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Microphysical approach to nonequilibrium dynamics of quantum fields

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We examine the nonequilibrium dynamics of a self-interacting $\lambda\phi^4$ scalar field theory. Using a real-time formulation of finite temperature field theory we derive, up to two loops and $O(\lambda^2)$, the effective equation of motion describing the approach to equilibrium. We present a detailed analysis of the approximations used in order to obtain a Langevin-like equation of motion, in which the noise and dissipation terms associated with quantum fluctuations obey a fluctuation-dissipation relation. We show that, in general, the noise is colored (time dependent) and multiplicative (couples nonlinearly to the field), even though it is still Gaussian distributed. The noise becomes white in the infinite temperature limit. We also address the effect of couplings to other fields, which we assume play the role of the thermal bath, in the effective equation of motion for ϕ. In particular, we obtain the fluctuation and noise terms due to a quadratic coupling to another scalar field.

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I. INTRODUCTION

The possibility that the Universe went through a series of phase transitions as it expanded and cooled from times close to the Planck scale has been actively investigated for the past 15 years or so [1]. It is hoped that by studying the nontrivial dynamics typical of the approach to equilibrium in complex systems, many of the current questions of cosmology, from the origin of the baryonic matter excess to the large-scale structure of the Universe, will be answered in the near future. As is well known, the origin of density perturbations that seed structure formation has been linked to either the existence of topological defects, such as strings or textures formed during a (GUT)-scale transition [2], or to inflation in one of its incarnations. In particular, the old, new, extended, and natural models of inflation all invoke a symmetry-breaking transition in which nonequilibrium conditions play a crucial role [3]. At the electroweak scale, the focus has been in generating the baryon number excess during a first-order phase transition [4]. Even though there are certain questions related to the reliability of the perturbative expansion for weak enough transitions [5] as well as to the mechanism by which weak first-order transitions complete [6], it is currently believed that nonequilibrium conditions are a crucial ingredient for baryogenesis.

Despite its relevance, not much has been done to understand nonequilibrium aspects of phase transitions in cosmology. (This situation is rapidly changing. We will soon refer to past and recent work on the subject.) Most of what has been done so far is related to the finite temperature effective potential (computed in general to one-loop order) which, by its very definition, is only adequate to describe equilibrium situations; the calculation is usually done in Euclidean time so that we can obtain the equilibrium partition function from a transition amplitude. The great advantage of using the effective potential is that it gives us information about static properties of the system such as its possible stable and metastable equilibrium states, and critical temperatures for phase transitions. The disadvantage is that we lose all information about real-time processes, which are crucial to understand the mechanism by which the system approaches equilibrium. In fact, the one-loop effective potential does carry, in a somewhat indirect way, information about unstable states in the system. These are states which are in the "spinodal" region, where the effective potential is concave. If we start with the system in thermal equilibrium above the critical temperature and then quench it to below the critical temperature so that its order parameter takes a value within the spinodal, the approach to equilibrium will be initially dominated by the growth of small amplitude long wavelength fluctuations, in the mechanism known as spinodal decomposition. Thus, the effective potential tells us that some states will be unstable, and that their final equilibrium state is at its global minimum, but it does not tell us how the system gets there. The reader is referred to the recent work of Boyanovsky, Lee, and Singh for details [7].

These limitations of the effective potential were pointed out by Mazenko, Unruh, and Wald, in work where they argued that for strong enough couplings, the slow-roll approximation necessary for successful inflation may not be adequate. Instead, the approach to equilibrium would proceed by the formation and growth of domains, typical of spinodal decomposition [8]. It was subsequently shown within the context of the new inflationary model, by both analytical [9] and phenomenological numerical methods [10] that due to the small couplings needed for the generation of density fluctuations, the slow-roll picture of inflation was correct.

This discussion of the validity of the slow-roll approximation in inflation raises some very interesting questions

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related to the way we picture the approach to equilibrium in field theories, which are quite independent of inflation. For example, the distinction between the "system," which is out of equilibrium, and the "thermal bath," which drives the system into equilibrium, is somewhat blurred in the context of nonlinear field theories, in contrast to the system-bath coupling in quantum mechanics [11]. In fact, for self-interacting field theories, the short wavelength modes can serve as the thermal bath driving the longer wavelength modes, which have slower dynamics, into equilibrium. In this sense, the field can be its own thermal bath. Of course, other fields coupled to the order parameter scalar field (henceforth the "system") may serve as the thermal (or, at $T = 0$, quantum) bath.

In one of the original works on this subject which was motivated by cosmology, Hosoya and Sakagami obtained an approximate dissipation term in the equation of motion satisfied by the thermal average of the scalar field, by invoking a small deviation from equilibrium in the Boltzmann equation for the number density operator. This calculation was then supplemented by a computation of transport coefficients using Zubarev's method for nonequilibrium statistical operators [12]. Using an approach which is closer to the one we will adopt here, Morikawa obtained the effective Langevin-like equation (that is, with both fluctuation and dissipation terms but not quite as simple as the Langevin equation) for a scalar field interacting with a fermionic bath using real-time field-theoretical techniques at zero and, very briefly, finite temperature [13]. More recently, Hu, Paz, and Zhang analyzed the case of a quantum bath given by a scalar field quadratically coupled to the system [14], while Lee and Boyanovsky considered the case of a thermal bath given by a scalar field linearly coupled to the system [15]. Some works dealing with nonequilibrium evolution within a cosmological framework can be found in Refs. [16,17]. Here we will only be concerned with dynamics in Minkowski spacetime.

Recently, and in particular in Refs. [14,15] the properties of the noise as being in general colored and multiplicative (unless the coupling between system and bath is linear) have been emphasized. This can have very important consequences to our understanding of phase transitions, as suggested by Habib, even though results at this point are preliminary [18]. The reason is that potentially, a multiplicative noise may sharply decrease the relaxation time scales in the system and thus accelerate the approach to equilibrium. Numerical simulations of the approach to equilibrium have so far employed a phenomenological Langevin equation, with white and additive noise to mimic the effects of the thermal bath. In 1+1 dimensions both the thermal nucleation of kink-antikink pairs [19] and the decay of metastable states [20] were studied, while in 2+1 dimensions the decay of metastable states was recently investigated [21]. The time scales measured in these simulations agree with the theoretical prediction for the decay rate, $\Gamma \sim \exp[-B(T)/T]$, as long as $B(T)$ is the classical (i.e., obtained with the classical potential) nucleation barrier given by the energy of the appropriate field configuration that saturates the path integral, the mass of the kink-antikink pair or the energy of the bounce configuration in the examples mentioned above.

The question then is if the phenomenological Langevin equation used in the above simulations is indeed reproducing the essential physics of the approach to equilibrium, or if we are dangerously oversimplifying things. The above discussion suggests that the effective equation which describes the approach to equilibrium of the slower moving modes can be quite different from the phenomenological Langevin equation with its white and additive noise. Two tasks are at hand then. First we must obtain the effective equation for a self-interacting scalar field which acts as its own bath and compare it with the equation obtained by having another field act as the bath. This should elucidate the nature of the thermal bath in these two situations, and also give us an answer as to whether the phenomenological Langevin equation is at all valid in some limit. The second task follows naturally the first. Once we have an effective equation we trust (in some limit), we should use it to simulate numerically the nonequilibrium dynamics, measure the relaxation time scales, and compare the results with the results obtained with the simplified phenomenological Langevin equation.

In this paper we will concentrate on the first task. Namely, we will obtain, within perturbation theory, the effective equation of motion describing the approach to equilibrium of a self-coupled scalar field. We will integrate out the short wavelength modes whose influence will be felt as a thermal bath through the nonlinear couplings to the longer wavelength modes, which we take as the system. The separation between bath and system is implemented by perturbation theory, since the effective action is obtained by *integrating* over small fluctuations about the state we are expanding about. We will include corrections up to two loops, as nonvanishing viscosity (and transport coefficients, in general) terms in finite temperature field theory only show up by considering higher order (loop) corrections to the field propagators and are dependent on the imaginary part (decay width) of the self-energy corrections [12,22–24]. Fluctuation terms are obtained by associating the imaginary terms in the effective action as coming from the interaction of φ with fluctuating (noise) fields, as done in [13] and [15]. We will also obtain the effective equation of motion in the presence of another scalar field quadratically coupled to the system, thus reproducing (even though we focus more on dynamical aspects) the analysis of Ref. [14] for finite temperatures. Our results could, for example, be used in the numerical investigation of symmetry restoration at finite temperature, although close to T_c our approximations will break down.

The paper is organized as follows: In Sec. II we derive the effective action for a nonuniform time-dependent background field configuration $\varphi(\mathbf{x}, t)$, up to two loops and order λ^2. In Sec. III we obtain the effective equation of motion for $\varphi(\mathbf{x}, t)$ and discuss the approximations involved in order that it obeys a Langevin-like equation. In Sec. IV we examine the effect of other fields interacting with the scalar field, by studying the case of a quadratically coupled scalar field and by evaluating its contributions to noise and dissipative terms. Conclusions are presented in Sec. V. Three Appendixes are included in order to obtain some technical results used in the paper.

II. THE TWO-LOOP FINITE TEMPERATURE EFFECTIVE ACTION

Consider the scalar field model with Lagrangian density

$$\mathcal{L}[\phi] = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 \qquad (2.1)$$

and with generating functional $Z[J]$, in terms of an external source J, given by

$$Z[J] = \int_c D\phi \exp\{iS[\phi, J]\} , \qquad (2.2)$$

where the classical action is given by

$$S[\phi, J] = \int_c d^4x \{\mathcal{L}[\phi] + J(x)\phi(x)\} . \qquad (2.3)$$

In (2.3) the time integration is along a contour suitable for real-time evaluations, which we choose as being Schwinger's closed-time path [25,23,26], where the time path c goes from $-\infty$ to $+\infty$ and then back to $-\infty$. The functional integration in (2.2) is over fields along this time contour. As with the Euclidean time formulation, the scalar field is still periodic in time, but now with $\phi(t, \mathbf{x}) = \phi(t - i\beta, \mathbf{x})$. Temperature appears due to the boundary condition, but now time is explicitly present in the integration contour.

