

## Topological defects as seeds for eternal inflation

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We investigate the global structure of an inflationary universe both by analytical methods and by computer simulations of stochastic processes in the early Universe. We show that the global structure of an inflationary universe depends crucially on the mechanism of inflation. In the simplest models of chaotic inflation with the effective potentials  $\phi^n$  or  $e^{\alpha\phi}$  the Universe looks like a sea of a thermalized phase, surrounding permanently self-reproducing inflationary domains. On the other hand, in the theories where inflation may occur near a local extremum of the effective potential corresponding to a metastable state, the Universe looks like de Sitter space surrounding islands of a thermalized phase. A similar picture appears even if the state  $\phi = 0$  is unstable but the effective potential has a discrete symmetry, e.g., the symmetry  $\phi \rightarrow -\phi$ . In this case the Universe becomes divided into domains containing different phases ( $\eta$  or  $-\eta$ ). These domains will be separated from each other by domain walls. However, unlike ordinary domain walls often discussed in the literature, these domain walls will inflate, and their thickness will exponentially grow. In the theories with continuous symmetries, inflation will generate exponentially expanding strings and monopoles surrounded by a thermalized phase. Inflating topological defects will be stable, and they will unceasingly produce new inflating topological defects. This means that topological defects may play a role of indestructible seeds for eternal inflation.

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### I. INTRODUCTION

Inflationary cosmology is gradually changing our point of view on the global structure of the Universe [1]. One of the most radical changes occurred when it was realized that in many versions of inflationary theory the process of inflation never ends. Originally this statement was shown to be correct for the old inflationary scenario [2] and for the new inflationary scenario [3,4]. The main idea is that the field near the top of the effective potential does not move. Therefore if the Universe expands fast enough, there always will be enough space where the field stays at the top (or occasionally jumps back to the top), and inflation continues. However, this conclusion did not attract much attention. Old inflation did not work anyway, and new inflation was also extremely problematic. It was plagued by the problem of initial conditions, and all its semirealistic versions looked very complicated and not very natural [1].

The chaotic inflation scenario [5] has brought two surprises. First of all, it was realized that inflation can occur even if there was no thermal equilibrium in the early Universe, and even if the effective potential  $V(\phi)$  does not

have any maximum at all, or if its maximum is not sufficiently flat. In particular, chaotic inflation scenario can be realized in the theories with potentials  $\frac{m^2}{2}\phi^2$ ,  $\frac{\lambda}{4}\phi^4$ ,  $\frac{\lambda}{4}(\phi^2 - \frac{m^2}{\lambda})^2$ , and  $e^{\alpha\phi}$ . But the most surprising realization was that inflation in these theories also goes on without end. Because of quantum fluctuations the scalar field  $\phi$  in some parts of the Universe perpetually climbs to higher and higher values of its potential energy  $V(\phi)$ , until it approaches the Planck density  $M_P^4$ . The existence of this regime may seem counterintuitive. Indeed, the probability that the field jumps up all the time is very small. However, those rare domains where it happens continue growing exponentially, much faster than the domains with small  $V(\phi)$ . This scenario was called “eternal inflation” [6].

An important feature of this scenario was the existence of domains where the field  $\phi$  may jump for a long time not far away from the Planck density. In these domains the Hubble constant is extremely large,  $H \sim M_P$ . This induces strong perturbations in all other scalar fields, which eventually leads to division of the Universe into exponentially large domains filled with matter with all possible types of symmetry breaking [1], and maybe even with different types of compactification of space-time [7]. This provides a physical justification of the weak anthropic principle. Under certain conditions, an eternally inflating universe enters a stationary regime, where the probability to find domains with given properties does not depend on time [8]. This is a considerable deviation of inflationary cosmology from the standard big bang paradigm. A

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detailed discussion of this scenario was given recently in [9].

In the simplest versions of the chaotic inflation scenario describing only one scalar field, the Universe looks like a sea of a thermalized phase, surrounding islands of inflating space [9]. A considerably different picture appears in the old inflationary theory, as well as in those versions of new inflation where the field  $\phi$  can stay near the top of the effective potential for a long time, being in a kind of metastable state. For example, if the probability of formation of bubbles of the new phase in the old inflationary universe scenario is sufficiently small, the distance between previously generated bubbles grows exponentially before any new bubbles appear. Thus, the new bubbles appear far away from the old ones. In such a situation the bubbles of a new phase do not percolate; they always remain surrounded by de Sitter space [10]. A similar conclusion was reached in [11] concerning the structure of the Universe in the new inflationary universe scenario. The authors performed a computer simulation of inflation and of quantum fluctuations in a simple theory with a potential which looked like a step function. It was equal to some positive constant  $V(0)$  for  $\phi < \phi_0$ , and it was equal to zero for  $\phi > \phi_0$ . This potential mimics many properties of realistic potentials used in the new inflation scenario. However, an important feature of this potential was its absolute flatness near  $\phi = 0$ , which effectively made the scalar field near  $\phi = 0$  metastable. The conclusion of Ref. [11] was that in the new inflationary universe scenario the Universe also consists of islands of thermalized phase surrounded by de Sitter space.

In the present paper we will report the results of our investigation of the global structure of the Universe in the theories with potentials of the type  $\frac{\lambda}{4}(\phi^2 - \frac{m^2}{\lambda})^2$ . Inflation in such models may occur in two different regimes. If it begins at  $\phi > M_P$ , all consequences will be the same as in the simple model  $\frac{\lambda}{4}\phi^4$ . This means that the inflationary domains will look like islands surrounded by the thermalized phase.

On the other hand, for  $m/\sqrt{\lambda} > M_P$ , inflation may occur near  $\phi = 0$  as well, as in the new inflationary universe scenario. We will argue that the global structure of the Universe in this case will depend on the properties of the theory. If we consider a theory of a real scalar field with a discrete symmetry  $\phi \rightarrow -\phi$ , the Universe will consist of islands of thermalized phase with  $\phi \sim m/\sqrt{\lambda}$ . However, if the scalar field has more than one component, for example, if it is a complex field  $\phi = \frac{1}{2}(\phi_1 + i\phi_2)$ , then the situation will be different. The Universe will be filled by the thermalized phase containing inflating strings. In the  $O(3)$ -symmetric theory where the scalar field is a vector  $(\phi_1, \phi_2, \phi_3)$ , the thermalized phase will surround inflating monopoles. This means that topological defects may play an extremely important role in the formation of the global structure of the Universe.

An investigation of this issue should help us to obtain a better understanding of a very interesting piece of physics which was missed in our previous studies of new inflation. Until very recently all experts in inflationary theory believed that primordial monopoles produced during inflation in the new inflationary scenario were ef-

fectively pointlike objects, which did not inflate themselves. For example, in the first version of this scenario based on the  $SU(5)$  Coleman-Weinberg theory [12] the Hubble constant during inflation was of the order  $10^{10}$  GeV, which is five orders of magnitude smaller than the mass of the  $X$  boson  $M_X \sim 10^{15}$  GeV. The size of a monopole estimated by  $M_X^{-1}$  is five orders of magnitude smaller than the curvature of the Universe given by the size of the horizon  $H^{-1}$ . It seemed obvious that such monopoles simply could not know that the Universe is curved.

This conclusion finds an independent confirmation in the calculation of the probability of spontaneous creation of monopoles during inflation. According to [13], this probability is suppressed by a factor of  $\exp(-2\pi m/H)$ , where  $m$  is the monopole mass. In the model discussed above this factor is given by  $\sim 10^{-10^6}$ , which is negligibly small. This result is rather general. In all (or almost all) realistic models of inflation the Hubble constant  $H$  at the end of inflation is smaller than  $10^{14}$  GeV [1]. Meanwhile most of the superheavy topological defects that may have interesting cosmological consequences appear in the theories with the scale of spontaneous symmetry breaking  $\eta \sim 10^{16}$  GeV, which is at least two orders of magnitude greater than  $H$ . The probability of creation of such topological defects by the mechanism described in [13] is extremely small. Even if there were no barriers for production of such objects, their density would have been suppressed by a factor<sup>1</sup>  $\sim \exp\left(-\frac{6\pi^2\eta^2}{H^2}\right) \sim 10^{-10^5}$  [14].

Despite all these considerations, in the present paper (see also [16,17]) we will show that in those theories where inflation is possible near a local maximum of the effective potential, topological defects expand exponentially and can be copiously produced during inflation. The main reason can be explained as follows. In the cores of topological defects the scalar field  $\phi$  always corresponds to the maximum of effective potential. When inflation begins, it makes the field  $\phi$  almost homogeneous. This provides ideal conditions for inflation inside topological defects. These conditions remain satisfied inside the topological defects even after inflation finishes outside of them. Moreover, as we are going to argue, each such topological defect will create many other inflating topological defects. We have called such configurations *fractal topological defects* [16].

The paper is organized in the following way. In Sec. II we will give a description of inflation of domain walls at the level of classical theory. In Sec. III we will briefly describe this process with an account of quantum fluctuations, and present the results of our computer simulations of this process. In Sec. IV we will describe similar processes in the case of inflating strings and monopoles. In Sec. V we will consider the problem of initial conditions for inflation near a local maximum of  $V(\phi)$ . In the concluding Sec. VI we will discuss our main results.

<sup>1</sup>Superheavy topological defects can be created, however, during inflationary phase transitions [15].

