Relationship between two theories of dissipative relativistic hydrodynamics applied to cosmology

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We compare Israel's relativistic causal theory of transient effects with the relativistic extended irreversible thermodynamics of Pavón, Jou, and Casas-Vasquèz, applying both to the case of cosmic fluid with bulk viscosity only. They are completely equivalent up to the first order in the parameter characterizing the departure from equilibrium. We show, if certain relations between the main variables of the two theories hold, that they are also equivalent up to the second order in this parameter. We also extend up to any order Israel's theory and give the relations permitting again the equivalence between the two theories in this general case. Some cosmological consequences are discussed.

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I. INTRODUCTION

The relativistic thermodynamics of dissipative fluids up to now remains divided between several theories [1–6], and the possible equivalence between them has not yet been examined. Our goal in this article is to show that in dissipative cosmology with bulk viscosity only, up to second-order irreversible effects, Israel's theory of transient effects (TE's) and the extended irreversible theory (EIT) of Pavón are equivalent. We use here the simplified denominations "Israel's theory" and "Pavón's theory" for brevity's sake. However, the causal theory of TE's as well as the EIT had their precursors [7–10], contributors, and co-authors, and their peculiarities are well reviewed in different works, e.g., [11,12]. We shall not discuss here the old, so-called "first-order theories" of Eckart [13], Landau [14], and Kluitenberg [15], and their pathological features put into evidence in the works of Hiscock and Lindblom [16,17], although many authors continue to use them, e.g., in recent papers [18,19].

The motivation of our study is a recent article by Hiscock and Salmonson [20] which suggests the idea that the EIT of Pavon *et al.* [2] is nothing more than the "truncated" Israel-Stewart theory, and consequently, when applied to the study of the dissipative Friedmann-Robertson-Walker (FRW) cosmology, gives discussible results that are studied by explicit comparative numerical computations in the case of a Boltzmann gas. The problem raised in this article [20] arises from a neglected term in the entropy production, with important consequences on the evolution of the Universe.

Contrary to many authors [21,22] we shall not discuss here the relevance of the Boltzmann gas model when applied in the context of the primordial cosmology, because it seems to us that the computations of Hiscock and Salmonson can give some relevant indications about the predictions of alternative theories even in this case, as they suggest themselves [20] (p. 3249 and p. 3258).

Our analysis here will focus on the thermodynamical aspect of the relativistic theory of dissipative fluids. We examine and compare with caution the two second-order theories of Israel [1] and of Pavón [2], and we show the following. (1) No term of entropy production is lacking in Pavón's theory; hence, it is not truncated but complete. So, its application to the study of bulk viscosity as the possible origin of the inflation seems relevant, at least as far as Israel's theory application, even in the considered Boltzmann gas case. (2) The two theories are compatible and equivalent up to the second order in the dissipative effects. (3) If we extend Israel's theory, the two theories are also equivalent up to any order.

The difference between the two theories lies simply in the definitions of the main intensive variables, i.e., the pressure and the temperature. Particular care must be taken while defining the pressure, which, as we shall see, seems to be the origin of the problem here. Thus, more generally, we stress again [6] the importance of the choice of the local equilibrium axiom in relativistic thermodynamics.

In Secs. II and III, we recall the main features of Israel's and Pavón's theories. In Sec. IV, we perform the comparison and show that they are compatible; then we show general relations, up to the second order, existing between the two theories. In Sec. V we give an extended version of Israel's theory, and discuss some general consequences on the bulk-viscosity-driven inflation.

II. ISRAEL'S THEORY

A. Balance equations

In Eckart's frame with four-velocity u^α, the fourteen independent primary variables of nonequilibrium, the number of particles flux N^α, and the energy-momentum tensor $T^{\alpha\beta}$, can be split as follows:

$$N^\alpha = n u^\alpha , \quad (2.1)$$

$$T^{\alpha\beta} = n(1+u)u^\alpha u^\beta - (P+\pi)\Delta^{\alpha\beta} - u^\alpha q^\beta - u^\beta q^\alpha + \pi^{\alpha\beta} , \quad (2.2)$$

giving 21 secondary variables [1]: u^α, the particle number density n, the internal specific energy u, the heat flux q^α, the bulk-viscous stress π, and the viscous stress $\pi^{\alpha\beta}$,

$$(N^\alpha, T^{\beta\gamma}) \equiv (n, u^\alpha, u, q^\alpha, \pi, \pi^{\alpha\beta}) , \quad (2.3)$$

constrained by the seven relations

$$u^\alpha u_\alpha = 1 , \quad u^\alpha q_\alpha = 0 , \quad \pi^\alpha_\alpha = 0 , \quad \pi^{\alpha\beta} u_\beta = 0 . \quad (2.3')$$

