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Radiative corrections to quark-quark-Reggeon vertex in QCD

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One loop corrections to the coupling of the Reggeized gluon to quarks are calculated in QCD. Combining this result with the known corrections to the gluon-gluon-Reggeon vertex, we check the self-consistency of the representation of the amplitudes with gluon quantum numbers and the negative signature in the t channel in terms of the Regge pole contribution.

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I. INTRODUCTION

It has been known for a long time [1] that, in the leading logarithmic approximation (LLA) for the Regge region, the total cross section $\sigma_{\rm tot}^{LLA}$ in the non-Abelian SU(N) gauge theories grows at large c.m. system (c.m.s.) energies $\sqrt{s}$:

$$\sigma_{\rm tot}^{\rm LLA} \sim \frac{s^{\omega_0}}{\sqrt{\ln s}} , \qquad (1)$$

where

$$\omega_0 = \frac{g^2}{\pi^2} N \ln 2 . \qquad (2)$$

Therefore the Froissart bound $\sigma_{\rm tot} < c \ln^2 s$ is violated in the LLA. The reason for this is the violation of the s-channel unitarity constraints for scattering amplitudes in the LLA.

The behavior (1) of the total cross section is determined by the position of the most right singularity in the complex momentum plane in the solution of the integral equation for t-channel partial waves with vacuum quantum numbers [1]. In order to find out the region in which the LLA can be applied, radiative corrections to the equation's kernel must be calculated. The calculation of these corrections was started by Lipatov and one of the authors (V.F.) in Ref. [2], where the calculation program was presented. The program makes strong use of the gluon Reggeization proven in the LLA [1]. As a necessary step in this program one needs to calculate one loop corrections to the particle-particle-Reggeon (PPR) vertices. Here the Reggeon is the Reggeized gluon and its trajectory in the LLA is given by

$$j(t) = 1 + \omega(t) ,$$

$$\omega(t) = \frac{g^2 t}{16\pi^3} N \int \frac{d^2k}{\bar{k}^2(\bar{q}-\bar{k})^2} , \qquad t = -\bar{q}^2 . \qquad (3)$$

The infrared divergence in the gluon trajectory (3) is canceled by the divergences in real gluon emission, so that the integral equation for the t-channel partial waves with vacuum quantum numbers [1] is free of singularities. In order to remove the infrared divergences at intermediate steps we use the dimensional regularization of Feynman integrals:

$$\frac{d^2k}{(2\pi)^2} \to \frac{d^{2+\varepsilon}k}{(2\pi)^{2+\varepsilon}}, \qquad \varepsilon = D - 4 , \qquad (4)$$

where D is the space-time dimension ($D = 4$ for the physical case). Then we get

$$\omega(t) = g^2 N \frac{2}{(4\pi)^{\frac{D}{2}}} (-t)^{\frac{D}{2}-2} \frac{\Gamma\left(2-\frac{D}{2}\right)\Gamma^2\left(\frac{D}{2}-1\right)}{\Gamma(D-3)} . \qquad (5)$$

In the case of pure gluodynamics one loop corrections to the gluon-gluon-Reggeon (GGR), as well as to the Reggeon-Reggeon-gluon vertices, were calculated by Lipatov and one of the authors (V.F.) [3, 4].

In the case of real QCD there is a quark contribution to the vertices; the quark loop contribution to the GGR vertex was calculated in Ref. [5]. In addition to that, in the real QCD an extra (compared to the pure gluodynamics case) vertex appears: the quark-quark-Reggeon (QQR) vertex. The existence of this vertex allows us to check the validity of the assumption that the high energy behavior of amplitudes with gluon quantum numbers in the t channel and negative signature is governed by the Regge pole contribution not only in the LLA, but beyond it as well. According to this assumption the amplitude $\mathcal{A}_{AB}^{A'B'}$ of a process $A + B \to A' + B'$ takes the factorized form

$$\mathcal{P}_8^- \mathcal{A}_{AB}^{A'B'} = \Gamma_{A'A}^{i} \frac{s}{t} \left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)} \right] \Gamma_{B'B}^{i} . \qquad (6)$$

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Here $\mathcal{P}_8^-$ is the projection operator into the octet color state with negative signature and i is the color index of the Reggeized gluon with the trajectory $j(t) = 1 + \omega(t)$, given by Eq. (3) in the lowest order of the perturbation theory. For the PPR vertex $\Gamma^i_{A'A}$ in the helicity basis we get, to lowest order,

$$\Gamma^i_{A'A} = g\langle A'|T^i|A\rangle\delta_{\lambda_A,\lambda_{A'}} , \tag{7}$$

where $\langle A'|T^i|A\rangle$ represents the matrix element of the group generator in the corresponding representation [i.e., fundamental for quarks $T_i = t_i = \frac{\lambda_i}{2}$ and adjoint for gluons $(T_i)_{ab} = -if_{iab}$] and λ_A is the helicity of particle A. We assume that the polarization states of the scattered particles are obtained from those of the initial particles by rotation around the axis orthogonal to the scattering plane. From Eq. (6) we may observe that the behavior of the three types of amplitudes (gluon-gluon, quark-quark, and quark-gluon elastic scattering amplitudes) is determined by two vertices GGR and QQR; therefore, one of the amplitudes can be expressed in terms of the others, thus giving a nontrivial test of the validity of representation (6).

Contrary to Eq. (7), in higher orders the PPR vertex Γ can contain another spin structure. Because of parity conservation it can be written in the form

$$\Gamma^c_{A'A} = g\langle A'|T^i|A\rangle[\delta_{\lambda_A,\lambda_{A'}}(1+\Gamma^{(+)}_{AA}) + \delta_{\lambda_A,-\lambda_{A'}}\Gamma^{(-)}_{AA}] , \tag{8}$$

if relative phases of states with opposite helicity are chosen appropriately [see Refs. [4, 5] for gluons and Eq. (36) below for quarks]. Here $\Gamma^{(+)}$ and $\Gamma^{(-)}$ respectively stand for helicity-conserving and -nonconserving loop contributions to the vertex.

One loop corrections to the GGR vertex were calculated in Refs. [3–5]. The contribution of the gluon loop can be written in the form (8) with

$$\Gamma^{(+)}_{GG}(\text{gluon loop}) = Ng^2\frac{(-t)^{\frac{D}{2}-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma\left(2-\frac{D}{2}\right)\Gamma^2\left(\frac{D}{2}-1\right)}{\Gamma(D-2)} \times\left\{(D-3)\left[\psi\left(3-\frac{D}{2}\right)-2\psi\left(\frac{D}{2}-2\right)+\psi(1)\right]-\frac{7}{4}-\frac{1}{4(D-1)}\right\}, \tag{9}$$

$$\Gamma^{(-)}_{GG}(\text{gluon loop}) = Ng^2\frac{(-t)^{\frac{D}{2}-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma\left(3-\frac{D}{2}\right)\Gamma^2\left(\frac{D}{2}-1\right)}{(D-1)\Gamma(D-2)}, \tag{10}$$

where ψ is the logarithmic derivative of the Γ function:

$$\psi(z) = \frac{\Gamma'(z)}{\Gamma(z)} . \tag{11}$$

For the quark loop contribution we have, in turn [5],

$$\Gamma^{(\pm)}_{GG}(\text{quark loop}) = \frac{2g^2}{(4\pi)^{\frac{D}{2}}}\sum_f V^{(f)}_{\pm}, \tag{12}$$

where

$$V^{(f)}_+ = -\Gamma\left(2-\frac{D}{2}\right)\left[\int_0^1\frac{dx\,x(1-x)}{\left[m_f^2-tx(1-x)\right]^{2-\frac{D}{2}}} + \int_0^1\int_0^1\frac{dx_1dx_2\theta(1-x_1-x_2)}{\left(m_f^2-tx_1x_2\right)^{2-\frac{D}{2}}}\left(\frac{(3-D)}{2}(2-x_1-x_2)+\left(2-\frac{D}{2}\right)\frac{x_1x_2(1-x_1-x_2)t}{\left(m_f^2-tx_1x_2\right)}\right)\right] \tag{13}$$

and

$$V^{(f)}_- = \Gamma\left(3-\frac{D}{2}\right)t\int_0^1\int_0^1\frac{dx_1dx_2\theta(1-x_1-x_2)x_1x_2(1-x_1-x_2)}{\left(m_f^2-tx_1x_2\right)^{3-\frac{D}{2}}} . \tag{14}$$

