and Λ is the cosmological term to be regarded as a function of the absolute Newtonian time t. We have assumed that the fluid is cold such that its pressure is negligible. Since we are interested in cosmological models with matter creation, we have modified the continuity equation and have included a source term Ψ.

Particle creation also modifies the Euler equation [13]. To show this, let us denote by δv an element of volume of the fluid. In an interval of time dt, momentum equal to $-\rho\delta v dt\nabla_r\Phi$ is added to this element by the gravitational force and $\Psi\delta v dt\mathbf{c}$ due to the creation of particles with velocity $\mathbf{c}$. The mass of the element of volume will change by $\delta m=\delta\rho\,\delta v=\Psi dt\,\delta v$. It is now straightforward to see that momentum conservation implies

$$(\rho+\delta\rho)(\mathbf{u}+\dot{\mathbf{u}}\,dt)-\rho\mathbf{u}=-(\nabla_r\Phi)\rho\,dt+\Psi\,dt\,\mathbf{c}\ ,$$

or

$$\left[\frac{\partial \mathbf{u}}{\partial t}\right]_r+(\mathbf{u}\cdot\nabla_r)\mathbf{u}=-\nabla_r\Phi+\frac{\Psi}{\rho}(\mathbf{c}-\mathbf{u})\ . \tag{4}$$

So, as can be seen from the above equation, we have implicitly assumed in Eq. (1) that the created particles have the same velocity as the already existing ones.

We now introduce [17,18] the comoving coordinate $\mathbf{x}$ related to the proper coordinate $\mathbf{r}$ by

$$\mathbf{x}=\frac{\mathbf{r}}{R}\ , \tag{5}$$

where $R=R(t)$ is the expansion factor. In terms of this coordinate, we rewrite the true fluid velocity and density as

$$\mathbf{u}=\dot{R}\mathbf{x}+R\dot{\mathbf{x}}=\dot{R}\mathbf{x}+\mathbf{v}(\mathbf{x},t) \tag{6}$$

and

$$\rho=\rho_0(t)[1+\delta(\mathbf{x},t)]\ , \tag{7}$$

where $\mathbf{v}$ and δ are first-order perturbations to the mean velocity $\dot{R}\mathbf{x}$ and density ρ_0, respectively. We shall assume that these quantities are small, i.e., $\delta\ll 1$ and $v\ll u$.

By changing variables from $(\mathbf{r},t)$ to $(\mathbf{x},t)$ and using

$$\nabla_x\equiv\nabla=R\nabla_r \tag{8}$$

and

$$\left[\frac{\partial}{\partial t}\right]_x\equiv\frac{\partial}{\partial t}=\left[\frac{\partial}{\partial t}\right]_r+\frac{\dot{R}}{R}\mathbf{x}\cdot\nabla_x\ , \tag{9}$$

Eqs. (1)–(3) are rewritten as

$$\ddot{R}\mathbf{x}+\frac{\partial \mathbf{v}}{\partial t}+\frac{\dot{R}}{R}\mathbf{v}=-\frac{\nabla\Phi}{R}\ , \tag{10}$$

$$\nabla\cdot\mathbf{v}=-R\left[\frac{\partial\delta}{\partial t}+\frac{\Psi\delta}{\rho_0}\right]\ , \tag{11}$$

$$\frac{1}{R^2}\nabla^2\Phi=4\pi G\rho_0(1+\delta)-\Lambda\ . \tag{12}$$

To obtain Eqs. (10) and (11), we have neglected second-order terms and have used that, in zero order,

$$\dot{\rho}_0+3\frac{\dot{R}}{R}\rho_0=\Psi\ . \tag{13}$$

By writing

$$\Phi=\phi(\mathbf{x},t)+\tfrac{2}{3}\pi G\rho_0R^2x^2-\tfrac{1}{6}\Lambda R^2x^2 \tag{14}$$

and using the zero-order equation

$$3\frac{\ddot{R}}{R}=-4\pi G\rho_0+\Lambda\ , \tag{15}$$

we rewrite Eqs. (10) and (12) as

$$\frac{\partial \mathbf{v}}{\partial t}+\frac{\dot{R}}{R}\mathbf{v}=-\frac{\nabla\phi}{R}\ , \tag{16}$$

$$\nabla^2\phi=4\pi GR^2\rho_0\delta\ . \tag{17}$$

Equations (11), (16), and (17) are the perturbed equations to be considered.

