PHYSICAL REVIEW D VOLUME 50, NUMBER 1 1 JULY 1994

Exclusive $W^+ + \gamma$ production in proton-antiproton collisions. I. General formalism

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(Received 18 February 1994)

We present a detailed computation of the fully exclusive cross section of $p+\bar{p}\rightarrow W^+ + \gamma + X$ with $X=0$ and 1 jet in the framework of the factorization theorem and dimensional regularization. Order α_S and photon bremsstrahlung contributions are discussed in the $\overline{\text{MS}}$ mass factorization scheme. The resulting expressions are ready to be implemented numerically using Monte Carlo techniques to compute single and double differential cross sections and correlations between outgoing pairs of particles.

PACS number(s): 13.85.Hd, 12.38.Bx, 14.70.Fm

I. INTRODUCTION

Ever since Mikaelian's discovery of a zero in the amplitude of the partonic subprocess $q+\bar{q}\rightarrow W+\gamma$ [1], radiative production of W bosons has been discussed as a way of testing the validity of the electroweak theory. The study of differential distributions in $p+\bar{p}\rightarrow W+\gamma+X$ may be the best way to place bounds on the magnitude of the magnetic dipole and electric quadrupole moments of the W boson. Deviations from the standard model could show up as a shift of the photon distributions near the dip that is a reflection of the partonic zero. The QCD corrections to the reaction $p+\bar{p}\rightarrow W+\gamma+X$ and its deviations from the standard model have been studied in [2–5] and other references therein. These papers have been mainly devoted to the analysis of single photon distributions, photon–W-boson pair mass correlations, and charge lepton-photon pseudorapidity correlations, and they either neglect or approximate the photon bremsstrahlung contributions.

When computing the photon inclusive process in [2] and [3], all the singularities associated with a jet emitted in a collinear or a soft region of phase space were regularized by analytically performing all the integrations associated with the jet and the W boson in n space-time dimensions. Although the numerical advantage of this procedure is obvious—one is left with only photon variables to be integrated over numerically—the predictive power of the whole computation is limited by the fact that one loses information about the energies and angles of the jet and the W boson.

In the present work we redo the exact first order calculation reported in [2] in an exclusive fashion. We present analytical results for the integrands needed in the computation of physical observables related to any of the outgoing particles in the reactions $p+\bar{p}\rightarrow W^+ + \gamma$ and $p+\bar{p}\rightarrow W^+ + \gamma +$jet. Using these results we will extend the studies of the electroweak and QCD sectors of the standard model by providing a complete set of single and double differential distributions and correlations including the W boson and, when applicable, the jet. Deviations of the experimental data from the theoretical predictions could not only mean new physics in the electroweak sector, but would also probe the QCD behavior and the underlying photon bremsstrahlung processes. In particular, an inadequate photon bremsstrahlung approximation would also result in deviations from the predicted photon single and double differential distributions and correlations.

The method that we employ for computing exclusive cross sections is based on the one used by Mele, Nason, and Ridolfi in the context of Z^0 pair production and production of heavy quarks [6]. This method allows for control of all soft and initial (final) state collinear singular regions of phase space in the framework of dimensional regularization and the factorization theorem.

We consider three different scenarios: (1) the two-body inclusive production of W^+ and γ, (2) the exclusive production of W^+, γ and one jet, and (3) the exclusive production of W^+ and γ accompanied by zero jets. In all three cases we take into account exact $O(\alpha_S)$ QCD contributions in the modified minimal subtraction ($\overline{MS}$) mass factorization scheme. Contributions arising from photon-quark and photon-gluon fragmentation functions (generically called "photon bremsstrahlung contributions") are also included in our discussion.

In Sec. II, we present a detailed review of the process $p+\bar{p}\rightarrow W^+ + \gamma + X$ for $X=0,1$ jet in the framework of the parton model and the factorization theorem. In Sec. III we show how the cancellation of singularities is performed in an exclusive fashion in each of the hard scattering channels of our process in the framework of n-dimensional regularization. Section IV is devoted to the definition of the three experimental scenarios and their corresponding cuts. We end our study in Sec. V with a discussion of the numerical implementation of the several expressions for the cross sections, together with a listing of the relevant formulas.

Results for total, single and double differential cross sections and correlations between pairs of outgoing particles are given in a separate paper [7].

II. THE PROCESS $p+\bar{p}\rightarrow W^+ + \gamma + X$, THE PARTON MODEL, AND THE FACTORIZATION THEOREM

A. Introduction

We are considering the hadronic processes given by

$$p+\bar{p}\to W^+ +\gamma \tag{2.1}$$

and

$$p+\bar{p}\to W^+ +\gamma+\text{jet} . \tag{2.2}$$

In what follows we will omit the charge index "+" when referring to the W^+ boson. In the framework of the parton model we can formally write the hadronic two-body inclusive differential cross section as

$$\sum_X \frac{D^2\sigma^H}{DQ_1DQ_2}[p(P_1)\bar{p}(P_2)\to W(Q_1)\gamma(Q_2)X]$$
$$= \sum_{i,j,k}\left[\int_0^1 du_1\int_0^1 du_2\int_0^1 du_3 D_{ip}(u_1)D_{j\bar{p}}(u_2)D_{\gamma k}(u_3)\sum_x \frac{D^2\sigma^P}{DQ_1DQ_2}\left[i(u_1P_1)j(u_2P_2)\to W(Q_1)k\left[\frac{Q_2}{u_3}\right]x\right]\right], \tag{2.3}$$

where $\sum_X$ is a sum over sets X of physical particles integrated over their phase space. $\sum_{i,j,k}$ denotes sums over partons i,j,k (by partons we mean quarks, antiquarks, gluons, and photons). $\sum_x$ is a sum over sets x of outgoing partons integrated over their phase space. In n-dimensional space-time DQ_1 and DQ_2 are generically given by

$$DQ_i=\frac{d^{n-1}\mathbf{Q}_i}{(2\pi)^{n-1}2Q_{i,0}}, \tag{2.4}$$

where Q_i is an n-momentum vector with space components $\mathbf{Q}_i$ and time component $Q_{i,0}$. $\sum_x D^2\sigma^P/DQ_1DQ_2$ formally denotes the "bare" partonic two-body inclusive differential cross section, which can be directly computed using perturbation theory in the standard model. $D_{iA}(u)$ for $A=p,\bar{p}$ are the bare partonic densities. The bare fragmentation function $D_{\gamma k}(u)$ gives the photon momentum fraction density when a parton of type k and momentum q fragments into a photon of momentum uq and any number of hadrons. After renormalization has been performed the bare partonic cross sections in (2.3) contain soft and collinear singularities coming from virtual corrections as well as integration over phase space of nonempty sets of outgoing partons x. Cancellation of soft singularities will occur after addition of the soft pole terms in the virtual corrections with the soft pole terms in the corresponding emission processes. By virtue of the factorization theorem [8] the bare partonic densities and the bare fragmentation function are defined to contain singularities that cancel against the remaining collinear singularities in the bare partonic cross sections so that the hadronic cross section on the left-hand side (LHS) of (2.3) is a finite quantity. This procedure is implemented at a specific mass factorization scale M. We will define this scale to equal the renormalization scale μ. To avoid complicating the subsequent formulas this scale dependence is not explicitly written, except where necessary for the discussion. According to the factorization theorem we can rewrite the singular bare partonic two-body inclusive cross section in terms of nonsingular "hard scattering cross sections":

$$\sum_x \frac{D^2\sigma^P}{DQ_1DQ_2}[i(p_1)j(p_2)\to W(Q_1)k(Q_2)x]$$
$$=\sum_{a,b,c,x_a,x_b,x_c}\left[\int_0^1 dv_1\int_0^1 dv_2\int_0^1 dv_3 d_{ai}^{x_a}(v_1)d_{bj}^{x_b}(v_2)d_{kc}^{x_c}(v_3)\right.$$
$$\left.\times\sum_y \frac{D^2\sigma}{DQ_1DQ_2}\left[a(v_1p_1)b(v_2p_2)\to W(Q_1)c\left[\frac{Q_2}{v_3}\right]y\right]\right], \tag{2.5}$$

where $\sum_y D^2\sigma/DQ_1DQ_2$ denotes a two-body inclusive hard scattering differential cross section.

$d_{ai}^{x_a}(v)$ denotes the splitting function of parton i into a parton a and a set of partons x_a with a carrying a momentum fraction v of its parent parton i. These splitting functions factorize the collinear singularities contained in the bare partonic cross section $\sum_x D^2\sigma^P/DQ_1DQ_2$ and they can be exactly computed order by order in perturbation theory. In this way (2.5) is solved perturbatively for the hard scattering cross sections $\sum_y D^2\sigma/DQ_1DQ_2$.

Using (2.5) in (2.3) we can rewrite the hadronic two-body inclusive differential cross section in terms of only nonsingular quantities:

$$\sum_X \frac{D^2\sigma^H}{DQ_1DQ_2}[p(P_1)\bar{p}(P_2)\to W(Q_1)\gamma(Q_2)X]$$
$$=\sum_{a,b,c}\left[\int_0^1 d\tau_1\int_0^1 d\tau_2\int_0^1 d\tau_3 f_{ap}(\tau_1)f_{b\bar{p}}(\tau_2)f_{\gamma c}(\tau_3)\sum_x \frac{D^2\sigma}{DQ_1DQ_2}\left[a(\tau_1P_1)b(\tau_2P_2)\to W(Q_1)c\left[\frac{Q_2}{\tau_3}\right]x\right]\right]. \tag{2.6}$$

The parton densities $f_{ap}(\tau), f_{b\bar{p}}(\tau)$ and the fragmentation function $f_{\gamma c}(\tau)$ are defined by

$$
\begin{aligned}
f_{ap}(\tau) &= \sum_{x_a,i} \int_\tau^1 du \frac{1}{u} d_{ai}^{x_a}\left[\frac{\tau}{u}\right] D_{ip}(u) ,\\
f_{b\bar{p}}(\tau) &= \sum_{x_b,j} \int_\tau^1 du \frac{1}{u} d_{bj}^{x_b}\left[\frac{\tau}{u}\right] D_{j\bar{p}}(u) , \qquad (2.7)\\
f_{\gamma c}(\tau) &= \sum_{x_c,k} \int_\tau^1 du \frac{1}{u} D_{\gamma k}(u) d_{kc}^{x_c}\left[\frac{\tau}{u}\right] ,
\end{aligned}
$$

and are obtained by fitting data of deep inelastic scattering to results of perturbation theory at a given mass factorization scale M. At present there are not enough data available to fit the photon fragmentation functions so one has to rely upon an approximation, for example, the so-called leading-log approximation [9,10].

