

Pair creation of extremal black holes and Kaluza-Klein monopoles

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(Received 30 December 1993)

Classical solutions describing charged dilaton black holes accelerating in a background magnetic field have recently been found. They include the Ernst metric of the Einstein-Maxwell theory as a special case. We study the extremal limit of these solutions in detail, both at the classical and quantum levels. It is shown that near the event horizon the extremal solutions reduce precisely to the static extremal black hole solutions. For a particular value of the dilaton coupling, these extremal black holes are five-dimensional Kaluza-Klein monopoles. The Euclidean sections of these solutions can be interpreted as instantons describing the pair creation of extremal black holes and/or Kaluza-Klein monopoles in a magnetic field. The action of these instantons is calculated and found to agree with the Schwinger result in the weak-field limit. For the Euclidean Ernst solution, the action for the extremal solution differs from that of the previously discussed wormhole instanton by the Bekenstein-Hawking entropy. However, in many cases quantum corrections become large in the vicinity of the black hole, and the precise description of the creation process is unknown.

PACS number(s): 04.70.Dy, 04.50.+h

I. INTRODUCTION

The creation of particle-antiparticle pairs in a background field is a common feature of quantum field theory. Schwinger [1] first studied this process for electrons and positrons in a uniform electric field. This was extended by Affleck and Manton [2] to the case of grand unified theory (GUT) monopole-antimonopole production in a background magnetic field. In general relativity, the analogue of a monopole is a magnetically charged black hole, and the question naturally arises as to whether black holes can be pair produced by a background magnetic field. Even though black holes and monopoles are both "solitons" in the sense of being static extended objects, there are important differences. First, the configuration of two black holes has a different spatial topology than the vacuum. So, unlike the monopole case, one cannot continuously deform one into the other. Pair production of black holes is necessarily a topology-changing process. A second difference is simply that the fundamental quantum theory is known for the case of monopoles (Yang-Mills-Higgs theory), but we still lack a quantum theory of gravity from which we can calculate black hole pair-creation rates from first principles. Nevertheless, the previous calculations were done in the context of an instanton approximation, and it seems reason-

able to hope that a similar approach will work for black holes in a sum-over-histories framework for quantum gravity.

Affleck and Manton used an approximate instanton to estimate the pair-creation rate for GUT monopoles. As Gibbons first realized [3], an exact instanton for the Einstein-Maxwell theory can be obtained by analytically continuing a solution found by Ernst almost 20 years ago [4]. The Ernst solution describes two oppositely charged black holes undergoing uniform acceleration in a background magnetic field.¹ It describes the evolution of the black holes after their creation. Regularity of the Euclidean instanton turns out to restrict the charge-to-mass ratio of the black holes. Gibbons believed that only extremal black holes could be created. But Garfinkle and Strominger [5] found a regular instanton for which the black holes were slightly nonextremal. Furthermore, the horizons of the two black holes were identified to form a wormhole in space.

For static charged black holes, the properties of the extremal solution are quite different from the nonextremal one. In particular, the spatial geometry of the extremal Reissner-Nordström metric resembles an infinite throat connected to an asymptotically flat region. If extremal black holes of this type can be pair created, one has the intriguing possibility that an infinite volume of space could be quantum-mechanically created in a finite time. One of the aims of this paper is to investigate this possibility. We explicitly exhibit an extremal Ernst instanton and study its properties. We will show that as one ap-

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¹Ernst actually considered electric fields, but by duality, these are equivalent to the magnetic fields we will consider here.

proaches the horizon, the solution reduces to the extremal Reissner-Nordström solution with its infinite throat.² Furthermore, we will see that, despite the infinite throats, the action is finite and agrees with Schwinger's result for weak magnetic fields. (This was also true for the nonextremal wormhole [5].) However, higher-order quantum corrections may become large down the throat and modify the geometry significantly.

The situation in low-energy string theory is similar. Extremal magnetically charged black holes have a spatial geometry which is identical to that of the Reissner-Nordström solution [7,8]. An effort was made to calculate the production rate for black holes in string theory [9] by constructing an instanton in a hybrid two- and four-dimensional theory, which was conjectured to approximate the full theory. The resulting configuration described nonextremal black holes with their horizons identified to form a wormhole. Arguments have been made that instantons corresponding to the pair creation of extremal black holes do not exist. We will see that this is incorrect; indeed, in the string case, the natural analogue of the wormhole instanton develops an infinite throat and becomes extremal. As in the Reissner-Nordström case, this reduces to the static solution far down the throat.

A convenient way to treat both the Einstein-Maxwell and low-energy string theories simultaneously is to consider the action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} [R - 2(\nabla\phi)^2 - e^{-2a\phi} F^2]. \quad (1.1)$$

For $a=0$, this is the standard Einstein-Maxwell theory coupled to a massless scalar field ϕ . By the no-hair theorems, ϕ must be constant for solutions describing static black holes. For $a=1$, S is part of the action describing the low-energy dynamics of string theory. In this case, ϕ is not constant outside a charged black hole. For some physical questions it is more appropriate to use the conformally rescaled metric $\tilde{g}_{\mu\nu} = e^{2\phi} g_{\mu\nu}$, which is called the string metric. It is with respect to this metric that the extremal black holes have infinite throats. The value $a=\sqrt{3}$ is also of special interest since this corresponds to standard Kaluza-Klein theory, i.e., (1.1) is equivalent to the five-dimensional vacuum Einstein action for geometries with a spacelike symmetry.

The equations of motion which follow from this action are

$$\begin{aligned} \nabla_\mu (e^{-2a\phi} F^{\mu\nu}) &= 0, \\ \nabla^2 \phi + \frac{a}{2} e^{-2a\phi} F^2 &= 0, \\ R_{\mu\nu} &= 2\nabla_\mu \phi \nabla_\nu \phi + 2e^{-2a\phi} F_{\mu\rho} F_\nu{}^\rho - \frac{1}{2} g_{\mu\nu} e^{-2a\phi} F^2. \end{aligned} \quad (1.2)$$

²This can be compared to an instanton found by Brill [6] which describes the splitting of one throat into many. The present instanton differs in that it includes the asymptotic region and does not require that a throat be present initially.

Recently, a solution to these equations was found (for all values of a) which generalizes the Ernst solution [10]. These dilaton Ernst solutions describe two oppositely charged black holes undergoing uniform acceleration in a background magnetic field. They contain a boostlike symmetry which allows one to analytically continue to obtain Euclidean instantons. Regular instantons describing the pair creation of nonextremal black holes with their horizons identified were constructed in [10] for $0 \leq a < 1$ and shown not to exist for $a \geq 1$. In this paper we will choose the parameters so that the black holes are extremal and study the resulting instantons for all values of a . As anticipated by Gibbons [3], for $a=0$ the instanton is completely regular. For $a=1$, the string metric corresponding to the extremal instanton is also regular, and for $a=\sqrt{3}$, the corresponding five-dimensional metric is again regular.

This paper is organized as follows. We begin in Sec. II, by describing the general features of the Lorentzian dilaton Ernst solution and investigate its extremal limit. In this limit, it is shown that as one approaches the black hole, the solution reduces exactly to the static black hole solution. Thus, for $a=0$ and $a=1$, the classical extremal accelerated black holes have infinite throats, just as in the static case. In fact, we will argue that there is a sense in which the extremal black hole is *not* accelerating despite the presence of the magnetic field. However, the region around the black hole is accelerating.

In Sec. III we study the corresponding Euclidean instanton. We will show that this describes the creation of a pair of extremal black holes for each value of a . The throats are not identified to form a wormhole. For fixed values of the physical charge \hat{q} and magnetic field \hat{B} , we compute the exact action for both the extremal and wormhole instantons. For weak fields, i.e., to leading order in $\hat{q}\hat{B}$, the action reduces to the action found by Schwinger, for all values of a and for both types of instantons. To the next order in $\hat{q}\hat{B}$ we find that, for $a=0$, the action of the wormhole instanton is less than that of the extremal instanton by the Bekenstein-Hawking entropy $A/4$ of the non-extremal black holes. We do not understand the physical significance of this intriguing result, which is reminiscent of [11] in which it was found that the action of the wormhole instanton is less than that of an instanton-describing pair creation of GUT monopoles by the same entropy term. This does suggest the suppression of extremal pair creation relative to that of wormholes, but quantum corrections can contribute at the same order in $\hat{q}\hat{B}$. For $a \neq 0$, we find that the actions of the extremal and wormhole instantons agree even to next order in $\hat{q}\hat{B}$. This is consistent with the $a=0$ result since, in this case, the area of the horizon shrinks to zero size in the extremal limit. (The area of the horizon in the nearly extremal wormhole instanton is nonzero, but higher order.)

Section IV contains a discussion of the effect of quantum fluctuations about the classical solution. This question is of interest in both the Lorentzian and Euclidean contexts. Since Hawking's discovery of black hole evaporation, there has been extensive discussion of quantum fields around static black holes and of observers ac-

celerated through the vacuum. When the black holes themselves are accelerating, these two subjects are combined in a fundamental way. By using results from these two areas, we will argue that the back reaction may become strong down the infinite throats. For the Euclidean instanton, the quantum fluctuations appear to affect the infinite throats as well, and may substantially alter the production rate.

The extremal dilaton Ernst solution is of special interest for $a = \sqrt{3}$, the Kaluza-Klein case. This is studied in detail in Sec. V. It is known that the extremal magnetically charged black hole for $a = \sqrt{3}$ is the Kaluza-Klein monopole [12,13]. So the Lorentzian solution describes two (oppositely charged) monopoles accelerating in a uniform magnetic field. The Euclidean instanton describes pair creation of Kaluza-Klein monopoles. Even though the four-dimensional metric describing an extremal magnetically charged black hole is singular, the corresponding five-dimensional metric turns out to be completely regular. So the five-dimensional instanton is also regular. Unlike the previous cases, the quantum corrections should remain small everywhere, so the action for the instanton should give a good approximation to the pair-creation rate. We conclude in Sec. VI with a discussion of some open problems.

Before proceeding to the accelerated black hole solutions, we briefly review the static dilaton black hole and background magnetic-field solutions to (1.2). The black hole is given by [7,8]

$$ds^2 = -\lambda^2 dt^2 + \lambda^{-2} dr^2 + R^2 (d\theta^2 + \sin^2 \theta d\varphi^2),$$

$$e^{-2a\phi} = \left[1 - \frac{r_-}{r} \right]^{2a^2/(1+a^2)}, \quad A_\varphi = q(1 - \cos\theta),$$
(1.3)

$$\lambda^2 = \left[1 - \frac{r_+}{r} \right] \left[1 - \frac{r_-}{r} \right]^{(1-a^2)/(1+a^2)},$$

$$R^2 = r^2 \left[1 - \frac{r_-}{r} \right]^{2a^2/(1+a^2)}.$$

$$ds^2 = (x-y)^{-2} A^{-2} \Lambda^{2/(1+a^2)} \{ F(x) [G(y) dt^2 - G^{-1}(y) dy^2] + F(y) G^{-1}(x) dx^2 \} + (x-y)^{-2} A^{-2} \Lambda^{-2/(1+a^2)} F(y) G(x) d\varphi^2,$$
(2.1)

$$e^{-2a\phi} = e^{-2a\phi_0} \Lambda^{2a^2/(1+a^2)} \frac{F(y)}{F(x)}, \quad A_\varphi = -\frac{2e^{a\phi_0}}{(1+a^2)B\Lambda} \left[1 + \frac{(1+a^2)}{2} Bqx \right] + k,$$

where the functions $\Lambda \equiv \Lambda(x, y)$, $F(\xi)$, and $G(\xi)$ are given by

$$\Lambda = \left[1 + \frac{(1+a^2)}{2} Bqx \right]^2 + \frac{(1+a^2)B^2}{4A^2(x-y)^2} G(x)F(x),$$

$$F(\xi) = (1+r_- A\xi)^{2a^2/(1+a^2)}, \quad (2.2)$$

$$G(\xi) = (1-\xi^2 - r_+ A\xi^3)(1+r_- A\xi)^{(1-a^2)/(1+a^2)},$$

If $r_+ > r_-$, the surface $r=r_+$ is the event horizon. For $a=0$, the surface $r=r_-$ is the inner Cauchy horizon; however, for $a>0$, this surface is singular. The parameters r_+ and r_- are related to the Arnowitt-Deser-Misner (ADM) mass m and total charge q by

$$m = \frac{r_+}{2} + \left[\frac{1-a^2}{1+a^2} \right] \frac{r_-}{2}, \quad q = \left[\frac{r_+ r_-}{1+a^2} \right]^{1/2}. \quad (1.4)$$

The extremal limit occurs when $r_+ = r_-$. As one approaches this extremal limit, the Hawking temperature goes to zero when $a < 1$, approaches a nonzero constant when $a = 1$, and diverges when $a > 1$.