Taking the divergence of (16) and substituting (11) and (17), we finally obtain

$$\frac{\partial^2\delta}{\partial t^2}+\left[2\frac{\dot{R}}{R}+\frac{\Psi}{\rho_0}\right]\frac{\partial\delta}{\partial t}-\left[4\pi G\rho_0-2\frac{\dot{R}\Psi}{R\rho_0}-\frac{\partial}{\partial t}\left[\frac{\Psi}{\rho_0}\right]\right]\delta=0\ . \tag{18}$$

Note that if $\Psi=0$ we recover the well-known time evolution equation for the mass density contrast in the linear approximation.

III. APPLICATION: THE CASE $\Lambda=3\beta H^2$

In order to integrate Eq. (18), it is first necessary to express Ψ in terms of the other quantities involved. In general, a physical principle must be used to direct this choice. For instance, if we were dealing with the steady-state cosmology, we should have to choose $\Psi=3(\dot{R}/R)\rho_0$ since, as can be seen from (13), only with this choice the mass density would always be constant as required by the perfect cosmological principle. We also remark that with this choice Eq. (18) reduces to Eq. (7) of Roxburgh and Saffman [13].

Here we are mainly interested in variable-Λ cosmological models with matter creation and we shall assume

$$\Psi=-\frac{1}{8\pi G}\frac{\partial\Lambda}{\partial t} . \tag{19}$$

Therefore, the balance equation (13) will assume the same form as the one we would obtain for the energy density, from the Bianchi identities, in the relativistic case [8].

By using Eqs. (13) and (19), we multiply Eq. (15) by $\dot{R}R$ (assuming $\dot{R}\neq 0$) and integrate it to obtain the Friedmann equation: namely,

$$\left[\frac{\dot{R}}{R}\right]^2=\frac{8\pi G}{3}\rho_0+\frac{\Lambda}{3}-\frac{k}{R^2} , \tag{20}$$

where k is an integration constant. It is worth mentioning that (20) is obtained independently of the time dependence of Λ. Furthermore, only with the (19) choice could we have obtained (20) from (13) and (15). In what follows we shall consider Newtonian analogues of the Friedman spatially flat models and take $k=0$.

We now assume that Λ varies as

$$\Lambda=3\beta H^2 , \tag{21}$$

where $H=\dot{R}/R$ is the Hubble parameter and β is a constant. The above variation for Λ was originally considered by Freese *et al.* [6] with β equal to the ratio of vacuum to the sum of vacuum and matter energy density. Some other properties of these models were also discussed in [9], [15], and [16]. Since the mass density is positive and we are interested in a non-negative cosmological term, it follows from (20) (with $k=0$) and (19) that β is restricted to the interval $0\leq\beta<1$.

In order to integrate Eq. (18), it will be convenient to change variables from t to R. So, with Ψ given by (19) and Λ by (21), after some simple algebra Eq. (18) is transformed into the equation

$$R^2\frac{\partial^2\delta}{\partial R^2}+\frac{3}{2}R(1+3\beta)\frac{\partial\delta}{\partial R}-\frac{3}{2}(1+\beta)(1-3\beta)\delta=0 . \tag{22}$$

By integrating the above equation we obtain

$$\delta=AR^{-3(1+\beta)/2}+BR^{(1-3\beta)} , \tag{23}$$

where $A(\mathbf{x})$ and $B(\mathbf{x})$ have to be determined by initial data. Note that we have a growing mode only if $\beta<\frac{1}{3}$. This means that structures cannot be formed by gravitational instability in cosmological models with $\beta\geq\frac{1}{3}$.