In the computation of hadronic quantities we use (2.6) with $a,b \in \{q,\bar{q},g\}$ and $c \in \{q,\bar{q},g,\gamma\}$ where q, $\bar{q}$, and g denote quark, antiquark, and gluon, respectively. The photon (γ) is treated in a dual way: it is a hadron, i.e., an observable final state particle, and it is also a parton of our Lagrangian. In our one-body inclusive computation in [3] we only considered contributions from $f_{\gamma\gamma}$ and neglected $f_{\gamma c}$ for $c \in \{q,\bar{q},g\}$. In the present work we include the four contributions keeping terms up to $O(\alpha_S \alpha \alpha_W)$, where α_S, α, and α_W are the strong, electromagnetic, and electroweak fine structure constants.

B. Contributions from $f_{\gamma\gamma}$

The leading photon-photon splitting function is given by $d_{\gamma\gamma}^{\{\}}(\tau/u) = \delta(1-\tau/u)$, i.e., when no partons are emitted from the photon. This leading order splitting function can be identified with the bare fragmentation function $D_{\gamma\gamma}(u)$ when zero hadrons are fragmented from the photon. Using this in (2.7) we obtain the leading contribution to the photon-photon fragmentation function:

$$
f_{\gamma\gamma}(\tau) = \delta(1-\tau) . \qquad (2.8)
$$

Setting $c=\gamma$ in (2.6) and keeping hard scattering contributions up to $O(\alpha_S \alpha \alpha_W)$ we obtain

$$
\begin{aligned}
&\sum_X \left[\frac{D^2\sigma^H}{DQ_1 DQ_2}\right]^{\gamma\gamma} [p(P_1)\bar{p}(P_2) \to W(Q_1)\gamma(Q_2)X]\\
&= \int_0^1 d\tau_1 \int_0^1 d\tau_2 \left\{ f_{qp}(\tau_1) f_{\bar{q}\bar{p}}(\tau_2) \left[\frac{D^2\sigma}{DQ_1 DQ_2}[q(\tau_1 P_1)\bar{q}(\tau_2 P_2) \to W(Q_1)\gamma(Q_2)] \right.\right.\\
&\qquad \left. + \frac{D^2\sigma}{DQ_1 DQ_2}[q(\tau_1 P_1)\bar{q}(\tau_2 P_2) \to W(Q_1)\gamma(Q_2) g] \right]\\
&\qquad + f_{qp}(\tau_1) f_{g\bar{p}}(\tau_2) \frac{D^2\sigma}{DQ_1 DQ_2}[q(\tau_1 P_1) g(\tau_2 P_2) \to W(Q_1)\gamma(Q_2) q]\\
&\qquad \left. + f_{gp}(\tau_1) f_{\bar{q}\bar{p}}(\tau_2) \frac{D^2\sigma}{DQ_1 DQ_2}[g(\tau_1 P_1)\bar{q}(\tau_2 P_2) \to W(Q_1)\gamma(Q_2)\bar{q}] + (q \leftrightarrow \bar{q}) \right\} . \qquad (2.9)
\end{aligned}
$$

Note that the lowest order hard scattering cross section is always equal to the corresponding lowest order bare partonic cross section, as we will verify in Secs. II F, II G, and II H.

C. Contributions from $f_{\gamma q}$ and $f_{\gamma\bar{q}}$

Setting $c=q$ and $c=\bar{q}$ in (2.6) we obtain, for these contributions,

$$
\begin{aligned}
&\sum_X \left[\frac{D^2\sigma^H}{DQ_1 DQ_2}\right]^{\gamma q} [p(P_1)\bar{p}(P_2) \to W(Q_1)\gamma(Q_2)X]\\
&= \int_0^1 d\tau_1 \int_0^1 d\tau_2 \int_0^1 d\tau_3 \left\{ f_{qp}(\tau_1) f_{g\bar{p}}(\tau_2) f_{\gamma q}(\tau_3) \frac{D^2\sigma}{DQ_1 DQ_2}\left[q(\tau_1 P_1) g(\tau_2 P_2) \to W(Q_1) q\left[\frac{Q_2}{\tau_3}\right]\right] \right.\\
&\qquad \left. + f_{gp}(\tau_1) f_{q\bar{p}}(\tau_2) f_{\gamma q}(\tau_3) \frac{D^2\sigma}{DQ_1 DQ_2}\left[g(\tau_1 P_1) q(\tau_2 P_2) \to W(Q_1) q\left[\frac{Q_2}{\tau_3}\right]\right] \right\} , \qquad (2.10)
\end{aligned}
$$

and

$$\sum_X \left[\frac{D^2\sigma^H}{DQ_1DQ_2}\right]^{\gamma\bar{q}}[p(P_1)\bar{p}(P_2)\to W(Q_1)\gamma(Q_2)X]$$

$$=\int_0^1 d\tau_1\int_0^1 d\tau_2\int_0^1 d\tau_3\left\{f_{gp}(\tau_1)f_{\bar{q}\bar{p}}(\tau_2)f_{\gamma\bar{q}}(\tau_3)\frac{D^2\sigma}{DQ_1DQ_2}\left[g(\tau_1P_1)\bar{q}(\tau_2P_2)\to W(Q_1)\bar{q}\left[\frac{Q_2}{\tau_3}\right]\right]\right.$$
$$\left.+f_{\bar{q}p}(\tau_1)f_{g\bar{p}}(\tau_2)f_{\gamma\bar{q}}(\tau_3)\frac{D^2\sigma}{DQ_1DQ_2}\left[\bar{q}(\tau_1P_1)g(\tau_2P_2)\to W(Q_1)\bar{q}\left[\frac{Q_2}{\tau_3}\right]\right]\right\}. \tag{2.11}$$

D. Contributions from $f_{\gamma g}$

Setting $c=g$ in (2.6) we have

$$\sum_X \left[\frac{D^2\sigma^H}{DQ_1DQ_2}\right]^{\gamma g}[g(P_1)\bar{p}(P_2)\to W(Q_1)\gamma(Q_2)X]$$

$$=\int_0^1 d\tau_1\int_0^1 d\tau_2\int_0^1 d\tau_3\left\{f_{qp}(\tau_1)f_{\bar{q}\bar{p}}(\tau_2)f_{\gamma g}(\tau_3)\frac{D^2\sigma}{DQ_1DQ_2}\left[q(\tau_1P_1)\bar{q}(\tau_2P_2)\to W(Q_1)g\left[\frac{Q_2}{\tau_3}\right]\right]+(q\leftrightarrow\bar{q})\right\}. \tag{2.12}$$

In B, C, and D sums over flavors of quarks q and antiquarks $\bar{q}$ satisfying the electric charge conservation for production of W^{+} are implicit. For a more detailed discussion on the way we treat this issue, see Sec. IV of [3]. The contributions from C and D will be referred to as "photon bremsstrahlung contributions."

E. The incoming hard scattering channels

We can now regroup all terms in B, C, and D according to three partonic channels in the incoming hard scattering state:

$$\sum_X \left[\frac{D^2\sigma^H}{DQ_1DQ_2}\right]^{q\bar{q}}[p(P_1)\bar{p}(P_2)\to W(Q_1)\gamma(Q_2)X]$$

$$=\int_0^1 d\tau_1\int_0^1 d\tau_2\left[f_{qp}(\tau_1)f_{\bar{q}\bar{p}}(\tau_2)\left\{\frac{D^2\sigma}{DQ_1DQ_2}[q(\tau_1P_1)\bar{q}(\tau_2P_2)\to W(Q_1)\gamma(Q_2)]\right.\right.$$
$$+\frac{D^2\sigma}{DQ_1DQ_2}[q(\tau_1P_1)\bar{q}(\tau_2P_2)\to W(Q_1)\gamma(Q_2)g]$$
$$+\int_0^1 d\tau_3 f_{\gamma g}(\tau_3)\frac{D^2\sigma}{DQ_1DQ_2}$$
$$\left.\left.\times\left[q(\tau_1P_1)\bar{q}(\tau_2P_2)\to W(Q_1)g\left[\frac{Q_2}{\tau_3}\right]\right]\right\}+(q\leftrightarrow\bar{q})\right], \tag{2.13}$$

$$\sum_X \left[\frac{D^2\sigma^H}{DQ_1DQ_2}\right]^{qg}[p(P_1)\bar{p}(P_2)\to W(Q_1)\gamma(Q_2)X]$$

$$=\int_0^1 d\tau_1\int_0^1 d\tau_2\left\{f_{qp}(\tau_1)f_{g\bar{p}}(\tau_2)\frac{D^2\sigma}{DQ_1DQ_2}[q(\tau_1P_1)g(\tau_2P_2)\to W(Q_1)\gamma(Q_2)q]\right.$$
$$\left.+f_{gp}(\tau_1)f_{q\bar{p}}(\tau_2)\frac{D^2\sigma}{DQ_1DQ_2}[g(\tau_1P_1)q(\tau_2P_2)\to W(Q_1)\gamma(Q_2)q]\right\}$$
$$+\int_0^1 d\tau_1\int_0^1 d\tau_2\int_0^1 d\tau_3\left\{f_{qp}(\tau_1)f_{g\bar{p}}(\tau_2)f_{\gamma q}(\tau_3)\frac{D^2\sigma}{DQ_1DQ_2}\left[q(\tau_1P_1)g(\tau_2P_2)\to W(Q_1)q\left[\frac{Q_2}{\tau_3}\right]\right]\right.$$
$$\left.+f_{gp}(\tau_1)f_{q\bar{p}}(\tau_2)f_{\gamma q}(\tau_3)\frac{D^2\sigma}{DQ_1DQ_2}\left[g(\tau_1P_1)q(\tau_2P_2)\to W(Q_1)q\left[\frac{Q_2}{\tau_3}\right]\right]\right\}, \tag{2.14}$$

