PHYSICAL REVIEW D VOLUME 50, NUMBER 4 15 AUGUST 1994

Geometric gauge fields, particle production, and time

Dhurjati Prasad Datta
Department of Mathematics, North Eastern Regional Institute of Science & Technology, Nirjuli, Itanagar-791 109, Arunachal Pradesh, India

(Received 29 December 1993)

The geometric magnetic- and electric-type fields are shown to have nontrivial effects in the form of semiclassical back reactions from quantized matter fields on the adiabatically evolving classical background geometry. As a consequence of the gauge invariance of the induced reaction forces it then follows that the matter vacuum polarization in a space-time emerging out from a flat simply connected superspace does not have a gravitational effect. However, the vacuum instability and the associated particle production do have a nonzero back reaction which gets encoded in the electric scalar potential. The relationship between the standard semiclassical definition of time and the existence of a nontrivial Berry phase is also explored. This offers an interesting constraint on the initial quantum state of the Universe.

PACS number(s): 04.60.Kz, 03.65.Bz

The study of the geometric gauge fields induced by a light quantum system on a relatively heavy (quasi) classical system has drawn much attention [1–5] recently. The induced gauge fields have their origin in the nontrivial geometric phase acquired by the light system for a parallel transport of the corresponding quantum state along a curve in the projective Hilbert space. The geometric forces generated by the induced gauge fields were shown [2] to have physically realizable effects on the heavy (quasi)classical system.

Some applications of the geometric magnetic-type field in the minisuperspace cosmology were studied in Refs. [4,5]. For a nondegenerate matter energy state the back reaction in the gravitational sector was shown to be determined by the corresponding U(1) adiabatic Berry connection. This involved a semiclassical reduction of the Wheeler-DeWitt (WD) equation of a fully quantized gravity-matter system. It was also shown how, for a noncyclic evolution, the total particle production rate was determined by the relative phase between the in and out (matter) vacua, i.e., the Pancharatnam phase [5]. The semiclassical Einstein's equations with back reaction were also obtained in this more general case. The induced electric-type field, being negligible under an adiabaticity condition, was, however, not considered in the above discussion. Recently, Berry and Robbins [2] also discussed the effects of such gauge reaction forces in some solvable classical models. The general framework of these studies is the following.

Let us consider an interacting system described by the Hamiltonian (density) of the form

$$H(Q,q)=\frac{1}{2M}G^{ij}P_iP_j+MV(Q)+h(q,Q)\ . \qquad (1)$$

Here G_{ij} is a metric in the configuration space of the quasiclassical system Q, and P denotes the corresponding conjugate momentum, $h(q,Q)$ is the fast Hamiltonian coupling the quantum (field) variables q to the slow quasiclassical system. We assume that the mass scale m of the q system is much less than that of the Q system: $m \ll M$. Thus m/M may be treated as an adiabatic parameter for the coupled system (1). We also use the summation convention.

Now an application of an improved Born-Oppenheimer-type adiabatic approximation shows that the heavy Q variables are governed by the effective Hamiltonian

$$H_{\text{eff}}(Q,P)=\frac{1}{2M}G^{ij}(P_i-A_i)(P_j-A_j)+MV(Q)+E_{\text{BO}}(Q)+\phi(Q)\ . \qquad (2)$$

The back reactions of the q system are encoded here in the form of the magnetic vector potential A_i and the electric scalar potential ϕ, apart from the usual Born-Oppenheimer scalar potential [2] $E_{\text{BO}}=E_n(Q)$, the energy eigenvalue of the normalized eigenstate $|n(Q)\rangle$ of the q system. The gauge invariant form of the potentials are given by

$$\text{magnetic field:}\quad B_{ij}=\partial_i A_j-\partial_j A_i\ , \qquad (3)$$

$$\text{electric field:}\quad E_i=-\partial_i\phi,\ \phi=G^{ij}g_{ij}\ , \qquad (4)$$

where

$$A_i=i\hbar\langle n|\partial_i n\rangle$$

and

$$g_{ij}=\frac{1}{2M}\hbar^2\langle\partial_i n|(1-|n\rangle\langle n|)|\partial_j n\rangle\ . \qquad (5)$$

Thus the gauge fields are completely determined by the geometry of the Q-configuration space and the q-energy state. Furthermore, the quantities B_{ij} and g_{ij} which influence the Q dynamics also have significance in the q-quantum system: B_{ij} is the "magnetic monopole" two-form, whose flux through a Q cycle gives the nonintegrable geometric phase and g_{ij} is a metric governing the separation of the quantum states in the space. Finally, A_i denotes the adiabatic connection one-form. The improved Born-Oppenheimer technique is expected to yield a more accurate description of the Q system [1,2].