For massless quarks the two vertices become, respectively [5],

$$\Gamma^{(+)}_{GG}(\text{quark loop}) = \frac{2g^2n_f}{(4\pi)^{\frac{D}{2}}}(-t)^{\frac{D}{2}-2}\frac{\Gamma\left(2-\frac{D}{2}\right)\Gamma^2\left(\frac{D}{2}\right)}{\Gamma(D)}, \tag{15}$$

$$\Gamma^{(-)}_{GG}(\text{quark loop}) = -\frac{2g^2n_f}{(4\pi)^{\frac{D}{2}}}(-t)^{\frac{D}{2}-2} \times\frac{\Gamma\left(3-\frac{D}{2}\right)\Gamma^2\left(\frac{D}{2}-1\right)}{\Gamma(D)}. \tag{16}$$

In this paper we calculate the QQR vertex in the one loop approximation and check the validity of describing amplitudes in terms of the Regge pole contribution, i.e., representation (6). In Sec. II we find the QQR vertex by calculating the quark-quark scattering amplitude. Combining the result obtained with the one for the GGR vertex, Eqs. (8)–(14), and the Regge trajectory (5), we get, with the help of Eq. (6), a prediction for the quark-gluon scattering amplitude. In Sec. III we perform an independent calculation of this amplitude and check the consistency of the approach by comparing the result we arrive at with the predicted one. Some conclusions are illustrated in Sec. IV.

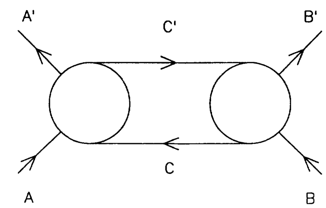

FIG. 1. Amplitude of the elastic scattering process $A + B \to A' + B'$ with the two particle intermediate state in the t channel.

II. QUARK-QUARK-REGGEON VERTEX

We will extract radiative corrections to the QQR vertex from the amplitude of the quark-quark elastic scattering. The calculation can be carried out through usual methods starting from the Feynman diagrams for the quark-quark scattering. However, for our purposes it is more convenient to use the method based on the t-channel unitarity relation. This method was used in Refs. [3–5] to calculate analogous corrections to the GGR vertex. Here its application allows us to demonstrate the factorization property of scattering amplitudes in the most economic way.

From the t-channel unitarity point of view, it is natural to decompose an amplitude according to intermediate states in the t channel. In the one loop approximation we need to consider the two particle intermediate state in the t channel. We will schematically represent in Fig. 1 the amplitude of the elastic scattering process $A(p_A)+B(p_B) \to A'(p_{A'})+B'(p_{B'})$ with the two particle intermediate state in the t channel and use the notation

$$s = (p_A + p_B)^2,\ u = (p_A - p_{B'})^2,\ q = p_A - p_{A'} = p_{B'} - p_B = p_{C'} - p_C,\ t = q^2,$$

$$s_A = (p_A + p_C)^2,\ u_A = (p_A - p_{C'})^2,\ s_B = (p_B + p_{C'})^2,\ u_B = (p_B - p_C)^2\ , \tag{17}$$

p_C and $p_{C'}$ being the momenta of the two intermediate particles.

Instead of calculating the t-channel discontinuities using $2\pi\delta(p^2 - m^2)$ for the intermediate particle line, we will calculate the contribution of diagram in Fig. 1 using full Feynman propagators for intermediate particles [3, 5]:

$$\mathcal{A}_{AB}^{A'B'} = \sum_{C,C'} \eta_{CC'} \int \frac{d^D p_C d^D p_{C'} \delta^{(D)}(p_C + q - p_{C'}) \mathcal{A}_{AC}^{A'C'} \mathcal{A}_{BC'}^{B'C}}{(2\pi)^D i \left(p_C^2 - m_C^2 + i\varepsilon\right)\left(p_{C'}^2 - m_C^2 + i\varepsilon\right)}\ . \tag{18}$$

Here the sum runs over kinds of intermediate particles, their polarization, color, and flavor states. As already mentioned in the Introduction, the space-time dimension D is not equal to 4, so that we can use the dimensional regularization for removing both infrared and ultraviolet divergences [4]. The numerical coefficient $\eta_{CC'}$ depends on the kind of intermediate state in the t channel, which can be a gluon-gluon or quark-antiquark state: in the first case $\eta_{GG} = \frac{1}{2}$ because of the identity of gluons, while in the second one $\eta_{Q\bar{Q}} = -1$ because of Fermi statistics. An arbitrary polynomial in t could be added to the result of integration in Eq. (18) because it does not change the t-channel discontinuity, but such terms would have a wrong asymptotic behavior incompatible with the renormalizability of the theory (cf. [1, 4]). Nevertheless, for massive quarks, some uncertainty still remains. We can add to the right-hand side (RHS) of Eq. (18) terms with the pole structure in t. In the case of pure gluodynamics such terms were absent in our regularization scheme because of a lack of appropriate values with mass dimensions. Evidently, these terms are connected with renormalization and will be considered at the end of this section.

A. Contribution of the quark-antiquark intermediate state

Let us first consider the simpler case of the quark-antiquark pair in the t channel. In this case the amplitudes $\mathcal{A}_{AC}^{A'C'}$ and $\mathcal{A}_{BC'}^{B'C}$ in Eq. (18) are the quark-quark scattering amplitudes taken in Born approximation. For such amplitudes we get

$$\mathcal{A}_{AB}^{A'B'} = \langle A'|t^c|A\rangle\langle B'|t^c|B\rangle \left(\frac{g^2}{t}\right) \bar{u}(p_{A'})\gamma^\mu u(p_A)\bar{u}(p_{B'})\gamma_\mu u(p_B)$$
$$-\delta_{AB}\langle B'|t^c|A\rangle\langle A'|t^c|B\rangle \left(\frac{g^2}{u}\right) \bar{u}(p_{B'})\gamma^\mu u(p_A)\bar{u}(p_{A'})\gamma_\mu u(p_B)\ , \tag{19}$$

where the second term contributes only for scattering of identical particles.

We are interested in the radiative correction to the QQR vertex when the Reggeon is a Reggeized gluon; therefore, one needs to project the amplitude (19) into the octet color state. First, we use the completeness relation for the generators of the fundamental representation of SU(N),

$$t^c_{\alpha\beta}t^c_{\gamma\delta} + \frac{1}{2N}\delta_{\alpha\beta}\delta_{\gamma\delta} = \frac{1}{2}\delta_{\alpha\delta}\delta_{\beta\gamma} \ , \tag{20}$$

and obtain

$$t^c_{\alpha\beta}t^c_{\gamma\delta} = -\frac{1}{N}t^c_{\alpha\delta}t^c_{\gamma\beta} + \frac{N^2-1}{2N^2}\delta_{\alpha\delta}\delta_{\beta\gamma} \ . \tag{21}$$

Successively, introducing the definition

$$\mathcal{P}_8^- \mathcal{A}^{A'B'}_{AB} = \langle A'|T^c|A\rangle\langle B'|T^c|B\rangle \left(\mathcal{A}_8\right)^{A'B'}_{AB} \ , \tag{22}$$

which will be used in the following, and using the result (21) we have

$$\left(\mathcal{A}_8\right)^{A'B'}_{AB} = \bar{u}(p_{A'})\gamma^\mu u(p_A)\left(\frac{g^2}{t}\right)\bar{u}(p_{B'})\gamma_\mu u(p_B) + \frac{\delta_{AB}}{N}\bar{u}(p_{B'})\gamma^\mu u(p_A)\left(\frac{g^2}{u}\right)\bar{u}(p_{A'})\gamma_\mu u(p_B) \ . \tag{23}$$

In order to apply the dispersion approach, it is convenient [4, 5] to decompose the amplitudes $\mathcal{A}^{A'C'}_{AC}$ and $\mathcal{A}^{B'C}_{BC'}$ entering Eq. (18) into the sum of two terms which are schematically shown in Fig. 2:

$$\mathcal{A}^{A'C'}_{AC} = \mathcal{A}^{A'C'}_{AC}(\text{as}) + \mathcal{A}^{A'C'}_{AC}(\text{na}) \tag{24}$$

and the analogous expression for $\mathcal{A}^{B'C}_{BC'}$. The first term on the RHS of Eq. (24) contains the asymptotic contribution for the Regge kinematics, $s_A \simeq -u_A \gg t$, while the nonasymptotic part contains the remaining amplitude terms. When performing the decomposition (24) on the RHS of (18), we are left with four contributions to $\mathcal{A}^{A'B'}_{AB}$, corresponding to the diagrams in Fig. 3. Only the first three of them are important; instead the contribution of diagram (d) can be disregarded. In fact, the essential values of variables s_A and s_B are small ($s_A \sim s_B \sim t$) for the contribution of this diagram. Consequently, only transverse [with respect to the (p_A, p_B) plane] components of momenta p_C and $p_{C'}$ can be taken in the propagators of intermediate particles, which means that integrals over s_A and s_B are factorized and can be evaluated by residues; as a result, the contribution of the diagram in Fig. 3(d) is purely imaginary in the Regge region and corresponds to positive signature partial waves [4]. Here

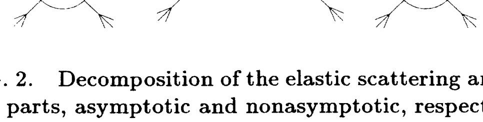

FIG. 2. Decomposition of the elastic scattering amplitude in two parts, asymptotic and nonasymptotic, respectively.