## II. INFLATING DOMAIN WALLS

To explain the basic idea of our work, we will begin with a discussion of inflating domain walls. The Lagrangian of the simplest model where such walls may appear is given by

$$L = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2. \quad (1)$$

Here  $\phi$  is a real scalar field. Symmetry breaking in this model leads to formation of domains with  $\phi = \pm\eta$ , where  $\eta = \frac{m}{\sqrt{\lambda}}$ . These domains are divided by domain walls which interpolate between the two minima. Neglecting gravitational effects, one can easily obtain a solution for a static domain wall in the  $yz$  plane:

$$\phi = \eta \tanh\left(\sqrt{\frac{\lambda}{2}} \eta x\right). \quad (2)$$

For small  $\eta$  our neglect of gravitational effects is reasonable. However, the situation becomes more complicated if  $\eta$  becomes comparable to the Planck mass  $M_P$ . The potential energy density in the center of the wall (2) at  $x = 0$  is equal to  $\frac{\lambda}{4}\eta^4$ ; the gradient energy is also equal to  $\frac{\lambda}{4}\eta^4$ . This energy density remains almost constant at  $|x| \ll m^{-1} \equiv \frac{1}{\sqrt{\lambda\eta}}$ , and then it rapidly decreases. Gravitational effects can be neglected if the Schwarzschild radius  $r_g = \frac{2M}{M_P^2}$  corresponding to the distribution of matter with energy density  $\rho = \frac{\lambda}{2}\eta^4$  and radius  $R \sim m^{-1}$  is much smaller than  $R$ . Here  $M = \frac{4\pi}{3}\rho R^3$ . This condition implies that gravitational effects can be neglected for  $\eta \ll \frac{3}{2\pi}M_P$ . In the opposite case,

$$\eta \gtrsim \frac{3}{2\pi}M_P, \quad (3)$$

gravitational effects can be very significant. A similar conclusion is valid for other topological defects as well. For example, recently it was shown that magnetic monopoles in the theory with the scale of spontaneous symmetry breaking  $\eta \gtrsim M_P$  form Reissner-Nordström black holes [20].

Now let us look at it from the point of view of inflationary theory. Inflation occurs at  $\phi \ll \eta$  in the model (1) if the curvature of the effective potential  $V(\phi)$  at  $\phi \ll \eta$  is much smaller than  $3H^2$ , where  $H = \sqrt{\frac{2\pi\lambda}{3}} \frac{\eta^2}{M_P}$  is the Hubble constant supported by the effective potential [1]. This gives  $m^2 \ll 2\pi\lambda\eta^4/M_P^2$ , which leads to the condition almost exactly coinciding with (3):  $\eta \gg M_P/\sqrt{2\pi}$ .

This coincidence by itself does not mean that domain walls and monopoles in the theories with  $\eta \gg M_P/\sqrt{2\pi}$  will inflate. Indeed, inflation occurs only if the energy density is dominated by the vacuum energy. As we have seen, for the wall (2) this was not the case: gradient energy density for the solution (2) near  $x = 0$  is equal to the potential energy density. However, this is correct only after inflation and only if gravitational effects are not taken into account.

At the initial stages of inflation the field  $\phi$  is equal

to zero. Even if originally there were any gradients of this field, they rapidly become exponentially small. Each time  $\Delta t = H^{-1}$  new perturbations with the amplitude  $H/\sqrt{2\pi}$  and the wavelength  $\sim H^{-1}$  are produced, but their gradient energy density  $\sim H^4$  is always much smaller than  $V(\phi)$  for  $V(\phi) \ll M_P^4$  [9]. Therefore originally the vacuum energy inside the walls dominated its gradient energy, and walls could easily expand. The reason why we did not understand this before is the same as the reason why we thought that the interior of the bubbles of the new phase cannot expand: We thought that the bubble walls during inflation were thin from the very beginning. Then we understood that this was wrong, and the new inflationary scenario was proposed. Here we encounter the same situation. Domain walls, just as the bubble walls, originally were thick, and they were exponentially expanding.

## III. SELF-REPRODUCTION OF THE UNIVERSE AND FRACTAL STRUCTURE OF DOMAIN WALLS

Previous description was purely classical. Meanwhile quantum fluctuations play extremely important role in this scenario.

The wavelengths of quantum fluctuations of the scalar field  $\phi$  grow exponentially in the expanding Universe. When the wavelength of any particular fluctuation becomes greater than  $H^{-1}$ , this fluctuation stops oscillating, and its amplitude freezes at some nonzero value  $\delta\phi(x)$  because of the large friction term  $3H\dot{\phi}$  in the equation of motion of the field  $\phi$ . The amplitude of this fluctuation then remains almost unchanged for a very long time, whereas its wavelength grows exponentially. Therefore, the appearance of such a frozen fluctuation is equivalent to the appearance of a classical field  $\delta\phi(x)$  that does not vanish after averaging over macroscopic intervals of space and time.

One can visualize fluctuations generated during the typical time  $H^{-1}$  as sinusoidal waves with average amplitude,

$$\delta\phi = \frac{H}{2\pi}, \quad (4)$$

and with a wavelength  $\sim H^{-1}$ . Since phases of each wave are random, the sum of all waves at a given point fluctuates and experiences Brownian jumps in all directions. As a result, the values of the scalar field in different points become different from each other, and the corresponding variance grows as  $\langle\phi^2\rangle = \frac{H^3}{4\pi^2} t$  [18,19].

In general, the Hubble constant  $H$  strongly depends on the value of the scalar field  $\phi$ . However, we consider the case when inflation occurs near a local maximum of the effective potential at  $\phi = 0$ . This gives  $H = \sqrt{\frac{2\pi\lambda}{3}} \frac{\eta^2}{M_P}$ . Consequently, the average amplitude of fluctuations generated during the time  $H^{-1}$  is given by  $\delta\phi = \sqrt{\frac{\lambda}{6\pi}} \frac{\eta^2}{M_P} = \frac{m^2}{\sqrt{6\pi\lambda}M_P}$ . These perturbations appear in the background of classically moving field  $\phi$ , which grows each time  $H^{-1}$  by  $\Delta\phi = \frac{V'(\phi)}{3H^2} = \frac{\phi\lambda M_P^2}{2\pi m^2}$ . Compar-

ison of these two quantities shows that  $\delta\phi > \Delta\phi$  (i.e., quantum jumps are more important than the classical rolling) for

$$\phi < \phi^* \equiv \frac{m^4 \sqrt{2}}{M_P^2 \lambda \sqrt{\lambda}}. \quad (5)$$

If from the very beginning the scalar field was sufficiently small,  $\phi \ll \phi^*$ , then the quantum jumps of the field  $\phi$  could always return the field back to even smaller values of  $\phi$ . The field  $\phi$  jumps back only in a half of domains with  $\phi \ll \phi^*$ . However, this is quite enough since each typical time interval  $H^{-1}$  the total volume of such domains grows approximately  $e^3/2 \sim 10$  times [4].

Let us consider fluctuations near  $\phi = 0$  in a more detailed way. Suppose that after the typical time  $H^{-1}$  quantum fluctuations of the scalar field  $\phi$  pushed it away from  $\phi = 0$ , and it acquired a positive value  $+H/2\pi$  inside a domain of a size  $H^{-1}$ . During the next period of time  $H^{-1}$  the original domain grows in size  $e$  times, its volume grows  $e^3 \sim 20$  times. Therefore it becomes divided into 20 domains of a size of the horizon  $H^{-1}$ . Evolution of the field inside each of them occurs independently of the processes in the other domains (no-hair theorem for de Sitter space). In each of these domains the scalar field with a probability  $\frac{1}{2}$  may jump back, or it may jump in the same direction. However, these jumps will occur on the scale which is  $e$  times smaller than the length scale of the previous fluctuation. In average those points which originally jumped to positive  $\phi$  will remain positive, and the value of the field  $\phi$  at these points will grow.

Suppose now that we paint white domains with positive  $\phi$ , paint grey domains with negative  $\phi$ , and black, the boundary between these domains. Then after the first step the domain will consist of two parts, one is homogeneously white, and another is homogeneously grey. After the second interval  $H^{-1}$  the size of each domain will grow  $e$  times. The white domain after expansion will contain some grey islands inside it, and the grey domain will contain some white islands. These domains will be separated by black domain walls corresponding to  $\phi = 0$ . Only at the domain walls does the Universe return to its state  $\phi = 0$ . Outside the walls the field  $\phi$  always moves down to the minima of its effective potential. After a while, the Universe becomes divided into white and grey islands separated from each other by black domain walls. These domain walls still continue to expand exponentially. Therefore, qualitatively the picture we obtain is very similar to the one which emerges in the old inflationary scenario: The islands of the thermalized phase are surrounded de Sitter space. However, the physical reason for this picture is somewhat different.

If the field  $\phi$  is in a metastable state, or if it is in a state of equilibrium for a certain sufficiently large range of its values, then the bubbles of the new phase always appear to be surrounded by the old phase. If the decay rate of the old phase is small enough, the thermalized phase will be always surrounded by de Sitter space, even if the field can roll only in one direction from its original position. On the other hand, the main reason for the

existence of the domain structure of the Universe in the model under consideration is the possibility of the field  $\phi$  falling down in two different directions from the maximum of the effective potential  $V(\phi)$ . In our model this was achieved due to the discrete symmetry  $\phi \rightarrow -\phi$  of the effective potential. We should emphasize, however, that in fact we do not need exact or even approximate symmetry. The same conclusions will remain valid for any one-component scalar field  $\phi$  which has a potential  $V(\phi)$  with a sufficiently flat local maximum. This maximum can be at any point  $\phi_0$ . The flatness condition reads  $V''(\phi_0) \ll H^2(\phi_0) = \frac{8\pi V(\phi_0)}{3M_P^2}$ .

As we have mentioned already, the jumps of the field  $\phi$  in our model can occasionally change its sign and create grey domains inside white surroundings. Simultaneously this forms new inflating domain walls. These new walls will be formed only in those places where the scalar field is sufficiently small for the jumps with the change of the sign of the field  $\phi$  to be possible. Therefore, the new walls will be created predominantly near the old ones (where  $\phi = 0$ ), thus forming a fractal domain wall structure.

As a part of our investigation, we made a series of computer simulations of this process in a two-dimensional slice of the Universe. All calculations were performed in comoving coordinates, which did not change during the expansion of the Universe. In such coordinates, expansion of the Universe results in an exponential shrinking of wavelengths of perturbations. We represent perturbations as sinusoidal waves in a two-dimensional universe:

$$\delta\phi(x, y) = \frac{H\sqrt{u}}{\sqrt{2\pi}} \sin [H e^{Ht} (x \cos \theta_n + y \sin \theta_n + \alpha_n)]. \quad (6)$$

Here  $u < 1$  is some small parameter which controls the time  $\Delta t = uH^{-1}$  between two consequent steps of our simulations,  $\theta_n$  and  $\alpha_n$  are random numbers. Equation (6) follows from the corresponding equations of our paper [9] in the case  $H = \text{const}$ . This is a very good approximation for describing inflation near the top of the effective potential. It fails in the thermalized regions, but the thermalized regions do not influence geometry of exponentially expanding part of the Universe at a distance greater than  $H^{-1}$  from the boundary between these regions [10]. This condition was satisfied during our simulations. Therefore, we expect that our simulations correctly represent the behavior of the field not too far away from the top of the effective potential. This is all we need. One should not take too seriously the distribution of the field  $\phi$  in the regions with  $V(\phi) \ll V(0)$  in our figures. Fortunately, this distribution for  $V(\phi) \ll V(0)$  is of no interest to us.