Two remarks are in order here.

(i) The Born-Oppenheimer potential is given by the energy eigenvalue $E_n(Q)$. However, in the semiclassical reduction of the WD equation, time is defined as an affine parameter of the integral curves of the vector field normal to the level surfaces of the Hamilton-Jacobi function in the gravitational sector of the superspace. Thus the concept of time is closely related to the emergence of the adiabatic Berry connection A_i (further clarifications will be discussed later). The Born-Oppenheimer potential, in this case, is shown [4] to be determined by the Berry connection itself. Thus, in a gravity-matter system, the back reaction of the quantum matter fields is totally determined by the induced gauge connection (fields).

(ii) Equation (2) is obtained under the adiabatic assumption that there are no transitions between energy states of the q system. However, the electric field contribution actually belongs to a higher order approximation because it arises due to the quantum fluctuations of the q state over an energy eigenstate. This is the reason that we have not considered this effect in Refs. [4,5].

In this paper, however, we show that the induced electric field does have nontrivial effects in semiclassical cosmology, e.g., in the vacuum instability and the related particle production issue. In fact, we obtain the semiclassical back-reaction equation in a more general form, indicating clearly the role played by the various gauge contributions. The present discussion also clarifies the exact forms of both effects due to the vacuum polarization and the particle production due to an instability in the back-reaction formula.

The issue of time in quantum gravity is still being debated enthusiastically in the literature. The recent discussions of Kiefer and Barbour [6] have made it clear that the method of defining time via a semiclassical reduction of the WD equation [3–5] is not fully justified. The semiclassical recovery of time demands using, for the semiclassical gravitational wave function, a single component complex WKB state $\cong\exp(iS/\hbar)$, S real (strictly with a negligible imaginary part), the justification of which is missing in the standard semiclassical analysis. We will, however, show that the proximity of the definition of time and the existence of a nontrivial adiabatic phase provides a *rationale* for the choice of the required complex WKB wave function.

Cosmological particle production. The WD equation in quantum cosmology has exactly the form (1) with the identifications $Q_i \rightarrow h_{jk}$, $P_i \rightarrow -i\hbar\partial/\partial h_{jk}$, and $G_{ij}\rightarrow G_{ijkl}$, where h_{ij} denotes the three-metric in the geomtrodynamic configuration space of compact three-geometries, P_i the associated conjugate momenta, G_{ijkl} the superspace metric, and $V=h^{1/2}(R-2\Lambda)$. Furthermore, $h=\det(h_{ij})$, R the scalar curvature for the three-geometry, Λ denotes a possible positive cosmological constant, and q stands for the matter fields. Here M corresponds to the Planck mass and we use the obvious double-indexed notation. Henceforth, H_m denotes the matter Hamiltonian.

Following the analysis of Refs. [3–5], the effective semiclassical Einstein equations (in fact, the Hamiltonian constraint) [7] with back reactions are obtained in the form

$$\frac{1}{2M}G^{ij}(P_i-\hbar A_i)(P_j-\hbar A_j)+MV+\frac{\langle\chi|H_m|\chi\rangle}{\langle\chi|\chi\rangle} = \frac{\hbar^2}{2M}G^{ij}\frac{\left\langle\chi\left|\left[\frac{\partial}{\partial Q_i}+iA_i\right]\left[\frac{\partial}{\partial Q_j}+iA_j\right]\right|\chi\right\rangle}{\langle\chi|\chi\rangle} . \tag{6}$$

Here the right-hand side (RHS) corresponds to the electric-type scalar potential $\phi(Q)$. It relates to the quantum fluctuations about the given matter state $|\chi\rangle$ and is expected to be small. Further, the Berry connection A is given here by

$$A_i=i\frac{\left\langle\chi\left|\frac{\partial}{\partial Q_i}\right|\chi\right\rangle}{\langle\chi|\chi\rangle} . \tag{7}$$

Note that the matter (vacuum) wave function(al) $|\chi\rangle$ is chosen in such a way that after a complete circuit in the parameter Q space, the final vacuum functional $|\chi_f\rangle$ comes back to the initial one, but with a phase shift,

$$|\chi\rangle_f=e^{i\gamma}|\chi\rangle_i , \tag{8}$$

where γ is the correction to the effective gravitational action [8]:

$$\gamma=\int_i^f\left[A^i\frac{dQ_i}{dt}+\phi(Q)\right]dt . \tag{9}$$

Thus the vacuum energy aside, the effective action determines the total back reaction from the matter states as a sum of the Berry phase and of an integral of the electric scalar potential. The higher order contribution of the electric type arises from the inclusion of the next to the lowest adiabatic approximation allowing for one intermediate transition to another eigenstate and then subsequently returning to the original eigenstate. The zeroth-order Berry phase correction comes from the states with the same quantum numbers.