As usual, the effective action $\Gamma[\varphi]$ is defined in terms of the connected generating functional $W[J]$ as

$$\Gamma[\varphi] = W[J] - \int_c d^4x J(x)\varphi(x) , \qquad (2.4)$$

with $\varphi(x)$ defined by $\varphi(x) \equiv \frac{\delta W[J]}{\delta J(x)}$, and

$$W[J] = -i \ln \int_c D\phi \exp\{iS[\phi, J]\} . \qquad (2.5)$$

The perturbative loop expansion for $\Gamma[\varphi]$ is obtained by writing the scalar field as $\phi \to \phi_0 + \eta$, where ϕ_0 is a field configuration which extremizes the classical action $S[\phi, J]$ and η is a small perturbation about this configuration. By using (2.4) and (2.5), we can relate ϕ_0 to φ ($\phi_0 = \varphi - \eta$) and write the effective action, for $J \to 0$, at one-loop order, as

$$\Gamma[\varphi] = S[\varphi] + \frac{i}{2}\mathrm{Tr}\ln[\Box + V''(\varphi)] , \qquad (2.6)$$

where

$$\frac{i}{2}\mathrm{Tr}\ln[\Box + V''(\varphi)] = -i \ln \int_c D\eta \exp\left\{-\frac{i}{2}\eta[\Box + V''(\varphi)]\eta\right\} . \qquad (2.7)$$

Neglecting contributions to (2.6) which are independent of φ, we can expand the logarithm in (2.6) as

$$\frac{i}{2}\mathrm{Tr}\ln[\Box + V''(\varphi)] = \frac{i}{2}\mathrm{Tr}_c \sum_{m=1}^{+\infty} \frac{(-1)^{m+1}}{m} G_\phi^m \left(\frac{\lambda}{2}\varphi^2\right)^m$$
$$= \frac{i}{2}\sum_{m=1}^{+\infty} \frac{(-1)^{m+1}}{m}\left(\frac{\lambda}{2}\right)^m \mathrm{Tr}\int d^4x_1 \cdots d^4x_m G_\phi^{n_1,l_1}(x_1 - x_2)\left[\varphi^2(x_2)\right]_{l_1,n_2} G_\phi^{n_2,l_2}(x_2 - x_3)\cdots$$
$$\times \cdots \left[\varphi^2(x_m)\right]_{l_{m-1},n_m} G_\phi^{n_m,l_m}(x_m - x_1)\left[\varphi^2(x_1)\right]_{l_m,n_{m+1}} . \qquad (2.8)$$

The matrix representation in (2.8) is a consequence of the time contour, since now we must identify field variables with arguments on the positive or negative directional branches of the time path, that we denote by φ_+ and φ_-, respectively. As a consequence of this doubling of field variables, we also have that $G_\phi^{n,l}(x - x')$, the real-time free-field propagators on the contour, are given by ($l, n = +, -$) (Refs. [23,27])

$$G_\phi^{++}(x - x') = i\langle T_+\phi(x)\phi(x')\rangle$$
$$G_\phi^{--}(x - x') = i\langle T_-\phi(x)\phi(x')\rangle$$
$$G_\phi^{+-}(x - x') = i\langle \phi(x')\phi(x)\rangle$$
$$G_\phi^{-+}(x - x') = i\langle \phi(x)\phi(x')\rangle , \qquad (2.9)$$

where T_+ and T_- indicate chronological and antichronological ordering, respectively. G_ϕ^{++} is the usual physical (causal) propagator. The other three propagators come as a consequence of the time contour and are considered as auxiliary (unphysical) propagators [27]. The explicit expressions for $G_\phi^{n,l}(x - x')$ in terms of its momentum space Fourier transforms are given by [23,26]

$$G_\phi(x - x') = i\int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \times \begin{pmatrix} G_\phi^{++}(\mathbf{k}, t - t') & G_\phi^{+-}(\mathbf{k}, t - t') \\ G_\phi^{-+}(\mathbf{k}, t - t') & G_\phi^{--}(\mathbf{k}, t - t') \end{pmatrix} , \qquad (2.10)$$

where

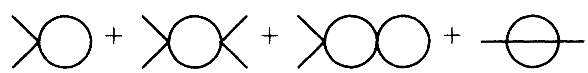
$$
\begin{aligned}
G_\phi^{++}(\mathbf{k},t-t') &= G_\phi^{>}(\mathbf{k},t-t')\theta(t-t') \\
&\quad + G_\phi^{<}(\mathbf{k},t-t')\theta(t'-t), \\
G_\phi^{--}(\mathbf{k},t-t') &= G_\phi^{>}(\mathbf{k},t-t')\theta(t'-t) \\
&\quad + G_\phi^{<}(\mathbf{k},t-t')\theta(t-t'), \\
G_\phi^{+-}(\mathbf{k},t-t') &= G_\phi^{>}(\mathbf{k},t-t'), \\
G_\phi^{-+}(\mathbf{k},t-t') &= G_\phi^{<}(\mathbf{k},t-t'),
\end{aligned} \tag{2.11}
$$

and, for free propagators at finite temperature,

$$
\begin{aligned}
G_\phi^{>}(\mathbf{k},t-t') &= \frac{1}{2\omega(\mathbf{k})} \\
&\quad \times \Big[[1+2n(\omega)]\cos[\omega(t-t')] \\
&\quad - i\sin[\omega(t-t')]\Big], \\
G_\phi^{<}(\mathbf{k},t-t') &= G_\phi^{>}(\mathbf{k},t'-t), ,
\end{aligned} \tag{2.12}
$$

where $n(\omega) = \left(e^{\beta\omega}-1\right)^{-1}$ is the Bose distribution and $\omega \equiv \omega(\mathbf{k})$ is the free particle energy, $\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2+m^2}$.

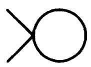
Let us now add to (2.6) contributions up to two loops and order λ^2. Graphically we have



$$+\,O(\lambda^3)\,, \tag{2.13}$$

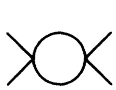
where, in the graphic representation, φ is in the external legs and the internal propagators are given by $G_\phi^{n,l}$. In terms of the field variables φ_+ and φ_-, the terms in Eq. (2.13) are given by (note that now time runs only forward)

$$
S[\varphi] = \int d^4x \left\{\mathcal{L}[\varphi_+] - \mathcal{L}[\varphi_-]\right\}\,, \tag{2.14}
$$



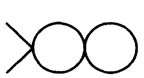
$$
\begin{aligned}
&= -\frac{\lambda}{4}\mathrm{Tr}\int d^4x \int \frac{d^3q}{(2\pi)^3}\begin{pmatrix} G_\phi^{++}(\mathbf{q},0) & G_\phi^{+-}(\mathbf{q},0) \\ G_\phi^{-+}(\mathbf{q},0) & G_\phi^{--}(\mathbf{q},0)\end{pmatrix}\begin{pmatrix}\varphi_+^2(x) & 0 \\ 0 & -\varphi_-^2(x)\end{pmatrix} \\
&= -\frac{\lambda}{4}\int d^4x\left[\varphi_+^2(x)-\varphi_-^2(x)\right]\int\frac{d^3q}{(2\pi)^3}\frac{1}{2\omega(\mathbf{q})}\left[1+2n(\omega)\right]\,,
\end{aligned} \tag{2.15}
$$

where $G_\phi^{n,l}(\mathbf{q},0)$ is given by (2.11) (for $t-t'=0$). Equation (2.15) gives just the finite temperature mass contribution to the effective action (renormalized by a proper mass counterterm δm^2 which we are not including here). The second graph in (2.13) is given by



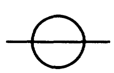
$$
\begin{aligned}
&= i\frac{\lambda^2}{16}\mathrm{Tr}\int d^4x d^4x'\int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\int\frac{d^3q}{(2\pi)^3}\begin{pmatrix} G_\phi^{++}(\mathbf{q},t-t') & G_\phi^{+-}(\mathbf{q},t-t') \\ G_\phi^{-+}(\mathbf{q},t-t') & G_\phi^{--}(\mathbf{q},t-t')\end{pmatrix} \\
&\quad\times\begin{pmatrix}\varphi_+^2(x') & 0 \\ 0 & -\varphi_-^2(x')\end{pmatrix}\begin{pmatrix} G_\phi^{++}(\mathbf{q}-\mathbf{k},t-t') & G_\phi^{+-}(\mathbf{q}-\mathbf{k},t-t') \\ G_\phi^{-+}(\mathbf{q}-\mathbf{k},t-t') & G_\phi^{--}(\mathbf{q}-\mathbf{k},t-t')\end{pmatrix}\begin{pmatrix}\varphi_+^2(x) & 0 \\ 0 & -\varphi_-^2(x)\end{pmatrix} \\
&= i\frac{\lambda^2}{16}\int d^4x d^4x'\int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\int\frac{d^3q}{(2\pi)^3}\Big[\varphi_+^2(x)G_\phi^{++}(\mathbf{q},t-t')G_\phi^{++}(\mathbf{q}-\mathbf{k},t-t')\varphi_+^2(x') \\
&\quad - \varphi_+^2(x)G_\phi^{+-}(\mathbf{q},t-t')G_\phi^{+-}(\mathbf{q}-\mathbf{k},t-t')\varphi_-^2(x') - \varphi_-^2(x)G_\phi^{-+}(\mathbf{q},t-t')G_\phi^{-+}(\mathbf{q}-\mathbf{k},t-t')\varphi_+^2(x') \\
&\quad + \varphi_-^2(x)G_\phi^{--}(\mathbf{q},t-t')G_\phi^{--}(\mathbf{q}-\mathbf{k},t-t')\varphi_-^2(x')\Big]\,.
\end{aligned} \tag{2.16}
$$