We performed our calculations using the grids containing  $300 \times 300$  and  $1000 \times 1000$  points. At each step of our calculations we added to the previous distribution of the field the wave (6) and also took into account the classical drift of the field by

$$\Delta\phi(x, y) = -\frac{uV'(\phi)}{3H^2}. \quad (7)$$

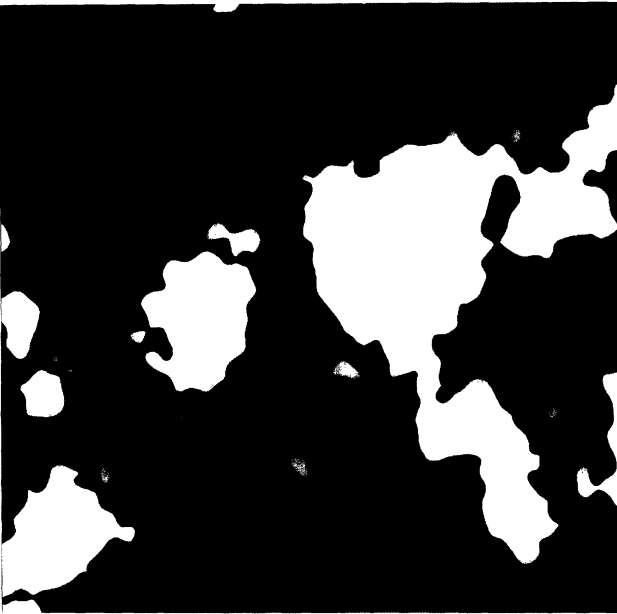
A more detailed description of our method can be

found in [9]. Here we will just briefly describe our results, which are shown in Fig. 1.

As we mentioned above, we paint black the regions corresponding to the domain walls. However, in our figures we included into the definition of a domain wall all points where  $|\phi| < \phi^*$ . Thus, the points in white and grey area ( $\phi > \phi^*$ , and  $\phi < \phi^*$ ) practically never change their color, since at  $|\phi| > \phi^*$  the amplitude of quantum jumps  $\delta\phi$  typically is much smaller than the classical drift  $\Delta\phi$ . Therefore, one can consider these regions as the regions containing thermalized matter. We begin our simulations in a domain of a typical size  $H^{-1}$  with

a field  $\phi \ll \phi^*$ . As we see, after a few steps white and grey islands appear inside the black area, Fig. 1(a). Then new islands become formed, Figs. 1(b)–1(d). The fractal structure of the domain walls is obvious from these simulations. These simulations are similar to those performed in an important paper by Aryal and Vilenkin [11]. However, the new method used in the present work allows us to reveal the physical nature of the exponentially expanding phase. This phase corresponds to the expanding domain walls dividing regions filled by *different* phases. Note that there are no black walls which separate white regions from white regions. (Such walls would exist in

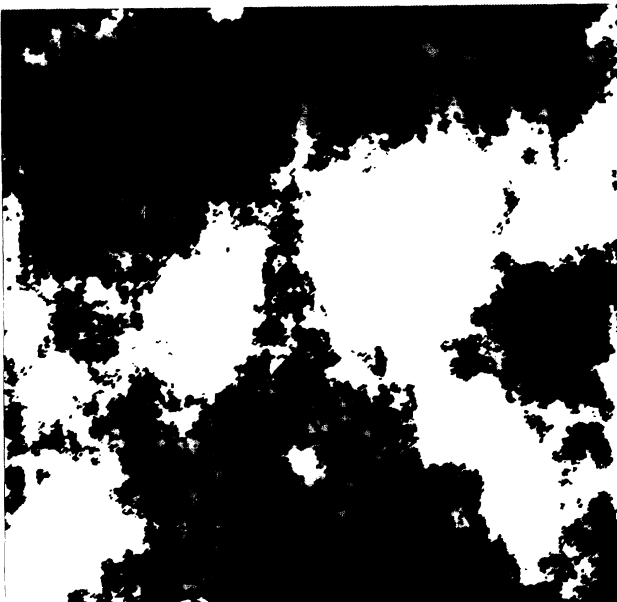
(a)



(b)



(c)



(d)



FIG. 1. The domain structure of the Universe in theory (1) with spontaneously broken discrete symmetry  $\phi \rightarrow -\phi$ .

the old inflationary scenario.) This demonstrates an important role topological defects can play in inflationary cosmology: they can determine the global structure of the Universe. This suggests also that in the models with different types of topological defects, the global structure of the Universe may look different.

#### IV. INFLATING STRINGS AND MONOPOLES

We will consider now more complicated models where instead of a discrete symmetry  $\phi \rightarrow -\phi$  we have a continuous symmetry. For example, instead of the model (1) describing a real scalar field one can consider a model

$$L = \partial_\mu \phi^* \partial_\mu \phi - \lambda \left( \phi^* \phi - \frac{\eta^2}{2} \right)^2, \quad (8)$$

where  $\phi$  is a complex scalar field,  $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$ . Spontaneous breaking of the U(1) symmetry in this theory may produce global cosmic strings. Each string contains a line with  $\phi = 0$ . Outside this line the absolute value of the field  $\phi$  increases and asymptotically approaches the limiting value  $\sqrt{\phi_1^2 + \phi_2^2} = \eta$ . This string will be topologically stable if the isotopic vector  $(\phi_1(x), \phi_2(x))$  rotates by  $2n\pi$  when the point  $x$  takes a closed path around the string.

The (global) monopole solutions for the first time appear in the theory with O(3) symmetry:

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4}(\phi^2 - \eta^2)^2, \quad (9)$$

where  $\phi$  is a vector  $(\phi_1, \phi_2, \phi_3)$ . The simplest monopole configuration contains a point  $x = 0$  with  $\phi(0) = 0$  surrounded by the scalar field  $\phi(x) \propto \mathbf{x}$ . Asymptotically this field approaches regime with  $\phi^2(x) = \eta^2$ .

The basic feature of all topological defects including strings and monopoles is the existence of the points where  $\phi = 0$ . Effective potential has an extremum at  $\phi = 0$ , and if the curvature of the effective potential is smaller than  $H^2 = \frac{8\pi V(0)}{3M_P^2}$ , space around the points with  $\phi = 0$  will expand exponentially, just as in the domain wall case considered above.

Now we can add gauge fields. We begin with the Higgs model, which is a direct generalization of the model (8):

$$L = D_\mu \phi^* D_\mu \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \lambda \left( \phi^* \phi - \frac{\eta^2}{2} \right)^2. \quad (10)$$

Here  $D_\mu$  is a covariant derivative of the scalar field, which in this case is given by  $\partial_\mu - ieA_\mu$ . In this model strings of the scalar field contain magnetic flux  $\Phi = 2\pi/e$ . This flux is localized near the center of the string with  $\phi(x) = 0$ , for the reason that the vector field becomes heavy at large  $\phi$ , see e.g. [21]. However, if inflation takes place inside the string, then the field  $\phi$  becomes vanishingly small not only at the central line with  $\phi(x) = 0$ , but even exponentially far away from it. In such a situation the flux of magnetic field will not be confined near the center of the string. The thickness of the flux will grow together

with the growth of the Universe. Since the total flux of magnetic field inside the string is conserved, its strength will decrease exponentially, and very soon its effect on the string expansion will become negligibly small. Therefore vector fields will not prevent inflation of strings.

The final step is to consider magnetic monopoles. With this purpose one can add non-Abelian gauge fields  $A_\mu^a$  to the O(3)-symmetric theory (9):

$$L = \frac{1}{2}|D_\mu \phi|^2 - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{\lambda}{4}(\phi^2 - \eta^2)^2. \quad (11)$$

Global monopoles of the theory (8) become magnetic monopoles in the theory (10). They also have  $\phi = 0$  in the center. Vector fields in the center of the monopole are massless ( $g\phi = 0$ ). During inflation these fields exponentially decrease, and therefore they do not affect inflation of the monopoles.

We should emphasize that even though the field  $\phi$  around the monopole during inflation is very small, its topological charge is well defined, it cannot change, and it cannot annihilate with the charge of other monopoles as soon as the radius of the monopole becomes greater than  $H^{-1}$ . However, an opposite process is possible. Just as domain walls can be easily produced by quantum fluctuations near other inflating domain walls, pairs of monopoles can be produced in the vicinity of an inflationary monopole. The distance between these monopoles grows exponentially, but the new monopoles will appear in the vicinity of each of them. We will show how it happens using computer simulations of this process.

Note that in the simple models discussed above inflation of monopoles occurs only if spontaneous symmetry breaking is extremely strong,  $\eta \gtrsim M_P$ . However, this is not a necessary condition. Our arguments remain valid for all models where the curvature of the effective potential near  $\phi = 0$  is smaller than the Hubble constant supported by  $V(0)$ . This condition is satisfied by all models which were originally proposed for the realization of the new inflationary universe scenario. In particular, the monopoles in the SU(5) Coleman-Weinberg theory also should expand exponentially. The reason why we thought that this is impossible was explained in the Introduction: The Hubble constant  $H$  during inflation in the SU(5) Coleman-Weinberg theory is much smaller than the mass of the vector field  $M_X$ , which is usually related to the size of the monopole. However, this argument is misleading. The effective mass of the vector field  $M_X \sim g\eta \sim 10^{15}$  GeV can determine the effective size of the monopole only *after* inflation. Effective mass of the vector field  $M_X(\phi) \sim g\phi$  is always equal to zero in the center of the monopole. Once inflation begins in a domain of a size  $O(H^{-1})$  around the center of the monopole, it expels vectors fields away from the center and does not allow them to penetrate back as far as inflation continues.