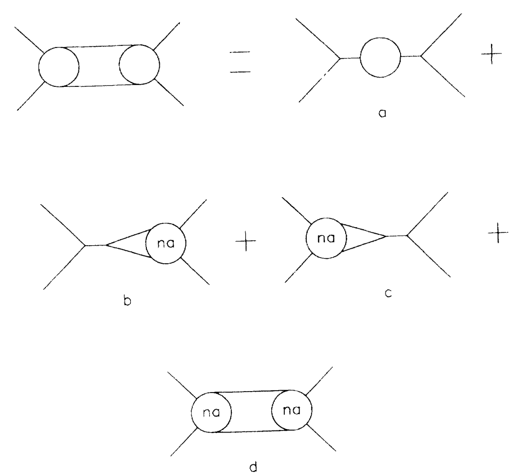

FIG. 3. Contributions to the amplitude of Fig. 1, coming from the product of (a) asymptotic-asymptotic parts, (b) asymptotic-nonasymptotic parts, (c) nonasymptotic-asymptotic parts, and (d) nonasymptotic-nonasymptotic parts.

we are interested in the radiative corrections to the QQR for the Reggeized gluons, i.e., for the case of negative signature; therefore, only diagrams in Figs. 3(a)–3(c) can contribute. The asymptotic contributions take the form

$$\left(\mathcal{A}_8^{(\text{as})}\right)^{A'C'}_{AC} = \frac{(2g^2)}{t}\bar{u}(p_{A'})\frac{\not{p}_B}{s}u(p_A)\bar{u}(p_{C'})\not{p}_A u(p_C) \tag{25}$$

and

$$\left(\mathcal{A}_8^{(\text{as})}\right)^{C'B}_{CB'} = \frac{(2g^2)}{t}\bar{u}(p_C)\not{p}_B u(p_{C'})\bar{u}(p_{B'})\frac{\not{p}_A}{s}u(p_B) \ . \tag{26}$$

We always take a very large value for s (in contrast to s_A and s_B, which are integration variables and can be small as well as large); therefore, we have, in the helicity basis,

$$\bar{u}(p_{A'})\frac{\not{p}_B}{s}u(p_A) = \delta_{\lambda_A,\lambda_{A'}} \ ,$$

$$\bar{u}(p_{B'})\frac{\not{p}_A}{s}u(p_B) = \delta_{\lambda_B,\lambda_{B'}} \ , \tag{27}$$

where λ_A is the helicity of particle A. It is assumed that the polarization states of the scattered particles are obtained from the ones of the initial particles by rotation around the axis orthogonal to the scattering plane. That implies

$$\varphi'_\lambda = \exp\left(i\bar{\nu}\bar{\sigma}\frac{\theta}{2}\right)\varphi_\lambda \ , \qquad \bar{\nu} = \frac{\bar{p}'\times\bar{p}}{|\bar{p}'\times\bar{p}|} \ , \tag{28}$$

where φ and φ' are, respectively, the polarization wave functions for initial and scattered particles and θ is the scattering angle.

Let us note that if we write the asymptotic contribution $(\mathcal{A}_8^{(\text{as})})^{A'C'}_{AC}$ [see Eq. (25)] in the helicity basis for particles A and A' using the first of Eqs. (27), we arrive at the same expression as for the process in which A and A' are gluons [5].

Taking into account the asymptotic parts (25) and (26) and decomposition (24), from Eq. (23) we obtain the following projections for the nonasymptotic parts into the octet color state:

$$\left(\mathcal{A}_8^{(\mathrm{na})}\right)_{AC}^{A'C'} = g^2\left[\bar{u}(p_{A'})\gamma^\mu u(p_A)\frac{g_\perp^{\mu\nu}}{t}\bar{u}(p_{C'})\gamma_\nu u(p_C) + \frac{\delta_{AC}}{N}\bar{u}(p_{C'})\gamma^\mu u(p_A)\frac{1}{u_A}\bar{u}(p_{A'})\gamma_\mu u(p_C)\right] , \tag{29}$$

$$\left(\mathcal{A}_8^{(\mathrm{na})}\right)_{C'B}^{CB'} = g^2\left[\bar{u}(p_C)\gamma^\mu u(p'_C)\frac{g_\perp^{\mu\nu}}{t}\bar{u}(p_{B'})\gamma_\nu u(p_B) + \frac{\delta_{CB}}{N}\bar{u}(p_{B'})\gamma^\mu u(p'_C)\frac{1}{u_B}\bar{u}(p_C)\gamma_\mu u(p_B)\right] , \tag{30}$$

where the decomposition

$$g^{\mu\nu} \simeq g_\perp^{\mu\nu} + \frac{2\left(p_A^\mu p_B^\nu + p_B^\mu p_A^\nu\right)}{s} \tag{31}$$

has been used and $\perp$ means transverse to the (p_A, p_B) plane. We neglect terms containing $\bar{u}(p_{A'})\not{p}_A u(p_A)$ or $\bar{u}(p_{B'})\not{p}_B u(p_B)$ because they are proportional to the corresponding masses and therefore cannot give a contribution of order s/t to the amplitude (18).

Inserting Eqs. (25) and (26) into Eq. (18) we get, for the contribution of the diagram in Fig. 3(a) [cf. Eqs. (20)–(22) of [5]],

$$\left(\mathcal{A}_8^{(a)}\right)_{AB}^{A'B'} = \delta_{\lambda_A,\lambda_{A'}}\delta_{\lambda_B,\lambda_{B'}}\left(\frac{2g^2}{t}\right)^2 p_A^\mu p_B^\nu \sum_f \mathcal{P}_{\mu\nu}^{(f)}(q) , \tag{32}$$

where the summation is performed over quark flavors and

$$\mathcal{P}_{\mu\nu}^{(f)}(q) = \frac{i}{2}\int \frac{d^Dp}{(2\pi)^D}\frac{\mathrm{tr}\left[\gamma^\mu(\not{p}+m_f)\gamma^\nu(\not{p}+\not{q}+m_f)\right]}{(p^2-m_f^2+i\varepsilon)[(p+q)^2-m_f^2+i\varepsilon]} = \frac{4\Gamma\left(2-\frac{D}{2}\right)}{(4\pi)^{\frac{D}{2}}}\left(-g_{\mu\nu}q^2+q_\mu q_\nu\right) \times \int_0^1 \frac{dx\,x(1-x)}{\left[m_f^2-q^2x(1-x)\right]^{2-\frac{D}{2}}} . \tag{33}$$

Here the calculations practically coincide with those of Ref. [5] because, as we previously noted, the asymptotic contribution $(\mathcal{A}_8^{(\mathrm{as})})_{AC}^{A'C'}$ [and $(\mathcal{A}_8^{(\mathrm{as})})_{BC'}^{B'C}$ as well] in the helicity basis for particles A and A' (correspondingly B and B') coincides with that of the case considered in Ref. [5], where particles A and A' (B and B') are gluons.