The parameter time t in Eq. (9) is defined intrinsically by the relation

$$\frac{d}{dt}=G^{ij}\frac{\partial S}{\partial Q_i}\frac{\partial}{\partial Q_j}\Longrightarrow\frac{dQ_i}{dt}=G^{ij}\frac{\partial S}{\partial Q_j} \tag{10}$$

where the real function $S(Q)$ is given by the WKB gravitational state $\cong\exp(-iS/\hbar)$. Finally the matter eigenstate is described by the curved space functional Schrödinger equation

$$i\hbar\frac{d}{dt}|\chi\rangle=-H_m|\chi\rangle . \tag{11}$$

As stated already, we restrict the present discussion, for definiteness, to the vacuum sector of the functional Schrodinger Eq. (11). Although the definition of a vacuum state functional is rather tricky in a curved background [9], however, we assume the existence of a suitable (nondegenerate) vacuum state, which should be compatible with one appearing in the standard description of the inflationary cosmological models based on a finite vacuum energy. The possibility of a degenerate state is considered elsewhere [10]. We note, incidentally, that the basic set of equations in an inflationary model are the semiclassical Einstein equations, a systematic and consistent recovery of which is, however, the main concern in this paper.

Now, as a consequence of Eqs. (10) and (11), one has the relation [4,5]

$$G^{ij}\frac{\partial S}{\partial Q_i}A_j = A_i\frac{dQ^i}{dt} = -\hbar^{-1}\langle\chi|H_m|\chi\rangle/\langle\chi|\chi\rangle$$
$$= -\hbar^{-1}\langle H_m\rangle\ . \qquad (12)$$

We note that S is a solution of the Hamiltonian-Jacobi equation corresponding to the source-free classical equation, i.e., the correspondence limit of Eq. (4) with $A=\phi=0$. It thus follows that the average energy from the matter state which acts as the Born-Oppenheimer potential in the effective Einstein's equations gets completely determined by the Berry connection. We also note that in the presence of a vacuum instability, the vacuum energy gets a small imaginary part: $\langle H_m\rangle = E_0 + i\Gamma$. The real part E_0 denotes the (nonzero) energy due to the vacuum polarization. The imaginary part Γ, on the other hand, gives the total particle production rate. The Berry phase thus has two components: (i) one due to the vacuum polarization and (ii) one due to the particle production, viz.,

$$\gamma_B = \gamma_V + \gamma_P \qquad (13)$$

where the vacuum polarization component γ_V is given by the real time integral

$$\gamma_V = -\hbar^{-1}\int_i^f E_0 dt \qquad (14)$$

and the component from the particle production γ_P is given by the imaginary (Euclidean) time integral [5]

$$\gamma_P = -\hbar^{-1}\int_i^f \Gamma d\tau\ ,\quad \tau = it\ . \qquad (15)$$

We note that although the Berry phase from the particle production effect can be made real by integrating along a path parametrized by the imaginary time, the corresponding Berry connection (7) will in general be complex. However, by a suitable gauge transformation, the Berry connection A_P for the particle production can be made to vanish [cf. Eq. (31) in Ref. [5]] in the required span of the real time evolution of the matter fields in the background of a slowly varying gravitational field. The final form of Einstein's equation with back reactions is thus obtained in the form

$$\frac{1}{2M}G^{ij}(P_i - \hbar A_i^V)(P_j - \hbar A_j^V) + MV + \mathrm{Re}\frac{\langle\chi|H_m|\chi\rangle}{\langle\chi|\chi\rangle}$$
$$= \frac{\hbar^2}{2M}G^{ij}\frac{\langle\chi|(\partial_i + iA_i^V)(\partial_j + iA_j^V)|\chi\rangle}{\langle\chi|\chi\rangle}\ , \qquad (16)$$