The local space projector is defined as

$$\Delta^{\alpha\beta} = g^{\alpha\beta} - u^\alpha u^\beta \quad (2.4)$$

(the signature of the metric $g^{\alpha\beta}$ is taken equal to -2). The pressure P appearing in the energy-momentum tensor (2.2) is only a dependent variable in the sense that it is not included among the fourteen independent secondary variables, whereas the bulk-viscous stress π is one of these independent variables [see (2.3)], which stresses the difference of treatment for the two variables P and π. For this reason, the pressure P cannot be *defined* as

$$\left[-\tfrac{1}{3}\Delta^\alpha_\rho T^{\rho\sigma}\Delta_{\sigma\alpha} - \pi \right] ,$$

as some authors often do (e.g., yet recently [23], p. 69). Therefore, one must find an alternative way to define the pressure P. Generally, it is defined thermodynamically from the axiom of local equilibrium, which imposes a new important independent variable to be considered: the entropy, which we shall introduce below. Most of the authors agree with the statement that the isotropy and homogeneity of the cosmological FRW model impose the bulk-viscous stress π in the energy-momentum tensor as the only dissipative phenomenon.[1] Therefore, in such a case, we have simply

$$q^\alpha \equiv 0 \equiv \pi^{\alpha\beta} ,$$

and the tensor (2.2) reduces to

$$T^{\alpha\beta} = n(1+u)u^\alpha u^\beta - (P+\pi)\Delta^{\alpha\beta} . \quad (2.5)$$

The conservation equations are

$$\nabla_\alpha N^\alpha = 0 , \quad (2.6a)$$

$$\nabla_\beta T^{\alpha\beta} = 0 , \quad (2.6b)$$

and lead, in particular, with (2.1) and (2.5), to the following expressions of the expansion rate and of the first principle of thermodynamics, respectively,

$$\nabla_\alpha u^\alpha = -\frac{\dot n}{n} , \quad (2.7)$$

$$n\dot u = (P+\pi)\frac{\dot n}{n} . \quad (2.8)$$

An overdot on a symbol means the derivative with respect to the proper time ω of Eckart's frame, e.g., $\dot n = dn/d\omega$, $d\omega^2 = g_{\alpha\beta}dx^\alpha dx^\beta$.

We suppose the existence of the four-vector entropy flux S^α for the system. The second principle of the thermodynamics is given by the relation

$$\nabla_\alpha S^\alpha \geq 0 . \quad (2.9)$$

In Eckart's frame, the nonequilibrium entropy density is given by

$$ns = S^\alpha u_\alpha . \quad (2.10)$$

The local isotropy of the system leads again to suppose that the entropy flux of conduction does vanish ($\Delta^{\alpha\beta} S_\beta \equiv 0$).

Most of the authors (including Pavón *et al*) agree on this common basis for any relativistic thermodynamics theory. The divergences appear when a thermodynamical local equilibrium axiom, and its associated Gibbs equation, is chosen. The pressure P to be considered in the energy-momentum tensor (2.5) depends on such an axiom.

B. Gibbs equation

Gibbs equation is founded here on the subtle notion of a "fictitious" [1,6], or "comparison's" [5], or "fiducial" [16] local equilibrium state. The specific entropy s_0 of the system in this local equilibrium state is supposed to be only a function of actual nonequilibrium values (n^{-1}, u), just as if it were in thermodynamical equilibrium:

$$s_0 = s_0(n^{-1}, u) . \quad (2.11)$$

Then, the associated Gibbs equation

$$ds_0 = T_0^{-1}du + P_0 T_0^{-1} d(n^{-1}) \quad (2.12)$$

enables us to define the temperature T_0 and the pressure P_0 of fictitious equilibrium by putting

$$T_0^{-1} = \left[\frac{\partial s_0}{\partial u} \right]_{n^{-1}} , \quad P_0 T_0^{-1} = \left[\frac{\partial s_0}{\partial n^{-1}} \right]_u . \quad (2.13)$$

In Israel's theory, the pressure introduced in (2.2) is the same as the one defined thermodynamically by (2.13) from the local equilibrium axiom (2.11)

$$P = P_0 . \quad (2.14)$$

But we note that the specific entropy of fictitious equilibrium s_0 must be *a priori* distinguished from the actual specific entropy s of nonequilibrium. At the second-order approximation considered by Israel, and for the only dissipative phenomenon considered, the bulk-viscous stress

[1] We do not consider here the case of (pure) matter creation [24,18,19]; i.e., we consider only the case of conserved number of particles, but dense enough for taking into account the bulk viscosity.