The time dependence of the expansion factor is given by [6,9]

$$R(t)=R_p[\tfrac{3}{2}H_p(1-\beta)t]^{2/3(1-\beta)} , \tag{24}$$

where R_p and H_p are the present values of the scale factor and the Hubble parameter, respectively. Then, by substituting $R(t)$ in Eq. (23), we can express the mass density contrast as a function of time: namely,

$$\delta=\delta_-(t_i)\left[\frac{t}{t_i}\right]^{-(1+\beta/1-\beta)}+\delta_+(t_i)\left[\frac{t}{t_i}\right]^{2(1-3\beta)/3(1-\beta)} , \tag{25}$$

where $\delta_+(t_i)$ and $\delta_-(t_i)$ denote the amplitude of the growing mode and the decreasing one at some initial time t_i, respectively. Note that by taking $\beta=0$ we recover the flat standard pressureless cosmological model results.

In order to roughly estimate an upper limit to the parameter β, we assume that the usual Sachs-Wolfe relation $\delta T/T\sim\phi/3$ is applicable for the considered models. Here $\phi\propto\delta_+(t_{\rm dec})$ is the potential at decoupling and $\delta T/T$ is the temperature fluctuation at large angular scales. The recent results from the Cosmic Background Explorer (COBE) strongly support the gravitational instability theory of structure formation and can be interpreted as providing evidence for the existence of small inhomogeneities generated in an early era. In the model of Freese *et al.*, this means that, if primordial inhomogeneities had grown and formed structures, the parameter β, according to (25), must be very small. So, although the above Sachs-Wolfe expression is only strictly valid if $\beta=0$, we expect no appreciable deviation from it for the range of small β we should consider.

From Eq. (23) we obtain

$$\frac{\delta_+(t_p,\beta)}{\delta_+(t_{\rm dec},\beta)}=(1+z_{\rm dec})^{1-3\beta} , \tag{26}$$

where t_p is the present time, $t_{\rm dec}$ is the decoupling time, and $z_{\rm dec}$ is the decoupling redshift. By assuming that during the matter-dominated era Λ decays only into nonrelativistic particles, it can be shown that $z_{\rm dec}$ in the model of Freese *et al.* is approximately equal to its corresponding value in the standard model ($\beta=0$). So, by assuming fixed amplitude at decoupling (COBE roughly fix this amplitude), $\delta_+(t_{\rm dec},\beta)=\delta_+(t_{\rm dec},\beta=0)$, we have

$$\frac{\delta_+(t_p)}{\delta_+(t_p,\beta=0)}=(1+z_{\rm dec})^{-3\beta}=\frac{\sigma_8(\beta)}{\sigma_8(\beta=0)} , \tag{27}$$

where σ_8 is the root-mean-square mass fluctuation in spheres of radius $8h^{-1}$ Mpc. The recent COBE measurements indicate that $\sigma_8(\beta=0)\simeq 1$ for the standard cold dark matter model [19] with $\Omega h=0.5$, and by requiring that structures form not too late, it is safe to assume $\sigma_8(\beta)\gtrsim\frac{1}{3}$. So, with $z_{\rm dec}\simeq 10^3$, Eq. (27) gives us $\beta\lesssim 0.053$.

Although small, the above upper limit for β is considerably higher than $\beta\lesssim 3\times 10^{-5}$ obtained by Freese *et al.* by using (18) with $\Psi=0$ and some assumptions on the observed isotropic γ-ray flux. Our result is closer to the one obtained by the same authors from primordial nucleosynthesis constraints and is two orders of magnitude higher than the result ($\beta\lesssim 0.001$) obtained recently by Overduin, Wesson, and Bowyer [16] by using a different method. In all cases, however, the conclusion is the same: The low-energy density and the age problems will not be solved for this kind of Λ variation with the above upper limits for the parameter β.