The solution describing the background magnetic field was found by Gibbons and Maeda [7] and is given by³

$$ds^2 = \Lambda^{2/(1+a^2)} [-dt^2 + dz^2 + d\rho^2] + \Lambda^{-2/(1+a^2)} \rho^2 d\varphi^2,$$

$$e^{-2a\phi} = \Lambda^{2a^2/(1+a^2)}, \quad A_\varphi = \frac{B\rho^2}{2\Lambda},$$
(1.5)

$$\Lambda = 1 + \frac{1+a^2}{4} B^2 \rho^2.$$

It is a generalization of Melvin's magnetic universe [14]. The square of the Maxwell field is $F^2 = 2B^2/\Lambda^4$, which is a maximum on the axis $\rho=0$ and decreases to zero at infinity. The parameter B labels the strength of the magnetic field. For $a > 0$, the dilaton is zero on the axis, but diverges to minus infinity as $\rho \rightarrow \infty$.

II. DILATON ERNST METRICS

A. General properties

The dilaton Ernst solutions to (1.2) constructed in [10] represent two oppositely charged dilaton black holes uniformly accelerating in a background magnetic field. They generalize the Einstein-Maxwell ($a=0$) solutions found by Ernst [4]. They are

and q is related to r_+ and r_- by (1.4). The constant k in the expression for the gauge field is introduced so that the Dirac string of the magnetic field of a monopole is confined to one axis. The constant ϕ_0 in the solution for

³The gauge potential given here differs from that in [10] by a gauge transformation so that A_μ is regular on the axis $\rho=0$.

the dilaton determines the value of the dilaton at infinity. Although one could keep this as a free parameter, we will fix it so that the dilaton vanishes on the axis at infinity in agreement with (1.5). The values of both k and ϕ_0 will be given below.

The solution (2.1) depends on four other parameters r_{\pm} , A, B . Defining m and q via (1.4), we can loosely think of these parameters together with A, B as denoting the mass, charge, and acceleration of the black holes and the strength of the magnetic field which is accelerating them, respectively. We emphasize, however, that this is heuristic since, for example, the mass and acceleration are not in general precisely defined and, further, we will see that q and B only approximate the physical charge \hat{q} and magnetic field \hat{B} in the limit $r_{\pm} A \ll 1$.

It is convenient to introduce the following notation. Define $\xi_1 \equiv -1/(r_- A)$ and let $\xi_2 \leq \xi_3 < \xi_4$ be the three roots of the cubic in G . The functions $F(\xi), G(\xi)$ then take the form

$$F(\xi) = (r_- A)^{2a^2/(1+a^2)} (\xi - \xi_1)^{2a^2/(1+a^2)}, \quad (2.3)$$

$$G(\xi) = -(r_+ A)(r_- A)^{(1-a^2)/(1+a^2)} (\xi - \xi_2)(\xi - \xi_3) \times (\xi - \xi_4)(\xi - \xi_1)^{(1-a^2)/(1+a^2)}.$$

We restrict the range of the parameters r_+ and A so that $r_+ A \leq 2/(3\sqrt{3})$, so that the ξ_i are all real. The limit $r_+ A = 2/(3\sqrt{3})$ corresponds to $\xi_2 = \xi_3$. We also restrict the parameter r_- so that $\xi_1 \leq \xi_2$.

The metric (2.1) has two Killing vectors: $\partial/\partial t$ and $\partial/\partial \varphi$. The surface $y = \xi_1$ is singular for $a > 0$, as can be seen from the square of the field strength. This surface is analogous to the singular surface (the “would-be” inner horizon) of the dilaton black holes. The surface $y = \xi_2$ is the black hole horizon, and the surface $y = \xi_3$ is the acceleration horizon; they are both Killing horizons for $\partial/\partial t$.

The coordinates (x, φ) in (2.1) are angular coordinates. To keep the signature of the metric fixed, the coordinate x is restricted to the range $\xi_3 \leq x \leq \xi_4$ in which $G(x)$ is positive. Because of the conformal factor $(x - y)^{-2}$ in the metric, spatial infinity is reached by fixing t and letting both y and x approach ξ_3 . Letting $y \rightarrow x$ for $x \neq \xi_3$ gives null or timelike infinity [15]. Since $y \rightarrow x$ is infinity, the range of the coordinate y is $-\infty < y < x$ for $a = 0$, and $\xi_1 < y < x$ for $a > 0$.

The norm of the Killing vector $\partial/\partial \varphi$ vanishes at $x = \xi_3$ and $x = \xi_4$, which correspond to the poles of the spheres surrounding the black holes. The axis $x = \xi_3$ points along the symmetry axis toward spatial infinity. The axis $x = \xi_4$ points toward the other black hole. The coordinates we are using only cover one region of spacetime containing one of the black holes. The Dirac string singularities attached to the monopoles will be taken to lie along the axis $x = \xi_4$; this is accomplished by fixing the constant k so that $A_{\varphi}(x = \xi_3) = 0$.

As discussed in [10], to ensure that the metric is free of conical singularities at both poles, $x = \xi_3, \xi_4$, we must impose the condition

$$G'(\xi_3)\Lambda(\xi_4)^{2/(1+a^2)} = -G'(\xi_4)\Lambda(\xi_3)^{2/(1+a^2)}, \quad (2.4)$$

where⁴ $\Lambda(\xi_i) \equiv \Lambda(x = \xi_i)$. It will be convenient to define

$$L \equiv \Lambda^{1/(1+a^2)}(\xi_3). \quad (2.5)$$

When (2.4) is satisfied, the spheres are regular as long as φ has a period

$$\Delta\varphi = \frac{4\pi L^2}{G'(\xi_3)}. \quad (2.6)$$

The condition (2.4) can be readily understood in the limit $r_+ A \ll 1$, which implies $r_- A \ll 1$. In this case one has the expansions

$$\begin{aligned} \xi_2 &= -\frac{1}{r_+ A} + r_+ A + \dots, \\ \xi_3 &= -1 - \frac{r_+ A}{2} + \dots, \\ \xi_4 &= 1 - \frac{r_+ A}{2} + \dots. \end{aligned} \quad (2.7)$$

Substituting the expressions (2.2) and (2.3) into (2.4) and expanding to leading order in $r_+ A$ gives Newton's law:

$$mA \approx qB, \quad (2.8)$$

where we have used (1.4) to replace r_{\pm} with m, q . This is true for all a . More generally, the condition (2.4) reduces the number of free parameters for the solution to three by relating the acceleration to the magnetic field, mass, and charge.

The appearance of Newton's law strongly suggests a regime in which the solution closely approximates a point particle moving in the Melvin background. Indeed, the point particle limit is given by $r_+ A \ll 1$, since this corresponds to a black hole small on the scale set by the magnetic field. In this limit, and taking $|r_+ Ay| \ll 1$, one finds that $G(\xi) \approx 1 - \xi^2, F(\xi) \approx 1$ and the solution (2.1) reduces to the form

$$ds^2 \approx \frac{\Lambda^{2/(1+a^2)}}{A^2(x-y)^2} \left[(1-y^2)dt^2 - \frac{dy^2}{1-y^2} + \frac{dx^2}{1-x^2} \right] + \Lambda^{-2/(1+a^2)} \frac{1-x^2}{(x-y)^2 A^2} d\varphi^2, \quad (2.9)$$

with

$$\Lambda \approx 1 + \frac{(1+a^2)B^2}{4} \frac{1-x^2}{A^2(x-y)^2}. \quad (2.10)$$

The coordinate transformation

$$\rho^2 = \frac{1-x^2}{(x-y)^2 A^2}, \quad \xi^2 = \frac{y^2-1}{(x-y)^2 A^2}, \quad (2.11)$$

⁴It follows from (2.2) that when x is equal to a root of $G(x)$, $\Lambda(x, y)$ is independent of y . So $\Lambda(\xi_i)$ are constants.

simplifies this to

$$ds^2 \approx \Lambda^{2/(1+a^2)}[-\xi^2 dt^2 + d\xi^2 + d\rho^2] + \Lambda^{-2/(1+a^2)} \rho^2 d\varphi^2, \tag{2.12}$$

with Λ given in (1.5). The dilaton and gauge fields are likewise found to be

$$A_\varphi \approx e^{a\phi_0} \frac{B\rho^2}{2\Lambda}, \quad e^{-2a\phi} \approx e^{-2a\phi_0} \Lambda^{2a^2/(1+a^2)}, \tag{2.13}$$

where k has been chosen so that A is regular on the axis $\rho=0$.

Equations (2.12) and (2.13) give the dilaton Melvin solution (1.5), expressed in Rindler coordinates, up to the arbitrary shift of the asymptotic value of the dilaton. The standard form follows using the coordinate transformation $\hat{t} = \xi \sinh t, z = \xi \cosh t$. Thus the coordinate t in the dilaton Ernst solution is the analogue of Rindler time. The subleading terms in (2.12) become important when $y \approx -1/r_+ A$, which, for small $r_+ A$, corresponds to $\rho \approx 0, \xi \approx 1/A$ —the trajectory of the black hole. The asymptotic limit (2.12) is obtained at a distance of order r_+ from the black hole, as expected. These features are illustrated in Fig. 1.

The relation to Melvin is not restricted to the point-particle limit; even away from this limit, (2.1) becomes

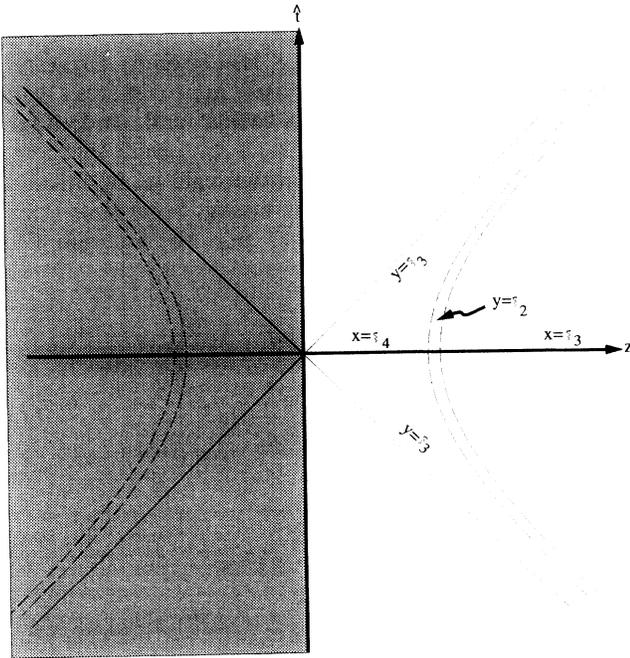


FIG. 1. The z, \hat{t} plane of the dilaton Ernst solution, in the limit $r_+ A \ll 1$. The dotted lines indicate a region of size $\sim r_+$ surrounding the black hole at $y \sim \xi_2$; inside this region the geometry is approximately that of the black hole. The black hole moves on a trajectory with acceleration A , and the acceleration horizon is given by $y = \xi_3$. The coordinates used in (2.1) cover only the unshaded part of the figure.