where A^V denotes the Berry connection from vacuum polarization. The imaginary part of the energy expectation value is neglected because of its exponential smallness in the adiabatic approximation. We remark, however, that the electric type-potential in the RHS of Eq. (16), although of a higher $O(\hbar^2/2M)$ correction, becomes significant in the region when the gravitational superspace becomes simply connected and flat. Indeed, for a simply connected, flat superspace (where a Lorentzian space-time can be realized) the Berry phase is trivial and hence the Berry connection A^V can be gauged away. It thus follows, in view of Eq. (12), that the vacuum polarization cannot produce a genuine gravitational effect. In other words, *the vacuum polarization in a space-time emerging from a simply connected flat superspace cannot gravitate.* Thus in most of the homogeneous models one does not expect the vacuum polarization to generate a reasonable back reaction (the situation might considerably alter when the matter Fock vacuum is prepared in a flat space-time with a nontrivial boundary surface/event horizon, e.g., the case of a moving mirror). However, the time-varying gravitational background is expected to induce an instability in the matter vacuum. The back reaction of the gravitational field due to the created particles should have a finite contribution in the form of the RHS in Eq. (16). In the presence of a nontrivial vacuum polarization the effect of the instability will however be negligible.

As an application of the above results, let us consider the simplest one-dimensional de Sitter minisuperspace with a scale scale factor a. Since the topology of the minisuperspace is trivial, $A^V=0$. However, the electric-type potential is given by

$$\frac{\hbar^2}{2M}\frac{\left\langle\chi\left|\frac{\partial^2}{\partial a^2}\right|\chi\right\rangle}{\langle\chi|\chi\rangle}$$
$$= -\frac{\hbar}{2M}\frac{d}{da}(\Gamma/S') + \frac{1}{2M}(\Gamma/S')^2,\quad S' = \frac{dS}{da}\ , \qquad (17)$$

where Γ is the total particle production rate in a de Sitter vacuum and S is the de Sitter action. By a choice of a suitable gauge, the vacuum energy is made to vanish [cf. Eq. (12)]. The electric contribution (17), although small, indeed plays a significant role in determining the back reaction from the produced particles. This point was overlooked in Ref. [5]. Note that the ratio Γ/S' is finite although Γ is exponentially small. [The change in action, S', is $O(\Gamma)$.] Thus the back reaction (17) is consistent with the approximations involved in obtaining Eq. (16).

Time. In the standard canonical geometrodynamic formalism, "time" is recovered from the full WD equation at a semiclassical regime where the pure gravitational wave function is assumed to be approximated by a WKB-type state $\cong\exp(-iS_g/\hbar)$. Here S_g denotes the source-free gravitational action. The WKB gravitational state happens to be single component and complex (S_g real). For a superposition of the WKB states of the form $A\exp(iS_g/\hbar) + B\exp(-iS_g/\hbar)$, the definition of time, Eq. (10), breaks down, indicating the special status of the choice of the single component, complex WKB state in recovering the semiclassical time. No justification is, however, offered for this special choice. The problem is all the more serious [6] once we note that the WD equation is *real.* Stated in other words, a dynamical principle needs to be invoked in selecting a very special complex wave function from a real WD equation.

From the discussion in the earlier paragraphs it turns out that the effective WD equation is essentially complex due to the adiabatic gauge coupling of matter to the gravitational modes. Further the definition of time is controlled by the appearance of a nontrivial Berry connection. Indeed, in the case of a trivial gauge coupling, Eq. (12) gives a constraint on the matter eigenstates, viz.,

$$H_m|\chi\rangle=0 \; . \tag{18}$$

This is however unphysical in the sense that the concept of time disappears even in the ordinary matter Schrödinger equation. Finally, in the definition (10) of time the action S must be either real or pure imaginary. A general mixing of WKB states is clearly incompatible with the induced gauge structure. We thus conclude that the occurrence of a nontrivial geometric phase in the semiclassical adiabatic regime singles out a well-defined complex WKB state in the gravitational sector, thus allowing for a consistent recovery of the concept of time.