π, Israel gives the explicit link between the two specific entropies:

$$s = s_0 + \varphi_0 \frac{\pi^2}{2} , \tag{2.15}$$

where

$$\varphi_0 = \varphi_0(n^{-1}, u) \tag{2.15$'$}$$

is a coefficient proportional to the relaxation time of transient effects [1]. Differentiating (2.15) yields the relation

$$ds = ds_0 + \frac{\pi^2}{2} d\varphi_0 + \varphi_0 \pi d\pi . \tag{2.16}$$

C. Entropy source

The four-vector entropy density is, in this case,

$$S^\alpha = nsu^\alpha , \tag{2.17}$$

so that the entropy source (2.9), with (2.17), (2.16), (2.12), and (2.7), can be written as

$$\nabla_\alpha S^\alpha = n\dot{s} = nT_0^{-1}\dot{u} + nP_0 T_0^{-1}(n^{-1})^{\cdot} + n\frac{\pi^2}{2}\dot{\varphi}_0 + n\varphi_0 \pi\dot{\pi} . \tag{2.18}$$

The first principle of thermodynamics (2.8) inserted in (2.18), together with (2.14), yields

$$\nabla_\alpha S^\alpha = \pi T_0^{-1}(n^{-1})\dot{n} + n\frac{\pi^2}{2}\dot{\varphi}_0 + n\varphi_0\pi\dot{\pi} . \tag{2.19}$$

According to Hiscock-Salmonson [20], the second term in the right hand side (rhs) of (2.19) is the term which has been neglected by different authors. Note that this term was repeatedly omitted by Israel [1,25].

III. PAVÓN'S THEORY

There are two (logically linked) main differences with respect to the precedent theory exposed above. Firstly, if we consider dissipative fluxes as new independent variables, it seems that we are necessarily led to a local equilibrium axiom in extended thermodynamics [2] of the form

$$s = s(n^{-1}, u, \pi) . \tag{3.1}$$

Then, the associated Gibbs equation

$$ds = T^{-1}du + PT^{-1}d(n^{-1}) + \left[\frac{\partial s}{\partial \pi}\right]_{n^{-1},u} d\pi \tag{3.2}$$

enables us to define thermodynamically the temperature T and the pressure P of extended local equilibrium by the relations

$$T^{-1} = \left[\frac{\partial s}{\partial u}\right]_{n^{-1},\pi} , \quad PT^{-1} = \left[\frac{\partial s}{\partial n^{-1}}\right]_{u,\pi} , \tag{3.3}$$

and Pavón [2] assumes that, in the second-order approximation,[2]

$$\left[\frac{\partial s}{\partial \pi}\right]_{n^{-1},u} = n^{-1}\alpha T^{-1}\pi , \tag{3.4}$$

where

$$\alpha = \alpha(n^{-1}, u, \pi) . \tag{3.4$'$}$$

Equation (3.2) gives the entropy production

$$\nabla_\alpha S^\alpha = n\dot{s} = nT^{-1}\dot{u} + nPT^{-1}(n^{-1})^{\cdot} + \alpha T^{-1}\pi\dot{\pi} . \tag{3.5}$$

The first principle of thermodynamics (2.8) in (3.5) yields

$$\nabla_\alpha S^\alpha = \pi T^{-1} n^{-1}\dot{n} + \alpha T^{-1}\pi\dot{\pi} . \tag{3.6}$$

The second difference, connected with the previous one, is that now the pressure P of nonequilibrium in (2.2) is no longer the pressure (2.13) of fictitious equilibrium (2.11), but the one defined by (3.3) for extended local equilibrium (3.1). Pavón had already noticed [2] (p. 87) that their variables (T, P) differed from the (T_0, P_0) introduced by Israel, except in the first-order approximation, where they coincide. This result, because of the fact that the functions s and s_0 are different, is true for all thermodynamical variables, except for n^{-1}, u, and u_α, which are, by definition, the same at the fictitious equilibrium and at the nonequilibrium.

We see from Eq. (3.6) that there is no neglected term of the (2.19) type, and therefore Pavón's formula is not a truncated Israel's formula.