The contribution of the diagram in Fig. 3(b) is expressed by the product $(\mathcal{A}^{(\mathrm{as})})_{AC}^{A'C'} \times (\mathcal{A}^{(\mathrm{na})})_{C'B}^{CB'}$. Using Eqs. (25) and (30) and keeping only terms of order s we find

$$\left(\mathcal{A}_8^{(b)}\right)_{AB}^{A'B'} = \delta_{\lambda_A,\lambda_{A'}}\frac{g^4 s}{Nt}\frac{p_A^\mu}{s}\bar{u}(p_{B'})\mathcal{V}_\mu(p_B,q)u(p_B) , \tag{34}$$

where

$$\mathcal{V}_\mu(p_B,q) = i\int\frac{d^Dp}{(2\pi)^D}\frac{\gamma^\nu(\not{p}+\not{q}+m_B)\gamma_\mu(\not{p}+m_B)\gamma_\nu}{(p^2-m_B^2+i\varepsilon)[(p+q)^2-m_B^2+i\varepsilon][(p_B-p)^2+i\varepsilon]} . \tag{35}$$

The integrals appearing in Eq. (35), as well as all the integrals appearing in Eq. (18), have been classified and calculated long ago [6] and are presented in the Appendix. Let us accept that, by definition, states with opposite helicity are connected by the relation

$$\varphi_{-\lambda} = \bar{\nu}\bar{\sigma}\varphi_\lambda , \quad \bar{\nu} = \frac{\bar{p}'\times\bar{p}}{|\bar{p}'\times\bar{p}|} , \tag{36}$$

which, in the helicity basis, leads to

$$\bar{u}(p_{A'})u(p_A) = 2m_A\delta_{\lambda_A,\lambda_{A'}} - i\sqrt{-t}\,\delta_{\lambda_A,-\lambda_{A'}} . \tag{37}$$

With such a definition we obtain

$$\frac{p_A^\mu}{s}\bar{u}(p_{B'})\mathcal{V}_\mu(p_B,q)u(p_B) = \frac{\Gamma\left(3-\frac{D}{2}\right)}{(4\pi)^{\frac{D}{2}}}\int_0^1\frac{dx}{\left[m_B^2-x(1-x)t\right]^{3-\frac{D}{2}}}\left\{\frac{2\delta_{\lambda_B,\lambda_{B'}}}{4-D}\left[t\left(\frac{1}{D-3}+(D-3)x(1-x)\right)-\frac{D-1}{D-3}m_B^2\right]-i\delta_{\lambda_B,-\lambda_{B'}}\frac{5-D}{D-3}m_B\sqrt{-t}\right\} . \tag{38}$$

Let us pay attention to the fact that the contribution given by Eqs. (34) and (36) does not depend on the nature of particle A.

Finally the contribution $(\mathcal{A}_8^{(c)})_{AB}^{A'B'}$ of the diagram in Fig. 3(c) can be obtained from Eqs. (34) and (38) by the substitution $A \leftrightarrow B$. Summing up the three contributions and comparing the sum $\mathcal{A}^{(a)} + \mathcal{A}^{(b)} + \mathcal{A}^{(c)}$ with the factorized form (6), where the vertices are defined through Eq. (8), we find

$$\Gamma^{(+)}_{QQ}(q\bar{q}\ \text{state}) = \frac{g^2}{(4\pi)^{\frac{D}{2}}}\Gamma\left(2-\frac{D}{2}\right)\left\{-2\sum_f\int_0^1 \frac{dx x(1-x)}{[m_f^2 - x(1-x)t]^{2-\frac{D}{2}}} + \frac{1}{2N}\int_0^1 \frac{dx}{[m_Q^2 - x(1-x)t]^{3-\frac{D}{2}}}\left[t\left(\frac{1}{D-3} + (D-3)x(1-x)\right) - m_Q^2\frac{(D-1)}{(D-3)}\right]\right\}, \tag{39}$$

$$\Gamma^{(-)}_{QQ}(q\bar{q}\ \text{state}) = \frac{-ig^2}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma\left(3-\frac{D}{2}\right)}{2N} m_Q\sqrt{-t}\frac{(5-D)}{(D-3)}\int_0^1 \frac{dx}{[m_Q^2 - x(1-x)t]^{3-\frac{D}{2}}}\ . \tag{40}$$

We expect that for massless quarks only the helicity-conserving part of the vertex survives and in fact in this case Eqs. (39) and (40) reduce to

$$\Gamma^{(+)}_{QQ}(q\bar{q}\ \text{state})|_{m_q=0} = \frac{g^2}{(4\pi)^{\frac{D}{2}}}(-t)^{\frac{D}{2}-2}\left[-2\Gamma\left(2-\frac{D}{2}\right)n_f\frac{\Gamma^2\left(\frac{D}{2}\right)}{\Gamma(D)} + \frac{\Gamma\left(3-\frac{D}{2}\right)}{2N}\frac{\Gamma^2\left(\frac{D}{2}-1\right)}{\Gamma(D-2)}\left(2+\frac{D}{2\left(\frac{D}{2}-2\right)^2}\right)\right], \tag{41}$$

$$\Gamma^{(-)}_{QQ}(q\bar{q}\ \text{state})|_{m_q=0} = 0\ . \tag{42}$$

Here n_f is the number of quark flavors.

B. Contribution of the two gluon intermediate state

Now let us consider the contribution of the two gluon intermediate state in the t channel to the QQR vertex. The general lines of consideration are the same as before, but now we need to take the quark-gluon scattering amplitude in the Born approximation for the amplitudes $\mathcal{A}^{A'C'}_{AC}$ and $\mathcal{A}^{CB'}_{C'B}$ in Eq. (18). Again, as in the quark contribution case, we are interested only in parts of the amplitudes which correspond to the octet color state in the t channel, because our Reggeon is the Reggeized gluon. Moreover, we need to keep only the F-type color octet for intermediate gluons, because only this color state survives in the Regge asymptotic regime. Consequently the asymptotic contribution $\mathcal{A}^{(\text{as})}$ in the decomposition (24) can contain only this color state, which is, therefore, the only state that contributes to the essential diagrams in Figs. 3(a)–3(c).

This part of the quark-gluon scattering amplitude $\mathcal{A}^{A'C'}_{AC}$ can be written as (cf. Ref. [5])

$$(\mathcal{A}_8)^{A'C'}_{AC} = \frac{(-g^2)}{2}\bar{u}(p_{A'})\left\{\not{e}_C\frac{\not{p}_A - \not{p}_{C'} + m_A}{u_A^2 - m_A^2}\not{e}^*_{C'} - \not{e}^*_{C'}\frac{\not{p}_A + \not{p}_C + m_A}{s_A^2 - m_A^2}\not{e}_C + \frac{2}{t}\left[(\not{p}_C + \not{p}_{C'})e^*_{C'}e_C + 2\not{e}_C(e^*_{C'}q) - 2\not{e}^*_{C'}(e_C q)\right]\right\}u(p_A)\ . \tag{43}$$

In contrast to Ref. [5], here gluons are intermediate particles, so we do not fix their gauge. Instead we write the asymptotic parts of the amplitudes in a form similar to the one used in Ref. [4] for the gluon-gluon scattering amplitude. The possibility of such a choice comes from the fact that the high energy behavior of both amplitudes is determined by the t-channel exchange of a gluon which Reggeizes. In the helicity basis [see Eq. (27)] we have

$$\left(\mathcal{A}_8^{(\text{as})}\right)^{A'C'}_{AC} = g^2\delta_{\lambda_A,\lambda_{A'}}e^{*\sigma'}_{C'}e^{\sigma}_C\Gamma_{\sigma\sigma'}(p_C,p_{C'},p_A)\ , \tag{44}$$

$$\left(\mathcal{A}_8^{(\text{as})}\right)^{CB'}_{C'B} = g^2\delta_{\lambda_B,\lambda_{B'}}e^{*\sigma}_C e^{\sigma'}_{C'}\Gamma_{\sigma'\sigma}(p_{C'},p_C,p_B)\ , \tag{45}$$

with

$$\Gamma^{\sigma\sigma'}(p_C,p_{C'},p_A) = -g^{\sigma\sigma'}\frac{(s_A-u_A)}{t} - 2\left(\frac{2}{t}+\frac{1}{s_A-m_A^2}\right)p_A^{\sigma}q^{\sigma'} + 2\left(\frac{2}{t}+\frac{1}{u_A-m_A^2}\right)p_A^{\sigma'}q^{\sigma} + 2\left(\frac{1}{s_A-m_A^2}-\frac{1}{u_A-m_A^2}\right)p_A^{\sigma}p_A^{\sigma'} \tag{46}$$

and

$$q = p_{C'} - p_C \ , \quad s_A = (p_A + p_C)^2 \ , \quad u_A = (p_A - p_{C'})^2 \ . \tag{47}$$

It is worth to note that the tensor $\Gamma^{\sigma\sigma'}$ can be considered as the generalization of the corresponding tensor used in Ref. [4], for the gluon-gluon scattering amplitude, to the case for which $p_A^2 = p_{A'}^2 = m_A^2 \neq 0$.