$$\sum_X \left[\frac{D^2\sigma^H}{DQ_1DQ_2} \right]^{g\bar{q}} [p(P_1)\bar{p}(P_2)\to W(Q_1)\gamma(Q_2)X]$$
$$= \int_0^1 d\tau_1 \int_0^1 d\tau_2 \left\{ f_{gp}(\tau_1) f_{\bar{q}\bar{p}}(\tau_2) \frac{D^2\sigma}{DQ_1DQ_2}[g(\tau_1P_1)\bar{q}(\tau_2P_2)\to W(Q_1)\gamma(Q_2)\bar{q}] \right.$$
$$\left. + f_{\bar{q}p}(\tau_1) f_{g\bar{p}}(\tau_2) \frac{D^2\sigma}{DQ_1DQ_2}[\bar{q}(\tau_1P_1)g(\tau_2P_2)\to W(Q_1)\gamma(Q_2)\bar{q}] \right\}$$
$$+ \int_0^1 d\tau_1 \int_0^1 d\tau_2 \int_0^1 d\tau_3 \left\{ f_{gp}(\tau_1) f_{\bar{q}\bar{p}}(\tau_2) f_{\gamma\bar{q}}(\tau_3) \frac{D^2\sigma}{DQ_1DQ_2} \left[g(\tau_1P_1)\bar{q}(\tau_2P_2)\to W(Q_1)\bar{q}\left[\frac{Q_2}{\tau_3}\right]\right] \right.$$
$$\left. + f_{\bar{q}p}(\tau_1) f_{g\bar{p}}(\tau_2) f_{\gamma\bar{q}}(\tau_3) \frac{D^2\sigma}{DQ_1DQ_2} \left[\bar{q}(\tau_1P_1)g(\tau_2P_2)\to W(Q_1)\bar{q}\left[\frac{Q_2}{\tau_3}\right]\right] \right\} . \tag{2.15}$$

F. Factorization in the $q\bar{q}$ channel

Let us write (2.5) for $i=q$, $j=\bar{q}$ and $k=\gamma$:

$$\sum_x \frac{D^2\sigma^P}{DQ_1DQ_2}[q(p_1)\bar{q}(p_2)\to W(Q_1)\gamma(Q_2)x]$$
$$= \sum_{a,b,c,x_a,x_b,x_c} \left\{ \int_0^1 dv_1 \int_0^1 dv_2 \int_0^1 dv_3 d_{aq}^{x_a}(v_1) d_{b\bar{q}}^{x_b}(v_2) d_{\gamma c}^{x_c}(v_3) \sum_y \frac{D^2\sigma}{DQ_1DQ_2} \left[a(v_1p_1)b(v_2p_2)\to W(Q_1)c\left[\frac{Q_2}{v_3}\right]y \right] \right\} . \tag{2.16}$$

We will solve (2.16) at $O(\alpha\alpha_W)$ and $O(\alpha_S\alpha\alpha_W)$ so only $x=\{\ \}$ and $x=\{g\}$ contribute on the LHS. By constraining the sums on the RHS so that $x_a\cup x_b\cup x_c\cup y\subseteq x$ we obtain, at $O(\alpha_S^0)$,

$$\frac{D^2\sigma^{(0)}}{DQ_1DQ_2}[q(p_1)\bar{q}(p_2)\to W(Q_1)\gamma(Q_2)] = \frac{D^2\sigma^{P(0)}}{DQ_1DQ_2}[q(p_1)\bar{q}(p_2)\to W(Q_1)\gamma(Q_2)] \tag{2.17}$$

and, at $O(\alpha_S)$,

$$\frac{D^2\sigma^{(1)}}{DQ_1DQ_2}[q(p_1)\bar{q}(p_2)\to W(Q_1)\gamma(Q_2)] + \frac{D^2\sigma^{(1)}}{DQ_1DQ_2}[q(p_1)\bar{q}(p_2)\to W(Q_1)\gamma(Q_2)g]$$
$$= \frac{D^2\sigma^{P(1)}}{DQ_1DQ_2}[q(p_1)\bar{q}(p_2)\to W(Q_1)\gamma(Q_2)] + \frac{D^2\sigma^{P(1)}}{DQ_1DQ_2}[q(p_1)\bar{q}(p_2)\to W(Q_1)\gamma(Q_2)g]$$
$$+ \frac{\alpha_S}{2\pi\bar{\epsilon}} \int_0^1 dv\, \bar{P}_{qq}(v) \frac{D^2\sigma^{(0)}}{DQ_1DQ_2}[q(vp_1)\bar{q}(p_2)\to W(Q_1)\gamma(Q_2)]$$
$$+ \frac{\alpha_S}{2\pi\bar{\epsilon}} \int_0^1 dv\, \bar{P}_{\bar{q}\bar{q}}(v) \frac{D^2\sigma^{(0)}}{DQ_1DQ_2}[q(p_1)\bar{q}(vp_2)\to W(Q_1)\gamma(Q_2)] , \tag{2.18}$$

and the corresponding equations for $p_1\leftrightarrow p_2$. In deriving (2.17) and (2.18) we have used the splitting functions

$$d_{ai}^{\{\ \}}(v) = \delta_{ai}\delta(1-v) ,$$
$$d_{aq}^{\{g\}}(v) = -\frac{\alpha_S}{2\pi\bar{\epsilon}}\bar{P}_{qq}(v)\delta_{aq} , \tag{2.19}$$
$$d_{b\bar{q}}^{\{g\}}(v) = -\frac{\alpha_S}{2\pi\bar{\epsilon}}\bar{P}_{\bar{q}\bar{q}}(v)\delta_{b\bar{q}} ,$$

and the definition

$$\bar{P}_{ij}(v) \equiv P_{ij}(v) - \bar{\epsilon}K_{ij}(v) \tag{2.20}$$

with

$$\frac{1}{\bar{\epsilon}} \equiv \frac{1}{\epsilon} - \gamma_E + \ln(4\pi) \tag{2.21}$$

and $\epsilon\equiv(4-n)/2$. The running strong fine structure constant is $\alpha_S = g_S^2(\mu)/4\pi$. In the $\overline{\text{MS}}$ mass factorization scheme $K_{ij}(v)=0$ for all relevant i,j in this and the following two subsections.

For $i=j=q(\bar{q})$ we have

$$P_{qq}(v)=P_{\bar{q}\bar{q}}(v)=C_F\left[(1+v^2)\left\{\frac{1}{1-v}\right\}_0+\tfrac{3}{2}\delta(v-1)\right]$$
$$=C_F\left[(1+v^2)\left\{\frac{1}{1-v}\right\}_{v_0}+[\tfrac{3}{2}+2\ln(1-v_0)]\delta(1-v)\right] \quad (2.22)$$

with

$$\int_0^1 dv\left\{\frac{1}{1-v}\right\}_{v_0}f(v)\equiv\int_0^{v_0}dv\frac{f(v)}{1-v}+\int_{v_0}^1 dv\frac{f(v)-f(1)}{1-v}\ , \quad (2.23)$$

where $0\le v_0<1$ and the color factor is given by $C_F=\frac{4}{3}$.

In an analogous way, setting $i=q$, $j=\bar{q}$, and $k=g$ in (2.5), we obtain

$$\frac{D^2\sigma^{(1)}}{DQ_1DQ_2}[q(p_1)\bar{q}(p_2)\to W(Q_1)g(Q_2)]=\frac{D^2\sigma^{P(1)}}{DQ_1DQ_2}[q(p_1)\bar{q}(p_2)\to W(Q_1)g(Q_2)]\ . \quad (2.24)$$

G. Factorization in the qg channel

Setting $i=q$, $j=g$, and $k=q$ in (2.5) and again keeping terms up to $O(\alpha_S\alpha\alpha_W)$ we can only have $x=\{\ \}$, thus constraining the sums on the RHS to $x_a\cup x_b\cup x_c\cup y=\{\ \}$. We obtain, at $O(\alpha_S)$,

$$\frac{D^2\sigma^{(1)}}{DQ_1DQ_2}[q(p_1)g(p_2)\to W(Q_1)q(Q_2)]=\frac{D^2\sigma^{P(1)}}{DQ_1DQ_2}[q(p_1)g(p_2)\to W(Q_1)q(Q_2)]\ . \quad (2.25)$$

Resetting $k=\gamma$ in (2.5) we can now only have $x=\{q\}$, thus constraining the sums on the RHS to $x_a\cup x_b\cup x_c\cup y=\{q\}$. We obtain, at $O(\alpha_S)$,

$$\frac{D^2\sigma^{(1)}}{DQ_1DQ_2}[q(p_1)g(p_2)\to W(Q_1)\gamma(Q_2)q]=\frac{D^2\sigma^{P(1)}}{DQ_1DQ_2}[q(p_1)g(p_2)\to W(Q_1)\gamma(Q_2)q]$$
$$+\frac{\alpha_S}{2\pi\bar{\epsilon}}\int_0^1 dv\,\bar{P}_{\bar{q}g}(v)\frac{D^2\sigma^{(0)}}{DQ_1DQ_2}[q(p_1)\bar{q}(vp_2)\to W(Q_1)\gamma(Q_2)]$$
$$+\frac{\alpha}{2\pi\bar{\epsilon}}\int_0^1 dv\,\bar{P}_{\gamma q}(v)\frac{D^2\sigma^{(1)}}{DQ_1DQ_2}\left[q(p_1)g(p_2)\to W(Q_1)q\left[\frac{Q_2}{v}\right]\right]\ , \quad (2.26)$$

and the corresponding equations for $p_1\leftrightarrow p_2$. The following splitting functions have been used when deriving (2.26):

$$d_{bg}^{\{q\}}(v)=-\frac{\alpha_S}{2\pi\bar{\epsilon}}\bar{P}_{\bar{q}g}(v)\delta_{b\bar{q}}\ ,\qquad d_{\gamma c}^{\{q\}}(v)=-\frac{\alpha}{2\pi\bar{\epsilon}}\bar{P}_{\gamma q}(v)\delta_{cq}\ , \quad (2.27)$$

with

$$P_{\bar{q}g}(v)=\frac{v^2+(1-v)^2}{2}\ ,\qquad P_{\gamma q}(v)=(\hat{e}_q)^2\frac{1+(1-v)^2}{v}\ , \quad (2.28)$$

where $\hat{e}_q=-\frac{1}{3}$ is the charge of the outgoing quark q on the LHS of (2.26), in units of e, and $\alpha=e^2(\mu)/4\pi$ is the running electromagnetic fine structure constant.