Melvin at large spacelike distances. This corresponds to $x, y \rightarrow \xi_3$. One way to show that the metric (2.1) approaches (1.5) asymptotically was given in [10]. A somewhat simpler approach is to change coordinates from (x, y, t, φ) to $(\rho, \xi, \eta, \bar{\varphi})$ using

$$\begin{aligned} x - \xi_3 &= \frac{4F(\xi_3)L^2}{G'(\xi_3)A^2} \frac{\rho^2}{(\rho^2 + \xi^2)^2}, \\ \xi_3 - y &= \frac{4F(\xi_3)L^2}{G'(\xi_3)A^2} \frac{\xi^2}{(\rho^2 + \xi^2)^2}, \\ t &= \frac{2\eta}{G'(\xi_3)}, \quad \varphi = \frac{2L^2}{G'(\xi_3)} \bar{\varphi}. \end{aligned} \tag{2.14}$$

Note that $\eta, \bar{\varphi}$ are related to t, φ by a simple rescaling and that $\bar{\varphi}$ has period 2π due to (2.6). For large $\rho^2 + \xi^2$, the dilaton Ernst metric reduces to

$$ds^2 \rightarrow \bar{\Lambda}^{2/(1+a^2)}(-\xi^2 d\eta^2 + d\xi^2 + d\rho^2) + \bar{\Lambda}^{-2/(1+a^2)} \rho^2 d\bar{\varphi}^2, \tag{2.15}$$

where

$$\begin{aligned} \bar{\Lambda} &= \left[1 + \frac{1+a^2}{4} \hat{B}^2 \rho^2 \right], \\ \hat{B}^2 &= \frac{B^2 G'(\xi_3)^2}{4L^{3+a^2}}. \end{aligned} \tag{2.16}$$

Again we recover the dilaton Melvin metric in Rindler coordinates.

The asymptotic form of the dilaton and gauge potential are

$$\begin{aligned} e^{-2a\phi} &\rightarrow L^{2a^2} \bar{\Lambda}^{2a^2/(1+a^2)} e^{-2a\phi_0}, \\ A_\varphi &\rightarrow L^{-a^2} e^{a\phi_0} \frac{\hat{B}\rho^2}{2\bar{\Lambda}}. \end{aligned} \tag{2.17}$$

This is equivalent to the standard background magnetic-field solution (1.5), provided we choose

$$e^{a\phi_0} = L^{a^2}. \tag{2.18}$$

We will take the constant ϕ_0 to be fixed at this value in the remainder of the paper. We can now see that the physical magnetic field is \hat{B} given by (2.16). Using (2.7), we note that in the limit $r_\pm A \ll 1, \hat{B} \approx B$.

The physical charge of the black hole is defined by $\hat{q} = 1/4\pi \int F$, where the integral is over any two-sphere surrounding the black hole. For the dilaton Ernst solution (2.1), one obtains

$$\hat{q} = q \frac{L^{(3+a^2)/2} (\xi_4 - \xi_3)}{G'(\xi_3) [1 + (1+a^2)qB\xi_4/2]}. \tag{2.19}$$

In the weak-field limit $r_\pm A \ll 1, \hat{q} \approx q$. Using (2.16), the product of the physical charge and magnetic field is

$$\hat{q}\hat{B} = \frac{qB(\xi_4 - \xi_3)}{2[1 + (1+a^2)qB\xi_4/2]}. \tag{2.20}$$

This will be useful shortly.

B. The limit $\xi_1 = \xi_2$: accelerating extremal black holes

Since $y = \xi_2$ is the event horizon and $y = \xi_1$ is an inner horizon ($a = 0$) or singularity ($a > 0$), it follows that the extremal limit of the dilaton Ernst solutions is given by choosing the parameter r_- so that $\xi_1 = \xi_2$. Recalling the regularity condition (2.4), it follows that the extremal solutions are described by two parameters which we can take to be the physical charge and magnetic field. In this section we will show that as $y \rightarrow \xi_2$ the extremal solutions become spherically symmetric and approach the static black hole solutions (1.3) with $r_- = r_+$. This surprising result has a number of consequences which we will discuss.

Since the derivation involves considerable algebra, we will simply indicate the main steps involved. The first step is to show that with $\xi_1 = \xi_2$, one can divide the condi-

tion for no nodal singularities (2.4) by $\xi_4 - \xi_3$ to obtain

$$1 + (1 + a^2)Bq\xi_2 + \frac{1}{4}(1 + a^2)^2 \times B^2q^2(\xi_2\xi_3 + \xi_2\xi_4 - \xi_3\xi_4) = 0. \quad (2.21)$$

Taking the limit $y \rightarrow \xi_2$ and using this equation, the function Λ in (2.1) becomes

$$\Lambda \rightarrow \alpha(x - \xi_2), \quad (2.22)$$

where

$$\alpha = (1 + a^2)Bq + \left[\frac{(1 + a^2)Bq}{2} \right]^2 (\xi_3 + \xi_4). \quad (2.23)$$

One can then show that the dilaton Ernst metric (2.1) tends to

$$ds^2 \rightarrow ds_0^2 = -(r_+ r_-) \alpha^{2/(1+a^2)} (\xi_4 - \xi_2)(\xi_3 - \xi_2)(y - \xi_2)^{2/(1+a^2)} dt^2 + \frac{\alpha^{2/(1+a^2)} r_-^{(3a^2-1)/(1+a^2)}}{A^{4/(1+a^2)} r_+} \times \left[\frac{dy^2}{(\xi_4 - \xi_2)(\xi_3 - \xi_2)(y - \xi_2)^{2/(1+a^2)}} + \frac{(y - \xi_2)^{2a^2/(1+a^2)}}{(x - \xi_2)^2} \left[\frac{dx^2}{(x - \xi_3)(\xi_4 - x)} + \frac{4(x - \xi_3)(\xi_4 - x)}{(\xi_4 - \xi_3)^2} d\tilde{\varphi}^2 \right] \right], \quad (2.24)$$

where we have used the coordinate $\tilde{\varphi}$ introduced in (2.14).

At this stage it is not obvious that the $x, \tilde{\varphi}$ part of the metric (2.24) corresponds to the round two-sphere, but the coordinate change

$$x = \frac{1}{2} \left[\xi_3 + \xi_4 - \frac{\xi_4 - \xi_3 + (\xi_4 + \xi_3 - 2\xi_2)\cos\theta}{\cos\theta + (\xi_4 + \xi_3 - 2\xi_2)/(\xi_4 - \xi_3)} \right], \quad (2.25)$$

puts it in the standard form, with polar coordinates $\theta, \tilde{\varphi}$. The final step consists of introducing a new variable

$$\hat{r}_+^2 = \frac{\alpha^{2/(1+a^2)} r_-^{(3a^2-1)/(1+a^2)} (-\xi_2)^{2a^2/(1+a^2)}}{A^{4/(1+a^2)} r_+ (\xi_4 - \xi_2)(\xi_3 - \xi_2)}, \quad (2.26)$$

and new coordinates

$$t' = \sqrt{r_+ r_- (\xi_4 - \xi_2)(\xi_3 - \xi_2)} (-\alpha\xi_2)^{1/(1+a^2)} t \quad (2.27)$$

$$y = \frac{\hat{r}_+}{r} \xi_2.$$

In the limit $y \rightarrow \xi_2$, these coordinates simplify (2.24) to

$$ds_0^2 = - \left[1 - \frac{\hat{r}_+}{r} \right]^{2/(1+a^2)} dt'^2 + \left[1 - \frac{\hat{r}_+}{r} \right]^{-2/(1+a^2)} dr^2 + \hat{r}_+^2 \left[1 - \frac{\hat{r}_+}{r} \right]^{2a^2/(1+a^2)} d\Omega^2. \quad (2.28)$$

The behavior of the other fields can also be worked out in the limit $y \rightarrow \xi_2$. One obtains

$$A_{\tilde{\varphi}} \rightarrow \hat{q}(1 - \cos\theta),$$

$$(2.29)$$

$$e^{-2a\phi} \rightarrow e^{-2a\phi_0} (-\alpha\xi_2)^{2a^2/(1+a^2)} \left[1 - \frac{\hat{r}_+}{r} \right]^{2a^2/(1+a^2)},$$

where

$$\hat{q} = \frac{\hat{r}_+}{\sqrt{1+a^2}} e^{a\phi_0} (-\alpha\xi_2)^{-a^2/(1+a^2)}. \quad (2.30)$$

This agrees with the extremal static dilaton black hole [given by (1.3) with $r_+ = r_-$] in the limit $r \rightarrow r_+$.⁵ Using (2.22) with $x = \xi_3, \xi_4$ (2.3) and (1.4), one can show that \hat{q} agrees with (2.19) when the black hole is extremal $\xi_1 = \xi_2$.

The fact that the extremal dilaton Ernst solution approaches the static black hole as $y \rightarrow \xi_2$ has several consequences. First, all the geometric properties of the extremal static solutions near the horizon carry over immediately to the accelerated case. In particular, for $a = 0$, a constant- t slice of the solution has an infinitely long throat. For $a = 1$, the string metric $d\tilde{s}^2 = e^{2\phi} ds^2$ also has an infinite throat in which the solution takes the form

⁵Equations (2.28) and (2.29) are exactly the extremal static solution except for a constant shift in the dilaton.

of the linear dilaton vacuum. For $a = \sqrt{3}$, the four-metric, dilaton, and gauge field together make up the five-dimensional metric of the Kaluza-Klein monopole (see Sec. V).

A second consequence is that there is a sense in which the extremal black holes are not accelerating. For $a = 0$, this is suggested by the fact that the event horizon is exactly spherical. But a more convincing argument comes from examining the acceleration of a family of observers near the horizon whose four-velocities are proportional to $\partial/\partial t$. For the static black hole, the acceleration of these observers approaches the finite limit $1/q$ as they approach the horizon. (This is related to the fact that the surface gravity vanishes for extremal black holes and is in contrast to the nonextremal case in which the acceleration diverges.) If one computes the acceleration of these observers for the Ernst solution, one again finds that it approaches $1/\hat{q}$ as $y \rightarrow \xi_2$ independent of direction. Although this particular argument cannot be extended to $a > 0$ since the acceleration (in the Einstein metric) now diverges for the static extremal solution near the horizon, other arguments can be made. For example, when $a = 1$ in the string metric the acceleration of these observers tends to zero down the throat. In addition, when $a = \sqrt{3}$, $y = \xi_2$ is a regular origin in the five-dimensional Kaluza-Klein solution, and one can show that its worldline is a geodesic.

Even though the black hole itself is not accelerating, the region around the black hole is. This is clear from the relation between the solution and the dilaton Melvin solution in accelerating coordinates discussed in the previous subsection. In terms of the infinite throats, one might say that the mouth of the throat is accelerating while the region down the throat is not.