IV. COMPARISON UP TO THE SECOND ORDER

A. Two bulk-viscous stresses

At this point the following important question arises: if the two points of view, i.e., Israel's and Pavón's, are complete and valid, why should they be incompatible? If the two theories describe the same intrinsic reality, they should start from the same primary variables: N^α, $T^{\alpha\beta}$, and S^μ. Since Israel and Pavón have chosen P_0 (2.13) and P (3.3), respectively, as pressures in the same energy-momentum tensor (2.5), where all other secondary variables are also identical, then it is necessary to distinguish two bulk-viscous stresses π_i (subscript i for "Israel") and π_p (subscript p for "Pavón"), respectively, but keeping only their sums equal:

$$P_0 + \pi_i = P + \pi_p . \tag{4.1}$$

We rewrite (2.16), with (2.12) and (2.15$'$), as

[2]This condition, in general, is unnecessary. The definition (3.4) of α holds at any order; in this sense it is general. It is only when we want to compare it, as we intend to do, with Israel's theory, that we limit ourselves to the second-order approximation only.

$$ds=\left[T_0^{-1}+\frac{\pi_i^2}{2}\frac{\partial\varphi_0}{\partial u}\right]du+\left[P_0T_0^{-1}+\frac{\pi_i^2}{2}\frac{\partial\varphi_0}{\partial n^{-1}}\right]\times d(n^{-1})+\varphi_0\pi_i d\pi_i \ , \qquad (4.2)$$

and compare (4.2) with (3.2):

$$ds=T^{-1}du+PT^{-1}d(n^{-1})+n^{-1}\alpha T^{-1}\pi_p d\pi_p \ . \qquad (4.3)$$

From now on, the comparison between the two theories makes sense up to the second-order approximation only.

Yet we cannot perform a direct term-to-term identification of (4.2) and (4.3), because the variable π_i is not independent of Pavón's variables (n^{-1},u,π_p). From (4.1) we have

$$d\pi_i=\left[\frac{\partial P}{\partial u}-\frac{\partial P_0}{\partial u}\right]du+\left[\frac{\partial P}{\partial n^{-1}}-\frac{\partial P_0}{\partial n^{-1}}\right]dn^{-1}+\left[1+\frac{\partial P}{\partial\pi_p}\right]d\pi_p \ . \qquad (4.4)$$

We can identify now (4.2) and (4.3), using (4.4), which yields the three relations

$$T^{-1}=T_0^{-1}+\frac{\pi_i^2}{2}\frac{\partial\varphi_0}{\partial u}+\pi_i\left[\frac{\partial P}{\partial u}-\frac{\partial P_0}{\partial u}\right]\varphi_0 \ , \qquad (4.5a)$$

$$PT^{-1}=P_0T_0^{-1}+\frac{\pi_i^2}{2}\frac{\partial\varphi_0}{\partial n^{-1}}+\pi_i\left[\frac{\partial P}{\partial n^{-1}}-\frac{\partial P_0}{\partial n^{-1}}\right]\varphi_0 \ , \qquad (4.5b)$$

$$n^{-1}\alpha T^{-1}\pi_p=\varphi_0\left[1+\frac{\partial P}{\partial\pi_p}\right]\pi_i \ . \qquad (4.5c)$$

So, we end up with four coupled differential equations (4.1) and (4.5) in the four Pavón's variables (T,P,π_p,α) as functions of the four Israel's variables $(T_0,P_0,\pi_i,\varphi_0)$. To solve such a system is not an easy task. Fortunately, the second order in the π_p approximation here considered enables us to simplify the problem: in this case, $(P-P_0)$ is necessarily at least of second order in π_p (or π_i); therefore, so is $(\pi_i-\pi_p)$, through (4.1). The same holds for the derivatives

$$\left[\frac{\partial P}{\partial u}-\frac{\partial P_0}{\partial u}\right],\quad\left[\frac{\partial P}{\partial n^{-1}}-\frac{\partial P_0}{\partial n^{-1}}\right],$$

which do not affect the π_p dependence of $(P-P_0)$, so that the third terms on the rhs of (4.5a) and (4.5b) are at least of third order in π_p (or π_i); therefore, we can neglect them in this approximation. So, we find

$$T^{-1}=T_0^{-1}+\frac{\pi_i^2}{2}\frac{\partial\varphi_0}{\partial u} \ , \qquad (4.6a)$$

$$PT^{-1}=P_0T_0^{-1}+\frac{\pi_i^2}{2}\frac{\partial\varphi_0}{\partial n^{-1}} \ . \qquad (4.6b)$$

