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Eikonal property of nuclear production of $J/\psi(3.1)$

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The production of J/ψ by p-nucleus and π-nucleus reactions is investigated using the optical model. It is found that the ψ-nucleon absorption cross section is energy independent, practically the same as for π^- and K^0 production, and the average $\bar{\sigma}_{abs}=12.7\pm5.1$ mb leads to a mean free path $\bar{\lambda}=5.2\pm2.0$ fm.

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I. INTRODUCTION

It has been found in a recent CERN Super Proton Synchrotron (SPS) experiment by the NA 38 Collaboration [1(a)] that the suppression of J/ψ production by heavy-ion (HI) collisions is more important than expected from the p-nucleus collision at the same energy. Thus arise the following questions: Is its mechanism different from π^- and K^0 production? How is it different from the p-nucleus collision?

We note that the A dependence of the hadron production cross sections is usually described by means of the power law $\sigma\sim A^\alpha$ of the naive quark model and that the mean free path (MFP) thus deduced is not free from ambiguity because, experimentally, the parametrization of α depends on the Feynman variable x_F. Furthermore, its estimate integrated over x_F is not A independent, but decreases with A.

As the cross section is essentially a geometrical concept, it is natural to use the eikonal model to investigate the nuclear production of J/ψ. We present in this paper an analysis of hadron production by high energy p-nucleus and π^--p collisions in terms of the geometrical model, equivalent to the Glauber theory [2]. We then compare σ_{abs} of J/ψ to those of π^- and K^0 and investigate the scaling property.

Remarks will be made on the Gaussian model cross-section formula, Eq. (9) below, for p-A collision and the relationship between the parameters α of the power law $\sigma\sim A^\alpha$ and σ_{abs} of the optical model. An empirical formula to approximate (9) will be proposed.

II. THE EIKONAL MODEL

Consider, to fix the ideas, the production of a J/ψ particle at the point B (Fig. 1) inside a target nucleus of radius $R=r_0A^{1/3}$ by an incident proton after a traversal of $AB=z$, at an impact parameter $b=OM$. If it survives after passing a distance $BN=z'$ inside the target nucleus, we get, for its cross section,

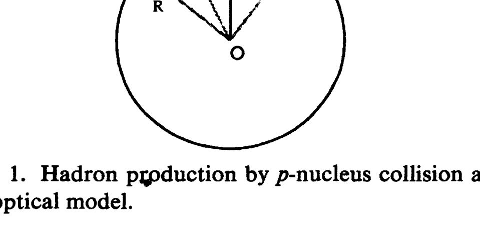

FIG. 1. Hadron production by p-nucleus collision according to the optical model.

$$\bar{n}_\psi\sigma=\pi\int_0^{+\infty}db^2\int_{-\infty}^{\infty}dz\,\sigma(pp\to\psi)\rho(b,z)\times\left[\exp(-\sigma_{abs}(\Psi,N)\times\int_z^{\infty}\rho(b,z')dz')\right], \quad (1)$$

where $\sigma(pp\to\psi)$ and $\sigma_{abs}(\psi,N)$ are the cross sections of J/ψ production by pp collision and its absorption by a nucleon of the target, and ρ is the nuclear density, which, for convenience sake, is assumed to be Gaussian, so that the limits of integration in (1) may be extended to ∞

$$\rho(r)=\frac{A}{(\sqrt{\pi}R)^3}e^{-r^2/R^2} \quad (2)$$

normalized to the mass number A of the target.