Equivalently, we get for the third graph in (2.13) the expression



$$
\begin{aligned}
&= -i\frac{\lambda^2}{8}\int\frac{d^3k}{(2\pi)^3}\frac{1+2n(\omega)}{2\omega(\mathbf{k})}\int d^4x\int dt'\int\frac{d^3q}{(2\pi)^3}\Big\{\varphi_+^2(x)\left[G_\phi^{++}(\mathbf{q},t-t')\right]^2 \\
&\quad - \varphi_+^2(x)\left[G_\phi^{+-}(\mathbf{q},t-t')\right]^2 - \varphi_-^2(x)\left[G_\phi^{-+}(\mathbf{q},t-t')\right]^2 + \varphi_-^2(x)\left[G_\phi^{--}(\mathbf{q},t-t')\right]^2\Big\}\,,
\end{aligned} \tag{2.17}
$$

whereas the fourth graph in (2.13) is given by



$$= i\frac{\lambda^2}{12}\int d^4x d^4x' \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\frac{d^3q_3}{(2\pi)^3}\delta(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3)$$
$$\times\Big\{\varphi_+(x)G_\phi^{++}(\mathbf{q}_1,t-t')G_\phi^{++}(\mathbf{q}_2,t-t')G_\phi^{++}(\mathbf{q}_3,t-t')\varphi_+(x')$$
$$-\varphi_+(x)G_\phi^{+-}(\mathbf{q}_1,t-t')G_\phi^{+-}(\mathbf{q}_2,t-t')G_\phi^{+-}(\mathbf{q}_3,t-t')\varphi_-(x')$$
$$-\varphi_-(x)G_\phi^{-+}(\mathbf{q}_1,t-t')G_\phi^{-+}(\mathbf{q}_2,t-t')G_\phi^{-+}(\mathbf{q}_3,t-t')\varphi_+(x')$$
$$+\varphi_-(x)G_\phi^{--}(\mathbf{q}_1,t-t')G_\phi^{--}(\mathbf{q}_2,t-t')G_\phi^{--}(\mathbf{q}_3,t-t')\varphi_-(x')\Big\}. \qquad (2.18)$$

Before continuing, it is advantageous to rewrite the field variables φ_+ and φ_- in (2.13) in terms of new field variables φ_c and φ_Δ, defined by

$$\varphi_+ = \frac{1}{2}\varphi_\Delta + \varphi_c\,,$$
$$\varphi_- = \varphi_c - \frac{1}{2}\varphi_\Delta\,. \qquad (2.19)$$

The physical meaning of these variables is suggested in Ref. [28], with φ_Δ being basically associated with a response field while φ_c is the physical field, which "feels" the fluctuations of the system. The change of variables (2.19) will allow us to identify, in the effective action, the terms responsible for the fluctuations in the system (the imaginary terms). The association of φ_c as the physical field imposes that we take $\varphi_\Delta = 0$ ($\varphi_+ = \varphi_-$) at the end of the calculation [23,13]. In terms of the new variables φ_c and φ_Δ, using (2.11) and (2.12), we get the following expression for the effective action (2.13), using the physical propagator $G_\phi^{++}(\mathbf{q},t-t')$ in the Feynman diagrams (2.14)–(2.18):

$$\Gamma[\varphi_\Delta,\varphi_c] = \int d^4x\Big\{\varphi_\Delta(x)\Big[-\Box - m^2 - \frac{\lambda}{2}\int\frac{d^3k}{(2\pi)^3}\frac{1+2n(\omega)}{2\omega(\mathbf{k})}$$
$$-\frac{\lambda^2}{2}\int dt'\int\frac{d^3q}{(2\pi)^3}\mathrm{Im}\left[G_\phi^{++}(\mathbf{q},t-t')\right]^2\theta(t-t')\int\frac{d^3k}{(2\pi)^3}\frac{1+2n(\omega)}{2\omega(\mathbf{k})}\Big]\varphi_c(x)$$
$$-\frac{\lambda}{4!}\left[4\varphi_\Delta(x)\varphi_c^3(x)+\varphi_\Delta^3(x)\varphi_c(x)\right]\Big\}$$
$$+\int d^4x d^4x'\int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\Big\{-\frac{\lambda^2}{8}\Big[\varphi_\Delta(x)\varphi_c(x)\varphi_\Delta^2(x')$$
$$+4\varphi_\Delta(x)\varphi_c(x)\varphi_c^2(x')\Big]\int\frac{d^3q}{(2\pi)^3}\mathrm{Im}\left[G_\phi^{++}(\mathbf{q},t-t')G_\phi^{++}(\mathbf{q}-\mathbf{k},t-t')\right]\theta(t-t')$$
$$-\frac{\lambda^2}{3}\varphi_\Delta(x)\varphi_c(x')\mathrm{Im}\left[\prod_{j=1}^{3}\int\frac{d^3q_j}{(2\pi)^3}G_\phi^{++}(\mathbf{q}_j,t-t')\right]\theta(t-t')\delta(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3)$$
$$+i\frac{\lambda^2}{4}\varphi_\Delta(x)\varphi_c(x)\varphi_\Delta(x')\varphi_c(x')\mathrm{Re}\int\frac{d^3q}{(2\pi)^3}\left[G_\phi^{++}(\mathbf{q},t-t')G_\phi^{++}(\mathbf{q}-\mathbf{k},t-t')\right]$$
$$+i\frac{\lambda^2}{12}\varphi_\Delta(x)\varphi_\Delta(x')\mathrm{Re}\left[\prod_{j=1}^{3}\int\frac{d^3q_j}{(2\pi)^3}G_\phi^{++}(\mathbf{q}_j,t-t')\right]\delta(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3)\Big\}. \qquad (2.20)$$

The last two terms in $\Gamma[\varphi_\Delta\varphi_c]$, Eq. (2.20), give the imaginary contributions to the effective action at the order of perturbation theory considered. It is straightforward to associate the imaginary terms in (2.20) as coming from functional integrations over Gaussian fluctuation fields ξ_1 and ξ_2 [13,15]:

$$\int D\xi_1 P[\xi_1]\int D\xi_2 P[\xi_2]\exp\Big\{i\int d^4x\left[\varphi_\Delta(x)\varphi_c(x)\xi_1(x)+\varphi_\Delta(x)\xi_2(x)\right]\Big\}$$
$$=\exp\Big\{i\int d^4xd^4x'\Big[i\frac{\lambda^2}{4}\varphi_\Delta(x)\varphi_c(x)\mathrm{Re}\left[G_\phi^{++}\right]^2_{x,x'}\varphi_\Delta(x')\varphi_c(x')$$
$$+i\frac{\lambda^2}{12}\varphi_\Delta(x)\mathrm{Re}\left[G_\phi^{++}\right]^3_{x,x'}\varphi_\Delta(x')\Big]\Big\}, \qquad (2.21)$$

where $P[\xi_1]$ and $P[\xi_2]$, the probability distributions for ξ_1 and ξ_2, respectively, are given by

$$P[\xi_1] = N_1^{-1} \exp\left\{ -\frac{1}{2} \int d^4x d^4x' \xi_1(x) \left(\frac{\lambda^2}{2} \mathrm{Re} \left[G_\phi^{++} \right]^2_{x,x'} \right)^{-1} \xi_1(x') \right\} , \tag{2.22}$$

$$P[\xi_2] = N_2^{-1} \exp\left\{ -\frac{1}{2} \int d^4x d^4x' \xi_2(x) \left(\frac{\lambda^2}{6} \mathrm{Re} \left[G_\phi^{++} \right]^3_{x,x'} \right)^{-1} \xi_2(x') \right\} , \tag{2.23}$$

where N_1^{-1} and N_2^{-1} are normalization factors, and in (2.21)–(2.23) we introduced the compact notation

$$\left[G_\phi^{++} \right]^2_{x,x'} = \int \frac{d^3k}{(2\pi)^3} \exp\left[i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}') \right] \int \frac{d^3q}{(2\pi)^3} G_\phi^{++}(\mathbf{q}, t-t') G_\phi^{++}(\mathbf{q} - \mathbf{k}, t - t') \tag{2.24}$$

and

$$\left[G_\phi^{++} \right]^3_{x,x'} = \int \frac{d^3k}{(2\pi)^3} \exp\left[i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}') \right] \left[\prod_{j=1}^{3} \int \frac{d^3q_j}{(2\pi)^3} G_\phi^{++}(\mathbf{q}_j, t - t') \right] \delta(\mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2 - \mathbf{q}_3) . \tag{2.25}$$

Therefore, using (2.21), Eq. (2.20) can be rewritten as

$$\Gamma[\varphi_\Delta, \varphi_c] = \frac{1}{i} \ln \int D\xi_1 P[\xi_1] \int D\xi_2 P[\xi_2] \exp\left\{ iS_{\text{eff}}[\varphi_\Delta, \varphi_c, \xi_1, \xi_2] \right\} , \tag{2.26}$$

where

$$S_{\text{eff}}[\varphi_\Delta, \varphi_c, \xi_1, \xi_2] = \mathrm{Re}\Gamma[\varphi_\Delta, \varphi_c] + \int d^4x \left[\varphi_\Delta(x)\varphi_c(x)\xi_1(x) + \varphi_\Delta(x)\xi_2(x) \right] , \tag{2.27}$$

and $\mathrm{Re}\Gamma[\varphi_\Delta, \varphi_c]$ is the real part of Eq. (2.20). In (2.27), the fields ξ_1 and ξ_2, with probability distributions given by (2.22) and (2.23), respectively, act as fluctuation sources for the scalar field configuration φ. ξ_1 couples with both the response field φ_Δ and with the physical field φ_c, leading to a coupled (multiplicative) noise term $(\varphi_c \xi_1)$ in the equation of motion for φ_c, while ξ_2 gives origin to an additive noise term. In the next section we examine the relevance of each of these noise terms in the equation of motion for the physical field φ_c and evaluate the dissipation coefficients associated with them.