In correspondence to the form we choose for the asymptotic term, the nonasymptotic contribution $\mathcal{A}^{(\mathrm{na})}$, taking into account Eq. (27), becomes

$$\left(\mathcal{A}_8^{(\mathrm{na})}\right)_{AC}^{A'C'} = \frac{(-g^2)}{2} e_{C'}^{*\sigma'} e_C^{\sigma} \bar{u}(p_{A'}) \chi_{\sigma\sigma'}(p_C, p_{C'}; p_A, p_B) u(p_A) \ , \tag{48}$$

$$\left(\mathcal{A}_8^{(\mathrm{na})}\right)_{C'B}^{CB'} = \frac{(-g^2)}{2} e_C^{*\sigma} e_{C'}^{\sigma'} \bar{u}(p_{B'}) \chi_{\sigma'\sigma}(p_{C'}, p_C; p_B, p_A) u(p_B) \ , \tag{49}$$

where

$$\chi^{\sigma\sigma'}(p_C, p_{C'}; p_A, p_B) = \left(\gamma^{\varrho} - \frac{2\not{p}_B p_A^{\varrho}}{s}\right) \left[2g^{\sigma\sigma'} \frac{(p_C + p_{C'})^{\varrho}}{t} + 2g^{\sigma\varrho} \left(\frac{2q^{\sigma'}}{t} - \frac{(p_A - q)^{\sigma'}}{s_A - m_A^2}\right) \right.$$
$$\left. -2g^{\sigma'\varrho} \left(\frac{2q^{\sigma}}{t} - \frac{(p_A - q)^{\sigma}}{u_A - m_A^2}\right)\right] - \frac{\gamma^{\sigma}\not{p}_C\gamma^{\sigma'}}{u_A - m_A^2} - \frac{\gamma^{\sigma'}\not{p}_{C'}\gamma^{\sigma}}{s_A - m_A^2} \ . \tag{50}$$

Let us pay attention to the fact that the nonasymptotic parts (48) and (49), as well as the whole amplitude (43), do not turn into zero under the substitution $e_C \to p_C$ ($e_{C'} \to p_{C'}$), but become proportional to the scalar product $e_{C'}p_{C'}$ ($e_C p_C$), i.e., they turn into zero for transverse polarization vectors $e_{C'}$ (e_C) only. It is well known that such properties of QCD amplitudes lead to the necessity of introducing the Faddeev-Popov ghosts in covariant gauges. But, contrary to the nonasymptotic parts, gauge invariance properties of the asymptotic parts (44) and (45) of the amplitudes on the mass shell $p_C^2 = p_{C'}^2 = 0$ are the same as in quantum electrodynamics: these parts turn into zero under the substitution $e_C \to p_C$ ($e_{C'} \to p_{C'}$) independent of the value $e_{C'}$ (e_C), due to the properties of the tensor $\Gamma_{\sigma\sigma'}(p_C, p_{C'}, p_A)$ on the mass shell $p_C^2 = p_{C'}^2 = 0$,

$$\Gamma_{\sigma\sigma'}(p_C, p_{C'}, p_A) p_C^{\sigma} = \Gamma_{\sigma\sigma'}(p_C, p_{C'}, p_A) p_{C'}^{\sigma'} = 0 \ . \tag{51}$$

The gauge invariance properties discussed above enable us to use the Feynman summation over polarization states of intermediate gluons,

$$\sum_{\lambda} e_{\mu}^{(\lambda)} e_{\nu}^{*(\lambda)} = -g_{\mu\nu} \ , \tag{52}$$

without introducing the Faddeev-Popov ghosts, when we calculate the t-channel discontinuities of the contributions of the diagrams in Figs. 3(a)–3(c). It is easy to see that an addition of extra terms, containing k_μ or k_ν, where k is a gluon momentum, to the RHS of Eq. (52) does not change the discontinuities, which are therefore gauge invariant, as they should be. We will use the Feynman summation (52) for calculating the whole contribution of diagrams of Figs. 3(a)–3(c). According to the discussion at the beginning of Sec. II, a gauge dependence can appear here only in terms connected with the renormalization, which will be considered at the end of the section.

In the case of the diagram in Fig. 3(a) we need to calculate the product of the asymptotic parts (44) and (45). The essential region of integration in Eq. (18) for large s and fixed t in this case is determined by the relations

$$p_C^2 \sim p_{C'}^2 \sim \frac{s_A s_B}{s} \sim t \ , \quad t \lesssim s_A \sim u_A \lesssim s \ , \quad t \lesssim s_B \sim u_B \lesssim s \ . \tag{53}$$

In this region, from Eq. (46), we find

$$\Gamma^{\sigma\sigma'}(p_C, p_{C'}, p_A) \Gamma_{\sigma'\sigma}(p_{C'}, p_C, p_B)$$
$$= \frac{(D-4)}{t^2}(u_A - s_A)(u_B - s_B) + \frac{4}{t^2}(s_A s_B + u_A u_B) + \frac{16 s}{t}$$
$$+ s^2 \left(\frac{1}{s_A - m_A^2} - \frac{1}{u_A - m_A^2}\right)\left(\frac{1}{s_B - m_B^2} - \frac{1}{u_B - m_B^2}\right)$$
$$+ 4s\left(\frac{1}{s_A - m_A^2} + \frac{1}{s_B - m_B^2} + \frac{1}{u_A - m_A^2} + \frac{1}{u_B - m_B^2}\right) - 2\left(\frac{u_B - m_B^2}{s_A - m_A^2} + \frac{s_B - m_B^2}{u_A - m_A^2}\right)$$
$$\times \left(1 - \frac{2m_A^2}{t}\right) - 2\left(\frac{u_A - m_A^2}{s_B - m_B^2} + \frac{s_A - m_A^2}{u_B - m_B^2}\right)\left(1 - \frac{2m_B^2}{t}\right) \ . \tag{54}$$

As to be expected, this expression differs from the corresponding one of Ref. [4] only by mass terms, as well as $\Gamma^{\alpha\beta}$ does.

The integrals appearing in Eq. (18), after substitution of Eqs. (44), (45), and (54), are presented in the Appendix. By means of these integrals one can give the contribution of the diagram in Fig. 3(a) the form

$$\left(\mathcal{A}_8^{(a)}\right)_{AB}^{A'B'} = \frac{Ng^4}{(4\pi)^{\frac{D}{2}}}\delta_{\lambda_A,\lambda_{A'}}\delta_{\lambda_B,\lambda_{B'}}\frac{2s}{t}\left[a(s,t)+\Delta a(m_A^2,t)+\Delta a(m_B^2,t)\right] . \tag{55}$$

Here $a(t)$ is the contribution for the massless case (it is the same as in Ref. [4]):

$$a(t) = \frac{\Gamma\left(2-\frac{D}{2}\right)\Gamma^2\left(\frac{D}{2}-1\right)}{(-t)^{2-\frac{D}{2}}\Gamma(D-2)}\left\{(D-3)\left[\ln\left(\frac{-s}{-t}\right)+\ln\left(\frac{-u}{-t}\right)\right.\right.$$
$$\left.\left.+2\psi\left(3-\frac{D}{2}\right)-4\psi\left(\frac{D}{2}-2\right)+2\psi(1)\right]+\frac{1}{2(D-1)}-\frac{4}{D-4}-\frac{9}{2}\right\} , \tag{56}$$

where $\psi(z)$ is defined in Eq. (11), while $\Delta a(m^2,t)$ is mass dependent and becomes zero at $m=0$:

$$\Delta a(m^2,t) = \Gamma\left(3-\frac{D}{2}\right)\int_0^1\int_0^1 dx_1dx_2\theta(1-x_1-x_2)$$
$$\times\left[\frac{t\,(1-x_1)^2}{x_1}\left(\frac{1}{\left[x_1^2m^2-x_2(1-x_1-x_2)t\right]^{3-\frac{D}{2}}}-\frac{1}{\left[(-t)\,(1-x_1-x_2)\right]^{3-\frac{D}{2}}}\right)\right.$$
$$\left.-\frac{2m^2x_1}{\left[x_1^2m^2-x_2\,(1-x_1-x_2t)\right]^{3-\frac{D}{2}}}\right] . \tag{57}$$

The essential point in Eq. (50) is the separation of dependences on m_A and m_B. Such separation is a vital necessity for the interpretation of the amplitude in terms of the Regge pole contribution [see Eq. (6)].