A final comment concerns the physical charge and magnetic field. Consider the product $\hat{q}\hat{B}$. This is small in the weak-field limit, which corresponds to ξ_2 being large and negative. What happens when, instead, ξ_2 approaches ξ_3 ? The two roots ξ_2 and ξ_3 can approach each other only if $\xi_2, \xi_3 \rightarrow -\sqrt{3}$, and $\xi_4 \rightarrow \sqrt{3}/2$. Using this Eq. (2.20), and the no-strut condition (2.21), one can show

$$\hat{q}\hat{B} \rightarrow \frac{1}{1+a^2}. \quad (2.31)$$

Thus there is an upper bound on the product of the charge and magnetic field. Roughly speaking, since the size of an extremal black hole is $\sim \hat{q}$ and the width of the Melvin flux tube is $\sim 1/(\hat{B}\sqrt{1+a^2})$, one of the consequences of (2.31) is that the black holes are moving in flux tubes wider than themselves. The limit $\xi_2 \rightarrow \xi_3$ corresponds to the event horizon approaching the acceleration horizon. Since we have assumed the black holes are extremal, $\xi_1 = \xi_2$, this corresponds to a ‘‘triple point’’ where three roots coincide. If one relaxes the condition $\xi_1 = \xi_2$, it appears that the bound on $\hat{q}\hat{B}$ is even lower, which is consistent with the fact that the event horizon is larger than the charge and hits the acceleration horizon at a smaller value of \hat{q} . What happens if one takes a charged black hole and turns up the magnetic field larger than the bound (2.31)? It would appear that this situation is no longer described by the class of solutions (2.1). The ques-

tion of what happens physically is currently under investigation.

III. DILATON ERNST INSTANTONS

Euclideanizing (2.1) by setting $\tau = it$, we find that another condition must be imposed on the parameters in order to obtain a regular solution. Two distinct ways that this may be achieved were discussed in [10] and are reviewed in the first subsection below. These include the wormhole instantons. There is a third way which leads to the extremal instantons and is described in the second subsection. The calculation of the action for the wormhole and extremal instantons is given in the third subsection.

A. Wormhole instantons

In the Lorentzian solutions, the vector $\partial/\partial t$ is timelike only for $\xi_2 < y < \xi_3$. In [10] the restriction $\xi_1 < \xi_2$ was made so that the Einstein metric had a regular horizon for $a > 0$. In this case, one must impose a condition on the parameters in order to eliminate conical singularities in the Euclidean solution at both the black hole ($y = \xi_2$) and acceleration ($y = \xi_3$) horizons with a single choice of the period of τ . This is equivalent to demanding that the Hawking temperature of the black hole horizon equal the Unruh temperature of the acceleration horizon.

In terms of the metric function $G(y)$ appearing in (2.1), the period of τ is taken to be

$$\Delta\tau = \frac{4\pi}{G'(\xi_3)}, \quad (3.1)$$

and the constraint is

$$G'(\xi_2) = -G'(\xi_3), \quad (3.2)$$

yielding

$$\left(\frac{\xi_2 - \xi_1}{\xi_3 - \xi_1} \right)^{(1-a^2)/(1+a^2)} (\xi_4 - \xi_2)(\xi_3 - \xi_2) = (\xi_4 - \xi_3)(\xi_3 - \xi_2). \quad (3.3)$$

With $\xi_1 < \xi_2$ there are two ways to satisfy this condition and correspondingly two types of instantons. The first one exists when $\xi_2 \neq \xi_3$ and only for $0 \leq a < 1$. It has topology $S^2 \times S^2 - \{pt\}$ and is interpreted as describing the creation of two oppositely charged dilaton black holes joined by a wormhole. These ‘‘wormhole’’ instantons generalize the Einstein-Maxwell instanton discussed in [5]. The reason these instantons only exist for $0 \leq a < 1$ can be understood by recalling the thermodynamic behavior of the dilaton black holes as extremality is approached: the Hawking temperature, as defined from the period of τ in the Euclidean section, goes to zero for $0 \leq a < 1$, approaches a constant for $a = 1$, and diverges for $a > 1$. Thus, for small magnetic fields and hence accelerations, we expect to be able to match the resultant Unruh temperature and the black hole temperature by a small perturbation of the black hole away from extremality only for $0 \leq a < 1$.

The second class of instantons we mention only for completeness since their interpretation is obscure. They are defined by $\xi_2 = \xi_3$, which is equivalent to $r_+ A = 2/(3\sqrt{3})$, and have topology $S^2 \times R^2$. They are related to the upper limit on $\hat{q}\hat{B}$ given by (2.31). Note that for these instantons one does not have to impose the condition (2.4) for regularity.

B. Extremal instantons

The wormhole-type instantons discussed above were made regular by the condition that the temperatures of the black hole and acceleration horizons should be equal. Gibbons [3] pointed out (for $a=0$) that there is another way that the temperatures of the black hole and acceleration horizons can be equal: that is, if the black hole is extremal. This might seem strange since extremal dilaton black holes have zero temperature in the sense that the Euclidean time coordinate need not be periodically identified to obtain a regular geometry. But, of course, we *can* periodically identify the Euclidean time and with any period we like (just as for flat space). In particular, we can choose the period forced on us by having to eliminate a conical singularity elsewhere in the spacetime.

For $a=0$, the extremal condition $\xi_1 = \xi_2$ does indeed lead to a smooth instanton. The coordinate y lies between ξ_2 and ξ_3 in the Euclidean section, and we must choose the period of τ to be again given by (3.1) in order that there be no conical singularities at the acceleration horizon $y = \xi_3$. We saw in Sec. II B that the Lorentzian solution near the back hole is just that of an extremal black hole. The same holds for the Euclidean solution. The horizon $y = \xi_2$ is infinitely far away (in every direction, since every direction is now spacelike) and gives no restriction on the period of τ . Thus we have obtained a regular geometry with internal infinities down the throats of the extremal black holes. The length of the $y = \text{constant}$ circles tends to zero as $y \rightarrow \xi_2$, as shown in Fig. 2, but the curvature remains bounded. Each point in Fig. 2 corresponds to a two-sphere, whose area approaches a constant near the event horizon and becomes large near the acceleration horizon. The figure is slightly misleading in the vicinity of the acceleration horizon, since the point corresponding to infinity ($x = y = \xi_3$) must be removed. The topology is $R^2 \times S^2 - \{pt\}$, and the $\tau=0, \Delta\tau/2$ zero-momentum slice is a spatial slice of a Melvin Universe with two infinite tubes attached. The latter is illustrated in Fig. 3.

The extremal case $\xi_1 = \xi_2$ also gives well-defined instan-

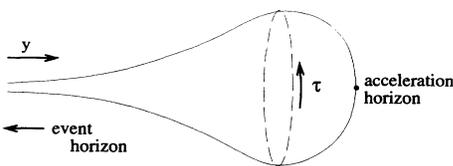


FIG. 2. The (y, τ) section of the extremal Euclidean $a=0$ solution.

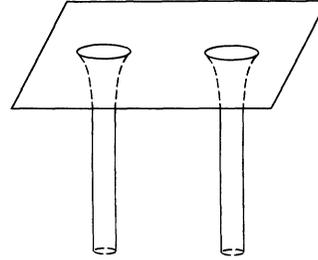


FIG. 3. The spatial slice $\tau=0, \Delta\tau/2$ through the instanton solution of Fig. 2. The geometry corresponds to an asymptotically Melvin region with two extremal throats attached. The solution may be continued to Lorentzian signature along this slice.

tons for $0 < a \leq 1$. Although the Einstein metric has a singularity, the so-called “total” metric [7], $ds_T = e^{2\phi/a} ds^2$, which is the same as the string metric for $a=1$, is perfectly regular. We saw in Sec. II B that the metric close to the singularity is that of the extremal black hole. In the total metric this looks like

$$ds_T^2 \propto -dt'^2 + \left[1 - \frac{\hat{r}_+}{r} \right]^{-4/(1+a^2)} dr^2 + \hat{r}_+^2 \left[1 - \frac{\hat{r}_+}{r} \right]^{2(a^2-1)/(1+a^2)} d\Omega^2. \tag{3.4}$$

For $0 < a < 1$, the total metric is geodesically complete and the spatial sections have the form of two asymptotic regions joined by a wormhole—one region being flat, the other having a deficit solid angle. Hence the corresponding extremal instantons are regular. For $a=1$, the geometry of the string metric is that of an infinitely long throat of constant radius, and thus the $a=1$ extremal instanton looks very much like that of the $a=0$ extremal instanton described above (see Fig. 4): the topology is the same, $R^2 \times S^2 - \{pt\}$, and the major difference is that the proper radius of $y = \text{constant}$ circles in the (y, τ) section tends to a finite limit as $y \rightarrow \xi_2$. The $\tau=0, \Delta\tau/2$ slice resembles the one shown in Fig. 3.

For $a > 1$, both the Einstein metric and the total metric have a naked singularity in the extremal limit. It has been argued in [16], however, that these “black holes” should be interpreted as elementary particles. The extremal instantons can then be interpreted as pair creating such objects. For $a = \sqrt{3}$, the instanton can also be un-

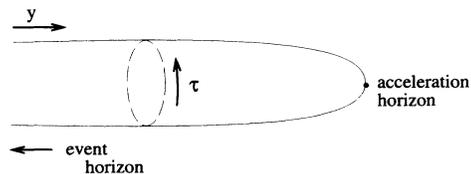


FIG. 4. The (y, τ) section of the Euclidean $a=1$ solution in the string metric.

derstood as describing the creation of a Kaluza-Klein monopole-antimonopole pair (see Sec. V).

Finally, we discuss how these different classes of instantons fit together in parameter space. The parameters of both the extremal and the wormhole-type instantons are restricted by the no-nodal singularities condition (2.4). The wormhole instantons are further restricted by (3.3), and this condition is plotted in Fig. 5 for various values of a . The extremal instantons satisfy $\xi_1 = \xi_2$, and this is also plotted in the figure. Note that in the limit $a \rightarrow 1$ the wormhole instantons approach the extremal instantons. It is also clear from the figure that for $r_{\pm} A \ll 1$ the constraint for both types of instantons is $r_+ = r_-$. The figure also includes the other class of instantons we discussed when $\xi_2 = \xi_3$, or equivalently $r_+ A = 2/3\sqrt{3}$.

C. The action

To leading semiclassical order, the pair production rate of nonextremal or extremal black holes is given by e^{-S_E} where S_E is the Euclidean action of the corresponding in-

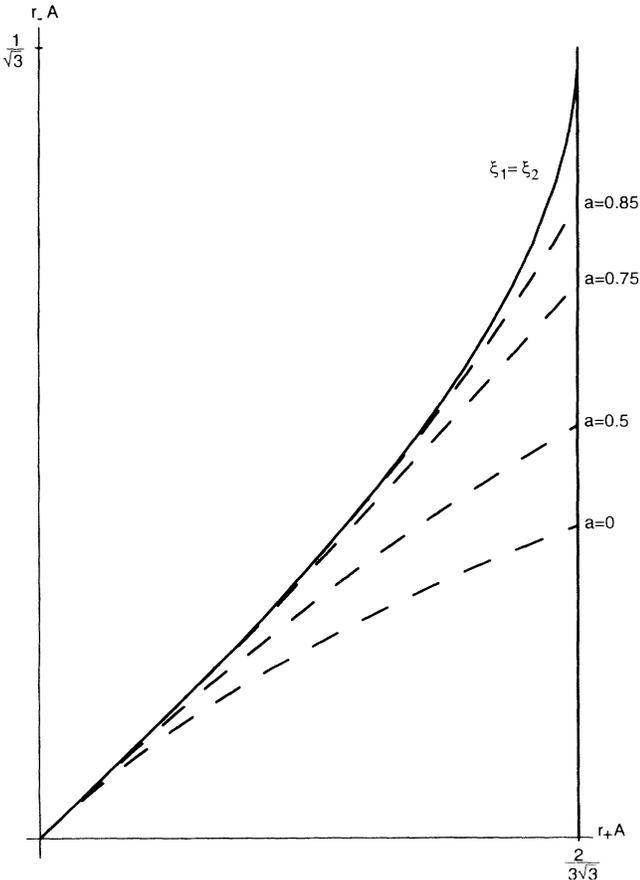


FIG. 5. Plot in parameter space of the wormhole-type instantons for various a (dashed lines), the extremal-type instantons, $\xi_1 = \xi_2$, and the curve $\xi_2 = \xi_3$ ($r_+ A = 2/3\sqrt{3}$) (solid lines).

stanton solutions. The Euclidean action including boundary terms is given by

$$S_E = \frac{1}{16\pi} \int_V d^4x \sqrt{g} [-R + 2(\nabla\phi)^2 + e^{-2a\phi} F^2] - \frac{1}{8\pi} \int_{\partial V} d^3x \sqrt{h} K, \quad (3.5)$$

where h is the induced three-metric and K is the trace of the extrinsic curvature of the boundary. Taking the trace of the metric equation of motion (1.2) yields $R = 2(\nabla\phi)^2$, so the first two terms in the action cancel. The dilaton equation of motion shows that the third term is a total derivative. Thus the action of any solution can be recast as a boundary term

$$S_E = -\frac{1}{8\pi} \int_{\partial V} d^3x \sqrt{h} e^{-(\phi/a)} \nabla_\mu (e^{\phi/a} n^\mu), \quad (3.6)$$

where n^μ is a unit outward-pointing normal to the boundary. Note that for $a=0$ (3.6) is still well defined for our solutions (2.1), since $\lim_{a \rightarrow 0} \phi/a$ is finite.