The above observation has an interesting implication in quantum cosmology. As we remarked earlier, in a simply connected flat superspace, a nontrivial Berry phase can emerge in connection with an instability and the associated particle production. Moreover, this nontrivial phase is realized in terms of a Euclidean time parameter. As a consequence, the initial state of the Universe may only be realized as a "tunneling" state peaked around a classical instanton, e.g., the de Sitter instanton. An accurate description of the initial state, however, asks for a suitable boundary condition proposal which may be implemented through either the no-boundary [11] or the tunneling boundary condition [12] proposal. In either of these boundary condition proposals the dominant contribution in the universe wave function comes from a *real* WKB state [13] of the form $\exp(\pm S_E/\hbar)$, S_E being the action for the de Sitter instanton. The classical evolution of the Universe in the physically allowed Lorentzian sector is governed by the analytically continued effective Einstein equation (16) with back reaction from the particles produced [14] in the (Euclidean) instanton regime.

An interesting constraint on the quantum state of the Universe can, therefore, be obtained from the adiabatic Berry phase. Indeed, the physical time is shown to have its origin in a periodic Euclidean parameter, inducing a thermal background in the initial Universe [14].

We conclude with the following remarks.

(i) When a Yang-Mills gauge group is included in the matter sector, the total Berry phase, Eq. (13), gets a contribution $\gamma_{\rm YM}$ corresponding to an ordinary gauge anomaly [8].

(ii) The present discussion is restricted, for simplicity, to the nondegenerate matter eigenstates. Thus the induced gauge group is Abelian. One might, however, generalize the above results to the case of a non-Abelian (induced) gauge group by considering suitable degenerate matter states [10,15]. Allowing for an induced gauge rotation on the gravitational quantum states one may then envision a possibility of realizing a fermionic state [16] in the cosmological context.

(iii) One expects to obtain similar, if not identical, results in the Ashtekar's canonical formalism [17]. The gauge structure in this case is larger. Apart from the usual diffeomorphism group invariance, the effective gravitational action must be invariant both under SO(3) and an induced geometric gauge group G.

(iv) Although the issue of renormalization is not considered explicitly, the main results obtained here are expected to run through when explicit renormalization is introduced, with obvious modifications due to the presence of higher derivative terms in the renormalized gravitational action [9]. Extra care is, however, needed in handling the electric-type potential in the effective gravitational action. In particular, we note that, the vacuum polarization being ineffective in generating a physically sensible back reaction, the initial inflationary epoch of the Universe may only be driven by the higher derivative terms in the renormalized gravitational action. This observation seems to offer a natural resolution of the cosmological constant problem.

It is a pleasure to thank Professor M. V. Berry and Professor G. Venturi for sending their works to me. I am also thankful to IUCAA, Pune for financial support.

[1] *Geometric Phases in Physics,* edited by A. Shapere and F. Wilczek (World Scientific, Singapore, 1993).

[2] M. V. Berry and J. M. Robbins, Proc. R. Soc. London **A442**, 641 (1993).

[3] G. Venturi, in *Differential Geometric Methods in Theoretical Physics,* edited by L. L. Chau and W. Nahm (Plenum, New York, 1990); R. Balbinot, A. Barletta, and G. Venturi, Phys. Rev. D **41**, 1848 (1990).

[4] D. P. Datta, Mod. Phys. Lett. A **8**, 191 (1993); **8**, 2523(E) (1993); **8**, 601 (1993); "Semiclassical back reaction from Wheeler-DeWitt equation," report (unpublished).

[5] D. P. Datta, Phys. Rev. D **48**, 5746 (1993).

[6] C. Kiefer, Phys. Rev. D **47**, 5414 (1993); J. B. Barbour, *ibid.* **47**, 5422 (1993).

[7] In a minisuperspace of a homogeneous cosmology, the momentum constraints are identically satisfied. In a general case, a similar analysis can be performed [3].

[8] R. Jackiw, Int. J. Mod. Phys. A **3**, 285 (1988); see also Ref. [3].

[9] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1989).

[10] D. P. Datta, "Can vacuum energy gravitate?," Gravity Research Foundation Essay, 1994 (unpublished).

[11] J. B. Hartle and S. Hawking, Phys. Rev. D **28**, 2960 (1983).

[12] A. Vilenkin, Phys. Rev. D **27**, 2848 (1983); **33**, 3560 (1986).

[13] A. Vilenkin, Phys. Rev. D **37**, 888 (1988).

[14] H. E. Kandrup and P. O. Mazur, Mod. Phys. Lett. A **5**, 1471 (1990).

[15] F. Wilczek and A. Zee, Phys. Rev. Lett. **52**, 2111 (1984).

[16] J. L. Friedman and R. D. Sorkin, Phys. Rev. Lett. **44**, 1100 (1980).

[17] A. Ashtekar, *Lectures on non Perturbative canonical Gravity* (World Scientific, Singapore, 1991).