Using (4.6a) in (4.6b) yields, to second order in π_i^2,

$$P-P_0=\gamma_0\pi_i^2 \ , \qquad (4.7)$$

with

$$\gamma_0:=\frac{T_0}{2}\left[\frac{\partial\varphi_0}{\partial n^{-1}}-P_0\frac{\partial\varphi_0}{\partial u}\right] . \qquad (4.7')$$

At the second order, (4.7) in (4.1) yields

$$\pi_p=\pi_i-\gamma_0\pi_i^2=\pi_i-\gamma_0\pi_p^2 \ . \qquad (4.8)$$

Then, with (4.8), (4.1), and (4.5a), (4.5c) becomes

$$\alpha T_0^{-1}\left[1+T_0\frac{\partial\varphi_0}{\partial u}\frac{\pi_p^2}{2}\right]=n\varphi_0\left[1+\frac{\partial P}{\partial\pi_p}\right](1+\varphi_0\pi_p) \ . \qquad (4.9)$$

In Pavon's theory, up to the second order, the function α must be expanded around $\pi_p=0$:

$$\alpha=\alpha_0+\alpha_0'\pi_p+\alpha_0''\frac{\pi_p^2}{2}+0(\pi_p^3) \ , \qquad (4.10)$$

where

$$\alpha_0':=\left[\frac{\partial\alpha}{\partial\pi_p}\right]_0,\quad\alpha_0'':=\left[\frac{\partial_\alpha^2}{\partial\pi_p^2}\right]_0 . \qquad (4.10')$$

With (4.10), (4.9) becomes

$$\alpha_0T_0^{-1}\left[1+\frac{\alpha_0'}{\alpha_0}\pi_p+\frac{\pi_p^2}{2}\left[\frac{\alpha_0''}{\alpha_0}+T_0\frac{\partial\varphi_0}{\partial u}\right]\right]=n\varphi_0\left[1+\gamma_0\pi_p+\frac{\partial P}{\partial\pi_p}+\gamma_0\pi_p\frac{\partial P}{\partial\pi_p}\right] . \qquad (4.11)$$

Differentiating the equation (4.7) leads to

$$\frac{\partial P}{\partial\pi_p}=2\gamma_0\pi_p \ . \qquad (4.12)$$

By identification of the π_p variable, and with (4.12), we obtain from (4.11) the three relations

$$\alpha_0T_0^{-1}=n\varphi_0 \ , \qquad (4.13a)$$

$$\frac{\alpha_0'}{\alpha_0}=3\gamma_0 \ , \qquad (4.13b)$$

$$\frac{\alpha_0''}{\alpha_0}+T_0\frac{\partial\varphi_0}{\partial u}=4\gamma_0^2 \ . \qquad (4.13c)$$

Replacing now α_0' and α_0'' in (4.10) by their values from (4.13b) and (4.13c), we can obtain α up to the second order as a function of π_p and of φ_0 and its derivatives:

$$\alpha=nT_0\varphi_0\left[1+3\gamma_0\pi_p+\left[4\gamma_0^2-T_0\frac{\partial\varphi_0}{\partial u}\right]\frac{\pi_p^2}{2}\right]+O(\pi_p^3) \ . \qquad (4.14)$$

Note that the condition $\alpha_0'\neq 0$ is essential for the generality of the result, with the consequence that the expansion (4.14) of α has generally a nonvanishing first-order term in π_p, while the expansions (4.6) of T and P, or (2.15) of s, do not have such a term. This is due to the definition

(3.4) of α, in which π_p is already factorized, the intensive variable considered in (3.4) being, in fact, $n^{-1}\alpha\pi_p$.

Because of differentiation (4.12), which lowers the expansion by one power, (4.13c) is meaningless and (4.14) plays the role up to the first order in π_p only: its last term becomes meaningless. We will run into the same problem again with Eqs. (4.25) and (4.26). In order to have the expansion for α up to the second order in π_p, we have to start from the expansion for s up to the third-order in π_p, which has not been considered in this section.

In this section we showed that the theories of Israel and of Pavón are equivalent up to the second-order approximation. Only the definitions of the intensive variables are different, but the formulas (4.6), (4.8), and (4.14) enable us to determine one set of variables from the other one. These formulas are equivalent to simultaneous changes of scales for T, P, and π_p.