For both the wormhole and extremal instantons there is a boundary at infinity, $x=y=\xi_3$, which contributes an infinite amount to the action. However, the action of the background magnetic-field solution is itself infinite. In the appendix we show how the infinite background contribution is subtracted to obtain the physical result. For the extremal instantons, there is also an additional boundary down the throats of the black holes, i.e., at $y=\xi_2$. The contribution to the action from this boundary vanishes.

Leaving the details of the calculations to the appendix, we quote the result here. The action is finite for both types of instantons and is given by

$$S_E = 2\pi\hat{q}^2 \frac{\Lambda(\xi_4)(\xi_3 - \xi_2)}{\Lambda(\xi_3)(\xi_4 - \xi_3)}. \quad (3.7)$$

Notice that the result is finite for the extremal instantons despite the infinite throats for $0 \leq a \leq 1$ and despite the fact that there are singularities in both the Einstein and the total metric for $a > 1$. The action can be expressed in terms of the physical charge \hat{q} and magnetic field \hat{B} by expanding out in the parameter $\hat{q}\hat{B}$. The action for the wormhole-type instantons is

$$S_E = \pi\hat{q}^2 \left[\frac{1}{\hat{q}\hat{B}} - \frac{1}{2} + \dots \right], \quad a=0, \quad (3.8)$$

$$S_E = \pi\hat{q}^2 \left[\frac{1}{(1+a^2)\hat{q}\hat{B}} + \frac{1}{2} + \dots \right], \quad 0 < a < 1,$$

while the action for the extremal type instantons for all a is given by

$$S_E = \pi\hat{q}^2 \left[\frac{1}{(1+a^2)\hat{q}\hat{B}} + \frac{1}{2} + \dots \right], \quad (3.9)$$

where dots denote higher order terms which may be fractional powers of $\hat{q}\hat{B}$. To leading order, these all give the Schwinger result, $\pi m^2/\hat{q}\hat{B}$, after using the relation between the mass and charge of extremal black holes,

$$(1+a^2)m^2=\hat{q}^2.$$

To next-to-leading order for $a=0$, the action of the extremal instanton is greater than the action of the wormhole instanton by $\pi\hat{q}^2=\frac{1}{4}A$, where A is the area of the horizon of an extremal black hole of charge \hat{q} . In fact, to this order it could also be the area of the horizon of the wormhole instanton. This difference is precisely the Bekenstein-Hawking entropy. For $0<a<1$, the difference is zero to this order, which is consistent with the difference being the area of the horizon of the extremal instanton, since that vanishes for $a>0$. The area of the horizon in the wormhole instanton is nonzero, but higher order in $\hat{q}\hat{B}$.

In [11] a comparison was made between the wormhole action for $a=0$ and the action of an instanton describing the creation of a monopole-antimonopole pair. It was found that the action of the monopole instanton was greater than that of the wormhole instanton by the black hole entropy. Our result thus suggests that, at least for $a=0$, the extremal black holes behave more like elementary particles than nonextremal ones. However, these conclusions neglect quantum corrections, to which we now turn.

IV. QUANTUM CONSIDERATIONS

Until now, investigation of the solutions has been carried out on the classical level. In this section we will discuss quantum corrections, and see that they have important effects. Let us begin with some qualitative observations. First consider the Lorentzian solutions with general m and q , and note that an observer traveling on a trajectory a fixed distance from the black hole will be accelerated⁶ and therefore would observe acceleration radiation if carrying a detector. This suggests that we should describe the black hole as being in contact with this approximately thermal radiation. If so, then the black hole would be expected to absorb energy, and the solution would then not be static. However, the black hole can also emit Hawking radiation, and therefore achieve a time-independent equilibrium state where the emission and absorption rates match. We have already seen evidence of this in the wormhole-type Euclidean solutions: for a regular solution, the periodic identification required for regularity at the acceleration horizon had to match that required at the black hole horizon. This corresponds to matching the Unruh and Hawking temperatures, and thus putting the black hole in equilibrium. There resulted a condition determining the mass of the black hole in terms of its charge and the magnetic field. We will investigate whether similar statements apply to the extremal case.

A quantitative study could be made by canonically quantizing fluctuations of the fields about the solutions and computing the Bogoliubov coefficients. Such a calcu-

lation has recently been done for the case of charged black holes in de Sitter space [17]. If the quantum state at \mathcal{J}^- is the Melvin analogue of the Minkowski vacuum, then these calculations should yield the above statement that the black hole is bathed in acceleration radiation. This is not in contradiction to our previous observation that the extremal black holes have zero proper acceleration. To see this, consider quanta of fixed frequency sent toward the black hole from infinity at two different times. The surroundings of the black hole at the two different times at which these quanta reach the black hole are related by a boost. Therefore in the instantaneous rest frame of the black hole the quanta will have different frequencies. Thus, in general, there is a time-dependent redshift or blueshift between infinity and the black hole resulting from the acceleration. In describing measurements made by observers near the black hole, one will encounter a corresponding Bogoliubov transformation similar to that for flat space modes in an accelerated frame, along with extra blueshift factors to account for the gravitational field of the black hole. This Bogoliubov transformation will describe the thermal flux of acceleration radiation into the black hole and could, in principle, be used to determine the quantum stress tensor.

In performing these calculations the forms of the effective potentials for fluctuations about the black holes are also crucial. In particular, note that the behavior of fluctuations about the extremal black holes [16,18] depend critically on the value of a . For $a<1$, there are potential barriers outside the black hole, but these vanish at the horizon and thus permit fluctuations there. However, for $a=1$, a barrier develops: all modes have a nonzero mass gap. This is even more pronounced in the case $a>1$, where the potentials grow as the horizon is approached. This suggests that fluctuations are effectively suppressed.

A detailed treatment of these perturbations and of their quantum effects is rather involved and will be left for future work. We will instead make two simplifications of the problem that we believe preserve the essential features. First, rather than considering the general perturbation of the graviton, Maxwell, and dilaton fields we will just consider perturbations of a free spectator field f , with the action

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} (\nabla f)^2. \quad (4.1)$$

We expect the dynamics of this field to be similar to that of a general perturbation.⁷

It should be noted that it is sometimes appropriate to extend the action (1.1) by explicitly adding other fields that do not have effective potential barriers. An example is at $a=1$, where one finds from string theory massless

⁶This is, of course, in addition to the usual acceleration needed to avoid falling into the black hole were it static.

⁷In a theory explicitly including N such fields, rigorous justification for neglecting the gravitational and electromagnetic fluctuations can be given in the large N limit.

modes with couplings of the form [19,18]

$$S = -\frac{1}{2} \int d^4x e^{2\phi} \sqrt{-g} (\nabla f)^2. \quad (4.2)$$

The extra coupling to the dilaton effectively removes the mass gap.

The second simplification is to work only in the s -wave sector of the theory. This assumption can be justified in a controlled approximation [19] for the $a=1$ near-extremal solutions with matter described by (4.2). The reason for this is that in the long throat of the $a=1$ solution, the potential for the nonspherical modes is constant and of order $1/q^2$. This gives an effective mass gap, and if we consider excitations below this energy, we can ignore these higher modes. In the case of the $a < 1$ throats it is less clear that such an approximation is strictly justified, since then the potential is not constant and vanishes down the throat. Nonetheless, we expect that treatment of the s -

wave modes should give us a reasonable picture of the role of quantum effects.

For illustration, we will focus on the cases $a=0$ and $a=1$. In both of these there is a two-dimensional effective action describing the throat region of the black hole. The gravitational part of these actions take the form

$$S_0 = \frac{1}{4} \int d^2x \sqrt{-g} \{ e^{-2D} [R + 2(\nabla D)^2] + 2 - 2q^2 e^{2D} \}, \quad (4.3)$$

$$S_1 = \frac{q^2}{2} \int d^2x \sqrt{-g} \left\{ e^{-2\phi} \left[R + 4(\nabla\phi)^2 + \frac{1}{2q^2} \right] \right\},$$

where, in the former, e^{-D} is the radius of the two-sphere cross section of the throat, and, in the latter, g is the two-dimensional reduction of the total (or string) metric. These have two-dimensional black hole solutions of the form

$$a=0: ds^2 = -\frac{(r_+ - r_-)^2}{4q^2} \sinh^2(x - x_h) dt^2 + q^2 dx^2, \quad e^{-D} = q, \quad (4.4)$$

$$a=1: ds^2 = -\tanh^2(x - x_h) dt^2 + 8q^2 dx^2, \quad e^{2\phi} = \frac{e^{2\hat{\phi}_0}}{\cosh^2(x - x_h)},$$

which correspond to the near-extremal Ernst solutions far down the throat, up to exponentially small corrections from effectively massive modes. In the extremal limits, $x_h \rightarrow -\infty$ and $\hat{\phi}_0 \rightarrow \infty$ (for details, see [18]) and these take the form

$$a=0: ds^2 = -\frac{e^{2x}}{q^2} dt^2 + q^2 dx^2, \quad e^{-D} = q, \quad (4.5)$$

$$a=1: ds^2 = -dt^2 + 8q^2 dx^2, \quad \phi = -x.$$

To the actions (4.3), must be added the reduced matter actions,

$$S_f = -\frac{1}{2} \int d^2x \sqrt{-g} (\nabla f)^2, \quad (4.6)$$

which arise from (4.1) in the case $a=0$ (where the small variations in D are neglected) and (4.2) in the case $a=1$. (Here f has been rescaled by a q -dependent constant.)

By working with the two-dimensional theory, we can compute the expectation value of the quantum stress tensor using the connection with the conformal anomaly [20,19]. Transforming to conformal coordinates,

$$ds^2 = e^{2\rho} (-dt^2 + dy^2) = -e^{2\rho} d\sigma^+ d\sigma^-, \quad (4.7)$$

this takes the form

$$T_{+-}^f = -\frac{1}{12} \partial_+ \partial_- \rho,$$

$$T_{++}^f = -\frac{1}{12} [\partial_+ \rho \partial_+ \rho - \partial_+^2 \rho + t_+ (\sigma^+)], \quad (4.8)$$

$$T_{--}^f = -\frac{1}{12} [\partial_- \rho \partial_- \rho - \partial_-^2 \rho + t_- (\sigma^-)],$$

where t_+ and t_- are to be determined by the boundary conditions. The leading quantum corrections to the solutions can be found by including these on the right-hand side of Einstein's equations. If we are looking for static solutions, we should demand that $t_+ = t_- = t_0$ is a constant.