III. THE EFFECTIVE EQUATION OF MOTION

The equation of motion for φ_c is defined by

$$\frac{\delta S_{\text{eff}}[\varphi_\Delta, \varphi_c, \xi_1, \xi_2]}{\delta \varphi_\Delta} |_{\varphi_\Delta = 0} = 0 \ . \tag{3.1}$$

Using (2.27) and (2.20), we obtain

$$\left[\Box + m^2 + \frac{\lambda}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1 + 2n(\omega)}{2\omega(\mathbf{k})} \left(1 + \lambda \int_{-\infty}^{t} dt' \int \frac{d^3q}{(2\pi)^3} \mathrm{Im} \left[G_\phi^{++}(\mathbf{q}, t - t') \right]^2 \right) \right] \varphi_c(x) + \frac{\lambda}{3!} \varphi_c^3(x)$$
$$+ \frac{\lambda^2}{2} \varphi_c(x) \int d^3x' \int_{-\infty}^{t} dt' \varphi_c^2(\mathbf{x}', t') \mathrm{Im} \left[G_\phi^{++} \right]^2_{x,x'} + \frac{\lambda^2}{3} \int d^3x' \int_{-\infty}^{t} dt' \varphi_c(\mathbf{x}', t') \mathrm{Im} \left[G_\phi^{++} \right]^3_{x,x'}$$
$$= \varphi_c(x)\xi_1(x) + \xi_2(x) , \tag{3.2}$$

where $\left[G_\phi^{++} \right]^2_{x,x'}$ and $\left[G_\phi^{++} \right]^3_{x,x'}$ are given by (2.24) and (2.25), respectively. In order to obtain a Langevin-like equation, a series of approximations must be performed in the above equation of motion. These approximations will certainly limit the scope of applicability of the final equation to be obtained (very much as in linear response theory), but on the other hand, will elucidate important aspects of the nonequilibrium physics. Strictly speaking, a Langevin-like equation can only be used to describe the nonequilibrium dynamics of slowly varying modes in near-equilibrium situations. To see this, we now focus on the last two terms on the left hand side of Eq. (3.2).

A. Dissipation coefficients

Let us first consider the term in the equation of motion dependent on $\left[G_\phi^{++} \right]^2_{x,x'}$. Inspecting (2.24), it is clear that the spatial nonlocality can be handled by considering only contributions with zero external momentum, as in the computation of linear response functions [24]. This is what is usually done in the computation of the one-loop effective potential as an expansion of vertex functions with zero external momentum, which is physically equivalent to considering only nearly spatially homogeneous fields. We thus obtain

$$\frac{\lambda^2}{2}\varphi_c(x)\int d^3x'\int_{-\infty}^{t} dt'\varphi_c^2(\mathbf{x}',t')\mathrm{Im}\left[G_\phi^{++}\right]^2_{x,x'} = \frac{\lambda^2}{2}\varphi_c(\mathbf{x},t)\int_{-\infty}^{t} dt'\left[\varphi_c^2(\mathbf{x},t')-\varphi_c^2(\mathbf{x},t)\right]\int\frac{d^3q}{(2\pi)^3}\mathrm{Im}\left[G_\phi^{++}(\mathbf{q},t-t')\right]^2 + \frac{\lambda^2}{2}\varphi_c^3(\mathbf{x},t)\int_{-\infty}^{t} dt'\int\frac{d^3q}{(2\pi)^3}\mathrm{Im}\left[G_\phi^{++}(\mathbf{q},t-t')\right]^2 , \tag{3.3}$$

where we have summed and subtracted in (3.3) the last term on the RHS. In order to handle the temporal nonlocality let us further assume that φ_c varies sufficiently slowly in time, so that we can expand the first term on the right-hand side (RHS) of (3.3) to first order around t. This is a valid assumption for systems near equilibrium, when φ_c is not expected to change considerably with time. (This has been called the quasiadiabatic approximation in Refs. [13,29]. We then obtain

$$\frac{\lambda^2}{2}\varphi_c(x)\int d^3x'\int_{-\infty}^{t} dt'\varphi_c^2(\mathbf{x}',t')\mathrm{Im}\left[G_\phi^{++}\right]^2_{x,x'} \simeq \lambda^2\varphi_c^2(\mathbf{x},t)\dot\varphi_c(\mathbf{x},t)\int_{-\infty}^{t} dt'(t'-t)\int\frac{d^3q}{(2\pi)^3}\mathrm{Im}\left[G_\phi^{++}(\mathbf{q},t-t')\right]^2 + \frac{\lambda^2}{2}\varphi_c^3(\mathbf{x},t)\int_{-\infty}^{t} dt'\int\frac{d^3q}{(2\pi)^3}\mathrm{Im}\left[G_\phi^{++}(\mathbf{q},t-t')\right]^2 . \tag{3.4}$$

The emergence of a time direction within this approximation is surely related to neglecting the faster moving modes in the description of the dynamics. This is an interesting question which deserves further study, but that we will not address in the present work. The last term on the left-hand side of Eq. (3.2) can also be worked out as in (3.3) and (3.4) and we obtain

$$\frac{\lambda^2}{3}\int d^3x'\int_{-\infty}^{t} dt'\varphi_c(\mathbf{x}',t')\mathrm{Im}\left[G_\phi^{++}\right]^3_{x,x'} \simeq \frac{\lambda^2}{3}\dot\varphi_c(\mathbf{x},t)\int_{-\infty}^{t} dt'(t'-t)\mathrm{Im}\left[\prod_{j=1}^{3}\int\frac{d^3q_j}{(2\pi)^3}G_\phi^{++}(\mathbf{q}_j,t-t')\right]\delta(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3) + \frac{\lambda^2}{3}\varphi_c(\mathbf{x},t)\int_{-\infty}^{t} dt'\mathrm{Im}\left[\prod_{j=1}^{3}\int\frac{d^3q_j}{(2\pi)^3}G_\phi^{++}(\mathbf{q}_j,t-t')\right]\delta(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3) . \tag{3.5}$$

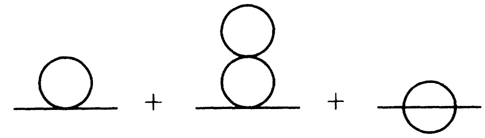
The first terms on the RHS of (3.4) and (3.5) are the corresponding dissipative terms associated with the fluctuation fields ξ_1 and ξ_2, respectively. The last term on the RHS of (3.4) is the one-loop finite temperature correction to the vertex [second graph in (2.13)], while the last term on the RHS of (3.5) is the contribution to the finite temperature two-loop correction to the mass coming from the "setting sun" diagram [the last graph in (2.13)]. The time integrations in (3.4) and (3.5) can be easily performed by using the expression for $G_\phi^{++}(\mathbf{q},t-t')$ given in (2.11), and by changing the time integration variable to $t-t'=t''$. However, if when computing the above dissipation terms we use the free propagator expressions given in (2.12), we would find that they both vanish, as can be explicitly checked. The results would be quite different if, instead of free propagators, we use dressed propagators. Self-energy corrections in the propagator introduce nontrivial effects (damping) due to the imaginary contributions to the self-energy: $\Sigma(q)=\mathrm{Re}\Sigma(q)+i\,\mathrm{Im}\Sigma(q)$. The first contribution to $\mathrm{Im}\Sigma$ comes from the "setting sun" diagram, being therefore of the order λ^2. Higher loop contributions to the self-energy are of higher order in λ and can be consistently neglected for weak couplings and within the order of perturbation on which we are working on. Writing the corrected propagator $G_\phi^{++}(\mathbf{q},t-t')$ in terms of the spectral function $\rho(\mathbf{q},q_0)$ (Appendix A), it is possible to show that due to the nonvanishing $\mathrm{Im}\Sigma$, $\rho(\mathbf{q},q_0)$ acquires a finite width, $\Gamma(q)$, which is $O(\lambda^2)$ for weak couplings. $\Gamma(q)$ will generate finite contributions to the terms proportional to $\dot\varphi_c$ in Eqs. (3.4) and (3.5), as we show next. This is essentially the same nonperturbative procedure adopted in Refs. [12,22,24] in the computation of dissipation coefficients. Although the introduction of the full propagator gives us a nonvanishing dissipation coefficient, its implementation must be done with some care. There have been suggestions recently that an improved finite temperature effective potential which includes leading infrared divergent terms could be obtained by a dressing of the field propagator, in such a way that daisy and superdaisy diagrams would be accounted for (see, e.g., the papers in [5]). It was then shown that if this resummation procedure was not implemented carefully, one would overcount diagrams, giving rise to wrong results, such as an effective potential with a linear term in the scalar field. Although our focus here is on dynamical issues, an improper introduction of full propagators may also generate an overcounting of terms in the final effective equation of motion that we derive. In order to deal with this problem in a self-consistent way, we apply the procedure of Parwani, Ref. [30], and of Arnold and Espinosa in Ref. [5], for the problem of resummation at high temperatures. We rewrite the Lagrangian density $\mathcal{L}[\phi]$ in (2.1) as

$$\mathcal{L}[\phi] \to \mathcal{L}[\phi] = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \frac{1}{2}\left(m^2+\Sigma\right)\phi^2 - \frac{\lambda}{4!}\phi^4 + \frac{1}{2}\Sigma\phi^2 \,, \tag{3.6}$$

where we have added and subtracted the self-energy-dependent term from $\mathcal{L}[\phi]$. We can now treat $\frac{1}{2}(m^2+\Sigma)\phi^2$ as part of the field propagator and $\frac{\lambda}{4!}\phi^4 - \frac{1}{2}\Sigma\phi^2$ as the interaction term. By doing this we can systematically and self-consistently rewrite our equations in terms of the dressed propagator and at the same time remove any extra overcounting generated by the dressing, through the modified interaction, as illustrated in Appendix B. We thus write the dressed propagator as

$$\frac{1}{q^2-m^2+i\epsilon} \to \frac{1}{q^2-m^2-\Sigma(q)+i\epsilon} \,, \tag{3.7}$$

where $\Sigma(q)$ is the self-energy contribution,

$\Sigma(q) =$ 

$$+\mathcal{O}(\lambda^3) \,. \tag{3.8}$$

In what follows, we will consider explicitly only contributions to $\Sigma(q)$ from diagrams up to $O(\lambda^2)$.