The integration over z is straightforward by making use of this property of the exponential function; namely, $dz\,\rho(b,z)$ is the differential up to a factor of σ_{abs}. This remarkable property leads to

$$n_\psi\sigma=\pi P\int_0^{+\infty}db^2(1-e^{-2\Omega})\,, \quad (3)$$

with

$$P=\sigma(pp\to J/\psi)/\sigma_{abs}(\psi,N) \quad (4)$$

and 2Ω the eikonal:

$$2\Omega(b,z)=\sigma_{abs}\int_{-\infty}^{\infty}\rho(b,z')dz'\,. \quad (5)$$

The radial variable in the Gaussian density $\rho(r)$ is $r^2=\vec{O}B^2=(z-\sqrt{R^2-b^2})^2$ so that

$$2\Omega(b,z)=\sigma_{abs}\frac{A}{\pi R^2}e^{-b^2/R^2}\,. \quad (6)$$

The integrals involved in the calculation of the cross section by (3) are elementary, if use is made of the series expansion

$$e^{-2\Omega}=\sum_0^{\infty}\frac{(-1)^\nu}{\nu!}(2\Omega)^\nu\,. \quad (7)$$

We get

$$\int_0^{+\infty} db^2(1-e^{-2\Omega})=R^2\sum_1^{\infty}\frac{(-1)^{\nu-1}}{\nu\times\nu!}\left[\frac{\sigma_{\rm abs}A}{\pi R^2}\right]^{\nu} . \quad (8)$$

The general expression for the production cross section of the inclusive $pA \to m + \cdots$ is

$$\sigma_{pA}(m)=\sigma_0(m)A\left[1-\frac{1}{2.2!}(a_m A^{1/3})+\frac{1}{3\times 3!}(a_m A^{1/3})^2-\cdots\right] , \quad (9)$$

where, for simplicity, we let

$$a_m=\frac{\sigma_{\rm abs}(m)}{\pi r_0^2} , \quad (10)$$

and $\sigma_0(m)$ is the coefficient. Note that the convergence condition, according to d'Alembert's rule, is $a_m A^{1/3}/\nu^2<1$, so that the model holds better for light nuclei.

III. HADRON PRODUCTION BY $p+A \to \pi^-, K^0, J/\psi$

We now use (9) to analyze $p+A \to J/\psi + \cdots$ with the recent Fermilab FNAL experiment E 772 at 800 GeV/c [3]. The cross sections (in μb) are estimated using their $Bd\sigma/dx$ distribution for $p+d$ production at the same energy and assuming the branching ratio $B(J/\psi \to \mu^+\mu^-)=5.97\%$. Their data are shown by triangles in Fig. 2.

As for the CERN SPS experiment NA 38 at 200 GeV/c with Cu and U targets [1(a)], as shown by crosses in Fig. 2, we associate with the results from a Fermilab experiment $n+A \to J/\psi$ at 400 GeV/c by the CFHS Collaboration [4] by scaling down to 200 GeV/c using the Cu point of NA 38. The cross sections thus obtained are shown by open circles in Fig. 2.

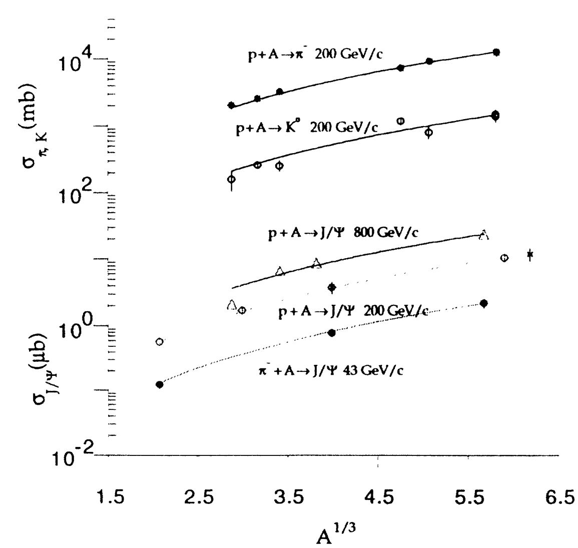

FIG. 2. Cross sections of J/ψ, π^-, and K^0 production by p-nucleus reactions vs $A^{1/3}$ of the target. The curves are fits with the optical model formula Eq. (9); the parameters are listed in Table I.

Next, for comparison, we consider the Serpukhov data of $\pi^-+p \to J/\psi$ at 43 GeV/c [5].