Of course, one may argue that there is no much reason to consider inflation generated by magnetic monopoles. If the inflaton field is not a gauge singlet, the density perturbations produced after inflation typically are too large [1]. However, there may be many different stages of inflation and the last one can be driven by a different mechanism. The main problem is how to obtain good

initial conditions for the first stage of inflation and (if possible) how to make it eternal. Here topological defects may be of some help.

An interesting feature of this scenario is that inflation of monopoles is eternal for purely classical (topological) reasons [16,17]. There is only one way for a monopole to stop inflating. Even though we have estimated the amplitude of quantum fluctuations around the monopole to be very small, eventually at some moment this amplitude may appear to be much larger than its typical value  $H/2\pi$ . The probability of large jumps of the scalar field  $\phi$  is exponentially small [9], but small probabilities can accumulate when we are speaking about eternity. If the gradients of the classical field  $\phi$  become sufficiently large because of the large fluctuation  $\delta\phi$ , the monopole may stop inflating. However, the probability of this event is much smaller than the probability of the monopole pair creation in the vicinity of an expanding monopole.

Therefore, quantum fluctuations which may destroy the inflation of the monopole simultaneously create many new inflationary monopoles. Moreover, even if quantum fluctuation can terminate inflation of a monopole, they certainly cannot do the same for inflating strings and domain walls.

We have performed computer simulations illustrating some of these issues. Our simulations were two dimensional, and analogues of the monopoles were the centers of the strings in the model (8). The centers of the monopoles should correspond to the points where  $\phi_1 = \phi_2 = 0$ . There are three series of figures in our simulations. Figure 2 shows the distribution of potential energy density in the two-dimensional Universe. In the beginning potential energy density is equal to  $V(0) = \frac{m^4}{4\lambda}$  in the whole domain of initial size  $H^{-1}$ . After a few steps of expansion, the surface  $V(\phi(x, y))$  shown in Fig. 2(a) bends a little, but still the value of the effective potential

(a)



(b)



(c)



(d)



FIG. 2. Energy density distribution during inflation in theory (8).

does not differ much from  $V(0)$ . [The box  $(x, y, V)$  is not shown in this figure.] Later it decreases everywhere except for some points where it remains equal to  $V(0)$ . These points are the peaks of the mountains surrounded by the thermalized phase in Fig. 2. In the beginning we see just a few such mountains, Fig. 2(b), but then they split and form new mountains separated from others by the thermalized phase, Figs. 2(c), 2(d). Note that all these mountains have equal height. It is instructive to compare this picture with a typical distribution obtained in the chaotic inflation scenario with the potential  $V(\phi) = \frac{m^2}{2}\phi^2$ , Fig. 3. In this case, mountains are also separated by the thermalized phase, but their height can be as large as  $M_P^4$ .

Our calculations have been performed with several different sets of parameters. The results shown in Fig. 2 correspond to  $m = 0.3 M_P$ ,  $\lambda = 0.09$ ,  $\eta = M_P$ . Inflationary condition  $m^2 \ll 3H^2$  is satisfied for these values of parameters. Of course, these parameters are far from their values in realistic models. Still they should give us a qualitatively correct picture of the process.

It is very tempting to identify the peaks of the mountains shown at Fig. 2 with monopoles. However, most of the mountains correspond to topologically trivial field configurations. Moreover, most of them do not even have  $\phi = 0$  in the center. Indeed, the only condition which is necessary for the self-reproduction of inflationary domains with large  $V(\phi)$  is that the absolute value of the field  $\phi$  should be smaller than  $\phi^*$  (5). This means that  $V(\phi)$  at the peaks of the mountains is somewhere in the interval  $V(\phi^*) \leq V(\phi) \leq V(0)$ . Typically this means that  $V(\phi)$  on the peaks of the mountains is very close to  $V(0)$ , but it may be slightly different from  $V(0)$ . Thus one should not overemphasize the role of topological defects in the eternal process of self-reproduction of the Universe. This process can occur without topological defects as well. Still the possibility of exponential expansion and self-reproduction of topological defects adds

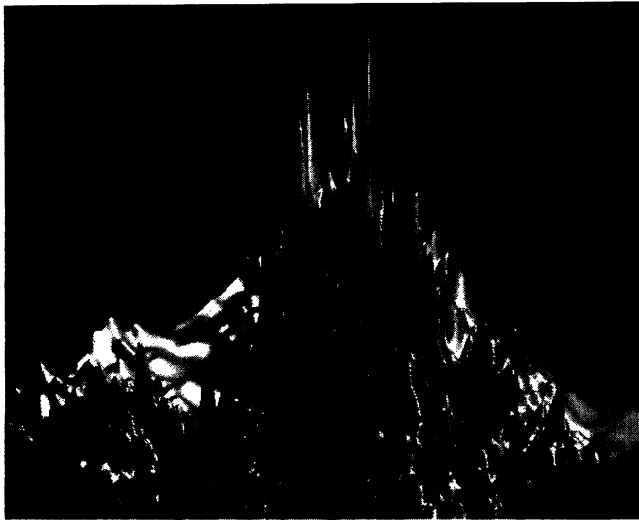


FIG. 3. Energy density distribution during inflation in the simplest chaotic inflation model with the effective potential  $\frac{m^2}{2}\phi^2$ .

some new dimension to this theory.

The field  $\phi$  should make many jumps back from  $\phi^*$  to  $\phi = 0$ . Consequently, the number of the monopoles produced due to these jumps will be suppressed by a factor  $\sim \exp(-4\pi^2\phi^2/H^2)$ . Monopoles will be copiously produced in this scenario, but only near the points where the field  $\phi$  is sufficiently small,  $|\phi_1|, |\phi_2| \lesssim H/2\pi$ , in particular, near other monopoles.

In order to identify those mountains which correspond to monopoles we performed another series of computer simulations. We used color to show the direction of the vector  $(\phi_1(x), \phi_2(x))$  in the isotopic space. Namely, we used white color if this vector was looking in the direction  $(1, 0)$  (i.e., positive  $\phi_0$  and vanishing  $\phi_2$ ), and then we gradually increased the level of darkness as the vector  $(\phi_1(x), \phi_2(x))$  rotated by the angle approaching  $2\pi$ . The point  $2\pi$  for obvious reasons corresponds to a discontinuity; the color is either white or black depending on the way we approach it. This discontinuity does not imply existence of any physical singularity. However, this color map allows us to identify the monopoles as the points where the boundary lines between black and white end in a grey area.

Figure 4 shows the distribution of the direction of the vector  $(\phi_1(x), \phi_2(x))$  using this color map. As we can see, monopoles are created in this process, and their distribution indeed looks like a fractal, which becomes more and more complicated in the course of time. [For the attentive reader: there are eight monopoles in Fig. 4(a) and sixteen monopoles in Fig. 4(b)]. However, if we impose these pictures on the distribution of the energy density  $V(\phi)$ , we will see that some mountains correspond to monopoles, and some do not, see Fig. 5. The stage of the process shown in Fig. 5(c) corresponds to the field distribution in Fig. 4(a). The first monopole can be seen in the upper right part of Fig. 5(a).

As we already mentioned, the centers of inflationary domains in Fig. 2 do not form walls surrounding the thermalized phase. On the contrary, inflating domains are surrounded by the thermalized phase. The reason for this behavior in the simplest versions of chaotic inflation scenario can be easily understood. Nothing prevents the field  $\phi$  at each particular point to roll down to the minimum of the effective potential. Only very rarely the field  $\phi$  jumps against the classical flow down. Those rare points where this happens form the peaks of mountains in Fig. 3. After a sufficiently long time these peaks become surrounded by the thermalized phase.

As we already mentioned, in the situation where the state  $\phi = 0$  is metastable with a sufficiently large lifetime, we would encounter an opposite regime. Independently of all topological considerations, we would obtain islands of thermalized phase surrounded by de Sitter space. Is there any strict boundary between these two regimes? Is it possible that topological defects will prevent rolling of the field  $\phi$  down to the minimum of  $V(\phi)$  in a considerable part of space even in the situations where the state  $\phi = 0$  is unstable?

One can get some insight by a more detailed investigation of the shape of domain walls (generalization to monopoles is straightforward) by using a slight extension



of the method of Ref. [17]. Let us assume that the field  $\phi$  initially is very small,  $\phi \ll \eta$ , and its configuration  $\phi(x, 0)$  is sufficiently smooth. Here  $x$  is a comoving coordinate of the point we consider. We will assume also that near the center of a domain wall one can write  $\phi(x, 0)$  in the first approximation as  $cx$ , where  $c$  is some small constant. In this case the amplitude of the scalar field at each particular point will grow exponentially [1]:

$$\phi(x, t) \approx cx \exp\left(\frac{m^2 t}{3H}\right), \quad (12)$$

Let us write this equation in terms of the physical distance  $X = xe^{Ht}$ :

$$\phi \approx cX \exp\left[-\left(H - \frac{m^2}{3H}\right)t\right]. \quad (13)$$

This equation means that at a physical distance  $X \sim \exp\left[\left(H - \frac{m^2}{3H}\right)t\right]$  from the center of the topological defect the value of the field  $\phi$  does not change in time. In other words, inflation stretches the domain wall without changing its shape at small  $\phi$ . However, the Universe