B. Phenomenological laws

Looking now at the entropy sources (2.19) and (3.6), we see that, using (2.8), we can rewrite, respectively,

$$\nabla_\alpha S^\alpha = \pi_i[T_0^{-1}n^{-1}\dot{n}(1-\gamma_0\pi_i)+n\varphi_0\dot{\pi}_i] , \tag{4.15}$$

and

$$\nabla_\alpha S^\alpha = \pi_p[T_0^{-1}n^{-1}\dot{n}+n\varphi_0(1+3\gamma_0\pi_p)\dot{\pi}_p] . \tag{4.16}$$

We notice that the term with $\dot{\varphi}_0$ of Israel's Eq. (2.19) gives, with (2.8), the corrective term $(-\gamma_0\pi_i)$ of $T_0^{-1}n^{-1}\dot{n}$ in (4.15), whereas the equivalent term in Pavón's formalism gives a corrective term $3\gamma_0\pi_p$ to $n\varphi_0\dot{\pi}_p$ in (4.16).

Comparing formally (4.16) and (3.6), we see that if we use in (3.6) the values of fictitious equilibrium T_0^{-1} and $\alpha_0 T_0^{-1}$ instead of the coefficients of the two rhs terms, the term $n\varphi_0 3\gamma_0\pi_p\dot{\pi}_p$ of (4.16) will disappear. Therefore, as Hiscock and Salmonson observed [20], (3.6) would become a "truncated" formula. Some authors [22] indeed use (wrongly) these fictitious values.[3] But some other authors do not use them [26].

The expressions (4.15) and (4.16) lead to the phenomenological laws

$$\pi_i = -\xi_i[(1-\gamma_0\pi_i)n^{-1}\dot{n}+n\varphi_0 T_0\dot{\pi}_i] , \tag{4.17}$$

$$\pi_p = -\xi_p[n^{-1}\dot{n}+n\varphi_0 T_0(1+3\gamma_0\pi_p)\dot{\pi}_p] , \tag{4.18}$$

where the positive coefficients of bulk viscosity are defined by the functions

$$\xi_i = \xi_i(n^{-1},u) , \quad \xi_p = \xi_p(n^{-1},u,\pi_p) . \tag{4.19}$$

C. Coefficients of bulk-viscosities

Inserting (4.18) in the expression (4.8) of π_i, we obtain

$$\pi_i = -\xi_p[n^{-1}\dot{n}(1+\gamma_0\pi_p) + n\varphi_0 T_0(1+4\gamma_0\pi_p+3\gamma_0^2\pi_p^2)\dot{\pi}_p] . \tag{4.20}$$

[3]Because they use a local equilibrium axiom of the (2.11) type.

The partial derivatives of (4.7) in (4.4) give, using also (2.8), an expression for $\dot{\pi}_i$ that we substitute in (4.17), which then becomes

$$\pi_i = -\xi_i\left\{n^{-1}\dot{n}\left[1-\pi_p\left[\gamma_0+\gamma_0^2\pi_p+\varphi_0 T_0 A_0\pi_p\right]\right] + n\varphi_0 T_0(1+2\gamma_0\pi_p)\dot{\pi}_p\right\} , \tag{4.21}$$

with

$$A_0 = \frac{\partial\gamma_0}{\partial n^{-1}} P_0 \frac{\partial\gamma_0}{\partial u} . \tag{4.21$'$}$$

Identifying (4.20) with (4.21), we get the two relations

$$\xi_i\left[1-\pi_p\left[\gamma_0+\gamma_0^2\pi_p+\varphi_0 T_0 A_0\pi_p\right]\right] = \xi_p(1+\gamma_0\pi_p) , \tag{4.22}$$

$$\xi_i(1+2\gamma_0\pi_p) = \xi_p(1+4\gamma_0\pi_p+3\gamma_0^2\pi_p^2) . \tag{4.23}$$