Let us evaluate the contribution of the diagram in Fig. 3(b). It is expressed in terms of the integral (18), where we need to use the amplitudes (44) and (49). The essential region of integration in Eq. (18) for this case is

$$p_C^2 \sim p_{C'}^2 \sim s_B \sim u_B \sim t, \qquad s_A \sim u_A \sim s. \tag{58}$$

In this region from Eqs. (46) and (50) we obtain

$$\Gamma^{\sigma\sigma'}(p_C,p'_C,p_A)\bar{u}(p'_B)\chi_{\sigma'\sigma}(p_{C'},p_C;p_B,p_A)u(p_B)$$
$$=\bar{u}(p'_B)\left\{\frac{s_A-u_A}{t}\left[\frac{2}{s_B-m_B^2}\left(m_B-\left(2m_B^2-t\right)\frac{\not{p}_A}{s}\right)\right.\right.$$
$$\left.-\frac{2}{u_B-m_B^2}\left(m_B-\left(2m_B^2-t\right)\frac{\not{p}_A}{s}\right)-(D-2)\left(\frac{\not{p}_{C'}}{u_B-m_B^2}+\frac{\not{p}_C}{s_B-m_B^2}\right)\right]$$
$$\left.+\frac{4}{t}\left[\frac{\not{q}\not{p}_{C'}\not{p}_A-\not{p}_A\not{p}_{C'}\not{q}}{u_B-m_B^2}+\frac{\not{p}_A\not{p}_C\not{q}-\not{q}\not{p}_C\not{p}_A}{s_B-m_B^2}\right]\right\} . \tag{59}$$

An important property of this expression is its independence on m_A. Remembering that the tensors $\Gamma^{\sigma\sigma'}(p_C,p_{C'},p_A)$ for the cases when particles A and A' are gluons or quarks differ only by mass terms, we conclude that the contribution of the diagram in Fig. 3(b) does not depend on the nature of particles A, A' if it is written in the helicity state basis for these particles. With the help of the Appendix, where integrals appearing in Eq. (18) after substituting Eqs. (44), (49), and (59) are presented, and using the relation

$$\bar{u}\,(p_{B'})\,(\not{q}\not{p}_B\not{p}_A-\not{p}_A\not{p}_B\not{q})\,u\,(p_B)=\bar{u}\,(p_{B'})\left[\left(t-4m_B^2\right)\not{p}_A+2sm_B\right]u\,(p_B) , \tag{60}$$

which is a result of a simple algebra, we obtain for this contribution

$$\left(A_8^{(b)}\right)_{AA'}^{BB'}=\delta_{\lambda_A,\lambda_{A'}}\frac{2s}{t}\frac{g^4N\Gamma\left(3-\frac{D}{2}\right)}{(4\pi)^{\frac{D}{2}}}\bar{u}(p_{B'})\int_0^1\int_0^1\frac{dx_1dx_2\theta\,(1-x_1-x_2)}{\left[x_1^2m_B^2-x_2(1-x_1-x_2)t\right]^{3-\frac{D}{2}}}$$
$$\times\left\{\frac{\not{p}_A}{s}\left(-2m_B^2x_1-\frac{D-2}{D-4}\left[m_B^2x_1^2-x_2\,(1-x_1-x_2)\,t\right]\right)+m_B\left[x_1-\left(\frac{D}{2}-1\right)x_1^2\right]\right\}u(p_B) . \tag{61}$$

The contribution of the diagram in Fig. 3(c) can be obtained from Eq. (61) by the substitution $A\leftrightarrow B$, $A'\leftrightarrow B'$. The total contribution of the two gluon intermediate state in the t channel to the asymptotic of the quark-quark scattering amplitude with the octet color state and negative signature in the t channel is given by the sum $\mathcal{A}^{(a)}+\mathcal{A}^{(b)}+\mathcal{A}^{(c)}$.

Comparing it with the representation (6), we are once more convinced that the Regge trajectory ω in the lowest order is given by Eq. (5) and find the two gluon intermediate state contribution to the quark-quark-Reggeon vertex. For the helicity conserving part of this contribution [see Eq. (8)], performing the decomposition

$$\Gamma_{QQ}^{(+)} = \Gamma_{QQ}^{(+)}\,|_{m_Q=0} + \Delta\Gamma_{QQ}^{(+)}\,, \tag{62}$$

with the help of Eqs. (27) and (37) we obtain

$$\Gamma_{QQ}^{(+)}\,(gg\text{ state})\,|_{m_Q=0} = \frac{g^2N}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma\left(2-\frac{D}{2}\right)\Gamma^2\left(\frac{D}{2}-1\right)}{(-t)^{2-\frac{D}{2}}\,\Gamma\,(D-2)}\left\{(D-3)\left[\psi\left(3-\frac{D}{2}\right)\right.\right.$$
$$\left.\left.-2\psi\left(\frac{D}{2}-2\right)+\psi\,(1)\right]+\frac{1}{4\,(D-1)}-\frac{2}{D-4}-\frac{7}{4}\right\} \tag{63}$$

and

$$\Delta\Gamma_{QQ}^{(+)}\,(gg\text{ state}) = \frac{g^2N}{(4\pi)^{\frac{D}{2}}}\Gamma\left(3-\frac{D}{2}\right)\int_0^1\int_0^1 dx_1dx_2\theta\,(1-x_1-x_2)$$
$$\times\left[t\left(\frac{(1-x_1)^2}{x_1}+\frac{(D-2)}{(D-4)}x_2\,(1-x_1-x_2)\right)\left(\frac{1}{[x_1^2m_Q^2-x_2\,(1-x_1-x_2)\,t]^{3-\frac{D}{2}}}\right.\right.$$
$$\left.\left.-\frac{1}{[-x_2\,(1-x_1-x_2)\,t]^{3-\frac{D}{2}}}\right)-\frac{m_Q^2\left(2x_1+(D-2)(D-3)\frac{x_1^2}{D-4}\right)}{[x_1^2m_Q^2-x_2\,(1-x_1-x_2)\,t]^{3-\frac{D}{2}}}\right]\,. \tag{64}$$

For the helicity nonconserving part we have

$$\Gamma_{QQ}^{(-)}\,(gg\text{ state}) = -i\frac{g^2N}{(4\pi)^{\frac{D}{2}}}\Gamma\left(3-\frac{D}{2}\right)m_Q\sqrt{-t}\int_0^1\int_0^1\frac{dx_1dx_2\theta\,(1-x_1-x_2)}{[x_1^2m_Q^2-x_2\,(1-x_1-x_2)\,t]^{3-\frac{D}{2}}}\left[x_1-\left(\frac{D}{2}-1\right)x_1^2\right]\,. \tag{65}$$

It vanishes in the zero mass limit, as it should.

C. Renormalization of the QQR vertex

It was explained at the beginning of this section that we can add a term with the pole structure in t to the RHS of Eq. (18) without changing the t-channel discontinuity and without spoiling the renormalizability of the theory. That means we can add an expression which is equal to the Born amplitude with some constant coefficient. In principle, we could put on the helicity conserving part of the QQR vertex some condition (of the type of $\Gamma_{QQ}^{(+)}\,|_{t=-\mu^2}= 0$) which would be a definition of the renormalized coupling constant. But it is useful to have a possibility to express the QQR vertex in terms of the coupling constant in commonly used renormalization schemes, such as the modified minimal subtraction ($\overline{\text{MS}}$) scheme. In order to have such a possibility we need to connect our results to those which are obtained by the usual approach, in terms of Feynman diagrams. In the diagrammatic approach the terms under discussion may come from quark-gluon vertices and gluon polarization operator at $t = 0$ and from self-energy insertions into external quark legs. It is easy to observe that the first two contributions in the Feynman gauge[1] are taken into account properly in Eqs. (32),(34) and (55),(61), respectively. So, we need only to consider the self-energy insertions into external quark legs. It means that in the one loop approximation we need to add to the helicity conserving part of the QQR vertex the value

$$\Gamma_{QQ}^{(+)}(\text{self-energy}) = -\frac{\partial\Sigma(\not p)}{\partial\not p}\,|_{\not p=m_Q}\,, \tag{66}$$

where $\Sigma(\not p)$ is the mass operator of the quark

$$\Sigma(\not p) = -g^2\frac{(N^2-1)}{2N}\times\int\frac{d^Dk}{(2\pi)^Di}\frac{\gamma^\mu\,(\not p-\not k+m_Q)\,\gamma_\mu}{(k^2+i\varepsilon)\,[(p-k)^2-m_Q^2+i\varepsilon]}\,. \tag{67}$$