H. Factorization in the $g\bar{q}$ channel

Analogously to the previous case, setting $i=g$, $j=\bar{q}$, and $k=\bar{q},\gamma$ in (2.5) and keeping up to $O(\alpha_S\alpha\alpha_W)$, we obtain

$$\frac{D^2\sigma^{(1)}}{DQ_1DQ_2}[g(p_1)\bar{q}(p_2)\to W(Q_1)\bar{q}(Q_2)]=\frac{D^2\sigma^{P(1)}}{DQ_1DQ_2}[g(p_1)\bar{q}(p_2)\to W(Q_1)\bar{q}(Q_2)] \quad (2.29)$$

and

$$\frac{D^2\sigma^{(1)}}{DQ_1DQ_2}[g(p_1)\bar{q}(p_2)\to W(Q_1)\gamma(Q_2)\bar{q}]=\frac{D^2\sigma^{P(1)}}{DQ_1DQ_2}[g(p_1)\bar{q}(p_2)\to W(Q_1)\gamma(Q_2)\bar{q}]$$
$$+\frac{\alpha_S}{2\pi\bar{\epsilon}}\int_0^1 dv\,\bar{P}_{\bar{q}g}(v)\frac{D^2\sigma^{(0)}}{DQ_1DQ_2}[q(vp_1)\bar{q}(p_2)\to W(Q_1)\gamma(Q_2)]$$
$$+\frac{\alpha}{2\pi\bar{\epsilon}}\int_0^1 dv\,\bar{P}_{\gamma\bar{q}}(v)\frac{D^2\sigma^{(1)}}{DQ_1DQ_2}\left[g(p_1)\bar{q}(p_2)\to W(Q_1)\bar{q}\left[\frac{Q_2}{v}\right]\right]\ , \quad (2.30)$$

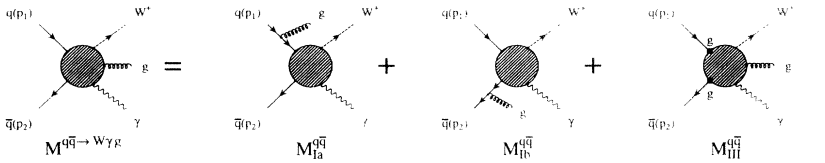

FIG. 1. Decomposition of the two- to three-body Feynman amplitude in the $q\bar{q}$ channel.

and the corresponding equations for $p_1 \leftrightarrow p_2$. In deriving (2.30) we have used the splitting functions

$$d_{ag}^{\{\bar{q}\}}(v) = -\frac{\alpha_S}{2\pi\bar{\epsilon}}\bar{P}_{qg}(v)\delta_{aq} ,$$
$$d_{\gamma c}^{\{\bar{q}\}}(v) = -\frac{\alpha}{2\pi\bar{\epsilon}}\bar{P}_{\gamma\bar{q}}(v)\delta_{c\bar{q}} , \tag{2.31}$$

with

$$P_{qg}(v) = \frac{v^2+(1-v)^2}{2} ,$$
$$P_{\gamma\bar{q}}(v) = (\hat{e}_{\bar{q}})^2\frac{1+(1-v)^2}{v} , \tag{2.32}$$

where $\hat{e}_{\bar{q}} = -\frac{2}{3}$ is the charge fraction of the outgoing antiquark $\bar{q}$ on the LHS of (2.30), in units of e.

III. EXCLUSIVE CANCELLATION OF SINGULARITIES

A. Introduction

In order to compute the hard scattering cross sections required in (2.13)–(2.15) using (2.17), (2.18), (2.24)–(2.26), (2.29) and (2.30), we first need the bare partonic cross sections $D^2\sigma^P/DQ_1DQ_2$ on the RHS of these equations evaluated in $n=4-2\epsilon$ space-time dimensions.

When computing phase space integrations of outgoing massless particles, singularities appear in regions of phase space where one of these particles is collinear to any other massless on-shell parton or where one of the outgoing massless gauge bosons is soft. Since we are tagging the outgoing photon, we do not have to worry about singularities associated with integration over the photon's phase space.

With this in mind we will classify the two- to three-body Feynman amplitudes according to the way the outgoing massless particle q, $\bar{q}$, or g which is integrated over is attached to the rest of the legs of the diagram. In Fig. 1 we have decomposed the two- to three-body Feynman amplitude for the partonic reaction $q+\bar{q}\to W+\gamma+g$ into three pieces: $M^{q\bar{q}\to W\gamma g} = M^{q\bar{q}}_{\mathrm{Ia}} + M^{q\bar{q}}_{\mathrm{Ib}} + M^{q\bar{q}}_{\mathrm{III}}$.

The labels g near the solid vertices at the end of the incoming quark and antiquark legs in the amplitude $M^{q\bar{q}}_{\mathrm{III}}$ mean that these vertices do not contain the outgoing gluon. Arrows show the direction of the fermionic charge and the W^+ charge. The momenta p_1 and p_2 are always incoming. The shaded blobs denote the inclusion of all possible Feynman diagrams (except for the mentioned constraints in $M^{q\bar{q}}_{\mathrm{III}}$.) If we integrate the squared matrix element over the phase space of the outgoing gluon summed over physical polarizations, the $\sum|M^{q\bar{q}}_{\mathrm{Ia}}|^2$ and $\sum|M^{q\bar{q}}_{\mathrm{Ib}}|^2$ pieces of the squared matrix element have collinear and soft singularities while the interference term $(\sum M^{q\bar{q}}_{\mathrm{Ia}}M^{*q\bar{q}}_{\mathrm{Ib}}+\mathrm{c.c.})$ has only soft singularities. Other pieces of the squared matrix element have no singularities.

For the partonic reaction $q+g\to W+\gamma+q$ we have $M^{qg\to W\gamma q} = M^{qg}_{\mathrm{Ib}} + M^{qg}_{\mathrm{II}} + M^{qg}_{\mathrm{III}}$, as shown in Fig. 2. In this case, only the $\sum|M^{qg}_{\mathrm{Ib}}|^2$ and $\sum|M^{qg}_{\mathrm{II}}|^2$ pieces of the squared matrix element have collinear singularities, with no soft singularities in this channel.

In Fig. 3 we show the analogous decomposition for the partonic reaction $g+\bar{q}\to W+\gamma+\bar{q}$: $M^{g\bar{q}\to W\gamma\bar{q}} = M^{g\bar{q}}_{\mathrm{Ia}} + M^{g\bar{q}}_{\mathrm{II}} + M^{g\bar{q}}_{\mathrm{III}}$. As in the previous case, only the $\sum|M^{g\bar{q}}_{\mathrm{Ia}}|^2$ and $\sum|M^{g\bar{q}}_{\mathrm{II}}|^2$ pieces of the squared matrix element have collinear singularities and no soft singularities are present in this channel.

We will develop the above decompositions in more detail for each channel in Secs. III B, III C, and III D.

The 2- to $(2+l)$-body partonic differential cross section for the $W+\gamma+X$ production process is defined in n dimensions of phase space by

$$d\sigma^P_{(2+l)} = \frac{1}{N}\frac{1}{2s}DQ_1DQ_2Dk_1\cdots Dk_l(2\pi)^n$$
$$\times\delta^n(p_1+p_2-Q_1-Q_2-k_1\cdots-k_l)$$
$$\times\sum|M|^2 , \tag{3.1}$$

where we have averaged over N possible incoming states of different polarizations and colors and the sum on the RHS is over polarizations and colors of all particles. Q_1 and Q_2 are the n-momenta of the W^+ boson and the photon, respectively. The variables in (3.1) are defined in the following way:

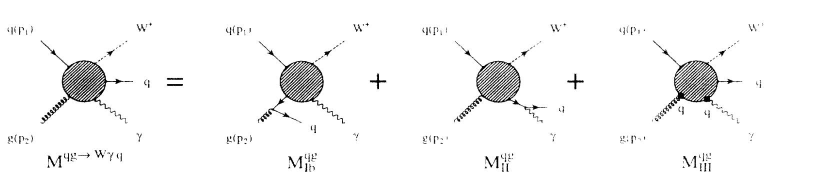

FIG. 2. Decomposition of the two- to three-body Feynman amplitude in the qg channel.