Finally, in order to investigate the properties of the parameters of the fits, we have analyzed, in addition, π^- and K^0 production by inclusive $p+A$ at 200 GeV/c by the NA 35 Collaboration [6(a)] and the NA 5 Collaboration [7], together with the FNAL experiment E 565/578 Collaboration [8]. Last but not least, we use the data of $\pi^-+A \to h^- + \cdots$ at 40 GeV/c of the Risk Collaboration [9], their data and the fit being omitted in Fig. 2 for simplicity.

In order to analyze these data in terms of the Gaussian model Eq. (9), let us recall this scaling property, similar to that of pp collisions, reported before [10]. For this purpose, we plot in Fig. 3 the reduced cross sections per nucleon $m^2\sigma_{pA}(m)/A$ multiplied by the fugacity f vs $A^{1/3}$ for π^- and J/ψ production at $P_{\rm lab}=200$ GeV/c, with $f=1$ for π and 2.6 for J/ψ corresponding to the chemical potential $\mu_\psi \approx 1.5$ GeV with respect to $\mu_\pi \approx 0$ (cf. below). As both sets of data fall into a single pattern, we may fit them simultaneously with (9) by considering successively $2,3,4,\ldots,8$ terms of the series (9), in order to search for the minimum of χ^2. We find the best fit corresponding to the case of four terms; the parameters are

$$a_m=0.35\pm0.03,\ \sigma_0(m)=1.98\pm0.65 .$$

The fit as shown by the solid curve in Fig. 3 is very satisfactory indeed, indicating that $\sigma_{\rm abs}(m)$ is practically the same for both π and J/ψ production.

We now proceed to try an overall fit to the data of π, K and J/ψ production in Fig. 2, assuming the same $\sigma_{\rm abs}(m)$. The results are shown by the solid curves in Fig. 2. The parameters are summarized in Table I. A comparison of these fits with the data of production of various particles by several h-nucleus reactions covering a

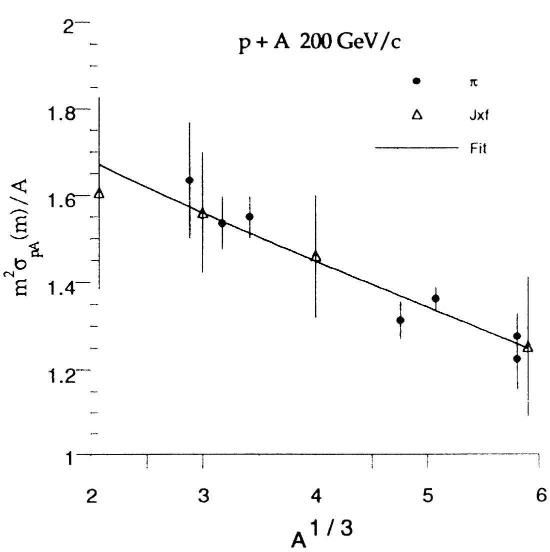

FIG. 3. Scaling property of $p+A \to \pi$ and J/ψ at 200 GeV/c. Plot of the reduced production cross section per nucleon $m^2\sigma_{pA}(m)/A$ scaled by the fugacity $f=1$ for π and $f=2.6$ for J/ψ against $A^{1/3}$. The curve is an overall fit with the Gaussian model, Eq. (9), corresponding to the minimum χ^2 by taking four terms of the series expansion.

TABLE I. Eikonal model parameters of partial cross sections for particle production by h-nucleus collisions: an overall fit with Eq. (9) assuming the same $\sigma_{\rm abs}/\pi r_0^2$. W^* is the energy available in the fireball system Eq. (14).