The boundary conditions of the two-dimensional theory are to be determined by matching correctly onto the four-dimensional theory in the region where the throat matches onto the asymptotic region. One obvious possibility is that the boundary conditions be chosen so that the two-dimensional quantum state is the vacuum annihilated by the positive frequency modes in the time variable t . This implies $t_0 = 0$. However, this is not a physically realizable state. To see this, note that in the context of the full four-dimensional theory, the state is that annihilated by the positive frequency modes defined with respect to the Killing vector. Asymptotically, this Killing vector corresponds to the boost symmetry of (2.15), and thus the state tends to a Rindler-like vacuum at infinity. As seen by an observer at rest with respect to the magnetic field, this vacuum has infinite stress tensor, and thus becomes singular on the acceleration horizon.

A more appropriate state at \mathcal{I}^- is the vacuum as defined by an observer stationary with respect to the asymptotic Melvin solution. From our earlier arguments, this state will not appear to be vacuum for an observer near the horizon. There will be particles arising from the nontrivial Bogoliubov transformation, and a flux of acceleration radiation is expected in the vicinity of the black hole. This corresponds to $t_0 \neq 0$; in general, one would expect t_0 to be proportional to the acceleration of the black hole. To determine the actual value of t_0 re-

quires knowing the details of the matching, and this is difficult. We can, however, see that t_0 will have a major effect on the solution. Consider, for example, $a=1$. In this case the string metric of the classical solution is perfectly regular, and tends to a product of the linear dilaton vacuum and the round two-sphere down the throat. However, the equations of motion from (4.3) and (4.6), with the quantum corrections (4.8), have been investigated both numerically and analytically in [21–23]. There it was found that for general t_0 the static solutions have singular horizons. These result from a nonvanishing stress tensor penetrating into the region where the theory is effectively strongly coupled. The exception to this is when the ingoing flux t_+ matches the outgoing flux due to Hawking radiation. This could happen only at a large definite acceleration with $r_+ A$ of order 1.

The story for $a=0$ is similar. The static equations were investigated in [24]. There it was found that the equations are singular for all t_0 . However, for $t_0=0$, the singularity is mild and quantum-corrected solutions were found. In contrast, at $t_0 \neq 0$, more serious singularities arise. This can be readily confirmed by writing the static equations in coordinates regular at the horizon, similar to the discussion in [21].

The preceding arguments are also expected to generalize to $0 < a < 1$. However, note that they do not apply to $a > 1$, as in this case the growing potentials mean that the action (4.6) is not a good approximation near the horizon—fluctuations are effectively suppressed in this region.

We therefore conclude that for $0 \leq a < 1$, or for $a=1$ with matter given by (4.2), quantum corrections become large and the semiclassical approximation fails near the black hole. The detailed construction of the fully quantum-mechanical solutions is therefore unknown and may depend on new physics. There should certainly exist *some* sensible solutions that closely resemble the classical solutions away from this region—one certainly hopes to be able to give a physical description of the equilibrium state of a charged black hole in a background electromagnetic field. It could be that the physical equilibrium solutions correspond to the Lorentzian version of the subextremal instantons of [5,11,10], or it could be that there are different physical solutions corresponding to the quantum-corrected extremal black holes of this paper. Note that although our arguments have been made with the f fields, we expect this instability to quantum corrections to persist with more general perturbations for $a < 1$. However, for $a=1$ without the fields in (4.2), this argument no longer applies.

Similar considerations apply to the Euclidean solutions. Indeed, the Euclidean solutions should be time-symmetric on the slice of constant Euclidean time along which we cut them to match to the Lorentzian solutions. As before, the role of quantum corrections can be inferred from the one-loop action of the matter field f . As in the Lorentzian case, the stress tensor for minimally coupled s -wave matter can be explicitly computed in conformal gauge, and the result is the analytic continuation of (4.8). Here one again expects t_0 to be nonzero when the four-dimensional and two-dimensional solutions are

matched. This has the unfortunate consequence of yielding large corrections to the equations of motion in the vicinity of the horizon—the back reaction becomes strong and the semiclassical approximation breaks down. This means that, without leaving the semiclassical approximation, the structure of the pair-produced objects cannot be determined near the horizon. It is plausible that once quantum corrections are included for $a < 1$, the corresponding geometry is similar to the black holes connected by Wheeler wormholes of [5,11,10], except in the immediate vicinity of the horizon. Furthermore, the rate cannot be calculated and may depend on new physics and, in particular, on the existence of fundamental charge in the theory.⁸ It is, however, reasonable to expect this production rate to be nonzero. One reason for believing this is that the objects we are considering are clearly in a different topological class from wormholes—the geometries are not connected through the throat—and it seems unlikely that the production rate would be zero in this sector. This belief is reinforced in the case $a=\sqrt{3}$ where, as we will describe, there is no infinite throat, the fluctuations do not make large contributions, and the Euclidean solution describes pair production of Kaluza-Klein monopoles. It is plausible that this type of production extends to $a \leq 1$.

It is also worth commenting on the issue of production rates for Reissner-Nordström black holes. If information is not lost in black hole formation and evaporation and it does not escape in Hawking radiation, this implies that a Reissner-Nordström black hole has an infinite number of states,⁹ and naïve effective-field-theory reasoning would then imply an infinite production rate. A possible resolution to this was suggested in [25], building on ideas in [27,9]: although Reissner-Nordström black holes do have infinite states, not all such states are produced by the Euclidean instantons. Reference [25] argued this for the case where the black holes are connected by a wormhole, although similar reasoning applies here as well. The basic point is that fluctuations of the infinite number of states near the black hole lead to a large quantum stress tensor and therefore a large back reaction. Indeed, when computing the amplitude for any process involving black holes, contributions of these states are summarized in the functional integral over fields in the black hole background; for example, in the case of f states,

$$\int \mathcal{D}f e^{iS[f]}. \quad (4.9)$$

When continued to Euclidean signature, this expression might at first sight be expected to include an overall infinite factor counting these states. However, as dis-

⁸This is in contrast to the case of Wheeler wormholes, where quantum corrections are not necessarily large [25] and fundamental charge is not required.

⁹This has been particularly convincingly argued in [26], using semiclassical techniques.

cussed above, the quantum stress tensor derived from this functional integral becomes large near the horizon, precisely because of these infinite states, and this signals a breakdown of the semiclassical approximation. Although this means that the rate cannot be calculated to find whether it is finite or infinite, it is also an indicator that the naïve effective-field-theory logic is breaking down. The nontrivial dynamical role of this functional integral is in contrast with a rate of the form

$$\Gamma \sim N e^{-S_{\text{instanton}}}, \quad (4.10)$$

that one would expect from an instanton that produced $N \rightarrow \infty$ states with comparable amplitudes. The failure to obtain a naïvely infinite rate of the form (4.10) can be viewed as a strong suggestion that a correct quantum calculation would in fact yield a finite answer, resolving the problem of infinite pair production. Whether such a result can be obtained in a type of effective theory [28] or lies entirely outside the domain of effective theory remains to be seen.

V. THE KALUZA-KLEIN CASE

As we have remarked several times, the value $a = \sqrt{3}$ is of special interest since, in this case, the action S is equivalent to Kaluza-Klein theory. In other words, if $(g_{\mu\nu}, A_\mu, \phi)$ is an extremum of S with $a = \sqrt{3}$, then one can construct a five-dimensional solution of the vacuum Einstein equations by

$$ds^2 = e^{-4\phi/\sqrt{3}}(dx_5 + 2A_\mu dx^\mu)^2 + e^{2\phi/\sqrt{3}}g_{\mu\nu}dx^\mu dx^\nu. \quad (5.1)$$

Since the fields do not depend on the fifth coordinate x_5 , this solution always has at least one translational symmetry. In this section we will explore the five-dimensional vacuum spacetimes associated with the dilaton Ernst solutions (2.1) in both the Lorentzian and Euclidean contexts.

We begin with the static magnetically charged black hole (1.3). Setting $a = \sqrt{3}$ and substituting the fields into (5.1) yields the five-dimensional metric [29]

$$ds^2 = - \left[1 - \frac{r_+}{r} \right] \left[1 - \frac{r_-}{r} \right]^{-1} dt^2 + \left[1 - \frac{r_+}{r} \right]^{-1} dr^2 + \left[1 - \frac{r_-}{r} \right] [dx_5 + 2q(1 - \cos\theta)d\varphi]^2 + r^2 \left[1 - \frac{r_-}{r} \right] d\Omega^2. \quad (5.2)$$

This spacetime has a horizon at $r = r_+$ and a singularity at $r = r_-$. In the extremal limit, $r_+ = r_-$, the metric becomes

$$ds^2 = -dt^2 + \frac{dr^2}{1 - r_+/r} + \left[1 - \frac{r_+}{r} \right] [dx_5 + 2q(1 - \cos\theta)d\varphi]^2 + r^2 \left[1 - \frac{r_+}{r} \right] d\Omega^2. \quad (5.3)$$

The horizon is no longer present. There appears to be a singularity at $r = r_+$, but if we set $\rho = 2r_+^{1/2}(r - r_+)^{1/2}$, then near $r = r_+$ the metric takes the form

$$ds^2 = -dt^2 + d\rho^2 + \frac{\rho^2}{4} \{ [d\psi + (1 - \cos\theta)d\varphi]^2 + d\theta^2 + \sin^2\theta d\varphi^2 \}, \quad (5.4)$$

where we have set $\psi = x_5/2q$ and used the fact (1.4) that $4q^2 = r_+^2$. If ψ is periodic with period 4π , then the quantity in brackets is just the metric on a three-sphere of radius two, expressed in terms of Euler angles. So the solution (5.3) is globally regular and free of singularities provided x_5 has period $8\pi q$. It is the Kaluza-Klein monopole [12,13]. At large r it asymptotically approaches the product of S^1 and four-dimensional Minkowski space. Globally, it is the product of time and the Taub-NUT instanton. Its topology is simply R^5 .

Next, we turn to the background magnetic-field solution (1.5). Setting $a = \sqrt{3}$ and substituting into (5.1) yields

$$ds^2 = -dt^2 + dz^2 + d\rho^2 + \Lambda \left[dx_5 + \frac{B\rho^2 d\varphi}{\Lambda} \right]^2 + \frac{\rho^2 d\varphi^2}{\Lambda}, \quad (5.5)$$

where $\Lambda = 1 + B^2\rho^2$. This metric is actually flat. It can be simplified to

$$ds^2 = -dt^2 + dz^2 + d\rho^2 + dx_5^2 + \rho^2(d\varphi + B dx_5)^2. \quad (5.6)$$

How can a flat five-dimensional space produce nontrivial four-dimensional fields? The point is that one is reducing to four dimensions not along the trivial translation in the fifth direction, but rather along a linear combination of that translation and a rotation [10]. This is why φ is shifted in (5.6). The result is not, in general, globally equivalent to the standard Kaluza-Klein vacuum. For almost all values of B , even though the metric on the two-dimensional torus of constant t, z , and $\rho \neq 0$ in (5.6) is flat, it is globally inequivalent to the metric with $B = 0$. Only if the period of Bx_5 is an integer multiple of 2π are the metrics equivalent. In this case one can start with (globally) the same five-dimensional spacetime, and reduce to obtain either the magnetic field or the trivial four-dimensional solution. However, in general, the five-dimensional flat space (5.6) is identified in a way which is different from the Kaluza-Klein vacuum.