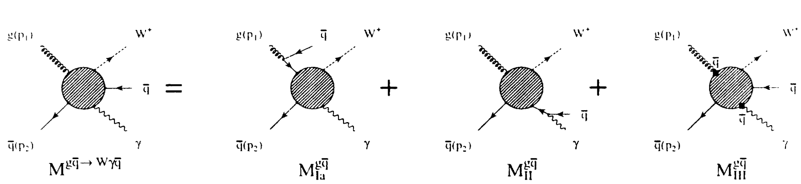

FIG. 3. Decomposition of the two- to three-body Feynman amplitude in the $g\bar q$ channel.

$$
\begin{aligned}
s&=2p_1\cdot p_2 ,\\
DQ_1&=\frac{1}{2(2\pi)^{n-1}}\frac{|\mathbf{Q}_1|^{n-2}}{\sqrt{|\mathbf{Q}_1|^2+M_W^2}}\\
&\quad\times d|\mathbf{Q}_1|d\Omega_{n-2}(\hat{\mathbf{Q}}_1(\theta_1,\ldots,\theta_{n-2})) ,\\
DQ_2&=\frac{1}{2(2\pi)^{n-1}}|\mathbf{Q}_2|^{n-3}d|\mathbf{Q}_2|\\
&\quad\times d\Omega_{n-2}(\hat{\mathbf{Q}}_2(\phi_1,\ldots,\phi_{n-2})) ,\\
Dk_1&=Dk=\frac{1}{2(2\pi)^{n-1}}|\mathbf{k}|^{n-3}\\
&\quad\times d|\mathbf{k}|d\Omega_{n-2}(\hat{\mathbf{k}}(\psi_1,\ldots,\psi_{n-2})) .
\end{aligned}
\tag{3.2}
$$

The angle differentials in (3.2) are generically given by

$$
\begin{aligned}
d\Omega_{n-2}(\hat{\mathbf{p}}(\alpha_1,\ldots,\alpha_{n-2}))&\equiv d\cos\alpha_1\sin^{n-4}\alpha_1\cdots\\
&\quad\times d\cos\alpha_{n-3}\sin^0\alpha_{n-3}d\alpha_{n-2} .
\end{aligned}
\tag{3.3}
$$

To account for the experimental cuts on the outgoing particles that define the experimental scenario under consideration, we have to include on the RHS of (3.1) an extra factor of $C(Q_1,Q_2,k_1,\ldots,k_l)$. These cuts may be expressed in a covariant way in terms of Θ step functions, as we will see in more detail in Sec. IV. In the rest of this section and the following sections we omit the charge index "+" when referring to the W^+ boson.

B. The $q\bar q$ channel

The two- to two-body total partonic cross section for the reaction $q\bar q\to W\gamma$ may be written in the following way:

$$
\begin{aligned}
&\int DQ_1DQ_2\frac{D^2\sigma^P}{DQ_1DQ_2}[q(p_1)\bar q(p_2)\to W(Q_1)\gamma(Q_2)]\\
&\quad=\frac{1}{4N_c^2}\frac{1}{2s}\Theta(\beta(s))\Phi(s)\\
&\qquad\times\int_{-1}^{1}d\cos\theta_1\sin^{-2\epsilon}\theta_1\sum|M^{q\bar q\to W\gamma}(s,b)|^2
\end{aligned}
\tag{3.4}
$$

with

$$
\begin{aligned}
N_c&=3 ,\\
\Phi(s)&=\frac{2^{2\epsilon}}{\Gamma(1-\epsilon)}\left[\frac{4\pi}{s}\right]^{\epsilon}\frac{1}{16\pi}\beta^{1-2\epsilon}(s) ,\\
\beta(s)&=1-\rho(s) ,\\
\rho(s)&\equiv\frac{M_W^2}{s} .
\end{aligned}
\tag{3.5}
$$

The only independent invariants in the squared matrix elements in (3.4) are s and

$$
b\equiv 2p_1\cdot Q_2=\frac{s}{2}\beta(s)(1+\cos\theta_1) . \tag{3.6}
$$

The rest of the invariants may be expressed in terms of b and s as

$$
\begin{aligned}
2p_1\cdot Q_1&=s-b ,\\
2p_2\cdot Q_1&=b+M_W^2 ,\\
2p_2\cdot Q_2&=s-M_W^2-b ,\\
2Q_1\cdot Q_2&=s-M_W^2 .
\end{aligned}
\tag{3.7}
$$

In (3.4) we chose the $(n-1)$th axis to be the one pointing in the direction of $\mathbf{p}_1$. The integration over the angles $\theta_2,\ldots,\theta_{n-2}$ has been performed because there is no dependence on them in the two- to two-body squared matrix element. To account for the experimental cuts on the outgoing particles we implicitly include an extra factor of $C(Q_1,Q_2,0)$ on the RHS of (3.4). In (3.5) and subsequent equations the unprimed variables refer to variables in the center of mass system of the incoming partons.

To obtain the two- to three-body partonic cross section for the reaction $q\bar q\to W\gamma g$ we define a primed reference frame in the $W\gamma$ center of mass system such that the n-momenta are given by

$$
\begin{aligned}
p_1'&=p_{1,0}'(1,0,\ldots,0,0,1) ,\\
p_2'&=p_{2,0}'(1,0,\ldots,0,\sin\eta',\cos\eta') ,\\
k'&=k_0'(1,0,\ldots,0,\sin\psi',\cos\psi') ,\\
Q_1'&=|\mathbf{Q}_1'|\left[\frac{Q_{1,0}'}{|\mathbf{Q}_1'|},\ldots,\sin\theta_1'\sin\theta_2'\cos\theta_3' ,\right.\\
&\qquad\left.\sin\theta_1'\cos\theta_2',\cos\theta_1'\right] ,\\
Q_2'&=|\mathbf{Q}_1'|(1,\ldots,-\sin\theta_1'\sin\theta_2'\cos\theta_3' ,\\
&\qquad-\sin\theta_1'\cos\theta_2',-\cos\theta_1') .
\end{aligned}
\tag{3.8}
$$

The two- to three-body total partonic cross section in the $q\bar q$ channel may thus be written as

$$\int DQ_1 DQ_2 \frac{D^2\sigma^P}{DQ_1 DQ_2}[q(p_1)\bar{q}(p_2)\to W(Q_1)\gamma(Q_2)g]$$
$$=\frac{1}{4N_c^2}\frac{1}{2s}(4\pi)^{\epsilon-2}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\frac{s^{1-\epsilon}}{2\pi}\int_{\rho(s)}^1 dx\,\Phi(sx)(1-x)^{1-2\epsilon}\int_{-1}^1 dy(1-y^2)^{-\epsilon}$$
$$\times\int_{-1}^1 d\cos\theta_1'\sin^{-2\epsilon}\theta_1'\int_0^\pi d\theta_2'\sin^{-2\epsilon}\theta_2'\sum|M^{q\bar{q}\to W\gamma g}(s,a,b,c,d)|^2\ , \tag{3.9}$$

where

$$y\equiv\cos\psi\ ,\qquad k_0=(1-x)p_{1,0}=(1-x)\frac{\sqrt{s}}{2}\ . \tag{3.10}$$

We implicitly include on the RHS of (3.9) the factor $C(Q_1,Q_2,k)$. We have chosen as independent invariants s and

$$\begin{aligned} a&\equiv 2p_1\cdot k=\frac{s}{2}(1-x)(1-y)\ ,\\ b&\equiv 2p_1\cdot Q_2=2p_{1,0}'|\mathbf{Q}_1'|(1+\cos\theta_1')\ ,\\ c&\equiv 2k\cdot Q_2\\ &=2k_0'|\mathbf{Q}_1'|(1+\sin\psi'\sin\theta_1'\cos\theta_2'+\cos\psi'\cos\theta_1')\ ,\\ d&\equiv 2p_2\cdot Q_2\\ &=2p_{2,0}'|\mathbf{Q}_1'|(1+\sin\eta'\sin\theta_1'\cos\theta_2'+\cos\eta'\cos\theta_1'), \end{aligned} \tag{3.11}$$

so we have the dependent invariants

$$\begin{aligned} 2p_1\cdot Q_1&=s-a-b\ ,\\ 2p_2\cdot Q_1&=M_W^2+a+b-c\ ,\\ 2Q_1\cdot Q_2&=b+d-c=sx-M_W^2\ ,\\ 2Q_1\cdot k&=s-M_W^2-b-d\ ,\\ 2p_2\cdot k&=s-M_W^2+c-a-b-d=\frac{s}{2}(1-x)(1+y)\ . \end{aligned} \tag{3.12}$$

All invariants can now be expressed in terms of s, x, y, $\cos\theta_1'$, and $\cos\theta_2'$ by solving for all the necessary quantities in the primed reference frame:

$$\begin{aligned} p_{1,0}'&=\frac{\sqrt{s}}{4}\left[\frac{1+x+(1-x)y}{\sqrt{x}}\right],\\ p_{2,0}'&=\frac{\sqrt{s}}{4}\left[\frac{1+x-(1-x)y}{\sqrt{x}}\right],\\ Q_{1,0}'&=\frac{\sqrt{sx}}{2}\left[1+\frac{M_W^2}{sx}\right],\\ Q_{2,0}'&=|\mathbf{Q}_1'|=\frac{\sqrt{sx}}{2}\left[1-\frac{M_W^2}{sx}\right]=\frac{\sqrt{sx}}{2}\beta(sx)\ ,\\ \cos\eta'&=1-\left[\frac{8x}{(1+x)^2-(1-x)^2y^2}\right],\\ \cos\psi'&=\frac{1-x+y(1+x)}{1+x+y(1-x)}\ ,\\ k_0'&=\frac{\sqrt{s}}{2}\left[\frac{1-x}{\sqrt{x}}\right]. \end{aligned} \tag{3.13}$$

Looking at (3.9) it is clear that the soft divergences will be present in those pieces of squared matrix element that contain a factor $(1-x)^{-2}$, while the collinear divergences will be due to factors of $(1-y)^{-1}$ or $(1+y)^{-1}$ in the squared matrix element. The squared matrix element for the $q\bar{q}$ channel can be written as

$$\sum|M^{q\bar{q}\to W\gamma g}|^2=\sum|M_{\rm Ia}^{q\bar{q}}|^2+\sum|M_{\rm Ib}^{q\bar{q}}|^2+\left[\sum M_{\rm Ia}^{q\bar{q}}M_{\rm Ib}^{q\bar{q}*}+\text{c.c.}\right]+\text{remain}\ . \tag{3.14}$$