In Appendix A we show that the physical propagator $G_\phi^{++}(\mathbf{q}, t-t')$ is then changed to

$$G_\phi^{++}(\mathbf{q},t-t') \simeq \frac{e^{-\Gamma(\mathbf{q})|t-t'|}}{2\omega(\mathbf{q})}\left[(1+2n)\cos[\omega|t-t'|] - i\sin[\omega|t-t'|] + 2\beta\Gamma(\mathbf{q})n(1+n)\sin[\omega|t-t'|] + O\left(\frac{\Gamma^2}{T^2}\right)\right] , \tag{3.9}$$

where $\Gamma(\mathbf{q})$ is the particle decay width [27]

$$\Gamma(q) = -\frac{\mathrm{Im}\Sigma(q)}{2\omega(q)} \tag{3.10}$$

and in (3.9) we used the approximation $\beta\Gamma \ll 1$ (see Appendix A), which is consistent with slow relaxation time scales. In (3.9), $\omega \equiv \omega(\mathbf{q})$ and $n(\omega)$ are now given in terms of the finite temperature effective mass m_T:

$$m_T^2 = m^2 + \mathrm{Re}\Sigma(m_T) \overset{T\gg m}{\simeq} m^2 + \frac{\lambda}{2}\left(\frac{T^2}{12} - \frac{mT}{4\pi}\right) - \frac{\lambda^2 T^3}{384\pi m} + \frac{\lambda^2 T^2}{192\pi^2}\ln\left(\frac{m^2}{T^2}\right) + \cdots \,, \tag{3.11}$$

where we have only written explicitly the main thermal contributions from each of the terms in (3.8) and we have neglected subdominant contributions. The second and third terms on the RHS of (3.11) are easily obtained. The last term on the RHS of (3.11), associated with the "setting sun" diagram, is explicitly evaluated in [30].

Using the dressed propagator (3.9) in the expression for the dissipation term (3.4) and performing the integration in t', we obtain, to order λ^2,

$$\begin{aligned} \frac{\lambda^2}{2}\varphi_c(x)\int d^3x' \int_{-\infty}^{t} dt' \varphi_c^2(\mathbf{x},t') \mathrm{Im}\left[G_\phi^{++}\right]^2_{x,x'} & \\ \simeq \frac{\lambda^2}{8}\varphi_c^2(\mathbf{x},t)\dot{\varphi}_c(\mathbf{x},t)\,\beta\int \frac{d^3q}{(2\pi)^3}\frac{n(1+n)}{\omega^2(\mathbf{q})\Gamma(\mathbf{q})} & \\ -\frac{\lambda^2}{2}\varphi_c^3(\mathbf{x},t)\int\frac{d^3q}{(2\pi)^3}\frac{1}{4\omega^2(\mathbf{q})}\left[\frac{1+2n}{2\omega(\mathbf{q})} + \beta n(1+n)\right] + O\left(\lambda^2\frac{\Gamma}{\omega}\right) \,. \end{aligned} \tag{3.12}$$

The first term on the RHS of (3.12) gives the dissipation term, $\eta_1\varphi_c^2\dot{\varphi}_c$, with dissipation coefficient η_1 given by

$$\eta_1 = \frac{\lambda^2}{8}\beta\int\frac{d^3q}{(2\pi)^3}\frac{n(\omega)\left[1+n(\omega)\right]}{\omega^2(\mathbf{q})\Gamma(\mathbf{q})} + \mathcal{O}\left(\lambda^2\frac{\Gamma}{\omega}\right) \,. \tag{3.13}$$

The second term on the RHS in (3.12) clearly gives just the one-loop finite temperature vertex correction. In order to obtain (3.12) we have performed an expansion to first order in powers of Γ/ω, consistent with slowly varying modes. Also, since $\Gamma \propto O(\lambda^2)$, we have omitted the

$O(\lambda^4)$ contributions. The expression for the dissipation coefficient can be further simplified if we consider the high temperature limit $T \gg m_T$. As shown in Refs. [12,30] the high temperature limit of $\Gamma(\mathbf{q})$ is

$$\Gamma \simeq \frac{\lambda^2 T^2}{1536\pi\omega(\mathbf{q})} . \tag{3.14}$$

Using (3.14) in (3.13), we obtain, for η_1, in the high temperature limit,

$$\eta_1 \overset{T\gg m_T}{\simeq} \frac{96}{\pi T} \ln\left(\frac{T}{m_T}\right) , \tag{3.15}$$

which shows that the dissipation coefficient associated with the multiplicative noise field ξ_1 is, in this limit, only weakly (logarithmically) dependent on the coupling constant λ.

We can proceed in an analogous way and evaluate Eq. (3.5) in order to obtain the expression for the dissipation coefficient associated with the second fluctuation (noise) field ξ_2, from the first term on the RHS of (3.5). From the second term we can obtain the two-loop mass correction coming from the fourth graph in (2.13). Substituting Eq. (3.9) for $G_\phi^{++}(\mathbf{q}_j, t-t')$ in (3.5) and performing the integration in t', it is possible to show (see Appendix C) that the dissipation coefficient associated with ξ_2 is at least of the order $\lambda^2\Gamma(\mathbf{q}_j) = O(\lambda^4)$. Therefore, in a weakly interacting model, the dominant contribution to dissipation in the equation of motion for φ_c comes from the dissipation term associated with the multiplicative noise field, ξ_1.

B. The effective Langevin-like equation of motion

Up to two loops and $O(\lambda^2)$, at zero external momentum and within the adiabatic approximation, we obtain, from (3.2), the following equation of motion for φ_c:

$$\left[\Box + m_T^2\right] \varphi_c(\mathbf{x},t) + \frac{\lambda_T}{3!}\varphi_c^3(\mathbf{x},t) + \eta_1\varphi_c^2(\mathbf{x},t)\dot{\varphi}_c(\mathbf{x},t) = \varphi_c(\mathbf{x},t)\xi_1(\mathbf{x},t) , \tag{3.16}$$

where η_1 is given by (3.15), m_T and λ_T are the renormalized finite temperature mass and coupling constant, respectively, obtained from the renormalized effective action, Eq. (2.27). The renormalization of $S_{\rm eff}$ can be defined by the usual introduction of counterterms in the initial Lagrangian, Eq. (2.1), by writing $\mathcal{L} \to \mathcal{L}+\delta\mathcal{L}$, where $\delta\mathcal{L} = \frac{1}{2}\mathcal{Z}(\partial_\mu\phi)^2 - \frac{1}{2}\delta m^2\phi^2 - \frac{\delta\lambda}{4!}\phi^4$, with $\mathcal{Z}$, δm^2, and $\delta\lambda$ being the wave function, mass, and vertex renormalization counterterms, respectively. $\delta\lambda$ cancels the logarithmic divergence of the one-loop vertex correction, while $\mathcal{Z}$ and δm^2 renormalize the self-energy contribution, Eq. (3.8). In the high temperature limit, m_T is given by Eq. (3.11) and λ_T is given by

$$\lambda_T \simeq \lambda - \frac{3\lambda^2}{2}\left\{\frac{T}{8\pi m} + \frac{1}{8\pi^2}\left[\ln\left(\frac{m}{4\pi T}\right) + \gamma\right] + O\left(\frac{m}{T}\right)\right\} + O(\lambda^3) . \tag{3.17}$$

Equation (3.16) can also be written in terms of a finite temperature effective potential $V_{\rm eff}(\varphi_c, T)$:

$$\Box\varphi_c + V'_{\rm eff}(\varphi_c,T) + \frac{96}{\lambda^2\pi T}\ln\left(\frac{T}{m_T}\right)\left[V^{(3)}(\varphi_c)\right]^2\dot{\varphi}_c = \varphi_c\xi_1 , \tag{3.18}$$

where $V^{(3)}(\varphi_c) = \frac{d^3V[\phi]}{d\phi^3}|_{\varphi_c}$.

Note that this equation, apart from the important multiplicative noise source on the RHS, is analogous to the one obtained by Hosoya and Sakagami, using quite different methods, for the evolution of the thermal average of the scalar field φ_c (Ref. [12]).