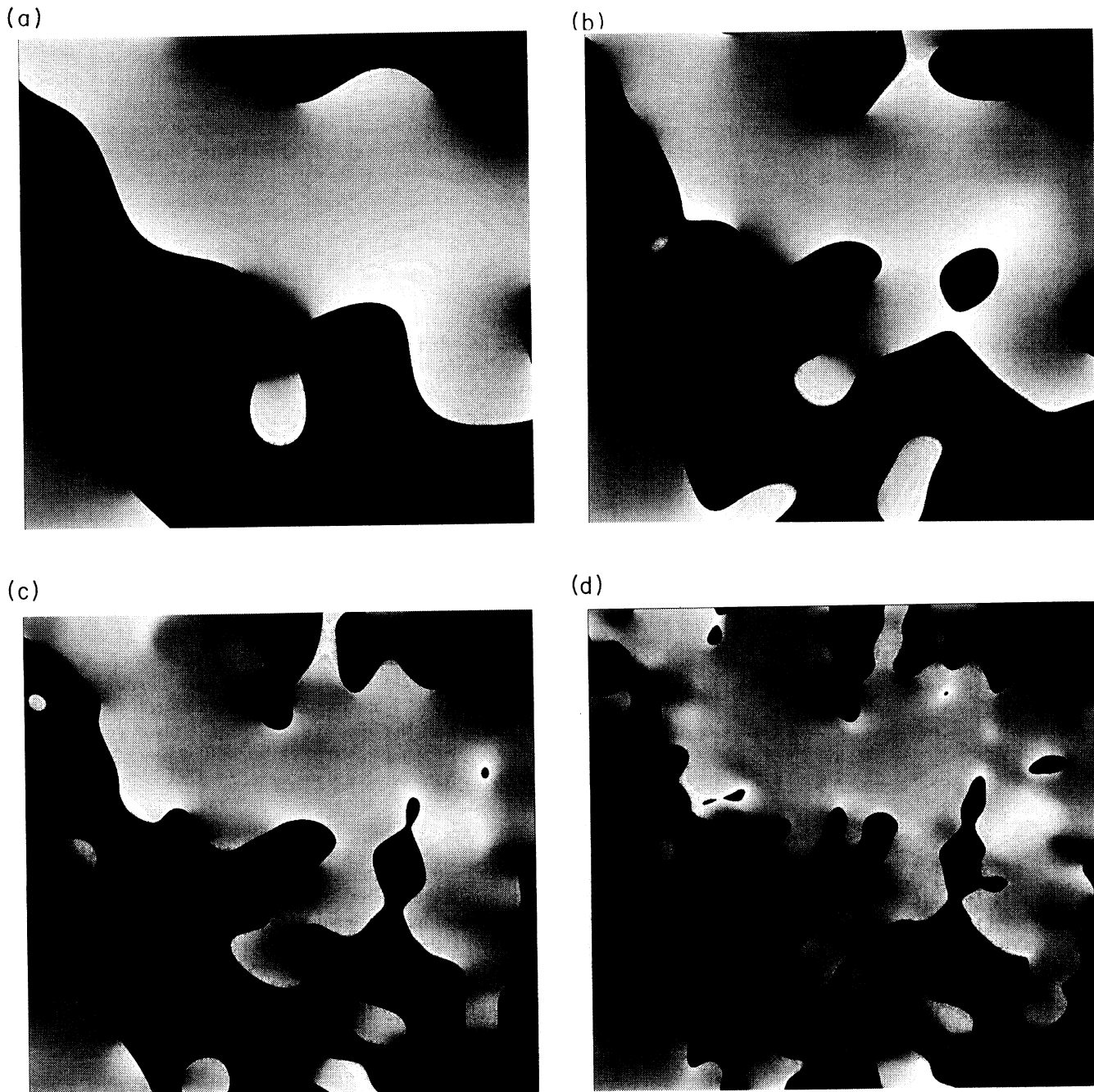


FIG. 4. Distribution of the field  $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$  during inflation in theory (8).

stretches domain walls in the  $x$  direction more slowly than it stretches itself. In the comoving coordinates the thickness of the wall exponentially decreases. Indeed, one can easily see that the value of the field (12) does not change for

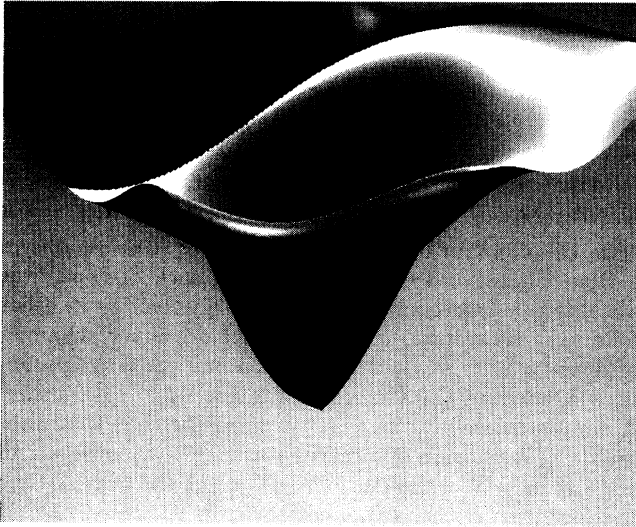
$$x \sim \exp\left(-\frac{m^2 t}{3H}\right). \quad (14)$$

This makes it easier to understand the difference between the topological structure of the Universe in the old inflation scenario and in the new one. In the old inflation scenario de Sitter phase decays due to spontaneous appearance of holes inside it, which leads to the forma-

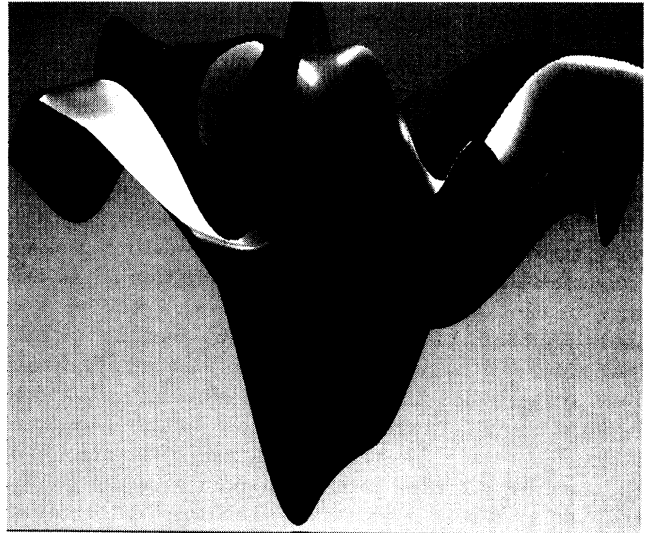
tion of islands of thermalized phase surrounded by de Sitter space. In our scenario the state  $\phi = 0$  is unstable and all space has a tendency to go to the thermalized phase. Inflation still continues near the regions with  $\phi = 0$ , but the comoving size of these regions exponentially decreases in some directions. In the case of domain walls this is not very important; they surround domains of the thermalized phase for topological reasons.<sup>2</sup> On the other hand, shrinking (in the comoving coordinates) strings and monopoles gradually become surrounded by the thermalized phase. This picture is consistent with the results of our calculations.

Note, however, that our last argument was based on the assumption that the effective potential is quadratic

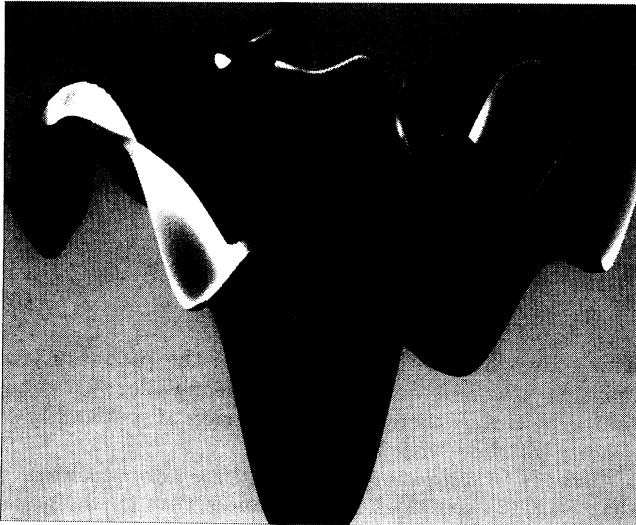
(a)



(b)



(c)



(d)

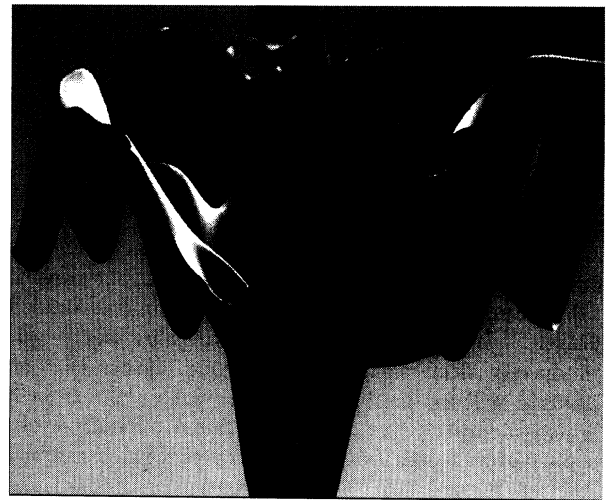


FIG. 5. These figures simultaneously show the energy density of the field  $\phi$  in theory (8), and its direction in the isotopic space.

<sup>2</sup>The possibility of percolation of thermalized domains in a three-dimensional space remains an open question [10,22].

near  $\phi = 0$ . Meanwhile, in the Coleman-Weinberg model the effective potential near the maximum looks like  $V(0) - \frac{\lambda}{4}\phi^4$ . The behavior of domain walls in this theory is more complicated. At small  $\phi$  the field decreases more slowly. This changes the shape of the domain wall, making it more flat near  $\phi = 0$ , which more closely resembles the situation in the old inflation scenario. On the other hand, one can argue that due to quantum fluctuations, the field  $\phi$  spends most of the time at  $\phi$  greater than  $H/2\pi$ , and therefore in average it rolls down at least as fast as the field in the theory (1) with  $m^2 \sim \lambda H^2$ . Therefore, we expect that our conclusions concerning the global structure of the Universe will remain qualitatively correct for the theories with the effective potentials  $\sim V(0) - \frac{\lambda}{4}\phi^4$ . However, this subject clearly requires further investigation.

## V. THE PROBLEM OF INITIAL CONDITIONS

The possibility of inflation of topological defects can lead to some improvement with the problem of initial conditions in the models where inflation occurs near a local maximum of  $V(\phi)$ . Initially the models of that type were introduced in the context of the new inflationary universe scenario [12]. The basic assumption of old and new inflation was that inflation begins in a state of thermal equilibrium at  $\phi = 0$ . This idea was not particularly successful and no realistic versions of new inflation have been suggested so far. Still it is possible for inflation to begin at  $\phi = 0$  in the context of chaotic inflation scenario, if for some reason the scalar field appears near the top of the effective potential inside a domain of a size greater than  $H^{-1}$ . But is it possible to achieve it in a natural way?