The type Ia matrix element (see Fig. 1) can be written as

$$M_{\rm Ia}[q(p_1,l_1,\lambda_1)\bar{q}(p_2,l_2)\to W(Q_1)\gamma(Q_2)g(k,c,\lambda)]$$
$$=g_S T_{l_1'l_1}^c\frac{[(2p_1^\rho-\not{k}\gamma^\rho)u_{\lambda_1}(p_1)]_\alpha}{2p_1\cdot k}\epsilon_\rho^\lambda(k)M[q(p_1-k,l_1',\alpha)\bar{q}(p_2,l_2)\to W(Q_1)\gamma(Q_2)]\ , \tag{3.15}$$

where l_1, l_2, and l_1' are quark color indices, λ_1 is the quark polarization index, c and λ are the outgoing gluon color and polarization indices, α and ρ are Lorentz indices, and g_S is the renormalized strong coupling constant. Other indices have been omitted because they are not necessary in what follows. The partial squared matrix element is given by

$$\sum|M_{\rm Ia}^{q\bar{q}}|^2=\frac{g_S^2}{(2p_1\cdot k)^2}C_F R_{q\bar{q},{\rm Ia}}^{\alpha\alpha'}\sum M[q(p_1-k,\alpha)\bar{q}(p_2)\to W(Q_1)\gamma(Q_2)]M^*[q(p_1-k,\alpha')\bar{q}(p_2)\to W(Q_1)\gamma(Q_2)]\ . \tag{3.16}$$

The Ib partial squared matrix element may be written in a similar fashion:

$$\sum |M^{q\bar{q}}_{\mathrm{Ib}}|^2 = \frac{g_S^2}{(2p_2\cdot k)^2} C_F R^{\beta\beta'}_{q\bar{q},\mathrm{Ib}} \sum M[q(p_1)\bar{q}(p_2-k,\beta)\to W(Q_1)\gamma(Q_2)] M^*[q(p_1)\bar{q}(p_2-k,\beta')\to W(Q_1)\gamma(Q_2)] \ . \tag{3.17}$$

The repeated indices on the RHS of (3.16) and (3.17) are contracted and the sum is over quark (antiquark) colors and polarizations as well as W and γ polarizations. In the $q\bar{q}$ center of mass frame, the tensors $R^{\alpha\alpha'}_{q\bar{q},\mathrm{Ia}}$ and $R^{\beta\beta'}_{q\bar{q},\mathrm{Ib}}$ can be written as

$$\begin{aligned} R^{\alpha\alpha'}_{q\bar{q},\mathrm{Ia}} &= -4p_1\cdot k \left[\left\{ \left[\frac{2-n}{2} + \frac{1+y}{2(1-x)} \right] \not{k} + \left[1 - \frac{1+y}{1-x} \right] \not{p}_1 - \frac{1-y}{2(1-x)} \bar{\not{k}} \right\} \gamma_0 \right]^{\alpha\alpha'} , \\ R^{\beta\beta'}_{q\bar{q},\mathrm{Ib}} &= -4p_2\cdot k \left[\gamma_0 \left\{ \left[\frac{2-n}{2} + \frac{1-y}{2(1-x)} \right] \not{k} + \left[1 - \frac{1-y}{1-x} \right] \not{p}_2 - \frac{1+y}{2(1-x)} \bar{\not{k}} \right\} \right]^{\beta'\beta} . \end{aligned} \tag{3.18}$$

In deriving (3.18) we have summed over physical gluon polarizations in the covariant gauge, where we may write

$$P_{\rho\rho'}(k) \equiv \sum_{\lambda=1}^{n-2} \epsilon^\lambda_\rho(k)\epsilon^\lambda_{\rho'}(k) = -g_{\rho\rho'} + \frac{k_\rho \bar{k}_{\rho'} + k_{\rho'}\bar{k}_\rho}{k\cdot\bar{k}} , \tag{3.19}$$

where if $k=(k_0,\mathbf{k})$ then $\bar{k}\equiv(k_0,-\mathbf{k})$. The factors $p_1\cdot k$ and $p_2\cdot k$ in front of the RHS of (3.18) will cancel similar factors in the denominators of the RHS of (3.16) and (3.17), respectively, leaving the type Ia and Ib squared matrix elements with singular terms proportional to $(1-x)^{-2}(1-y)^{-1}$, $(1-x)^{-1}(1-y)^{-1}$, and $(1-x)^{-2}(1+y)^{-1}$, $(1-x)^{-1}(1+y)^{-1}$, respectively, i.e., both terms will contribute to the soft and collinear singularities. In a similar way the remain in (3.14) can be shown to have no collinear or soft singularities while the interference term $M_{\mathrm{Ia}}M^*_{\mathrm{Ib}}$ has only a soft singularity. It is thus convenient to define the nonsingular function $F^{q\bar{q}}$,

$$F^{q\bar{q}}(s,x,y,\cos\theta_1',\theta_2') \equiv 4(p_1\cdot k)(p_2\cdot k) \sum |M^{q\bar{q}\to W\gamma g}|^2 , \tag{3.20}$$

so that we can now rewrite the two- to three-body total partonic cross section in (3.9) as

$$\begin{aligned} \sigma^P[q(p_1)\bar{q}(p_2)\to W\gamma g] = {}& \frac{1}{4N_c^2}\frac{1}{2s}(4\pi)^{\epsilon-2}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\frac{s^{-1-\epsilon}}{2\pi} \\ & \times \int_{\rho(s)}^1 dx\, \Phi(sx)(1-x)^{-1-2\epsilon} \int_{-1}^1 dy (1-y^2)^{-1-\epsilon} \int_{-1}^1 d\cos\theta_1' \sin^{-2\epsilon}\theta_1' \\ & \times \int_0^\pi d\theta_2' \sin^{-2\epsilon}\theta_2' F^{q\bar{q}}(s,x,y,\cos\theta_1',\theta_2') \ . \end{aligned} \tag{3.21}$$

Now that the singular factors in x and y have been isolated we can rewrite them as distributions for $\epsilon<0$:

$$\begin{aligned} (1-x)^{-1-2\epsilon} &\sim -\frac{1}{2\epsilon}(1-x_0)^{-2\epsilon}\delta(1-x) + \left\{\frac{1}{1-x}\right\}_{x_0} \\ &\quad -2\epsilon\left\{\frac{\ln(1-x)}{1-x}\right\}_{x_0} + O(\epsilon^2) , \\ (1-y^2)^{-1-\epsilon} &\sim -\frac{(2y_0)^{-\epsilon}}{2\epsilon}[\delta(1-y)+\delta(1+y)] \\ &\quad +\frac{1}{2}\left\{\frac{1}{1-y}\right\}_{y_0} + \frac{1}{2}\left\{\frac{1}{1+y}\right\}_{y_0} + O(\epsilon) , \end{aligned} \tag{3.22}$$

with $\zeta(2)=\pi^2/6$. We have introduced the definitions

$$\begin{aligned} \int_{\rho(s)}^1 dx\, f(x)\left\{\frac{1}{1-x}\right\}_{x_0} &\equiv \int_{\rho(s)}^{x_0} dx \frac{f(x)}{1-x} + \int_{x_0}^1 dx \frac{f(x)-f(1)}{1-x} , \\ \int_{-1}^1 dy\, f(y)\left\{\frac{1}{1-y}\right\}_{y_0} &\equiv \int_{-1}^{1-y_0} dy \frac{f(y)}{1-y} + \int_{1-y_0}^1 dy \frac{f(y)-f(1)}{1-y} , \\ \int_{-1}^1 dy\, f(y)\left\{\frac{1}{1+y}\right\}_{y_0} &\equiv \int_{-1+y_0}^1 dy \frac{f(y)}{1+y} + \int_{-1}^{-1+y_0} dy \frac{f(y)-f(-1)}{1+y} . \end{aligned} \tag{3.23}$$

The parameters x_0 and y_0 are arbitrary as long as they satisfy the conditions $\rho(s)\le x_0<1$ and $0<y_0\le 2$. The symbol $\sim$ in (3.22) means that the equality only holds under an integration over x ranging from $\rho(s)$ to 1 for the first expression in (3.22) and under an integration over y ranging from -1 to 1 for the second expression. When x and y are not integrated over their whole range care has to be taken when defining x_0 and y_0 so as not to introduce unphysical dependences into the quantities we want to compute. We will discuss this in more detail in Sec. IV.