$P_{\rm lab}$ (GeV/c)	Sec.	$\frac{\sigma_{\rm abs}}{\pi r_0^2}$	σ_0 (mb)	W^* (GeV)
200 p-A	π^-	0.28 ± 0.13	92.6 ± 1.0	9.32
	K^0	Same	10.5 ± 0.7	8.60
40 π^--A	h^-	Same	112.1 ± 0.7	5.05
800 p-A	J/ψ	Same	$(1.82\pm0.09)\times10^{-4}$	11.67
200 n-A	J/ψ	Same	$(7.34\pm0.06)\times10^{-5}$	6.50
43 π-A	J/ψ	Same	$(1.66\pm0.03)\times10^{-5}$	2.28

wide range of energy gives us a strong indication that the agreement is, in general, very good.

The properties of these parameters will be discussed in Sec. IV. Here, let us note especially that, assuming $r_0=1.2$ fm, we estimate the absorption cross section of secondary particles by nucleons of the target nucleus to be

$$\sigma_{\rm abs}=12.7\pm5.1 \text{ mb} , \tag{11}$$

corresponding to a mean free path

$$\lambda=5.0\pm2.0 \text{ fm} . \tag{12}$$

We note that this estimate agrees with that using an empirical and closed formula to be discussed in Sec. V.

IV. EIKONAL PROPERTY AND SCALING

If we compare the nuclear production of J/ψ with π^- and K^0 by pA and πp collisions, we find a striking feature; namely, the parameter $\bar{\sigma}_{\rm abs}/\pi r_0^2$ remains practically constant, independent of energy and the nature of particles we are dealing with, the average being 0.28 ± 0.12. It is interesting to note that the corresponding average $\bar{\lambda}$ is comparable to the radius of the heaviest nuclei; this is expected, since the produced hadrons are strongly interacting particles just like the colliding nucleons.

This important eikonal property, namely $\sigma_{\rm abs}(m)/\pi r_0^2 \approx$const, implies that at a given energy, the A dependence part is a common factor for all partial cross sections of any secondary production. In other words, the cross sections of the inclusive $p+A\to m+\cdots$, as is well known, may be scaled to the inclusive $p+p\to m+\cdots$ at the same energy, as discussed previously [10(a)].

As for the behavior of the parameter $\sigma_0(m)$ from the fits (Table I) we expect it to equal the cross section for pp at the same energy. In this regard, it is interesting to note that, in the case of $p+A\to\pi+\cdots$ at 200 GeV/c, the coefficient $\sigma_0(\pi)=92.6\pm1.0$ mb is consistent with that of $p+p\to\pi+\cdots$:

$$\sigma_{pp}(\pi)=(32.5\pm0.5)(2.96\pm0.03)=96.2\pm1.8 \text{ mb}$$

by the NA 5 Collaboration [7]. As for other particle production by pp, we recall that the process $p+p\to m+\cdots$ is found to behave like the bremsstrahlung, so that

$$\sigma_{pA}(m)\sim W^*(m)/m^2 , \tag{13}$$

$W^*(m)$ being the energy available in the rest frame of secondaries, referred to as the fireball (FB) system; its velocity is the same as that of pp at the same energy, namely $\beta^*=1-2/\gamma_{\rm c.m.}$, $\gamma_{\rm c.m.}=\sqrt{s}/2m_p$ being the Lorentz factor of pp collision, as reported elsewhere [10], so that

$$W^*(m)=2m_p[2(\sqrt{s}/2m_p-1)^{1/2}-1]-\Delta m . \tag{14}$$

Here Δm is the threshold energy: $\Delta m=m_J$ for J/ψ and $\Delta m=2m$ for π or K production. The values of $W^*(m)$ (in GeV) are listed in Table I.

Finally, we note that the large difference between the partial cross sections of $J/\psi(c\bar{c})$ and $\pi^-(\bar{u}d)$ produced by $p+A$ at 200 GeV/c, namely, $\sigma_\psi/\sigma_\pi\approx7.78\times10^{-7}$, is due to the large mass difference between the charm c and the light u/d quarks. Indeed, if Θ denotes the temperature characteristic of the emission of a quark of mass μ with probability $e^{-\mu/\Theta}$, then, according to the statistical model,

$$\frac{\sigma_\psi}{\sigma_\pi}=e^{-2(\mu_c-\mu_u)/\Theta} . \tag{15}$$

With $\mu_u\approx0$ and $\mu_c\approx1.5$ GeV, we find

$$\Theta\approx0.214 \text{ GeV} , \tag{16}$$

comparable to the latest estimate $\sim$200 GeV of the Hagedorn temperature [11]. That $\Theta\gg140$ MeV, the conventional temperature corresponding to $\langle P_\perp\rangle\approx350$ MeV/c of π production, is due to the fact that the production of J/ψ is a hard process in contrast with the soft process of π production.