Finally, we turn to the dilaton Ernst solution. The five-dimensional metric is free of the fractional powers present in the four-dimensional solution. It is most conveniently described in terms of functions \bar{F}, \bar{G} , which are

simplified versions of the functions F, G which appeared in the four-dimensional metric:

$$\begin{aligned}\bar{F}(\xi) &= (1 + r_- A \xi), \\ \bar{G}(\xi) &= [1 - \xi^2 - r_+ A \xi^3].\end{aligned}\quad (5.7)$$

Substituting the solution (2.1) with $a = \sqrt{3}$ into (5.1) yields

$$\begin{aligned}ds^2 &= \frac{e^{-4\phi_0/\sqrt{3}} \Lambda \bar{F}(y)}{\bar{F}(x)} (dx_5 + 2A_\phi d\varphi)^2 + \frac{e^{2\phi_0/\sqrt{3}}}{A^2(x-y)^2} \\ &\times \left[\bar{F}(x)^2 \left[\frac{\bar{G}(y) dt^2}{\bar{F}(y)} - \frac{dy^2}{\bar{G}(y)} \right] \right. \\ &\left. + \bar{F}(y) \left[\frac{\bar{F}(x) dx^2}{\bar{G}(x)} + \frac{\bar{G}(x) d\varphi^2}{\Lambda} \right] \right],\end{aligned}\quad (5.8)$$

where Λ and A_ϕ are given by

$$A_\phi = -\frac{e^{\sqrt{3}\phi_0}}{2B\Lambda} (1 + 2Bqx) + k \quad (5.9)$$

$$\Lambda = (1 + 2Bqx)^2 + \frac{B^2 \bar{G}(x) \bar{F}(x)}{A^2(x-y)^2}.$$

Since \bar{G} is just the cubic part of G , its roots are the same as in our earlier discussion, ξ_2, ξ_3, ξ_4 , and $\xi_1 = -1/r_- A$ is the root of \bar{F} . The nonextremal case, $\xi_1 < \xi_2$, has a structure similar to the four-dimensional solution. There is an acceleration horizon at $y = \xi_3$, a black hole horizon at $y = \xi_2$, and a singularity at $y = \xi_1$. The ranges of the coordinates are $\xi_1 < y < x$ and $\xi_3 \leq x \leq \xi_4$. In the extremal limit $\xi_1 = \xi_2$ the situation is different. One can see immediately from (5.8) that in this case g_{tt} approaches a constant as $y \rightarrow \xi_2$. In fact, we showed in Sec. II that the extremal dilaton Ernst solution approaches the extremal static black hole as $y \rightarrow \xi_2$ with a constant shift in the dilaton. If the dilaton was not shifted, we could use the relation between the extreme black hole and the monopole to immediately conclude that the metric (5.8) is nonsingular at $y = \xi_2$, provided we identify x_5 with period $8\pi\hat{q}$ where \hat{q} is the physical charge. It turns out that the constant shift in the dilaton does not affect this conclusion. One way to see this is to rewrite the five-dimensional metric in the form

$$\begin{aligned}ds^2 &= e^{2\phi/\sqrt{3}} [(e^{-\sqrt{3}\phi} dx_5 + 2e^{-\sqrt{3}\phi} A_\mu dx^\mu)^2 \\ &+ g_{\mu\nu} dx^\mu dx^\nu].\end{aligned}\quad (5.10)$$

To satisfy the field equations (1.2), when a constant is added to ϕ , the gauge field must also be rescaled in such a way that $e^{-\sqrt{3}\phi} A_\mu$ is invariant. So, if we start with the metric (5.3) with periodicity $8\pi q$ for x_5 , and add a constant ϕ_0 to ϕ , then regularity requires $e^{-\sqrt{3}\phi_0} x_5$ to have the same period $8\pi q$. Thus x_5 has period $8\pi q e^{\sqrt{3}\phi_0}$. But $q e^{\sqrt{3}\phi_0}$ is just the physical charge after the dilaton has been shifted.

The solution (5.8) can thus be viewed as describing a pair of oppositely charged Kaluza-Klein monopoles ac-

celerating in a background magnetic field. This is not strictly accurate, since the origin of each monopole is not accelerating: one can show that the worldline $y = \xi_2$ is a geodesic. (This is analogous to the fact that the horizon of the extremal Ernst solution is not accelerating, which was discussed in Sec. II B.) However, all points away from the center are accelerating, and the monopole is not spherically symmetric.

We showed in Sec. II A that the dilaton Ernst solution approaches the background magnetic field at large distances. Since the five-dimensional metric associated with this background field is flat, we conclude that (5.8) is asymptotically flat. We have seen that even though the magnetic field solution is flat, it is generally not equivalent to the standard Kaluza-Klein vacuum. It will be globally equivalent only if the period of x_5 is an integer multiple of $2\pi/\hat{B}$. But in (5.8) the period of x_5 is fixed by regularity at the center of the monopole to be $8\pi\hat{q}$. Thus (5.8) approaches the standard Kaluza-Klein vacuum only if $\hat{q}\hat{B} = n/4$ for some integer n . However, as discussed in Sec. II B, there is an upper limit on $\hat{q}\hat{B}$ coming from the fact that $\xi_2 < \xi_3$. Setting $a^2 = 3$ in Eq. (2.31) yields $\hat{q}\hat{B} < \frac{1}{4}$. So the asymptotic region is never equivalent to the standard Kaluza-Klein vacuum, but instead includes an identification involving a rotation as well as a translation.

Even with the nontrivial identifications at infinity, it is interesting that (5.8) is a globally regular, nontrivial, asymptotically flat solution of the five-dimensional vacuum Einstein equations. It is also dynamical in the sense that the Killing vector $\partial/\partial t$ is not asymptotically a time translation and is spacelike in some regions. This is difficult to achieve in four dimensions. In fact, to the best of our knowledge, there is no analogous solution known in that case. However, it is easier to achieve in five dimensions. Another solution of this type was previously found by Witten [30]. By taking the five-dimensional Schwarzschild solution and analytically continuing in both t and θ , he obtained

$$\begin{aligned}ds^2 &= -r^2 dt^2 + \left[1 - \frac{2M}{r^2} \right]^{-1} dr^2 \\ &+ \left[1 - \frac{2M}{r^2} \right] d\chi^2 + r^2 \cosh^2 t d\Omega^2.\end{aligned}\quad (5.11)$$

This solution describes a bubble undergoing uniformly accelerated expansion in spacetime. Like (5.8), it is nonsingular, dynamical, and asymptotically flat. In fact, it has another feature in common with (5.8). Before describing it, let us recall that the positive energy theorem does not hold in Kaluza-Klein theory if surfaces of different topology are allowed: there are regular initial data with negative energy [31]. Witten's bubble (5.11) has zero ADM energy and can be interpreted as a possible outcome for the decay of the Kaluza-Klein vacuum. Our solution (5.8) also has zero ADM energy. This follows from the fact that there is a boost symmetry, and a timelike ADM energy-momentum vector would not be invariant under such a symmetry, and corresponds to the statement that the solution has the same energy as the

corresponding Kaluza-Klein Melvin solution (5.6). One can thus view (5.8) as a possible outcome for the decay of this solution.

The corresponding instanton, obtained by replacing t with $i\tau$ in (5.8), can be viewed as creating a pair of Kaluza-Klein monopoles. The metric is positive definite if the coordinate y is restricted to lie in the range $\xi_2 \leq y \leq \xi_3$. The period of τ is fixed by regularity at the acceleration horizon $y = \xi_3$. There is no restriction on the period coming from regularity at $y = \xi_2$ since the metric approaches (5.3) in this region.

The topology of this instanton is $S^5 - S^1$. To see this, consider slicing the manifold into two pieces along $y = (\xi_3 - \xi_2)/2$, say. The piece that contains $y = \xi_2$ has topology $D^4 \times S^1$, with the S^1 being the Euclidean time. The piece containing $y = \xi_3$ has topology $S^3 \times D^2 - S^1$: the D^2 comes from the y, τ part of the metric, and the subtracted S^1 is $x = y = \xi_3$. The instanton is obtained by gluing these pieces along their common boundary $S^3 \times S^1$ by the obvious diffeomorphism. Using the fact that

$$S^5 = \partial(D^6) = \partial(D^4 \times D^2) = D^4 \times S^1 \cup_{S^3 \times S^1} S^3 \times D^2,$$

we deduce that the topology of the instanton is indeed $S^5 - S^1$. We can also show that the topology of the zero-momentum slice $\tau = 0, \Delta\tau/2$ is given by $S^4 - S^1$. Consider slicing this four manifold along $y = (\xi_3 - \xi_2)/2$ as before. The piece that contains $y = \xi_2$ is simply two copies of D^4 , while the piece containing $y = \xi_3$ has topology $S^3 \times D^1 - S^1$. Gluing these along the common boundary $S^3 \cup S^3$ gives $S^4 - S^1$.

The exact action for this instanton is given by (3.7) with $a^2 = 3$. Expanded in powers of $\hat{q}\hat{B}$, the result is

$$S_E = \pi \hat{q}^2 \left[\frac{1}{4\hat{q}\hat{B}} + \frac{1}{2} + \dots \right]. \quad (5.12)$$

The semiclassical pair-creation rate is thus $\Gamma = e^{-S_E}$. As discussed in the previous section, unlike the situation for $a = 0$ or $a = 1$ extremal instantons, the quantum corrections should remain small and the instanton approximation should be valid. This is because fluctuations near $y = \xi_2$ should be suppressed by the large potential barriers, or equivalently from the regularity of the five-dimensional solution. Indeed, for weak magnetic fields and large charge, the curvature is small everywhere and the quantum corrections will be small.

VI. DISCUSSION

As we have seen, the extremal limit of the dilaton Ernst solutions found in [10] have a number of interesting properties. These include the fact that the Lorentzian solutions near the horizon reduce exactly to the static dilaton black holes. Analytic continuation yields a finite action instanton which describes the pair creation of Kaluza-Klein monopoles when $a = \sqrt{3}$, or extremal black holes for $0 \leq a \leq 1$. These extremal black holes contain infinite throats (in an appropriate metric) and are topologically different from the wormhole originally discussed in [5] and generalized in [10]. We have also considered possible quantum corrections to this leading-order semi-

classical approximation, and found that in certain cases they can become large. These corrections can affect both the geometry down the throat and the physical pair-creation rate.

Many open problems remain. Some have been mentioned earlier, and include a better understanding of the limit on $\hat{q}\hat{B}$ (2.31) and the fact that the difference of the actions for the wormhole and extremal instantons for $a = 0$ is the Bekenstein-Hawking entropy. One of the most important is to develop a better understanding of the quantum corrections to the instanton approximation and their effects on the geometry and pair-creation rate. It is particularly important to understand the calculation of the rate, as a finite answer may indicate that such black holes serve as a model for black hole remnants [19,32,18,27,9,26,25]. It is notable that because of the higher-order quantum effects, one does not immediately recover the naïve estimate of an infinite rate arising from the infinite number of states. A better understanding of these corrections will also help to resolve the question of whether an infinite volume of space can really be created in a finite amount of time. If so, there would appear to be problems with causality, unless the state down the throats were fixed uniquely.

Another interesting problem is to understand the behavior of charged black holes when the background fields are turned on or off in a finite time. Suppose one starts with an extremal black hole and slowly turns on a magnetic field. Will it stay extremal? We have seen that the solution right near the horizon is independent of the magnetic field. But there will certainly be an effect on the solution farther from the black hole, which can propagate toward the horizon. The outcome seems to depend on a . The large potential barriers [16] for $a > 1$, or for $a = 1$ with any matter other than (4.2), indicate that the perturbations never reach the horizon. The black holes stay extremal. In particular, a Kaluza-Klein monopole should not turn into a magnetically charged black hole if a magnetic field is turned on. On the other hand, for $a < 1$, the potential barriers vanish at the horizon. This together with the second law of black hole thermodynamics strongly suggests that any time-dependent perturbation will raise the mass of the black hole away from extremality. (One could perhaps produce an extremal accelerating black hole with $a < 1$ by first accelerating any charged black hole and then adding charged particles with $q > m$.)