From the equation for the probability distribution for the fluctuation field ξ_1, $P[\xi_1]$, Eq. (2.22), the two-point correlation function for $\xi_1(x)$ is given by

$$\langle\xi_1(x)\xi_1(x')\rangle = \frac{\lambda^2}{2}\mathrm{Re}\left[G_\phi^{++}\right]^2_{x,x'} . \tag{3.19}$$

Using (2.24) and (3.9), we obtain for the two-point correlation function (3.19) the expression (at zero external momentum)

$$\langle\xi_1(x)\xi_1(x')\rangle = \frac{\lambda^2}{2}\delta^3(\mathbf{x}-\mathbf{x}')\int\frac{d^3q}{(2\pi)^3}\frac{1}{4\omega^2(\mathbf{q})}\left\{2n(\omega)\left[1+n(\omega)\right] + \left[1+2n(\omega)+2n^2(\omega)\right]\cos\left[2\omega|t-t'|\right] + 2\beta\Gamma(\mathbf{q})n(\omega)[1+n(\omega)][1+2n(\omega)]\sin[2\omega|t-t'|]\right\}e^{-2\Gamma(\mathbf{q})|t-t'|} + O\left(\lambda^2\frac{\Gamma^2}{T^2}\right) , \tag{3.20}$$

which shows that the noise is colored (time dependent), although it is Gaussian distributed. Up to order λ^2 and for $\Gamma/\omega \ll 1$, $\Gamma/T \ll 1$, we obtain the fluctuation-dissipation relation

$$\eta_1 = \frac{1}{T}\int d^4x'\langle\xi_1(x)\xi_1(x')\rangle\theta(t-t') . \tag{3.21}$$

We can also obtain the Markovian limit of (3.20), that is, the limit in which the noise is uncorrelated (white). Note that as $T\to\infty$, $\Gamma\to\infty$, and thus the integrand becomes sharply peaked at $|t-t'|\sim 0$. In this limit, we can approximate (3.20) by

$$\langle\xi_1(x)\xi_1(x')\rangle \overset{T\to\infty}{\longrightarrow} \frac{\lambda^2}{2}\delta(\mathbf{x}-\mathbf{x}')\delta(t-t') \times\int\frac{d^3q}{(2\pi)^3}\frac{n(\omega)[1+n(\omega)]}{2\omega^2(\mathbf{q})\Gamma(\mathbf{q})} = 2T\eta_1\delta(\mathbf{x}-\mathbf{x}')\delta(t-t') , \tag{3.22}$$

where η_1 is given by (3.13). Equation (3.22) is the standard expression of the fluctuation-dissipation theorem for a Gaussian white noise.

IV. COUPLING THE SCALAR FIELD TO OTHER FIELDS

The previous computation of the effective equation of motion for the field configuration φ_c can be generalized to include the effects of interactions with other fields. As an example, consider the Lagrangian density for the scalar field ϕ interacting quadratically with another scalar field χ:

$$\mathcal{L}[\phi,\chi] = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}(\partial_\mu\chi)^2 - V[\phi,\chi]\,, \tag{4.1}$$

with potential $V[\phi,\chi]$ given by

$$V[\phi,\chi] = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{\mu^2}{2}\chi^2 + \frac{f}{4!}\chi^4 + \frac{g^2}{2}\phi^2\chi^2\,, \tag{4.2}$$

where m^2 and μ^2 are positive. This model is a good toy model for several physical cases of interest. For example, for some relations among the values of the coupling constants λ, f, and g^2 (e.g., $\lambda = O(g^4)$, $f = \mathcal{O}(g^2)$ (Ref. [31])), Eq. (4.1) exhibits the properties of Coleman-Weinberg models, for which the quantum corrections coming from integrating out the χ field break the symmetry in the potential for the scalar field ϕ (corrected by the χ-loop quantum corrections), modifying the original vacuum structure of the model. Also, as pointed out by Hu, Paz, and Zhang [14], Eq. (4.1) can mimic, at lowest order (one loop) in the χ-loop quantum corrections, a coarse-grained effective model for the scalar field ϕ, after integrating out the χ field. In this case, the ϕ field would represent the field with components containing the long wavelength modes, while χ would contain the short wavelength modes, with a cutoff determined by some scale Λ. In inflationary models, ϕ would behave as a classical field, while χ would represent the subhorizon high frequency modes [32,33,14]. The authors in [14] thus consider the field χ as the quantum bath (at $T=0$), allowing them to obtain an effective action for the scalar field ϕ (the classical action corrected by the χ field one loop quantum corrections), where the scalar field is coupled to a noise field, very much like the multiplicative noise field ξ_1 in Eq. (2.27). Following the results of the last section, the generalization of their results to $T \neq 0$ is relatively simple. Up to one-loop in the χ field, the effective action $\Gamma[\varphi]$ in (2.13) (also called the influence functional by some authors), will be given by

$$\Gamma[\varphi] \to \Gamma[\varphi] + \frac{1}{2} i \mathrm{Tr}_c \ln\left[\Box + \mu^2 + g^2\varphi^2\right]\,. \tag{4.3}$$

Expanding the logarithm in (4.3) as in (2.8), up to order g^4, we will get expressions analogous to the ones in (2.15) and (2.16), with φ_+, φ_- in the external legs and internal propagators for the χ field, $G_\chi^{n,l}(x-x')$ $(n,l=+,-)$, with expressions just as in (2.10) and (2.11). By changing the field variables φ_+, φ_- to φ_Δ, φ_c as before, the contribution from the χ field to the effective action for ϕ, $\Gamma[\varphi_\Delta,\varphi_c]$, Eq. (2.20), will be

$$\begin{aligned}\Gamma[\varphi_\Delta,\varphi_c] \to \Gamma[\varphi_\Delta,\varphi_c] &- g^2\int d^4x\varphi_\Delta(x)\varphi_c(x)\int\frac{d^3k}{(2\pi)^3}\frac{1+2n_\chi}{\omega_\chi}\\ &-\frac{g^4}{2}\int d^4x d^4x'\left[\varphi_\Delta(x)\varphi_c(x)\varphi_\Delta^2(x') + 4\varphi_\Delta(x)\varphi_c(x)\varphi_c^2(x')\right]\mathrm{Im}\left[G_\chi^{++}\right]^2_{x,x'}\theta(t-t')\\ &+ig^4\int d^4x d^4x'\varphi_\Delta(x)\varphi_c(x)\varphi_\Delta(x')\varphi_c(x')\mathrm{Re}\left[G_\chi^{++}\right]^2_{x,x'}\,,\end{aligned} \tag{4.4}$$

where $\left[G_\chi^{++}\right]^2_{x,x'}$ is given by an expression analogous to (2.24).

The imaginary term in (4.4), coming from integrating out the χ field (at one-loop order) can be rewritten by redefining the fluctuation field ξ_1 in (2.27), such that its probability distribution in (2.22) is changed to

$$P[\xi_1] = N_1^{-1}\exp\left\{-\frac{1}{2}\int d^4x d^4x'\xi_1(x)\left[\frac{\lambda^2}{2}\mathrm{Re}\left[G_\phi^{++}\right]^2_{x,x'} + 2g^4\mathrm{Re}\left[G_\chi^{++}\right]^2_{x,x'}\right]^{-1}\xi_1(x')\right\} \tag{4.5}$$

and the two-point correlation function for ξ_1 is now given by

$$\langle\xi_1(x)\xi_1(x')\rangle = \frac{\lambda^2}{2}\mathrm{Re}\left[G_\phi^{++}\right]^2_{x,x'} + 2g^4\mathrm{Re}\left[G_\chi^{++}\right]^2_{x,x'}\,. \tag{4.6}$$

In the equation of motion for φ_c, from (4.4), we will have an additional contribution to the dissipation coefficient η_1, obtained from a term analogous to (3.4):

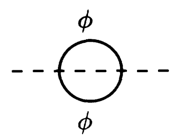
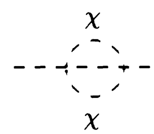
$$\begin{aligned}2g^4\int d^4x'\varphi_c^2(x')\mathrm{Im}\left[G_\chi^{++}\right]^2_{x,x'}\theta(t-t') \simeq\; &4g^4\varphi_c^2(\mathbf{x},t)\dot\varphi_c(\mathbf{x},t)\int_{-\infty}^{t}dt'(t'-t)\int\frac{d^3q}{(2\pi)^3}\mathrm{Im}\left[G_\chi^{++}(\mathbf{q},t-t')\right]^2\\ &+2g^4\varphi_c^3(\mathbf{x},t)\int_{-\infty}^{t}dt'\int\frac{d^3q}{(2\pi)^3}\mathrm{Im}\left[G_\chi^{++}(\mathbf{q},t-t')\right]^2\,.\end{aligned} \tag{4.7}$$

The first term on the RHS in (4.7) gives the contribution to the dissipation coefficient η_1, due to the interaction of the scalar field ϕ with the χ field. The second term in (4.7), together with the second term on the RHS in (4.4), gives the corrections of the order g^4 and g^2 to the scalar field ϕ vertex and mass, respectively, due to the χ-loop quantum corrections.

As in (3.4), in order to obtain a nonvanishing contribution to the dissipation coefficient coming from (4.7), we must consider the dressed propagator G_χ^{++} for the χ field, instead of the free propagator. As in the last section, the introduction of the dressed propagator has to be done self-consistently in order to avoid any overcounting problem. $G_\chi^{++}(\mathbf{q}, t-t')$ has an expression similar to the one given for the scalar field ϕ, Eq. (A8) [or (3.9), for $\beta\Gamma_\chi \ll 1$], where, at $O(g^4,\ f^2)$, the decay width Γ_χ can be written as

$$\Gamma_\chi(q) = -\frac{\mathrm{Im}\Sigma_\chi(\mathbf{q}, \omega_\chi)}{2\omega_\chi(q)}\ , \tag{4.8}$$

with the imaginary part of the χ-field self-energy, from (A7), given by the imaginary part of the two two-loop contributions below:

$\mathrm{Im}\Sigma_\chi = \mathrm{Im}\Big[$  $\Big] + \mathrm{Im}\Big[$  $\Big]$

$$= -\left(1 - e^{-\beta q_0}\right)\left[\prod_{j=1}^{3}\int \frac{d^4k_j}{(2\pi)^4}\left[1 + n(k_j^0)\right]\right]$$
$$\times \left(g^4 \rho_\phi(k_1)\rho_\chi(k_2)\rho_\phi(k_3) + \frac{f^2}{12}\rho_\chi(k_1)\rho_\chi(k_2)\rho_\chi(k_3)\right)$$
$$\times (2\pi)^4 \delta^4(q - k_1 - k_2 - k_3)\ , \tag{4.9}$$

where $\rho_\phi(k)$ is given by (A2), with m^2 and λ corrected by the χ-loop ($T \neq 0$) quantum corrections. $\rho_\chi(k)$ is the spectral function for the scalar field χ, with expression analogous to the one for the scalar field ϕ, given by Eq. (A2), but now with Γ_χ given by (4.8) and ω_χ given by the solution of $\omega_\chi^2(q) = \mathbf{q}^{\,2} + \mu^2 + \mathrm{Re}\Sigma_\chi(\mathbf{q}, \omega_\chi)$.