An elementary calculation gives

$$\Gamma_{QQ}^{(+)}(\text{self-energy}) = g^2\frac{(N^2-1)}{2N}\frac{\Gamma\left(2-\frac{D}{2}\right)}{(4\pi)^{\frac{D}{2}}}m_Q^{D-4}\times\left(\frac{1-D}{D-3}\right)\,. \tag{68}$$

The total one loop correction to the helicity conserving part of the QQR vertex is given by the sum

$$\Gamma_{QQ}^{(+)} = \Gamma_{QQ}^{(+)}(q\bar q\text{ state})+\Gamma_{QQ}^{(+)}(gg\text{ state})+\Gamma_{QQ}^{(+)}(\text{self-energy})\,, \tag{69}$$

where the first term on the RHS is given by Eq. (39), the

[1] A Feynman gauge did appear here because we have used the Feynman summation over polarization states of intermediate gluons (52) for calculating the contributions of Figs. 3(a)–3(c). As explained after Eq. (52), a gauge dependence appears in terms connected with the renormalization (and in these terms only).

second by Eq. (62), and the third by Eq. (68). For the helicity nonconserving part we have only contributions of the first two types, which are given by Eqs. (40) and (65) correspondingly.

In all the above formulas we used the nonrenormalized coupling constant g. Therefore expression (69) for $D \to 4$ contains singularities coming from ultraviolet as well as infrared divergences of Feynman integrals. Let us note here that the helicity nonconserving part of the vertex does not contain the ultraviolet divergences. We can remove the ultraviolet divergences expressing g in terms of the renormalized coupling constant, for example, in the $\overline{\text{MS}}$ scheme:

$$g = g_\mu \mu^{\frac{4-D}{2}} \left\{ 1 + \left(\frac{11}{3} N - \frac{2}{3} n_f \right) \frac{g_\mu^2}{(4\pi)^{\frac{D}{2}} (D-4)} \times \left[1 + \frac{D-4}{2} \left(\ln \frac{1}{4\pi} - \psi(1) \right) \right] + \cdots \right\} , \quad (70)$$

where g_μ is the renormalized coupling constant at the normalization point μ and $\psi(z)$ has been already defined [see Eq. (11)].

III. CHECK OF THE APPROACH CONSISTENCY

Now we have both the gluon-gluon-Reggeon vertex Γ^R_{GG} and quark-quark-Reggeon vertex Γ^R_{QQ} in the one loop approximation. The first of them was calculated by using the gluon-gluon scattering amplitude as a tool and the second via the quark-quark scattering amplitude. The process of quark-gluon scattering has not been considered up to now. But the Reggeon contribution to this amplitude [see Eq. (6)] is expressed in terms of the GGR vertices and QQR vertices and the gluon trajectory $1+\omega(t)$, which is given in the one loop approximation by the formula (5). It allows us to check the validity of representation (6) for the high energy behavior of the amplitudes with gluon quantum numbers in the t channel and negative signature.

It can be done by calculating the quark-gluon scattering amplitude and comparing the results of calculation with the expression given by formula (6) in terms of the known trajectory $1+\omega(t)$ and vertices Γ^R_{QQ} and Γ^R_{GG}. But really we need not perform any new calculation. The method of calculations used for gluon-gluon and quark-quark scattering amplitudes allows us to check the validity of the expression (6) simply by keeping an eye on the calculations we performed. Our starting point in calculating any amplitude is Eq. (18). In each case we have two gluon and quark-antiquark intermediate states in the t channel. An essential step in the calculation is the decomposition of the amplitude entering in the integrand of Eq. (18) into the sum (24) (see Fig. 2) of asymptotic and nonasymptotic parts. An important fact is that a product of nonasymptotic parts in the integrand in Eq. (18) cannot give a contribution of order s to an amplitude with gluon quantum numbers in the t channel and negative signature which we are interested in, so we are left with contributions presented schematically in Figs. 3(a)–3(c).

The next important fact is that the asymptotic parts of the amplitudes $(A_8^{(\text{as})})^{A'C'}_{AC}$ and $(A_8^{(\text{as})})^{B'C'}_{BC'}$ can be chosen in a factorized form. Of course, this fact is strictly related to the case in which the high energy behavior of the amplitudes is determined by gluon exchange in the t channel. We choose the asymptotic parts for the case of a quark-antiquark intermediate state (CC') in the t channel in the form

$$\left(A_8^{(\text{as})}\right)^{A'C'}_{AC} = \frac{2g^2}{t} \delta_{\lambda_A \lambda_{A'}} \bar{u}(p_{C'}) \not{p}_A u(p_C) \quad (71)$$

in the helicity basis for particles A and A', independently of whether they are gluons [see Eqs. (17) and (13) of Ref. [5]] or quarks [see Eqs. (25) and (27) of this paper]. Therefore, in this intermediate channel the contribution of the diagram in Fig. 3(a) does not depend on the kind of particles A, A' and B, B' [see Eq. (32)]. For the same reason the contribution of the diagram in Fig. 3(b) [Fig. 3(c)] depends only on the kind of the particles B, B' (A, A'). Taking into account that these diagrams contribute only to the vertices $\Gamma^R_{BB'}$ and $\Gamma^R_{AA'}$ correspondingly, we conclude that the contribution of the quark-antiquark intermediate state in the t channel to an amplitude of any of the processes under consideration can be put in the form of Eq. (6), where the vertices $\Gamma^R_{AA'}$ $(\Gamma^R_{BB'})$ do not depend on the kind of particles B, B' (A, A').

For the case of the two gluon intermediate state the conclusion is the same, although the properties of the asymptotic contributions are slightly changed. We choose these contributions in the form of Eqs. (44)–(46) and here the dependence on the kind of the particles A, A' (B, B') enters through the masses of these particles.[2] Of course in the Regge asymptotic limit for the amplitude $A^{AC'}_{AC}$ $(A^{B'C}_{BC'})$, which means for $s_A \sim u_A \gg t$ $(s_B \sim u_B \gg t)$, this dependence becomes negligible, but we need to integrate over s_A (u_A) in Eq. (18). Therefore we choose these asymptotic contributions in such a form which conserves the analytic properties of the exact amplitudes.

An essential property of asymptotic parts (44) and (45) is that the contribution of the diagram in Fig. 3(a), being calculated in terms of these parts, is presented in the form of Eq. (55), where the dependences on the masses m_A and m_B are separated. Therefore, all the dependence on the kind of particles A, A' (B, B') coming from this contribution is included into the vertices $\Gamma^R_{AA'}$ $(\Gamma^R_{BB'})$. The same is true for the case of the contributions of the diagrams in Figs. 3(b) and 3(c) as well, because the contribution of the diagram in Fig. 3(b) [Fig. 3(c)] depends only on the kind of particles B, B' (A, A') [see Eq. (61)], just as in the case of the quark-antiquark intermediate state. Consequently we come to the same conclusion as for this case. Since the contributions of the quark-antiquark and two gluons intermediate states enter into the PPR vertices additively, it means that the high energy behavior of all QCD elastic scattering amplitudes with gluon quantum numbers in the t channel and negative signature are presented by the Regge pole contribution (6).

[2]Let us note that, on the contrary, the dependence on $p_A^2 = m_A^2$ is negligible in Eq. (66).

IV. CONCLUSIONS

We calculated one loop corrections to the quark-quark-Reggeon vertex in the QCD, where the Reggeon is a Reggeized gluon. Taking into account this vertex together with the gluon-gluon-Reggeon vertex calculated before, we get Regge pole contributions to gluon and quark elastic scattering processes. Since nonlogarithmic terms of these contributions to the amplitudes of the three processes (gluon-gluon, quark-quark, and quark-gluon elastic scattering) are expressed in terms of the two vertices, these amplitudes have to satisfy nontrivial relations if the Regge pole only contributes to the large s behavior of the amplitudes with gluon quantum numbers and negative signature in the t channel. We have checked that these relations are satisfied; i.e., the representation (6) of these amplitudes in terms of the Regge pole contribution is applicable beyond the leading logarithmic approximation for the Regge region. The results obtained are needed for the next step in the calculation program [2] for corrections to the LLA: calculation of two loop corrections to the gluon-Regge trajectory. In the two loop approximation a part of the corrections to the trajectory comes from two particle intermediate states in the two channels. The calculation of this part can be performed by using the results presented. It will be done in a subsequent publication.