In order to analyze this question let us imagine that we are witnessing the moment of the Universe creation ("Planck time"), when the first domain of classical space-time with the Planck energy density  $M_P^4$  emerged from the space-time foam. It seems extremely unlikely that this first domain is infinite from the very beginning. In this case we would face the horizon problem: How was it possible for the same event (the appearance of matter with the Planck density) to be correlated in infinitely many causally disconnected domains?

The only natural length scale in general relativity theory is the Planck length  $M_P^{-1}$ . Therefore, the most natural assumption is that the initial domain has the Planck length.<sup>3</sup> If inflation of this domain does not begin immediately after that, there is a good chance that such a domain will momentarily collapse within the time  $M_P^{-1}$ . This is definitely the case if this domain locally looks like a part of a closed universe, but even if the domain looks

like a part of an open universe of a size  $M_P^{-1}$  immersed into space-time foam, the only obvious way for it to avoid collapse and to evolve into a large homogeneous universe would be to begin inflation instantaneously.

This is not a problem at all for the simplest versions of chaotic inflation, where inflation can easily begin at  $V(\phi) \sim M_P^4$  [1]. However, in all models where inflation occurs near the top of the effective potential, the value of  $V(0)$  appears to be at least ten orders of magnitude smaller than the Planck density and typically it is even much smaller than that. Inflation in such models can begin only at a much later stage of the evolution of the Universe, at a time  $t \sim M_P/\sqrt{V(0)} \gtrsim 10^5 M_P^{-1}$ . The size of initial domain of inflationary universe at that time should be greater than  $\Delta x \sim H^{-1} \sim M_P/\sqrt{V(0)}$ . Suppose for simplicity that the Universe from the very beginning was dominated by ultrarelativistic matter. Then its scale factor expanded as  $\rho^{-4}$ , where  $\rho$  is the energy density at the pre-inflationary stage. Therefore at the Planck time the size of the part of the Universe which later evolved into inflationary domain was not  $M_P/\sqrt{V(0)}$ , but somewhat smaller:  $\Delta x \sim V^{-1/4}(0)$ . This whole scenario can work only if at the Planck time the domain of this size was sufficiently homogeneous,  $\frac{\delta\rho}{\rho} \ll 1$ . However, at the Planck time this domain consisted of  $M_P^3 V^{-3/4}(0)$  domains of a Planck size, and energy density in each of them was absolutely uncorrelated with the energy density in other domains. Therefore, *a priori* one could expect changes of density  $\delta\rho \sim \rho$  when going from one causally disconnected parts of the Universe of a size  $M_P^{-1}$  to another. A simple combinatorial analysis suggests that the probability of formation of a reasonably homogeneous part of the Universe of a size  $\Delta x \sim V^{-1/4}(0)$  at the Planck time is suppressed by the exponential factor

$$P \sim \exp\left(-\frac{C M_P^3}{V^{3/4}(0)}\right), \quad (15)$$

where  $C \sim 1$ . To get a numerical estimate, one can take  $V(0) \sim 10^{-10} M_P^4$ . This gives  $P \lesssim 10^{-10^7}$ . For the original SU(5) Coleman-Weinberg model, this number is even much smaller. Note that this estimate is very similar to the estimate of the probability of a direct quantum creation of inflationary universe with the vacuum energy density  $V(\phi)$  [24],

$$P \sim \exp\left(-\frac{3M_P^4}{8V(\phi)}\right), \quad (16)$$

which gives an even smaller value of the probability of inflation at  $\phi = 0$  than Eq. (15). Meanwhile, this equation tells us that there is no suppression of probability of chaotic inflation with  $V(\phi) \sim M_P^4$ .

One of the differences between these two estimates is that Eq. (15) still does not guarantee that the homogeneous part of the Universe will inflate. Inflation begins near the local maximum of the effective potential only if the field  $\phi$  in this domain appears in a state with  $\phi \ll \eta$ , and is sufficiently homogeneous. Meanwhile, in theory (1) the field  $\phi$  initially can take any value in the interval from  $-\lambda^{-1/4} M_P$  to  $+\lambda^{-1/4} M_P$ . In realistic models with

<sup>3</sup>Of course, this size might be much larger if there was a preceding stage of evolution of the Universe, for example, something like stringy pre-inflation [23]. This possibility is extremely interesting, but its discussion is outside the scope of the present paper.

$\lambda \sim 10^{-12}$  this means that the typical initial value of  $\phi$  would be of the order of  $10^3 M_P$ . Then it will participate in inflation and roll down to  $\phi = \eta$ , just as in the simplest versions of chaotic inflation scenario. The probability to obtain a domain containing homogeneous field  $\phi \ll \eta$  (assuming that  $\eta \ll 10^3 M_P$ ) will be even smaller than  $\exp\left(-\frac{C M_P^2}{V^{3/4}(0)}\right)$ . At this stage topology may help [17]. Once we have a sufficiently large and homogeneous domain, it is most probable that the field  $\phi$  in the model (1) will take both positive and negative values in its different parts. Consequently, there will be domain walls. Since in this model domain walls can be stretched by inflation, they will be even more easily stretched at the pre-inflationary stage, because at that time the Hubble constant was even greater. This naturally creates good conditions for inflation inside the domain wall. However, Eqs. (15) and (16) clearly indicate that the probability to obtain inflation beginning at large  $\phi$  is much better.

Does this mean that we should abandon the idea of chaotic inflation near the local maximum of effective potential? In our opinion, this would be incorrect. First of all, it might happen that in a future theory of elementary particles inflation cannot occur anywhere else except for a local maximum of  $V(\phi)$ . Still it will be much better than no inflation at all. On the other hand, there exist several different ways to create good conditions for inflation at  $\phi = 0$ . The simplest way is to add to the theory some heavy field  $\Phi$  with a simple effective potential for which inflation may begin at  $V(\Phi) \sim M_P^4$ . This stage of inflation initiated by the field  $\Phi$  will force the field  $\phi$  to jump to the top of the effective potential  $V(\phi)$ , at least in some part of the Universe. Initially the part of the volume of the Universe where the field  $\phi$  stays at the top of the effective potential will be relatively small, but later these regions will become increasingly important since they will eternally inflate [9].

Another way is to consider potentials of the new inflationary type in the context of the Brans-Dicke theory. In this case the Planck mass depends on the value of the Brans-Dicke field  $\Phi$ , and the condition  $V(\phi) \sim M_P^4(\Phi)$  can be satisfied at the local maximum of  $V(\phi)$  [25,26].

There is also another interesting possibility [27,9]. The wave function of the Universe should describe all possible initial conditions and all possible outcomes. However, we are interested only in the conditional probability to obtain particular observational data under an obvious but very nontrivial condition of our own existence. There may be many branches of the wave function of the Universe which may seem natural from the point of view of initial conditions, but most of them describe the Universe where intelligent observers cannot live. In our calculation of the probability (15) we simply counted all trajectories, even those which correspond to “virtual” universes collapsing within the Planck time. But why should we count them? Perhaps we should see where most of the observers can live, and we should call the corresponding trajectories “typical.” There will be many problems with such an approach, in particular the problem of introducing a proper measure on the set of all such trajectories [26]. However, it seems plausible that with any reason-

able choice of measure the trajectories corresponding to eternal inflation will always win being compared to the trajectories which do not possess this property.

The only real problem appears if we should compare many different possibilities corresponding to different realizations of the eternal inflation scenario. In this case one should take into account that it is much more difficult for inflation to begin at  $V(\phi) \ll M_P^4$  than at  $V(\phi) \sim M_P^4$ .

## VI. DISCUSSION

When we began this investigation, our main purpose was to study the difference between the global structure of the Universe in the models of two different classes: those models where inflation occurs near a local maximum of the effective potential and those models where inflation begins at large  $\phi$ , outside the equilibrium. However, during our work we recognized that some other important features of inflation in the models of the first class theories have not been properly analyzed. For more than ten years we knew that inflation solves the primordial monopole problem, but we did not know that monopoles and other topological defects can inflate.

Now it appears that under certain conditions topological defects do inflate and their inflation never ends [16,17]. According to this scenario, our part of the Universe could be formed from what initially was an interior of an inflating topological defect. The first attempt to investigate this question was made in our paper [9], where we have shown that in accordance to the most natural realization of the “natural inflation” scenario [28] we should live in the remnants of an inflating domain wall. Now we understand that this situation is much more general.

The structure of space-time near inflating topological defects is very complicated; it should be studied by the methods developed in [29] for description of a bubble of de Sitter space immersed into vacuum with vanishing energy density. Depending on initial conditions, many possible configurations may appear. For example, under certain initial conditions an inflating monopole may evolve into a magnetically charged Reissner-Nordström black hole [20]. However, it will contain a part of exponentially expanding space inside it. This will be a wormhole configuration similar to those studied in [29–32]. On the other hand, an observer which originally participated in inflation near a monopole (or near some other topological defect), eventually will find himself in a post-inflationary part of the Universe, exponentially far away from the inflating monopole. Such an observer may argue that he does not see any monopoles since the exponentially expanding space around them expels them from our part of the Universe.

At the quantum level the whole scenario becomes even more interesting. Fluctuations of the field  $\phi$  near the center of a monopole are strong enough to create new regions of space with  $\phi = 0$ , some of which will become monopoles. After a while, the distance between these monopoles becomes exponentially large, so that they cannot annihilate. This process of monopole-antimonopole pair creation produces a fractal structure consisting of

monopoles created in the vicinity of other monopoles.

One of the original motivations for the development of inflationary cosmology was a desire to get rid of primordial magnetic monopoles and dangerous domain walls produced in the theories with spontaneous breaking of discrete symmetries. For a long time topological defects and inflation were opposed to each other as two almost incompatible sources of density perturbations in the early Universe. Now we see that the interplay between inflationary theory and the theory of topological defects can

be very constructive. According to our scenario, inflation can produce inflating topological defects which in turn can serve as seeds for eternal inflation.