Now ξ_p, also a function (4.19) of π_p, can be expanded up to the second order as follows:

$$\begin{gathered}\xi_p = \xi_{p0}+\xi_0'\pi_p+\xi_0''\frac{\pi_p^2}{2}+0(\pi_p^3) , \\ \xi_0' = \left[\frac{\partial\xi_p}{\partial\pi_p}\right]_0 , \quad \xi_0'' = \left[\frac{\partial^2\xi_p}{\partial\pi_p^2}\right]_0 ,\end{gathered} \tag{4.24}$$

in the rhs of (4.22) and (4.23). Identifying the two polynomials (4.22) and (4.23) for the variable π_p, we obtain

$$\xi_i = \xi_{p0} , \tag{4.25a}$$

$$-2\xi_i\gamma_0 = \xi_0' , \tag{4.25b}$$

and, instead of the third equation (identification of π_p^2 terms), we have two contradictory equations that we shall not write down. This comes from the fact [already seen about Eq. (4.14)] that we did not take into account the π_p^2 term in the second term of the LHS of (4.23), which comes from the derivative $\partial P/\partial\pi_p$, which, in turn, from (4.7), can give a π_p term only. For this reason we can obtain a relation between ξ_i and ξ_p up to the first order in π_p only:

$$\xi_p = \xi_{p0}+\xi_0'\pi_p = \xi_i(1-2\gamma_0\pi_p) . \tag{4.26}$$

D. Relaxation times

We can evaluate the relaxation time of Pavón's theory, which is defined by

$$\tau = \alpha\xi_p , \tag{4.27}$$

and compare it with the relaxation time of Israel's theory

$$\tau_0 = \alpha_0\xi_i , \tag{4.28}$$

where α_0 is given by (4.13a). Using (4.14), (4.26), and (4.28) in (4.27) leads to

$$\tau = \tau_0(1+\gamma_0\pi_p) . \tag{4.29}$$

We see that the two theories use two distinct time scales as a consequence of the scale's changes for the intensive variables T, P, and α (or π) found in (4.6), (4.14) [or (4.8)]. Though our formulas are valid up to the second order only (or first order for ξ and τ), they show clearly that τ *a fortiori* will differ from τ_0 up to any order in π_p.

V. ON THE EXPONENTIAL INFLATION DRIVEN BY THE BULK VISCOSITY

Some authors [18,27] distinguish qualitatively two types of bulk viscosities: a "real" or a "usual classical" one, and an "effective" one or "viscous pressure." However, these authors did not try to give any relation or link between these two types of bulk viscosities, and devoted their attention to the second type only, which, according to them, originates from "matter creation," and is the only possible candidate for the inflation agent. Indeed, we know that the stationary bulk viscosity cannot drive inflation, as has been recognized by Lima *et al.* [22] after Pacher *et al.* [28], from kinetic theory considerations applied to the conditions of the early Universe. Here we make a different distinction, namely, the distinction between π_i and π_p [see (4.1) and (4.8)], which is valid in the case when the matter is conserved, but our approach leads to similar conclusions. The only explicit link we have found in the literature is an interesting relation proposed by Lima *et al.* [[22], Eq. (3.2)], which connects two coefficients of bulk viscosity:

$$\xi=\xi_{qs}\left[1+\tau\frac{\dot{\gamma}}{\gamma}\right], \tag{5.1}$$

where ξ_{qs} is the quasistationary bulk-viscosity coefficient.

Lima *et al.* consider also a conservation law for the "bosonic primeval charge," so that the conditions are in fact comparable to ours. Formula (5.1) has the same form as our formula (4.26), but presents two essential differences: (i) It is not an approximation up to the second order, whereas ours is; (ii) it is considered in the particular case of exponential inflation and for a given state equation, which is not our case. In addition, the ways of obtaining them are quite different. Their Eq. (5.1) is the rewriting of the phenomenological law only, which we can have from (3.6); namely,

$$\pi_p=\xi_p(n^{-1}\dot{n}+\alpha\dot{\pi}_p) . \tag{5.2}$$

Moreover, our relation (4.26) comes from the comparison of the two bulk-viscous stresses (4.17) and (4.18) [which is (5.2) with (4.14)] of the two theories. So, we cannot directly compare (5.1) and (4.26).

A. Extended Israel's theory

Now we want to apply our results to the problem of bulk-viscosity-driven inflation. So, great magnitudes of the bulk-viscous stress are needed. Hence we must consider an extension of Israel's theory up to any order in π_i. In this case, instead of the function (2.15), we can set, more generally,

$$s(n^{-1},u,\pi_i)=s_0(n^{-1},u)+\phi(n^{-1},u,\pi_i) , \tag{5.3}$$

with the conditions

$$\phi_0=\phi(n^{-1},u,0)\equiv 0 , \quad \phi_0'=\left[\frac{\partial\phi}{\partial\pi_i}\right]_0\equiv 0 ,$$

$$\phi_0''=\left[\frac{\partial^2\phi}{\partial\pi_i^2}\right]_0\equiv\varphi_0 , \tag{5.4}$$

recovering (2.15) while expanding the ϕ function.