The high temperature limit of (4.8) is analogous to the one for the case with one scalar field ϕ, Eq. (3.14):

$$\Gamma_\chi(\mathbf{q}) \simeq \frac{T^2}{128\pi\omega_\chi(\mathbf{q})}\left(g^4 + \frac{f^2}{12}\right)\ , \tag{4.10}$$

with $\mu_T^2 = \mu^2 + \mathrm{Re}\Sigma_\chi(\mu)$. Using these in (4.7), we obtain an equation of motion for φ_c still written as in (3.16), up to two loops and order λ^2 in the scalar field ϕ and up to one loop[1] and order g^4 in the scalar field χ. The dissipation coefficient η_1 is given by

$$\eta_1 = \frac{\lambda^2}{8}\beta \int \frac{d^3q}{(2\pi)^3}\frac{n_\phi(1+n_\phi)}{\omega_\phi^2(\mathbf{q})\Gamma_\phi(\mathbf{q})}$$
$$+\frac{g^4}{2}\beta \int \frac{d^3q}{(2\pi)^3}\frac{n_\chi(1+n_\chi)}{\omega_\chi^2(\mathbf{q})\Gamma_\chi(\mathbf{q})}$$
$$+O\left(\lambda^2\frac{\Gamma_\phi}{\omega_\phi}\right) + O\left(g^4\frac{\Gamma_\chi}{\omega_\chi}\right)$$
$$\simeq \frac{96}{\pi T}\left[\ln\left(\frac{T}{m_T}\right) + \frac{4g^4}{12g^4 + f^2}\ln\left(\frac{T}{\mu_T}\right)\right]\ , \tag{4.11}$$

with the second correction for η_1 coming from the χ-ϕ interaction in (4.1). Associated with this modified dissipation term there is a modified multiplicative fluctuation (noise) field ξ_1, with probability distribution given by (4.5). For a Coleman-Weinberg potential, we have that $\lambda = O(g^4)$ and $f = O(g^2)$, so that the dissipation coefficient is, as in (3.15), weakly dependent on the coupling constants within our approximations. Using the expression for the two-point correlation function for ξ_1, Eq. (4.6), both the fluctuation-dissipation relation, Eq. (3.21), and the Markovian limit expression, Eq. (3.22), still hold.

V. CONCLUSIONS

In this work we have studied the nonequilibrium dynamics of a self-coupled scalar field. Even though our formalism is in principle applicable in situations far from equilibrium, the effective Langevin-like equation we obtained is only adequate to study the approach to equilibrium if the initial conditions are not too far from equilibrium. This limitation is essentially due to the use of perturbation theory and should come as no surprise. However, this approach clarifies many important issues concerning nonequilibrium fields and the nature of the system-bath coupling. By integrating over fluctuations in order to obtain the effective action, it becomes clear in what sense the short wavelength modes can function as the thermal bath that drives the longer wavelength modes into equilibrium. In this sense, the approximations employed in order to obtain a Langevin-like

[1]Up to two loops in χ the situation would be identical as discussed for the scalar field ϕ, with results similar to the last section and that of Appendix C.

equation are consistent with this system-bath separation; longer wavelength modes have slower dynamics and are responsible for the large-distance coherent behavior observed during the approach to equilibrium both in the laboratory and in numerical simulations. By going to higher order in perturbation theory, we were able to obtain the contributions to the noise and dissipation terms coming from different diagrams and their relevance to the nonequilibrium dynamics.

We found that the Langevin-like equation describing the approach to equilibrium both for a self-coupled scalar field and for quadratic coupling with other fields is quite different from the usual phenomenological form with Gaussian white noise used so far in numerical simulations of the approach to equilibrium in field theory. There are basically three differences. The first is that the dominant contribution to the noise is multiplicative; it couples quadratically to the field, acting as a "noisy" source to the mass term in the equation of motion. The second difference is that even though this noise is still Gaussian distributed, it is now non-Markovian; the correlation times depend on the decay width of the fluctuations generating the noise. As we show in the text, only in the limit of very high temperatures the noise becomes white, as one would naively expect. The final difference has to do with the way the dissipation term appears in the equation of motion. Instead of the simple $\eta\dot{\phi}$ term, we find instead that the dissipation "coefficient" depends quadratically on the amplitude of the field, $\eta(T)\phi^2\dot{\phi}$. In the high temperature limit for a single scalar field we obtained that $\eta(T) \sim (1/T)\ln(T/m_T)$, being thus weakly dependent on the coupling constant. This result is in agreement with the work of Ref. [12], which assumed a small departure from equilibrium within a kinetic approach. Both results are consistent with linear response theory commonly used to obtain transport coefficients in field theories [24].

By studying the effects of another scalar field quadratically coupled to ϕ, we were able to obtain their different contributions to the noise and dissipation terms in the effective equation of motion. Now, the coefficient of the dissipation term depends on ratios of couplings, as one would expect in more realistic situations, while the noise is still Gaussian and multiplicative. In both cases, we showed that one can recover a fluctuation-dissipation relation. It will be interesting to investigate the implications of this Langevin-like equation to the equilibration time scales during phase transitions, by employing it in numerical simulations. Apart from studying the approach to equilibrium from near-equilibirum initial conditions, it is possible to use this equation in the study of finite temperature symmetry restoration, if one takes into account the effects of expanding about the broken-symmetric vacuum. In this connection, it is interesting to note that the coefficient of the dissipation term, in the high-temperature limit, displays the typical critical slowing down (poor infrared behavior) observed in many second-order phase transitions; since $\eta \sim \ln(T/m_T)$, as the critical temperature is approached from below, the temperature corrected mass vanishes and the viscosity diverges logarithmically. We leave as an open question the potential impact that a better understanding of nonequilibrium dynamics of field theories will have on our current modeling of primordial phase transitions and their possible observational consequences. However, we believe that interesting physics is lurking behind our present level of understanding of nonequilibrium physical processes that took place in the early Universe.

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APPENDIX A

In this appendix we obtain the expression for the dressed scalar field propagator, Eq. (3.9). The finite temperature, real-time propagator $G_\phi^{++}(\mathbf{q}, t-t')$, can be written in terms of the spectral function $\rho(\mathbf{q}, q_0)$ [27,24],

$$G_\phi^{++}(\mathbf{q}, t-t') = \int_{-\infty}^{+\infty} \frac{dq_0}{2\pi} e^{-iq_0(t-t')} \rho(\mathbf{q}, q_0) \times \{[1+n(q_0)]\,\theta(t-t') + n(q_0)\theta(t'-t)\}\,, \tag{A1}$$

where $n(q_0) = \frac{1}{e^{\beta q_0}-1}$ and the spectral function, for the dressed propagator (3.7), is

$$\rho(\mathbf{q}, q_0) = i\left[\frac{1}{(q_0+i\Gamma)^2-\omega^2(q)} - \frac{1}{(q_0-i\Gamma)^2-\omega^2(q)}\right], \tag{A2}$$

where $\omega(q)$ is the solution of $\omega^2(q) = \mathbf{q}^2 + m^2 + \mathrm{Re}\Sigma(\mathbf{q}, \omega)$ and $\Sigma(q)$ is the scalar field self-energy, given by (3.8), up to two loops. The spectral function (A2) has a peak at $q_0 = \omega(q)$, with a width given by $\Gamma \equiv \Gamma(q)$:

$$\Gamma(q) = -\frac{\mathrm{Im}\Sigma(\mathbf{q}, \omega)}{2\omega(q)}\,. \tag{A3}$$

For the free propagator,

$$\rho(\mathbf{q}, q_0) = i\left[\frac{1}{(q_0+i\epsilon)^2-\mathbf{q}^2-m^2} - \frac{1}{(q_0-i\epsilon)^2-\mathbf{q}^2-m^2}\right] \tag{A4}$$

and $\rho(\mathbf{q}, q_0) \stackrel{\epsilon\to 0}{\longrightarrow} 2\pi\varepsilon(q_0)\delta(q^2-m^2)$, where $\varepsilon(q_0) = \theta(q_0) - \theta(-q_0)$. Substituting (A4) in (A1), we obtain the free propagator expressions in (2.11) and (2.12).

Equation (A2) has four poles in the complex q_0 plane: $\omega \pm i\Gamma$ and $-\omega \pm i\Gamma$. Using (A2) in (A1) and performing the q_0 integration, we obtain

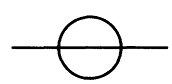
$$G_\phi^{++}(\mathbf{q}, t-t') = G_\phi^{>}(\mathbf{q}, t-t')\theta(t-t') + G_\phi^{<}(\mathbf{q}, t-t')\theta(t'-t) \, , \tag{A5}$$

where

$$\begin{aligned} G_\phi^{>}(\mathbf{q}, t-t') &= \frac{1}{2\omega}\left\{[1+n(\omega - i\Gamma)]\, e^{-i(\omega-i\Gamma)(t-t')} + n(\omega+i\Gamma)e^{i(\omega+i\Gamma)(t-t')}\right\} , \\ G_\phi^{<}(\mathbf{q}, t-t') &= G_\phi^{>}(\mathbf{q}, t'-t) \, . \end{aligned} \tag{A6}$$

The expressions for G_ϕ^{--}, G_ϕ^{+-} and G_ϕ^{-+} are the same as in (2.11), but with $G_\phi^{>,<}$ given now by (A6).

$\Gamma(q)$ is given in terms of the imaginary part of the self-energy [with first nontrivial contribution, $O(\lambda^2)$, coming from the third graph in (3.8)] by (A3), where $\mathrm{Im}\Sigma(q)$ is [22,24]

$\mathrm{Im}\Sigma(q) = \mathrm{Im}\Big[$  $\Big]$

$$= -\frac{\lambda^2}{12}\left(1-e^{-\beta q_0}\right)\left[\prod_{j=1}^{3}\int \frac{d^4k_j}{(2\pi)^4}\rho(k_j)\left[1+n(k_j^0)\right]\right](2\pi)^4\delta^4(q-k_1-k_2-k_3) \, . \tag{A7}$$

The high-temperature limit of (A3) is given in Refs. [12,30] and we have just quoted the final result in the text.