V. REMARKS

From the analysis of the nuclear production of J/ψ by p-nucleus and π-nucleus collisions in terms of the optical model, we find the absorption MFP $\lambda\approx5.0\pm2.0$ fm, comparable to that of the nucleon [12], but considerably shorter than the 16±8 fm estimated by Gerschel and Hufner [13], whereas our estimate $\sigma_{\rm abs}\approx12.7$ mb exceeds the 6 mb reported by Lesnick [14].

As $\sigma_{\rm abs}$ is practically the same for J/ψ and π,K production, it follows that the group velocity determined by the eikonal phase of these particles is the same for all the secondaries; i.e., the index of refraction is independent of the nature of the produced hadron, so that the secondaries have the same rest frame, as they should, since we are dealing with strongly interacting particles. This property has actually been observed in the case of hadron production by pp collisions [10(a)]. For $p+A \rightarrow J/\psi$ at $P_{\rm lab}=800$ GeV/c, we have found from Fermilab experiment E 722 [3] $\beta^*_\psi=0.961\pm0.012$, and from the latest E 789 [15] we get 0.883 ± 0.067, leading to an average $\langle\beta^*_\psi\rangle=0.897\pm0.064$ in agreement with $\beta^*=1-2/\gamma_{\rm c.m.}=0.903$ as expected from the scaling. Here, $\gamma_{\rm c.m.}$ is the Lorentz factor of the c.m. system (c.m.s.) of the incident proton and a quasifree nucleon of the target nucleus, just like the case of $p+p\rightarrow m$.

As regards the optical model for p-nucleus production, Eq. (9), we note that it may be approximated as

$$\sigma_{pA}(m)=\sigma_0\exp\left[-\frac{\sigma_{\rm abs}A^{1/3}}{4\pi r_0^2}\right]. \tag{17}$$

The parameters thus estimated are listed in Table II. Note that they are consistent within narrow uncertainities with those of the overall fit using (9); in particular, the average

$$\frac{\sigma_{\rm abs}}{\pi r_0^2}=0.29\pm0.08$$

is in good accord with 0.28 ± 0.13 in Table I.

We may relate (17) to the power law A^α, which is commonly used in the analyses of the hadrons from nuclear production, as follows:

$$\alpha=1-\frac{\sigma_{\rm abs}}{4\pi r_0^2}\frac{A^{1/3}}{\ln A}. \tag{18}$$

We note that its application needs another assumption to approximate the ratio $A^{1/3}/\ln A$, which decreases from 0.950 for $A=9$ to a minimum 0.905 for $A\approx19$, then increases to 1.130 for $A=238$. It is therefore rather ambiguous to estimate the parameter α assuming a constant ratio $A^{1/3}/\ln A\approx1$, as in [13].

Finally, as regards the mechanism of hadron production by nuclear reactions, we note that the hadron temperature increases slightly with the mass m of the secondary and that the entropy density of π, $E(\pi)\approx3T_\pi+m_\pi$, is about twice that of other hadrons heavier than π, $E(m)\approx\frac{3}{2}T_m$. Consequently, pions serve as a heat bath for thermalization with other hadrons; they are emitted at the surface of the nucleus, whereas for J/ψ, they are produced by a hard process, in the deep region of the colliding nucleon. Its velocity $\beta_\psi\approx\sqrt{3T_\psi/m_\psi}<0.4$ is rather small for the reactions we are dealing with in the present work. When the J/ψ reaches the surface of the nucleon to be emitted as a secondary particle, the other nucleons around the wounded nucleon which produced the J/ψ are already separated apart. Consequently, because of the space-time evolution of the process, the nuclear absorption of J/ψ behaves like that of nucleon constituents of the nucleus in collision.