Instead of (4.5), by comparison with Eq. (4.3), we then obtain

$$T^{-1}=T_0^{-1}+\left[\frac{\partial P}{\partial u}-\frac{\partial P_0}{\partial u}\right]\frac{\partial\phi}{\partial\pi_i}+\frac{\partial\phi}{\partial u} , \tag{5.5a}$$

$$PT^{-1}=P_0T_0^{-1}+\left[\frac{\partial P}{\partial n^{-1}}-\frac{\partial P_0}{\partial n^{-1}}\right]\frac{\partial\phi}{\partial\pi_i}+\frac{\partial\phi}{\partial n^{-1}} , \tag{5.5b}$$

$$n^{-1}\alpha T^{-1}\pi_p=\frac{\partial\phi}{\partial\pi_i}\left[\frac{\partial P}{\partial\pi_p}+1\right] . \tag{5.5c}$$

The system of equations (5.5) has to be completed by Eq. (4.1). We observe that we can no longer use the simplifying hypothesis leading to (4.6).

The energy conservation (2.8) can be rewritten as [[22], p. 2756 Eq. (2.6)].

$$\dot{\rho}+(\rho+P+\pi_p)\theta=0 , \quad (\theta=\nabla_\alpha u^\alpha\neq 0) , \tag{5.6}$$

where

$$\rho=n(1+u)=\rho(n^{-1},u) . \tag{5.7}$$

B. Exponential inflation

Let us consider the case of exponential inflation. Then, by definition, we have the Hubble parameter $H:=\dot{R}/R$ (where R is the scale factor of FRW metric) such as

$$H=C^{te}=H_0 , \tag{5.8}$$

so that the first Einstein equation [see, e.g., [22], p. 2757, Eq. (2.15)] for the FRW cosmological model yields

$$\dot{\rho}=0 . \tag{5.9}$$

(5.9) in (5.6) leads to [with (4.1)]

$$P+\pi_p=-\rho=P_0+\pi_i . \tag{5.10}$$

Accordingly, it follows from (5.10) and (5.7) that

$$\pi_i\equiv\text{const} \tag{5.11}$$

and

$$\frac{\partial P}{\partial\pi_p}+1\equiv 0 \tag{5.12}$$

are satisfied, because n^{-1},u and, π_i (or π_p) are independent variables.

Relation (5.11) leads to the cancellation of the transient effects in Israel's theory:

$$\dot{\pi}_i\equiv 0 , \tag{5.13}$$

and the relation (5.12) in (5.5c) leads to the same result in Pavón's theory:

$$\alpha=0 . \quad (5.14)$$

We can compare now our results to those of Lima *et al.* [22]. The result (5.13) leads to $\dot{\gamma}=0$ in Eq. (5.1), so that we find again a bulk viscosity which cannot drive exponential inflation ([22], note 32).

Equation (5.14) in (5.2) and in (3.4) gives also

$$\pi_p=\xi_p\frac{\dot{n}}{n} , \quad (5.15)$$

and s independent of π_p, or, by using (5.3), ϕ independent of π_i. Then (5.4) implies that

$$\phi\equiv 0 . \quad (5.16)$$

The two theories become equivalent and reduce themselves to Eckart's theory.

VI. CONCLUSION

The comparison of two theories of relativistic irreversible processes limited to bulk-viscosity considerations only led us to a necessary distinction between two bulk-viscous stresses connected via the relation (4.1). We emphasize explicitly the distinction between these bulk-viscous stresses, which seems to confirm the qualitative distinction made by other authors [18,27]. The difference between the two theories originates essentially from the consideration of two distinct thermodynamical local equilibria, and we give the relations (4.5) or (5.5) up to the second order or at any order in π, respectively, between the intensive variables, namely, pressure, temperature, and coefficient of transient effects of the two theories.

When these results are applied to the study of a possible bulk-viscosity-driven exponential inflation in early cosmology, the field equations for the FRW metric (with $k=0$) inexorably lead to the disappearance of the transient effects. Such a drastic consequence is involved by the peculiarly simple form of the field equations, in the case of the exponential inflation, which permits us to separate the variables (n^{-1},u) and π_i [see (5.10)].

Of course, if we want to take into account the full impact of dissipative effects, we have now to consider the specific role of matter creation [24]. This more general scenario will be considered in a forthcoming paper.

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