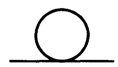
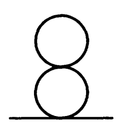
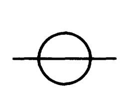
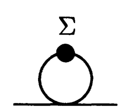
The expression for $G_\phi^{++}(\mathbf{q}, t-t')$ in (A5) can also be explicitly written as

$$\begin{aligned} G_\phi^{++}(\mathbf{q}, t-t') = & \frac{e^{-\Gamma|t-t'|}}{2\omega\left[\cosh(\beta\omega)-\cos(\beta\Gamma)\right]}\left\{\sinh(\beta\omega)\cos(\omega|t-t'|)\right. \\ & \left. + \sin(\beta\Gamma)\sin(\omega|t-t'|) + i\left[\cos(\beta\Gamma)-\cosh(\beta\omega)\right]\sin(\omega|t-t'|)\right\} \, . \end{aligned} \tag{A8}$$

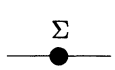
Expanding (A8) for $\beta\Gamma \ll 1$, we obtain Eq. (3.9).

APPENDIX B

Let us show that the use of (3.6) correctly accounts for the problem of overcounting of terms when the full propagator (3.7) is used in our expressions. The thermal mass in the effective equation of motion in (3.16) is given by

$m_T^2 = m^2 +$  $+$  $+ \mathrm{Re}\Big[$  $\Big] +$  , (B1)

where the last term in (B1) comes from the "interaction" term $\frac{1}{2}\Sigma\phi^2$ in (3.6), with "interaction vertex" denoted by

$\frac{1}{2}\Sigma\phi^2 \equiv$  (B2)

and the propagators in (B1) are expressed in terms of the full propagator (3.7). The imaginary part of the setting sun diagram is handled in Sec. II and it is associated with the fluctuation field ξ_2 in (2.21). Equation (B1) can be derived from the expressions given in Secs. II and III, in terms of the real time field propagator, Eq. (A8) or (3.9), or directly in terms of the usual imaginary time method. The results are the same and (B1), at high temperatures, can be expressed by

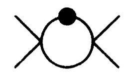
$$\begin{aligned} m_T^2 = m^2 &+ \frac{\lambda}{2}\left\{\frac{T^2}{12}\frac{T}{4\pi}\left[m^2+\mathrm{Re}\Sigma\right]^{\frac{1}{2}}\right\} \\ &- \frac{\lambda^2T^3}{384\pi\left[m^2+\mathrm{Re}\Sigma\right]^{\frac{1}{2}}} + \Sigma\frac{\lambda T}{16\pi\left[m^2+\mathrm{Re}\Sigma\right]^{\frac{1}{2}}} \\ &+ \mathcal{O}\left(\ln\frac{m^2+\mathrm{Re}\Sigma}{T^2}\right) \, . \end{aligned} \tag{B3}$$

Using (3.11) for $m^2 + \mathrm{Re}\Sigma$ in (B3) and expanding in λ, we get

$$\begin{aligned} m_T^2 = m^2 &+ \frac{\lambda}{2}\left(\frac{T^2}{12}-\frac{Tm}{4\pi}\right) - \lambda^2\frac{T^3}{384\pi m} \\ &+ O\left(\ln\frac{m^2}{T^2}\right) + \mathcal{O}(\lambda^3) \, , \end{aligned} \tag{B4}$$

which is the correct result for the thermal mass. Terms of the order λ^3 and higher in (B4) must be handled by the introduction of higher loop terms (three loops and beyond), in which the last term in (B1) and the consistent introduction of higher order graphs with the interaction vertex (B2) would cancel any extra contributions. Without the last term in (B1), we can easily check that we would overcount higher order diagrams, beginning with the "figure 8" two-loop diagram, which would be counted twice. A similar overcounting, in particular the one associated with the "figure 8" diagram, leads to the problem of linear terms (in the field-dependent mass) in the finite temperature effective potential improved improperly by dressed propagators. As shown by Arnold and Espinosa (in Ref. [5]), the introduction of the interaction term (B2) cancels the linear terms and the procedure used here gives the correct finite-temperature effective potential in terms of the thermal mass.

In the effective equation of motion, additional overcounting terms can also arise from the dressed one-loop vertex correction and these are accounted for by a term such as (at lowest order)

 ;

however, these terms are higher order in λ, $O(\lambda^3)$, and we will not deal explicitly with them here.

APPENDIX C

We estimate here the dissipation coefficient η_2 associated with the fluctuation field ξ_2, obtained from Eq. (3.5), with $G_\phi^{++}(\mathbf{q}, t-t')$ given by (A9) and show that it is subdominant. From the first term on the RHS in (3.5), we get that η_2 is given by

$$\eta_2 = \frac{\lambda^2}{3}\int_{-\infty}^{t} dt'(t'-t)\int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\mathrm{Im}\left[G_\phi^{++}(\mathbf{q}_1, t-t')G_\phi^{++}(\mathbf{q}_2, t-t')G_\phi^{++}(-\mathbf{q}_1-\mathbf{q}_2, t-t')\right] . \tag{C1}$$

From (A8), we can write $G_\phi^{++}(\mathbf{q}_j, t-t')$ as

$$G_\phi^{++}(\mathbf{q}_j, t-t') = a_j + i b_j , \tag{C2}$$

where $a_j \equiv a(\mathbf{q}_j, t-t')$ and $b_j \equiv b(\mathbf{q}_j, t-t')$ are given by the real and imaginary terms of Eq. (A8), respectively $[\Gamma_j \equiv \Gamma(\mathbf{q}_j)$ and $\omega_j \equiv \omega(\mathbf{q}_j)]$:

$$\begin{aligned} a_j &= \frac{e^{-\Gamma_j|t-t'|}}{2\omega_j\left[\cosh(\beta\omega_j)-\cos(\beta\Gamma_j)\right]}\left[\sinh(\beta\omega_j)\cos(\omega_j|t-t'|)+\sin(\beta\Gamma_j)\sin(\omega_j|t-t'|)\right] , \\ b_j &= -\frac{e^{-\Gamma_j|t-t'|}\sin(\omega_j|t-t'|)}{2\omega_j} . \end{aligned} \tag{C3}$$

Using (C2) in (C1), we get (with $\mathbf{q}_3 = -\mathbf{q}_1 - \mathbf{q}_2$)

$$\eta_2 = \frac{\lambda^2}{3}\int_{-\infty}^{t} dt'(t'-t)\int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\left(a_1a_2b_3 + a_1a_3b_2 + a_2a_3b_1 - b_1b_2b_3\right) . \tag{C4}$$

Using Eq. (C3) for a_j and b_j, we can perform the time integration in (C4), by changing the time integration variable t' to $t-t'=t''$, and obtain for (C4) the expression

$$\begin{aligned} \eta_2 = \; & \frac{\lambda^2}{3}\int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\frac{1}{\omega_1\omega_2\omega_3}\left\{\frac{\Gamma_1+\Gamma_2+\Gamma_3}{2}\left[\frac{2(1+2n_1)(n_2-n_3)+(1+2n_2)(1+2n_3)-1}{(\omega_1-\omega_2+\omega_3)^3}\right.\right. \\ & + \frac{2(1+2n_2)(n_3-n_1)+(1+2n_1)(1+2n_3)-1}{(\omega_1+\omega_2-\omega_3)^3} \\ & + \frac{2(1+2n_3)(n_1-n_2)+(1+2n_1)(1+2n_2)-1}{(-\omega_1+\omega_2+\omega_3)^3} \\ & \left. - \frac{(1+2n_1)(1+2n_2)+(1+2n_1)(1+2n_3)+(1+2n_2)(1+2n_3)-1}{(\omega_1+\omega_2+\omega_3)^3}\right] \\ & + \beta\Gamma_1 n_1(1+n_1)\left[(n_2-n_3)\left(\frac{1}{(\omega_1-\omega_2+\omega_3)^2}-\frac{1}{(\omega_1+\omega_2-\omega_3)^2}\right)\right. \\ & \left. + (n_2+n_3)\left(\frac{1}{(\omega_1+\omega_2+\omega_3)^2}-\frac{1}{(-\omega_1+\omega_2+\omega_3)^2}\right)\right] \end{aligned}$$

$$+ \beta\Gamma_2 n_2(1+n_2)\left[(n_1-n_3)\left(\frac{1}{(-\omega_1+\omega_2+\omega_3)^2}-\frac{1}{(\omega_1+\omega_2-\omega_3)^2}\right)\right.$$
$$\left.+\ (n_1+n_3)\left(\frac{1}{(\omega_1+\omega_2+\omega_3)^2}-\frac{1}{(\omega_1-\omega_2+\omega_3)^2}\right)\right]$$
$$+ \beta\Gamma_3 n_3(1+n_3)\left[(n_1-n_2)\left(\frac{1}{(-\omega_1+\omega_2+\omega_3)^2}-\frac{1}{(\omega_1-\omega_2+\omega_3)^2}\right)\right.$$
$$\left.\left.+\ (n_1+n_2)\left(\frac{1}{(\omega_1+\omega_2+\omega_3)^2}-\frac{1}{(\omega_1-\omega_2+\omega_3)^2}\right)\right]\right\} + O\left(\lambda^2\frac{\Gamma_j^3}{\omega_j^3}\right) . \quad \text{(C5)}$$

The above expression is at least of the order $\lambda^2\Gamma_i$ and, since $\Gamma_i = O(\lambda^2)$, we have that $\eta_2 = O(\lambda^4)$. We are thus justified in neglecting its contribution to the effective equation of motion to $O(\lambda^2)$.

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