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APPENDIX

All integrals appearing in Eq. (18) were classified and calculated earlier by 't Hooft and Veltman [6]. We present them here in our notation for the reader's convenience only.

Let us first consider the case of a quark-antiquark intermediate state. To obtain the contribution of the diagrams in Figs. 3(a) and 3(b) we need the integrals

$$I_{Q2} = -i\int \frac{d^Dp}{(p^2-m^2+i\varepsilon)\left[(p+q)^2-m^2+i\varepsilon\right]} = \pi^{\frac{D}{2}}\Gamma\left(2-\frac{D}{2}\right)\int_0^1 \frac{dx}{\left[m^2-x(1-x)t\right]^{2-\frac{D}{2}}}\,, \tag{A1}$$

$$I^{\mu}_{Q2} = -i\int \frac{d^Dp\,(-p^\mu)}{(p^2-m^2+i\varepsilon)\left[(p+q)^2-m^2+i\varepsilon\right]} = \frac{q^\mu}{2}I_{Q2}\,, \tag{A2}$$

$$I^{\mu\nu}_{Q2} = -i\int \frac{d^Dp\,p^\mu p^\nu}{(p^2-m^2+i\varepsilon)\left[(p+q)^2-m^2+i\varepsilon\right]} = \pi^{\frac{D}{2}}\Gamma\left(2-\frac{D}{2}\right)\int_0^1 \frac{dx}{\left[m^2-x(1-x)t\right]^{2-\frac{D}{2}}} \times\left[q^\mu q^\nu x^2-\frac{g^{\mu\nu}\left[m^2-x(1-x)t\right]}{2-D}\right] \tag{A3}$$

and

$$I_{Q3B} = -i\int \frac{d^Dp}{(p^2-m^2+i\varepsilon)\left[(p+q)^2-m^2+i\varepsilon\right]\left[(p_B-p)^2+i\varepsilon\right]} = \pi^{\frac{D}{2}}\frac{\Gamma\left(2-\frac{D}{2}\right)}{2}\int_0^1 \frac{dx}{\left[(m^2-x(1-x)t\right]^{3-\frac{D}{2}}}\,, \tag{A4}$$

$$I^{\mu}_{Q3B} = -i\int \frac{d^Dp\,p^\mu}{(p^2-m^2+i\varepsilon)\left[(p+q)^2-m^2+i\varepsilon\right]\left[(p_B-p)^2+i\varepsilon\right]} = \pi^{\frac{D}{2}}\frac{\Gamma\left(2-\frac{D}{2}\right)}{2(D-3)}\int_0^1 \frac{dx}{\left[m^2-x(1-x)t\right]^{3-\frac{D}{2}}}\left[p_B^\mu-(D-4)xq^\mu\right]\,, \tag{A5}$$

$$I^{\mu\nu}_{Q3B} = -i\int \frac{d^Dp\,p^\mu p^\nu}{(p^2-m^2+i\varepsilon)\left[(p+q)^2-m^2+i\varepsilon\right]\left[(p_B-p)^2+i\varepsilon\right]} = \frac{\pi^{\frac{D}{2}}\Gamma(2-\frac{D}{2})}{2(D-2)(D-3)}\int_0^1 \frac{dx}{\left[m^2-x(1-x)t\right]^{3-\frac{D}{2}}}\{2p_B^\mu p_B^\nu-(D-4)x(p_B^\mu q^\nu+q^\mu p_B^\nu) +(D-4)(D-3)x^2q_\mu q^\nu+(D-3)g^{\mu\nu}[m^2-x(1-x)t]\}\,, \tag{A6}$$

where $q=p_A-p_{A'}=p_{B'}-p_B$ and $t=q^2$.

Let us now consider the case of two gluon intermediate state. To get the contribution of the diagram in Fig. 3(a), in addition to the integrals I_{Q2}, I^{μ}_{Q2}, and $I^{\mu\nu}_{Q2}$, we need

$$I_{G3B} = -i\int \frac{d^Dk}{(k^2+i\varepsilon)\,[(k+q)^2+i\varepsilon]\,[(p_B-k)^2-m^2+i\varepsilon]}$$
$$= -\pi^{\frac{D}{2}}\Gamma\left(3-\frac{D}{2}\right)\int_0^1\int_0^1 \frac{d\rho_2}{{R_{G3}}^{3-\frac{D}{2}}}\ , \quad (A7)$$

$$I^{\mu}_{G3B} = -i\int \frac{d^Dk k^{\mu}}{(k^2+i\varepsilon)\,[(k+q)^2+i\varepsilon]\,[(p_B-k)^2-m^2+i\varepsilon]}$$
$$= -\pi^{\frac{D}{2}}\Gamma\left(3-\frac{D}{2}\right)\int_0^1\int_0^1 \frac{d\rho_2}{R_{G3}^{3-\frac{D}{2}}}\,(x_1p_B^{\mu}-x_2q^{\mu})\ , \quad (A8)$$

$$I^{\mu\nu}_{G3B} = -i\int \frac{d^Dk k^{\mu}k^{\nu}}{(k^2+i\varepsilon)\,[(k+q)^2+i\varepsilon]\,[(p_B-k)^2-m^2+i\varepsilon]}$$
$$= -\pi^{\frac{D}{2}}\Gamma\left(3-\frac{D}{2}\right)\int_0^1\int_0^1 \frac{d\rho_2}{R_{G3}^{3-\frac{D}{2}}}\left[(x_1p_B^{\mu}-x_2q^{\mu})\,(x_1p_B^{\nu}-x_2q^{\nu})-\frac{g^{\mu\nu}}{4-D}R_{G3}\right]\ , \quad (A9)$$

where

$$R_{G3} = m^2x_1^2 - x_2(1-x_1-x_2)t\ , \quad (A10)$$

and integrals I_{G3A}, I^{μ}_{G3A}, and $I^{\mu\nu}_{G3A}$ which are obtained from Eqs. (A7)–(A9) by replacing p_B and q with p_A and $-q$, respectively.

We also have to consider the more complicated integral

$$I_{G\Box} = -i\int \frac{d^Dk}{(k^2+i\varepsilon)\,[(k+q)^2+i\varepsilon]\,[(k+p_A)^2-m_A^2+i\varepsilon]\,[(k-p_B)^2-m_B^2+i\varepsilon]}\ , \quad (A11)$$

which in the asymptotic region

$$s=(p_A+p_B)^2 \gg -t \sim m_A^2 \sim m_B^2$$

can be written as the sum of three terms:

$$I_{G\Box} \approx I_{\Box} + \triangle_B + \triangle_A\ . \quad (A12)$$

Here $I_{\Box}$ is $I_{G\Box}$ in the massless case

$$I_{\Box} = -2(D-3)\left[\ln\left(\frac{-s}{-t}\right) - 2\psi\left(\frac{D}{2}-2\right) + \psi\left(3-\frac{D}{2}\right) + \psi(1)\right]\frac{I_2}{st}\ , \quad (A13)$$

where $\psi(z)=\frac{\Gamma'(z)}{\Gamma(z)}$ and $I_2 = I_{Q2}\,(m=0)$. $\triangle_B$ is given by

$$\triangle_B \approx \frac{\pi^{\frac{D}{2}}\Gamma\left(3-\frac{D}{2}\right)}{-s}\int_0^1\frac{du}{u}\int_0^{1-u}dx\left[\frac{1}{[-x(1-x-u)t+u^2m_B^2]^{3-\frac{D}{2}}} - \frac{1}{[-x(1-x-u)t]^{3-\frac{D}{2}}}\right]\ , \quad (A14)$$

and $\triangle_A$ can be obtained from $\triangle_B$ by substituting m_B with m_A.

Finally, substitution $p_B \leftrightarrow -p_{B'} = -(p_B+q)$ in Eq. (A11) leads to the last integral we need: it can be obtained from $I_{G\Box}$ simply by changing s with $u \approx -s$. At last, in calculating the contribution of the diagram in Fig. 3(b) in the case of two gluon intermediate state we meet again the integrals I_{G3B}, I^{μ}_{G3B}, and $I^{\mu\nu}_{G3B}$.

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