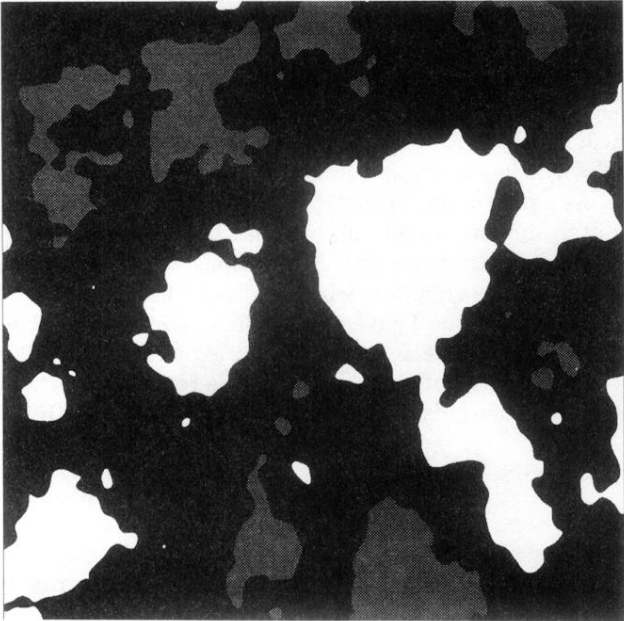
#### ACKNOWLEDGMENTS

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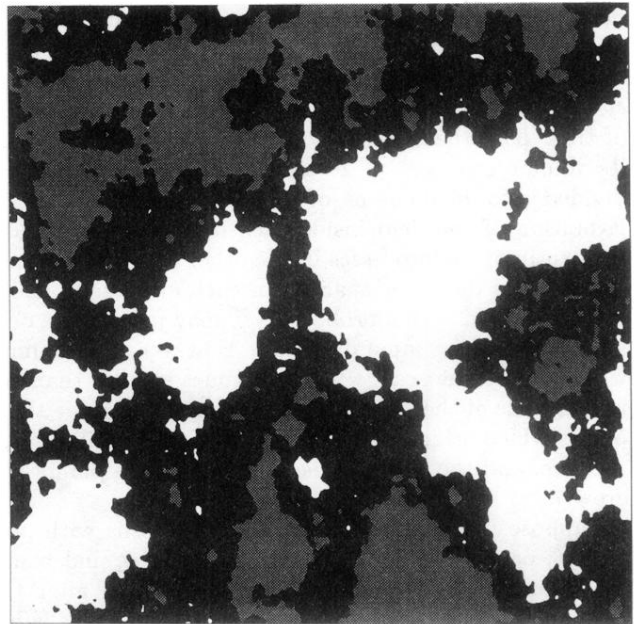
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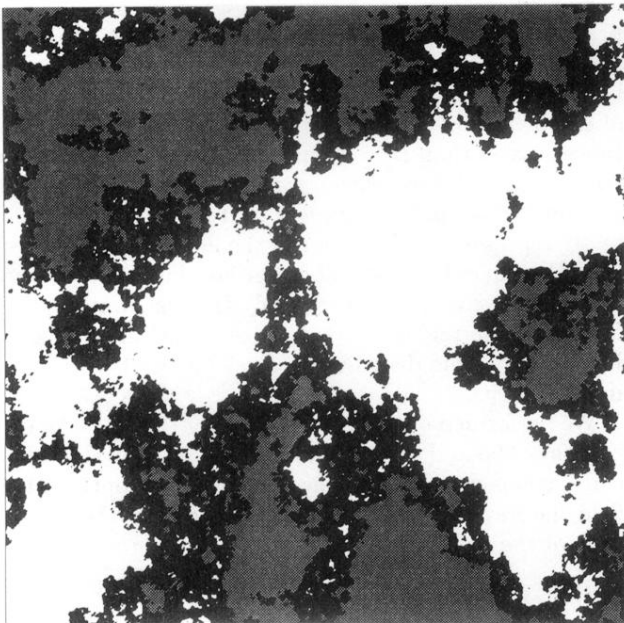
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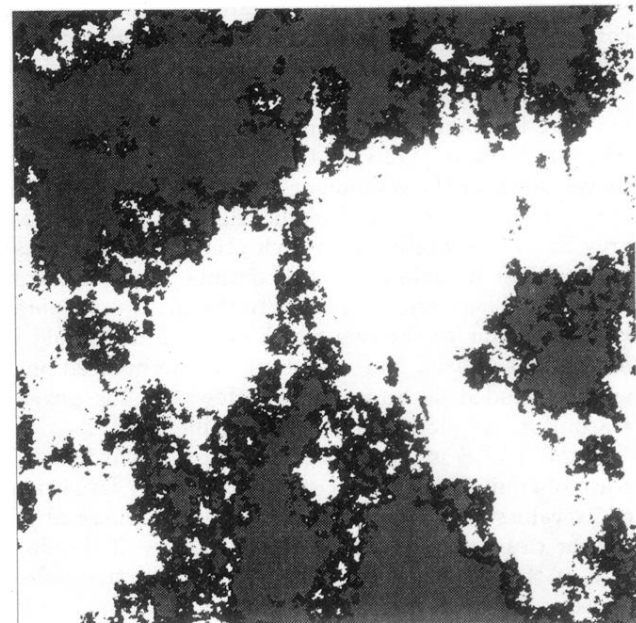
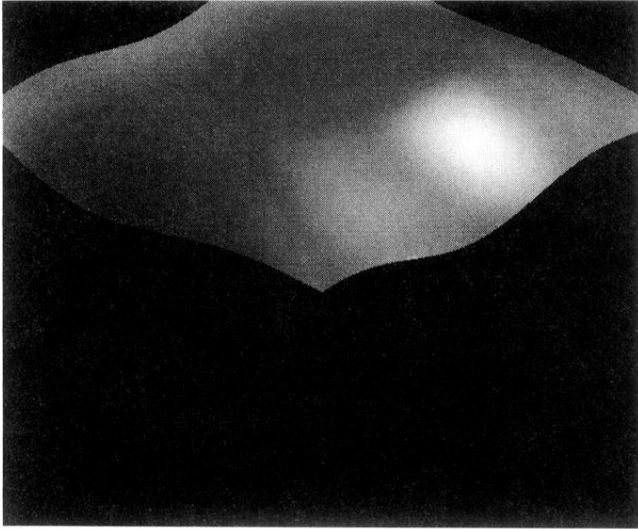
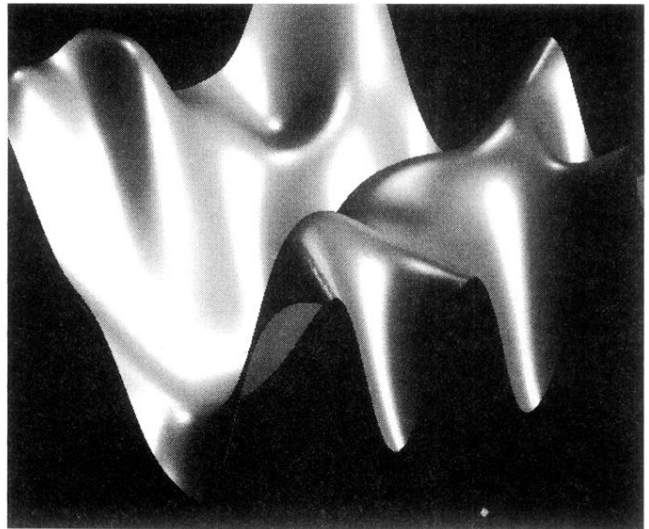


FIG. 1. The domain structure of the Universe in theory (1) with spontaneously broken discrete symmetry  $\phi \rightarrow -\phi$ .

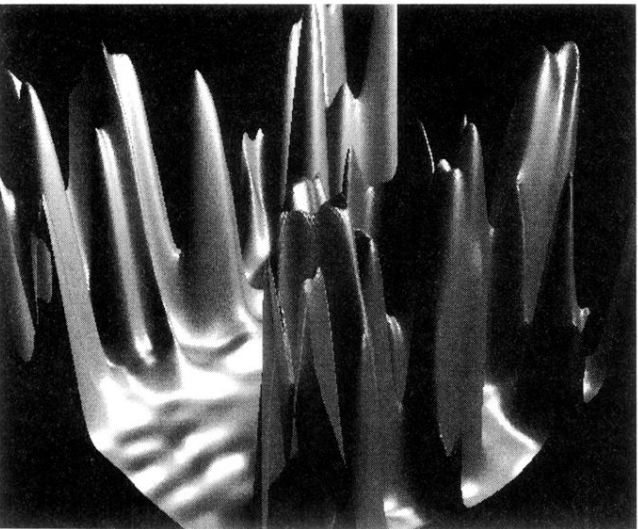
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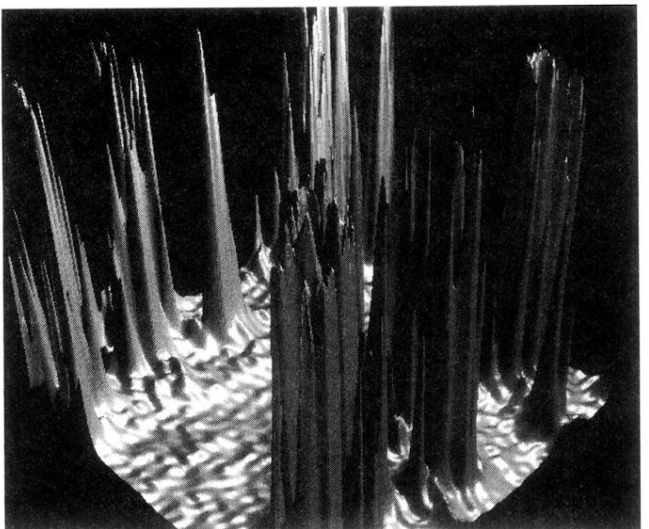


FIG. 2. Energy density distribution during inflation in theory (8).