This dependence on a is further supported by considerations of black hole thermodynamics. Recall that for the static dilaton black holes, the Hawking temperature vanishes in the extremal limit only for $a < 1$. It reaches a constant for $a = 1$ and diverges for $a > 1$. Thus, if one turns on a weak magnetic field, one could match the Unruh temperature of the acceleration with the Hawking temperature of a slightly nonextremal black hole only if $a < 1$. We have already encountered the Euclidean analogue of this statement in Sec. III. Wormhole-type instantons require the same matching of temperatures and hence only exist for $a < 1$. A particularly puzzling case is $a = 1$ with the special matter (4.2). Since the potential barriers vanish near the horizon, one

might expect the accelerated black hole to become slightly nonextremal, but the temperatures cannot be matched in this case. It is not clear what the equilibrium solution is.

In the quantum theory, other important questions are related to observations by observers at infinity and the issue of energy balance. Suppose as suggested above, that an accelerating black hole is continually emitting Hawking radiation to stay in equilibrium with the acceleration radiation that it absorbs. One would expect this Hawking radiation to be observed at infinity. But the stationary observer does not see the acceleration radiation. Where does she say that the energy is coming from to maintain equilibrium? It may be that, as in the case of an accelerating charge, to describe energy balance requires understanding the details of the switching on and off of the background field [33]. One more intriguing question is whether the black hole continually and indefinitely swallows quantum information in this process, producing an arbitrary amount of entropy in the outgoing state.

We close with one final issue. There is presumably a rotating analogue of the dilaton Ernst solution (2.1). (A rotating analogue of the $a=0$ C metric is already known [34].) It is likely that this could be analytically continued to discuss pair creation of rotating black holes. There is reason to believe that this will provide a wormhole instanton for $a=1$. One piece of evidence comes from [9], where an approximate wormhole instanton was constructed which includes rotation. Another comes from the fact that, for the rotating black hole with $a=1$, the Hawking temperature goes to zero in the extremal limit whenever the angular momentum is nonzero [35]. Thus

one could match the Unruh temperature at the acceleration horizon by a slightly nonextremal rotating black hole.

ACKNOWLEDGMENTS

We would like to thank T. Banks, M. Britton, J. Friedman, D. Garfinkle, R. Geroch, J. Harvey, D. Kastor, M. O'Loughlin, E. Martinec, R. Sorkin, A. Strominger, R. Wald, and S. Weinberger for useful conversations. F.D. and G.T.H. were supported in part by NSF Grant No. PHY-9008502. J.P.G. was supported by a grant from the Mathematical Discipline Center of the Department of Mathematics, University of Chicago. S.B.G. was supported in part by NSF PYI Grant No. PHY-9157463 and by DOE Grant No. DOE-91ER40618.

APPENDIX A: CALCULATING THE ACTION

We present here more details of the calculation of the action of the instantons discussed in Sec. III C. As shown there, the action of any solution can be reduced to a surface term

$$S_E = -\frac{1}{8\pi} \int_{\partial V} d^3x \sqrt{h} e^{-\phi/a} \nabla_\mu (e^{\phi/a} n^\mu). \quad (\text{A1})$$

For both the extremal and wormhole instantons there is a boundary at infinity, $x=y=\xi_3$. We will evaluate the action on the surface $y=x-\epsilon$ and then take the limit $\epsilon \rightarrow 0$. This will enable us to properly subtract off the infinite contribution of the background magnetic-field spacetime.

Performing the trivial integrations over φ and τ yields

$$S_E = -\frac{1}{8\pi} \Delta\varphi \Delta\tau \int_{\xi_3}^{\xi_3+\epsilon} dx \sqrt{h} e^{-\phi/a} \frac{1}{\sqrt{g}} \partial_\mu (e^{\phi/a} \sqrt{g} n^\mu) \Big|_{y=x-\epsilon}, \quad (\text{A2})$$

where $\Delta\varphi$ is the range of φ (2.6), and $\Delta\tau$ is the range of τ which is given by (3.1) for both types of instantons. The individual terms that appear in this integral are given as follows. The unit outward-pointing normal to the surface $y=x-\epsilon$ has components

$$n^y = -\frac{A(x-y)G(y)F(y)^{1/2}}{\Lambda^{1/(1+a^2)} F(x)^{1/2} [F(x)G(x) - F(y)G(y)]^{1/2}}, \quad (\text{A3})$$

$$n^x = -\frac{A(x-y)G(x)F(x)^{1/2}}{\Lambda^{1/(1+a^2)} F(y)^{1/2} [F(x)G(x) - F(y)G(y)]^{1/2}}.$$

The induced three metric h on the surface can be constructed and used to obtain

$$\sqrt{h} = A^{-3} \epsilon^{-3} \Lambda^{1/(1+a^2)} F(x)^{1/2} F(y)^{1/2} \times [F(x)G(x) - F(y)G(y)]^{1/2}. \quad (\text{A4})$$

Expanding in powers of ϵ and integrating, (A2) becomes

$$S_E = \frac{\pi L^2}{A^2 G'(\xi_3)} \left[-\frac{3F(\xi_3)}{\epsilon} + \frac{F'(\xi_3)}{a^2} + O(\epsilon) \right]. \quad (\text{A5})$$

The first term diverges as $\epsilon \rightarrow 0$. In terms of the asymptotic Melvin coordinates ρ, ξ introduced in (2.14), this term is simply $-3\pi(\rho^2 + \xi^2)/4$. It is independent of the black hole charge and is precisely the action of the Euclidean analogue of the dilaton Melvin solution (2.15). Notice that the dependence on \hat{B} cancels. Since we are only interested in the difference between the Euclidean Ernst and Melvin actions, we subtract the leading term in (A5).

For the extremal instantons there is an additional boundary down the throats of the black holes $y=\xi_2$. A similar calculation to that above, but much simpler, shows that this boundary does not contribute to the action, and hence the action of the extremal instantons is also given by (A5). Thus the action for both types of instantons is finite and given by

$$S_E = \frac{\pi L^2 F'(\xi_3)}{a^2 A^2 G'(\xi_3)} = 2\pi\hat{q}^2 \frac{\Lambda(\xi_4)(\xi_3 - \xi_2)}{\Lambda(\xi_3)(\xi_4 - \xi_3)}, \quad (\text{A6})$$

where we have used the expression (2.19) for the physical charge \hat{q} .

For general a , we have not been able to express S_E exactly in terms of the physical charge \hat{q} and physical value of the background magnetic field \hat{B} given by (2.16). Using (2.20), one can, however, establish that in the case of the $a=0$ wormhole instantons, (A6) agrees with the exact result obtained in [11]: namely,

$$S_E = 4\pi\hat{q}^2 \frac{(1 - \hat{q}\hat{B})^2}{1 - (1 - \hat{q}\hat{B})^4}. \quad (\text{A7})$$

A necessary condition for the instanton approximation to be valid is that $\hat{q}\hat{B}$ be small. Thus we would like to expand S_E in $\hat{q}\hat{B}$. We will expand the quantities that appear in (A6) in terms of $\delta = 1/\xi_2$. Then we expand $\hat{q}\hat{B}$ in terms of δ and invert the relation. Finally, we will use these expansions to find the leading-order behavior of S_E and the next-order correction. The calculation will be different in the two cases of the wormhole and extremal instantons since the conditions on the parameters differ.

First we give some relations which hold for both the wormhole and extremal cases. From the definition of the roots of the cubic in the function G we deduce

$$\xi_2\xi_3\xi_4 = \frac{1}{r_+ A},$$

$$\xi_2\xi_3 + \xi_3\xi_4 + \xi_2\xi_4 = 0, \quad (\text{A8})$$

$$\xi_2 + \xi_3 + \xi_4 = -\frac{1}{r_+ A},$$

and hence that

$$\xi_3 = -1 + \frac{1}{2}\delta - \frac{5}{8}\delta^2 + \dots, \quad (\text{A9})$$

$$\xi_4 = 1 + \frac{1}{2}\delta + \frac{5}{8}\delta^2 + \dots.$$

1. Wormhole

We first consider the wormhole instantons for $0 \leq a < 1$ defined by the constraints (2.4) and (3.2). From (3.3) and (A9), we deduce

$$\xi_1 = \frac{1}{\delta} [1 + (-2\delta)^{(1+a^2)/(1-a^2)} + \dots]. \quad (\text{A10})$$

Next using (2.4) and (2.20)

$$\hat{q}\hat{B} = -\delta \left[1 + \frac{3}{2}\delta + \dots \right], \quad a=0, \quad (\text{A11})$$

$$(1+a^2)\hat{q}\hat{B} = -\delta \left[1 + \frac{1}{2}\delta + \dots \right], \quad 0 < a < 1.$$

Expressing δ in terms of $\hat{q}\hat{B}$ and using (A6), we obtain the action for the wormhole-type instantons:

$$S_E = \pi\hat{q}^2 \left[\frac{1}{\hat{q}\hat{B}} - \frac{1}{2} + \dots \right], \quad a=0, \quad (\text{A12})$$

$$S_E = \pi\hat{q}^2 \left[\frac{1}{(1+a^2)\hat{q}\hat{B}} + \frac{1}{2} + \dots \right], \quad 0 < a < 1.$$

2. Extremal

We now turn to the extremal instantons defined by $\xi_1 = \xi_2$ and (2.21). Using (A9) and (2.21), we first deduce that

$$(1+a^2)qB = -\delta + O(\delta^3), \quad (\text{A13})$$

and from (2.20) that

$$(1+a^2)\hat{q}\hat{B} = -\delta(1 + \frac{1}{2}\delta) + O(\delta^3). \quad (\text{A14})$$

Expressing δ in terms of $\hat{q}\hat{B}$ and using (A6), we obtain the action for the extremal type instantons for all a

$$S_E = \pi\hat{q}^2 \left[\frac{1}{(1+a^2)\hat{q}\hat{B}} + \frac{1}{2} + \dots \right]. \quad (\text{A15})$$

The mass of a static extremal black hole is given by (1.4) with $r_+ = r_-$, so $m = \hat{q}/\sqrt{1+a^2}$. Thus we see that the leading-order term in the action in all cases is $S_0 = \pi m^2/\hat{q}\hat{B}$, the Schwinger result. The difference between the extremal and the wormhole actions is $\pi\hat{q}^2$ for $a=0$, and zero for $0 < a < 1$ as $\hat{q}\hat{B} \rightarrow 0$.

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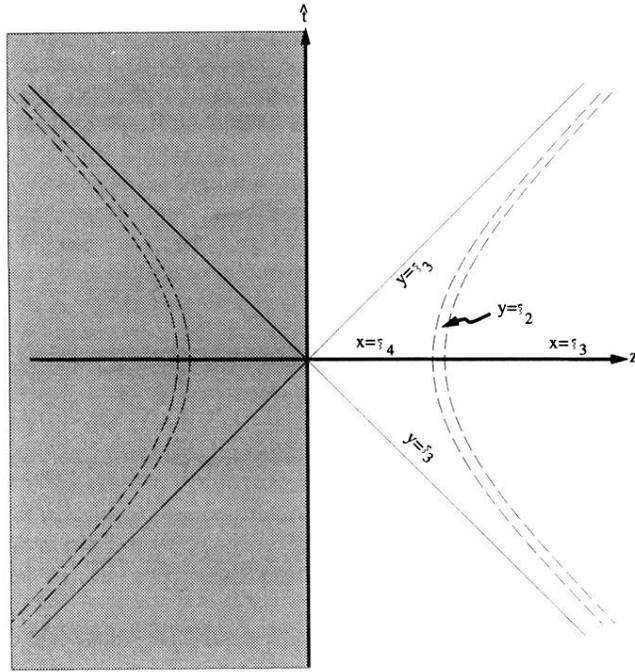


FIG. 1. The z, \hat{z} plane of the dilaton Ernst solution, in the limit $r_+ A \ll 1$. The dotted lines indicate a region of size $\sim r_+$ surrounding the black hole at $y \sim \xi_2$; inside this region the geometry is approximately that of the black hole. The black hole moves on a trajectory with acceleration A , and the acceleration horizon is given by $y = \xi_3$. The coordinates used in (2.1) cover only the unshaded part of the figure.