In summary, in this paper we analyzed the growth of density perturbations in Newtonian cosmological models with a matter creation term Ψ. We obtained a new time evolution equation for the mass density contrast that generalizes the usual one. The special case where matter is created by the decay of a cosmological term varying as $\Lambda = 3\beta H^2$ was investigated. We showed that the recent results of COBE strongly constrain the possible values of the parameter β.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge Josh Frieman for discussions on the recent COBE results and their possible implications in constraining the parameter β. We also thank Alexandre Tort and José Ademir Sales Lima for reading carefully the manuscript and for several useful suggestions. Partial financial support from the Brazilian agencies FINEP and CNPq is also acknowledged.

[1] P. J. E. Peebles, Astrophys. J. **284**, 439 (1984); M. S. Turner, G. Steigman, and L. M. Krauss, Phys. Rev. Lett. **52**, 2090 (1984).
[2] H. J. Blome and W. Priester, Astrophys. Space Sci. **117**, 327 (1985); H. P. Klapdor and K. Grotz, Astrophys. J. **301**, L39 (1986); R. J. Tayler, Q. J. R. Astron. Soc. **27**, 367 (1986); T. S. Olson and T. F. Jordan, Phys. Rev. D **35**, 3258 (1987); M. Moles, Astrophys. J. **382**, 369 (1991).
[3] S. M. Carrol, W. H. Press, and E. L. Turner, Annu. Rev. Astron. Astrophys. **30**, 499 (1992).
[4] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
[5] M. Ozer and M. O. Taha, Phys. Lett. B **171**, 363 (1987); Nucl. Phys. **B287**, 776 (1987); W. Chen and Y. S. Wu, Phys. Rev. D **41**, 695 (1990); J. M. Salim and I. Waga, Class. Quantum Grav. **10**, 1767 (1993); J. A. S. Lima, and J. M. F. Maia, Mod. Phys. Lett. A **8**, 591 (1993).
[6] K. Freese, F. C. Adams, J. A. Frieman, and E. Mottola, Nucl. Phys. **B287**, 797 (1987).
[7] P. J. E. Peebles and B. Ratra, Astrophys. J. **325**, L17 (1988); B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3407 (1988).
[8] J. C. Carvalho, J. A. S. Lima, and I. Waga, Phys. Rev. D **46**, 2404 (1992).
[9] I. Waga, Astrophys. J. **414**, 436 (1993).
[10] H. Bondi and T. Gold, Mon. Not. R. Astron. Soc. **108**, 252 (1948); F. Hoyle, *ibid.* **108**, 372 (1948); **109**, 365 (1949).
[11] W. H. McCrea, Proc. R. Soc. London **A206**, 562 (1951).
[12] W. B. Bonnor, Mon. Not. R. Astron. Soc. **117**, 104 (1957).
[13] M. Harwit, Mon. Not. R. Astron. Soc. **122**, 47 (1961); **123**, 257 (1961); I. W. Roxburgh and P. G. Saffman, *ibid.* **129**, 181 (1965).
[14] D. W. Sciama, Mon. Not. R. Astron. Soc. **115**, 3 (1955); Q. J. R. Astron. Soc. **5**, 196 (1964).
[15] T. M. Abdalla and A. M. M. Abdel-Rahman, Phys. Rev. D **46**, 5675 (1992).
[16] J. M. Overdiun, P. S. Wesson, and S. Bowyer, Astrophys. J. **404**, 1 (1993).
[17] P. J. E. Peebles, *The Large-Scale Structure of the Universe* (Princeton University Press, Princeton, 1980).
[18] P. J. E. Peebles, *Principles of Physical Cosmology* (Princeton University Press, Princeton, 1993).
[19] A. R. Liddle and D. H. Lyth, Phys. Rep. **231**, 1 (1993); J. A. Frieman, in *Some Recent Developments in Cosmology: Dark Matter, Inflation, and COBE, Proceedings of the First Symposium on Nuclear Physics in the Universe,* edited by M. W. Guidry and M. R. Strayer (IOP, Bristol, 1993).