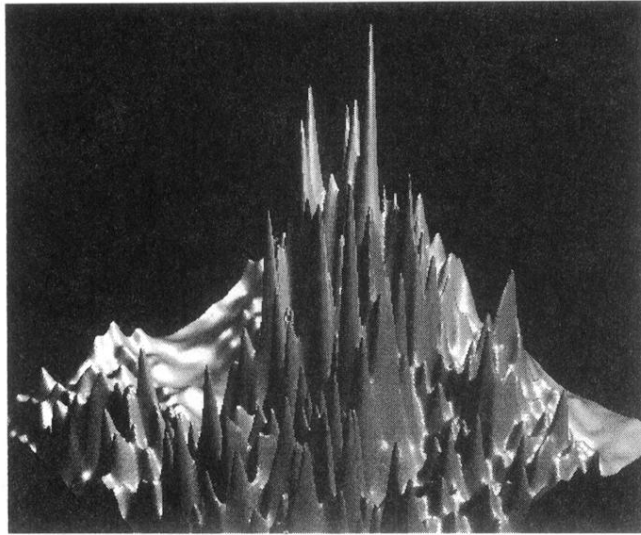
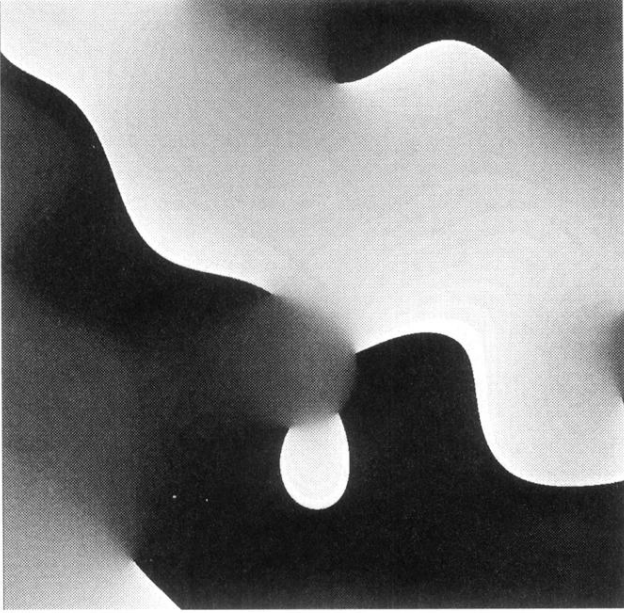


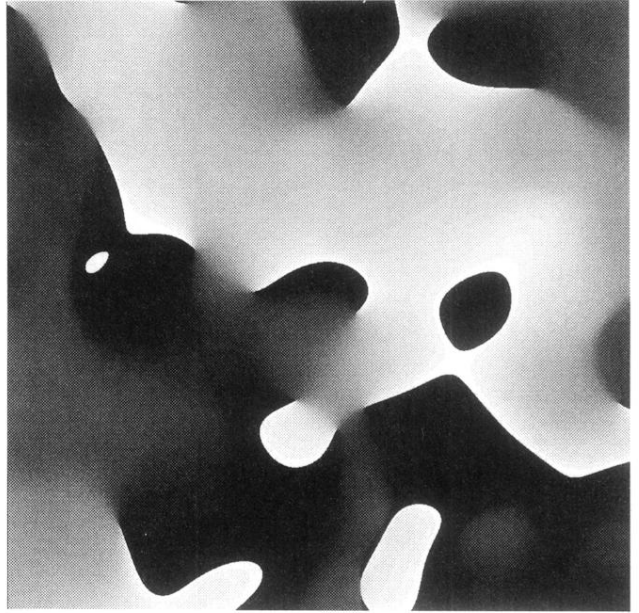
FIG. 3. Energy density distribution during inflation in the simplest chaotic inflation model with the effective potential  $\frac{m^2}{2}\phi^2$ .



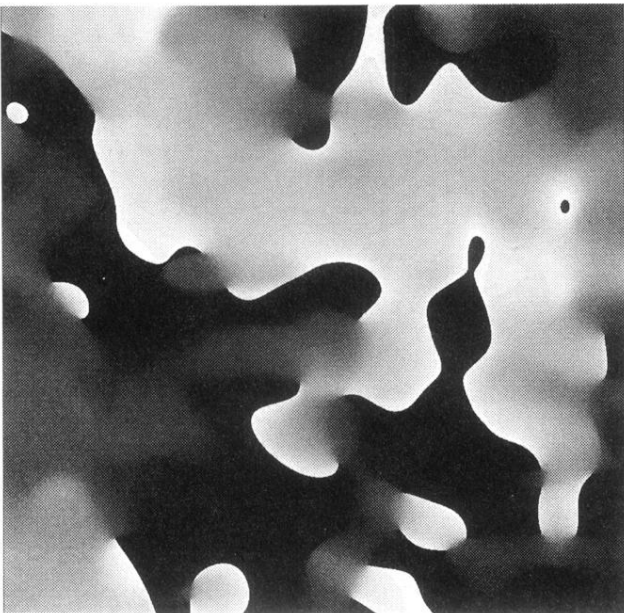
(a)



(b)



(c)



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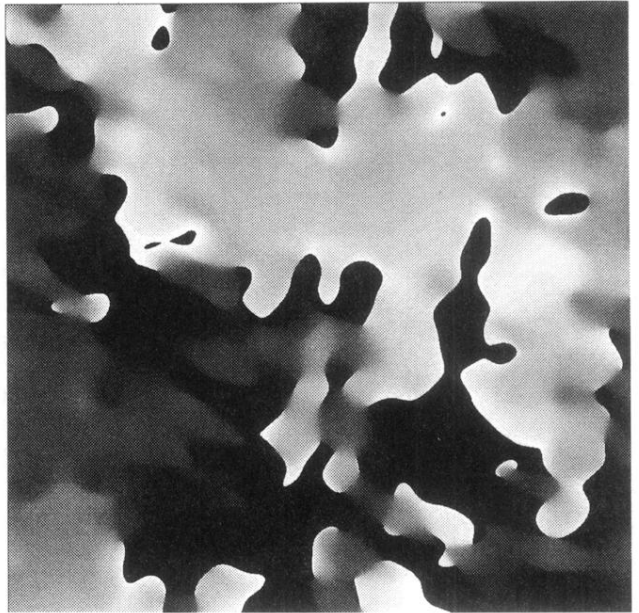
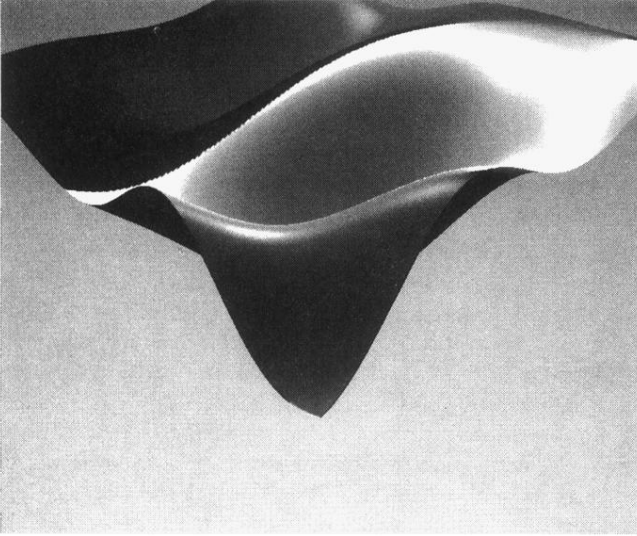
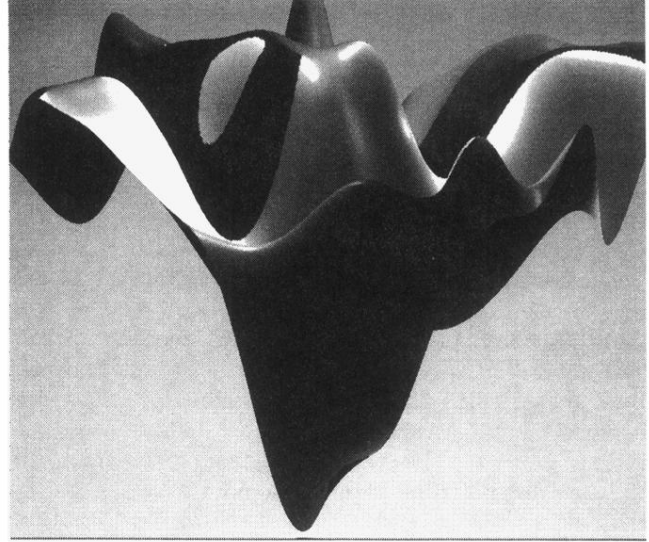


FIG. 4. Distribution of the field  $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$  during inflation in theory (8).

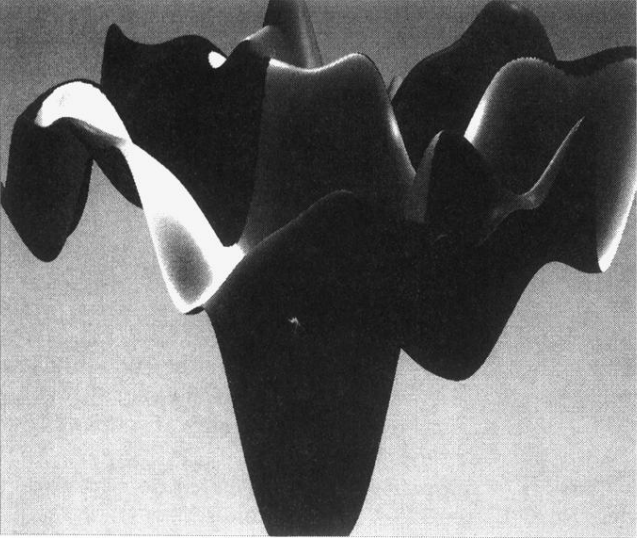
(a)



(b)



(c)



(d)

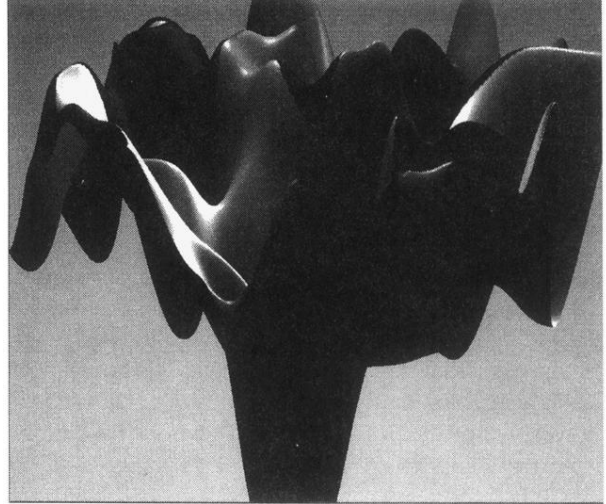


FIG. 5. These figures simultaneously show the energy density of the field  $\phi$  in theory (8), and its direction in the isotopic space.