TABLE II. Parameters of the empirical formula (17) for nuclear absorption cross section.

$P_{\rm lab}$ (GeV/c)	Sec.	$\frac{\sigma_{\rm abs}}{\pi r_0^2}$	σ_0 (mb)
200 p-A	π^-	0.32 ± 0.07	102.3 ± 9.9
	K^0	0.32 ± 0.49	11.2 ± 0.74
40 π^--A	h^-	0.32 ± 0.02	122.7 ± 2.90
800 p-A	J/ψ	0.28 ± 0.25	$(1.91\pm0.14)\times10^{-4}$
200 n-A	J/ψ	0.31 ± 0.10	$(7.97\pm0.14)\times10^{-5}$
43 π-A	J/ψ	0.17 ± 0.04	$(1.50\pm0.53)\times10^{-5}$

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[1] NA 38 Collaboration, C. Baglin *et al.*, (a) Phys. Lett. B **270**, 105 (1991); (b) **251**, 472 (1990).
[2] R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin *et al.* (Interscience, New York, 1959), Vol. I.
[3] E 772 Collaboration, D. M. Alde *et al.*, Phys. Rev. Lett. **66**, 133 (1991).
[4] CFHS Collaboration, M. Binskley *et al.*, Phys. Rev. Lett. **37**, 571 (1976).
[5] Superkhov Experiment, Y. M. Antipov *et al.*, Phys Lett. **72B**, 278 (1977).
[6] NA 35 Collaboration, (a) J. Bucher *et al.*, Z. Phys. C **51**, 157 (1991); (b) A. Bamberger *et al.*, Phys. Lett. B **205**, 583 (1988); (c) J. Bartke *et al.*, Z. Phys. C **48**, 191 (1990).
[7] NA 5 Collaboration, C. de Marzo *et al.*, Phys. Rev. D **45**, 734 (1982).
[8] E 565/570 Collaboration, D. H. Brick *et al.*, Phys. Rev. D **45**, 734 (1992).
[9] Brick Collaboration, E. G. Boss *et al.*, Z. Phys. C **26**, 43 (1984).
[10] T. F. Hoang, (a) Z. Phys. C **56**, 673 (1992); (b) **57**, 655 (1993).
[11] G. H. Bergers *et al.*, Z. Phys. C **46**, 465 (1990).
[12] J. W. Negele and K. Yazaki, Phys. Rev. Lett. **47**, 71 (1981).
[13] C. Gerschel and J. Hufner, Phys. Lett. B **207**, 253 (1988).
[14] L. Lesnick, Phys. Lett. B **302**, 140 (1993).
[15] E 789 Collaboration, (a) M. S. Kowitt *et al.*, Phys. Rev. Lett. **72**, 1318 (1994); (b) M. S. Kowitt, Ph.D. thesis, University of California, Berkeley. We have used the covariant Boltzmann factor to estimate the temperature and the velocity β^* of the rest frame of secondaries for p-A collision as described in [10]. For their x_F distribution of the Cu target in the interval $0.325<x_F<0.675$, we fit with $\exp(-ax_F)$ to get $a\equiv\sqrt{s}(1-\beta^*)/2T=10.96\pm0.17$, and from the parameter $P_0=2.70\pm0.09$ of their power law fit to the $P_\perp$ distribution we estimate $\langle P_\perp^2\rangle=P_0^2/4=1.823$ GeV/c^2, leading to $T_\psi\approx\langle P_\perp^2\rangle/2m_\psi\approx0.294\pm0.034$ GeV. Thus $\beta^*_\psi=0.833\pm0.064$, compared to $\beta^*=1-2/\gamma_{\rm c.m.}=0.903$ according to the scaling.