Using (3.22) and (3.23) in (3.21), we write the $O(\alpha_S)$ two- to three-body cross section as

$$\sigma^{P(1)}[q(p_1)\bar{q}(p_2)\to W\gamma g] = \sigma^P_{q\bar{q}(\text{finite})}+\sigma^P_{q\bar{q}(\text{col}+)}+\sigma^P_{q\bar{q}(\text{col}-)}+\sigma^P_{q\bar{q}(\text{soft})}+O(\epsilon) \tag{3.24}$$

with

$$\sigma^P_{q\bar{q}(\text{finite})}=\frac{1}{4N_c^2}\frac{1}{2s}2^{-10}\pi^{-4}s^{-1}\int_{\rho(s)}^1 dx\,\beta(sx)\left\{\frac{1}{1-x}\right\}_{x_0} \times\int_{-1}^1 dy\left[\left\{\frac{1}{1-y}\right\}_{y_0}+\left\{\frac{1}{1+y}\right\}_{y_0}\right]\int_{-1}^1 d\cos\theta_1'\int_0^\pi d\theta_2' F^{q\bar{q}}(s,x,y,\cos\theta_1',\theta_2')\ ,$$

$$\sigma^P_{q\bar{q}(\text{col}\pm)}=-\frac{1}{4N_c^2}\frac{1}{2s}(4\pi)^{\epsilon-2}\frac{1}{\Gamma(1-\epsilon)}\frac{s^{-1-\epsilon}}{2\pi}\frac{\pi}{2\epsilon}\left[\frac{2}{y_0}\right]^\epsilon \times\int_{\rho(s)}^1 dx\,\Phi(sx)\left[\left\{\frac{1}{1-x}\right\}_{x_0}-2\epsilon\left\{\frac{\ln(1-x)}{1-x}\right\}_{x_0}\right]\int_{-1}^1 d\cos\theta_1'\sin^{-2\epsilon}\theta_1' F^{q\bar{q}(\text{col}\pm)}(s,x,\cos\theta_1')\ , \tag{3.25}$$

$$\sigma^P_{q\bar{q}(\text{soft})}=\frac{1}{4N_c^2}\frac{1}{2s}(4\pi)^{\epsilon-2}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\frac{s^{-1-\epsilon}}{2\pi}\frac{\pi}{2\epsilon^2}(1-x_0)^{-2\epsilon}\Phi(s)\int_{-1}^1 d\cos\theta_1'\sin^{-2\epsilon}\theta_1' F^{q\bar{q}(\text{soft})}(s,\cos\theta_1')\ ,$$

where

$$F^{q\bar{q}(\text{col}\pm)}(s,x,\cos\theta_1')\equiv F^{q\bar{q}}(s,x,y=\pm1,\cos\theta_1',\theta_2')\ ,$$
$$F^{q\bar{q}(\text{soft})}(s,\cos\theta_1')\equiv F^{q\bar{q}}(s,x=1,y,\cos\theta_1',\theta_2')\ . \tag{3.26}$$

To compute the quantities in (3.26) we first note that the following relations hold for $y=1$:

$$p_1-k=xp_1$$
$$\not{k}=(1-x)\not{p}_1\ . \tag{3.27}$$

Using these relations in (3.16)–(3.18) and noting that in the limit $y\to1$ (-1) only $|M^{q\bar{q}}_{\text{Ia}}|^2$ $(|M^{q\bar{q}}_{\text{Ib}}|^2)$ contributes to $F^{q\bar{q}}$ in (3.20) we obtain the covariant expression

$$F^{q\bar{q}(\text{col}+)}(s,x,\cos\theta_1')=8sg_S^2C_F\frac{1+x^2-\epsilon(1-x)^2}{x} \times\sum|M^{q\bar{q}\to W\gamma}(xs,xb^+)|^2\ . \tag{3.28}$$

We implicitly include in (3.28) an overall factor of $C(Q_1,Q_2,(1-x)p_1)$. An analogous computation yields

$$F^{q\bar{q}(\text{col}-)}(s,x,\cos\theta_1')=8sg_S^2C_F\frac{1+x^2-\epsilon(1-x)^2}{x} \times\sum|M^{q\bar{q}\to W\gamma}(xs,b^-)|^2\ . \tag{3.29}$$

In the latter we implicitly include an overall factor of $C(Q_1,Q_2,(1-x)p_2)$. The collinear limits of $F^{q\bar{q}}$ have thus been reduced to two- to two-body squared matrix elements. We checked that the same expressions for the collinear limits of $F^{q\bar{q}}$ are obtained when the sum of gluon polarizations is taken in the axial gauge.

Now we obtain the soft residues of each of the contributing terms in the squared matrix element:

$$\lim_{x\to1}4(p_1\cdot k)(p_2\cdot k)\sum|M^{q\bar{q}}_{\text{Ia}}|^2 =4sg_S^2C_F(1+y)^2\sum|M^{q\bar{q}\to W\gamma}(s,b^{\text{soft}})|^2\ ,$$

$$\lim_{x\to1}4(p_1\cdot k)(p_2\cdot k)\sum|M^{q\bar{q}}_{\text{Ib}}|^2 =4sg_S^2C_F(1-y)^2\sum|M^{q\bar{q}\to W\gamma}(s,b^{\text{soft}})|^2\ ,$$

$$\lim_{x\to1}4(p_1\cdot k)(p_2\cdot k)\left[\sum M^{q\bar{q}}_{\text{Ia}}M^{*q\bar{q}}_{\text{Ib}}+\text{c.c.}\right] =8sg_S^2C_F(1-y^2)\sum|M^{q\bar{q}\to W\gamma}(s,b^{\text{soft}})|^2\ . \tag{3.30}$$

After summing the above three contributions we obtain

$$F^{q\bar{q}(\text{soft})}(s,\cos\theta_1')=16sg_S^2C_F\sum|M^{q\bar{q}\to W\gamma}(s,b^{\text{soft}})|^2\ . \tag{3.31}$$

This contribution contains an implicit factor of $C(Q_1,Q_2,0)$. Note that the dependence on y cancels after all terms in (3.30) are added together. We have checked that the same result is obtained if the gluon polarization sum is taken in the axial gauge. In (3.28), (3.29), and (3.31) we define

$$b^+ \equiv b(s,x,y=1,\cos\theta_1') = \frac{s}{2}\beta(sx)(1+\cos\theta_1') ,$$
$$b^- \equiv b(s,x,y=-1,\cos\theta_1') = \frac{sx}{2}\beta(sx)(1+\cos\theta_1') , \quad (3.32)$$
$$b^{\rm soft} \equiv b(s,x=1,y,\cos\theta_1') = \frac{s}{2}\beta(s)(1+\cos\theta_1') .$$

Note that in the soft limit the variable $\cos\theta_1'$ is equivalent to $\cos\theta_1$ of a two- to two-body kinematics.

Noting that the squared matrix elements in (3.28)–(3.31) are of the two- to two-body type we can now rewrite the soft and collinear terms in a more convenient way:

$$\sigma^P_{q\bar q(\rm soft)} = \frac{\alpha_S}{\pi} C_F \left[-2V - \frac{1}{\epsilon}[\tfrac{3}{2} + 2\ln(1-x_0)] + \tfrac{3}{2}\ln\frac{s}{\mu^2} + 2\ln^2(1-x_0) + 2\ln(1-x_0)\ln\frac{s}{\mu^2} + 2\zeta(2) - 4 \right] \sigma^{(0)}[q(p_1)\bar q(p_2) \to W\gamma] ,$$

$$\sigma^P_{q\bar q(\rm col+)} = -\frac{\alpha_S}{2\pi\bar\epsilon}\int_{\rho(s)}^1 dx \left[C_F(1+x^2)\left\{\frac{1}{1-x}\right\}_{x_0} + \epsilon C_F \left[\ln\left\{\frac{2\mu^2}{s y_0}\right\}(1+x^2)\left\{\frac{1}{1-x}\right\}_{x_0} - 2(1+x^2)\left\{\frac{\ln(1-x)}{1-x}\right\}_{x_0} + x - 1 \right]\right] \sigma^{(0)}[q(xp_1)\bar q(p_2) \to W\gamma] , \quad (3.33)$$

$$\sigma^P_{q\bar q(\rm col-)} = -\frac{\alpha_S}{2\pi\bar\epsilon}\int_{\rho(s)}^1 dx \left[C_F(1+x^2)\left\{\frac{1}{1-x}\right\}_{x_0} + \epsilon C_F \left[\ln\left\{\frac{2\mu^2}{s y_0}\right\}(1+x^2)\left\{\frac{1}{1-x}\right\}_{x_0} - 2(1+x^2)\left\{\frac{\ln(1-x)}{1-x}\right\}_{x_0} + x - 1 \right]\right] \sigma^{(0)}[q(p_1)\bar q(xp_2) \to W\gamma] .$$

In the previous formulas we neglected terms of $O(\epsilon)$ and we used

$$V \equiv -e^{-[\gamma_E - \ln(4\pi)]\epsilon}\left[\frac{1}{2\epsilon^2} - \frac{1}{2\epsilon}\left[\ln\frac{s}{\mu^2} - \frac{3}{2}\right] + \frac{1}{4}\ln^2\frac{s}{\mu^2} - \frac{3}{4}\ln\frac{s}{\mu^2} - \tfrac{7}{4}\zeta(2) + 2 \right] . \quad (3.34)$$

We also made the replacement $g_S^2 = 4\pi\alpha_S\mu^{2\epsilon}$.

The $O(\alpha_S)$ corrections to the two- to two-body partonic cross section for $q\bar q \to W\gamma$ were computed in [2] and they are given by

$$\sigma^{P(1)}[q(p_1)\bar q(p_2) \to W\gamma] = \frac{\alpha_S}{\pi} C_F \sigma^{(0)}[q(p_1)\bar q(p_2) \to W\gamma] 2V + \frac{1}{4N_c^2}\frac{1}{2s}\beta(s) C_W^2 \frac{\alpha}{9}\frac{\alpha_S}{\pi} N_c C_F \int_{-1}^1 d\cos\theta_1 \frac{(2(s-M_W^2-b)-b)(2F_1(s,b)-F_2(s,b))}{s-M_W^2} \times C(Q_1,Q_2,0) , \quad (3.35)$$

where

$$F_1(s,b) \equiv F(-b,-(s-M_W^2-b),s,M_W^2) ,$$
$$F_2(s,b) \equiv F(-(s-M_W^2-b),-b,s,M_W^2) , \quad (3.36)$$
$$C_W \equiv M_W \left[\frac{G_F}{\sqrt{2}}\right]^{1/2} ,$$

and G_F is the Fermi coupling constant, b was defined in (3.6), and $F(t_1,t_2,s,M_W^2)$ was defined in (3.7) of [2].

Using (3.24), (3.25), (3.33), and (3.35) in the RHS of (2.18) we can write the $O(\alpha_S)$ hard scattering cross section on the LHS of (2.18) after performing an integration over Q_1 and Q_2:

$$\sigma^{(1)}[q(p_1)\bar{q}(p_2)\to W\gamma]+\sigma^{(1)}[q(p_1)\bar{q}(p_2)\to W\gamma g]$$

$$=\sigma^P_{q\bar{q}(\text{finite})}+\sigma^P_{q\bar{q}(\text{soft})}+\sigma^{P(1)}[q(p_1)\bar{q}(p_2)\to W\gamma]+\sigma^P_{q\bar{q}(\text{col}+)}+\frac{\alpha_S}{2\pi\bar{\epsilon}}\int_0^1 dv\,\bar{P}_{qq}(v)\sigma^{(0)}[q(vp_1)\bar{q}(p_2)\to W\gamma]$$

$$+\sigma^P_{q\bar{q}(\text{col}-)}+\frac{\alpha_S}{2\pi\bar{\epsilon}}\int_0^1 dv\,\bar{P}_{q\bar{q}}(v)\sigma^{(0)}[q(p_1)\bar{q}(vp_2)\to W\gamma]$$

$$=\sigma^P_{q\bar{q}(\text{finite})}+\sigma^P_{q\bar{q}(\text{SV})}-\frac{\alpha_S}{2\pi}C_F\int_{\rho(s)}^1 dx\left[(1+x^2)\ln\left[\frac{2\mu^2}{sy_0}\right]\left\{\frac{1}{1-x}\right\}_{x_0}-2(1+x^2)\left\{\frac{\ln(1-x)}{1-x}\right\}_{x_0}+x-1\right]$$

$$\times\{\sigma^{(0)}[q(xp_1)\bar{q}(p_2)\to W\gamma]+\sigma^{(0)}[q(p_1)\bar{q}(xp_2)\to W\gamma]\}\ , \tag{3.37}$$

where we have defined the soft-plus-virtual contributions

$$\sigma^P_{q\bar{q}(\text{SV})}\equiv\sigma^P_{q\bar{q}(\text{soft})}+\sigma^{P(1)}[q(p_1)\bar{q}(p_2)\to W\gamma]$$

$$=\frac{\alpha_S}{\pi}C_F\left[\frac{3}{2}\ln\frac{s}{\mu^2}+2\ln^2(1-x_0)+2\ln(1-x_0)\ln\frac{s}{\mu^2}+2\zeta(2)-4\right]$$

$$\times\sigma^{(0)}[q(p_1)\bar{q}(p_2)\to W\gamma]+\frac{1}{4N_c^2}\frac{1}{2s}\beta(s)C_W^2\frac{\alpha}{9}\frac{\alpha_S}{\pi}N_cC_F$$

$$\times\int_{-1}^1 d\cos\theta_1\frac{(2(s-M_W^2-b)-b)(2F_1(s,b)-F_2(s,b))}{s-M_W^2}C(Q_1,Q_2,0)\ . \tag{3.38}$$

Summarizing all the contributions in the incoming $q\bar{q}$ partonic channel, we have

$$\int DQ_1DQ_2\sum_X\left[\frac{D^2\sigma^H}{DQ_1DQ_2}\right]^{q\bar{q}}[p(P_1)\bar{p}(P_2)\to W(Q_1)\gamma(Q_2)X]$$

$$=\int_0^1 d\tau_1\int_0^1 d\tau_2\left\{f_{qp}(\tau_1)f_{\bar{q}\bar{p}}(\tau_2)\left[\sigma^{(0)}[q(p_1)\bar{q}(p_2)\to W\gamma]\right.\right.$$

$$+\sigma^{(1)}[q(p_1)\bar{q}(p_2)\to W\gamma]+\sigma^{(1)}[q(p_1)\bar{q}(p_2)\to W\gamma g]$$

$$\left.\left.+\int_0^1 d\tau\,f_{\gamma g}(\tau)\int DQ_1Dq_2\frac{D^2\sigma^{(1)}}{DQ_1Dq_2}[q(p_1)\bar{q}(p_2)\to W(Q_1)g(q_2)]\right]+(q\leftrightarrow\bar{q})\right\}$$

$$\tag{3.39}$$

with $p_1=\tau_1P_1$, $p_2=\tau_2P_2$, and $q_2=Q_2/\tau$. Since there are no singularities left all the necessary squared matrix elements in (3.39) can now be safely evaluated in $n=4$ dimensions. On the RHS of (3.39) we still need the squared matrix elements

$$\sum|M^{q\bar{q}\to W\gamma}(s,b)|^{2(0)}$$

$$=\frac{2^7}{9}\pi N_cC_W^2\alpha(2(s-M_W^2-b)-b)^2$$

$$\times\frac{sM_W^2-b(s-M_W^2-b)+\frac{1}{2}(s-M_W^2)^2}{b(s-M_W^2-b)(s-M_W^2)^2}\ ,$$

$$\sum|M^{q\bar{q}\to Wg}(s,\hat{b})|^{2(1)}$$

$$=2^6\pi N_cC_FC_W^2\alpha_S\left[\frac{\hat{b}^2+(s-M_W^2-\hat{b})^2+2sM_W^2}{\hat{b}(s-M_W^2-\hat{b})}\right]\ , \tag{3.40}$$

where $\hat{b}\equiv b/\tau$. The integrand of the last term in (3.39) carries an implicit factor of $C(Q_1,\tau q_2,(1-\tau)q_2)$. In (3.38) and (3.40) the invariant b is evaluated in the unprimed frame as defined in (3.6).

The expression for $\sum|M^{q\bar{q}\to W\gamma g}|^2$ in $n=4$ needed when evaluating $\sigma^P_{q\bar{q}(\text{finite})}$ is too long to be presented here, but it may be obtained upon request. We note here that γ_5 is never needed in $n\neq4$ dimensions: we obtained the cancellation of singularities before fully computing any squared matrix element where we had to explicitly evaluate γ_5 and whatever remained after this cancellation could be safely computed in $n=4$ dimensions.

Since we have integral expressions of all quantities on the RHS of (3.39) in terms of the variables x, y, θ_1', and θ_2' which define all the independent invariants of the system, we can compute these integrals using numerical Monte Carlo techniques and make a histogram of any physical variable of interest that can be expressed in terms of these invariants. We will reexamine these issues in more detail in Secs. IV and V.

C. The qg channel

In this channel the singularities are not both of initial state (type I), as in the case of the $q\bar{q}$ channel, but we have now one piece in the initial state (type I) and another in the final state (type II), as shown in Fig. 2. In this case it is thus more convenient to integrate each term separately and write for the total partonic cross section in this channel

$$\begin{aligned}\sigma^P[q(p_1)g(p_2)\to W\gamma q] &= \sigma^{P,\mathrm{I}}[q(p_1)g(p_2)\to W\gamma q] \\ &\quad + \sigma^{P,\mathrm{II}}[q(p_1)g(p_2)\to W\gamma q] \\ &\quad + \tilde{\sigma}^P[q(p_1)g(p_2)\to W\gamma q]\ ,\end{aligned} \tag{3.41}$$

where the first two terms on the RHS of (3.41) contain only the partial squared matrix elements $\sum|M^{qg}_{\mathrm{Ia}}|^2$ and $\sum|M^{qg}_{\mathrm{II}}|^2$, respectively, while the third term contains $\sum|M^{qg}_{\mathrm{III}}|^2$ and all interference terms of the squared matrix element. If the gluon and the photon are summed over physical polarizations only, then the third term on the RHS of (3.41) is free of singularities while the first two terms contain only collinear singularities.

If we define the same kinematics in $\sigma^{P,\mathrm{I}}_{qg\to W\gamma q}$ as we did with the $q\bar{q}$ channel, the collinear pieces of integrand contain a factor $(1+y)^{-1}$. To isolate the singularity in this term it is therefore enough to define the nonsingular function

$$F^{qg}_{\mathrm{I}}(s,x,y,\cos\theta_1',\theta_2') \equiv -2p_2\cdot k \sum |M^{qg}_{\mathrm{Ia}}|^2\ , \tag{3.42}$$

so that the first term on the RHS of (3.41) may be written

$$\begin{aligned}\sigma^{P,\mathrm{I}}[q(p_1)g(p_2)\to W\gamma q] = &-\frac{1}{8N_c^2C_F(1-\epsilon)}\frac{1}{2s}(4\pi)^{\epsilon-2}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\frac{s^{-\epsilon}}{\pi} \\ &\times\int_{\rho(s)}^1 dx\,\Phi(sx)(1-x)^{-2\epsilon}\int_{-1}^1 dy(1-y)^{-\epsilon}(1+y)^{-1-\epsilon} \\ &\times\int_{-1}^1 d\cos\theta_1'\sin^{-2\epsilon}\theta_1'\int_0^\pi d\theta_2'\sin^{-2\epsilon}\theta_2' F^{qg}_{\mathrm{I}}(s,x,y,\cos\theta_1',\theta_2')\ .\end{aligned} \tag{3.43}$$

We now rewrite the factor $(1+y)^{-1-\epsilon}$ as a distribution:

$$(1+y)^{-1-\epsilon}\sim -\frac{y_0^{-\epsilon}}{\epsilon}\delta(1+y)+\left\{\frac{1}{1+y}\right\}_{y_0}+O(\epsilon)\ . \tag{3.44}$$

Using (3.44) in (3.43) we can write the corresponding contribution to the $O(\alpha_S)$ two- to three-body partonic cross section as

$$\sigma^{P,\mathrm{I}(1)}[q(p_1)g(p_2)\to W\gamma q] = \sigma^{P,\mathrm{I}}_{qg,\mathrm{finite}}+\sigma^{P,\mathrm{I}}_{qg,\mathrm{col}-}+O(\epsilon)\ , \tag{3.45}$$

where

$$\begin{aligned}\sigma^{P,\mathrm{I}}_{qg,\mathrm{finite}} &= -\frac{1}{8N_c^2C_F}\frac{1}{2s}(4\pi)^{-4}\int_{\rho(s)}^1 dx\,\beta(sx)\int_{-1}^1 dy\left\{\frac{1}{1+y}\right\}_{y_0}\int_{-1}^1 d\cos\theta_1'\int_0^\pi d\theta_2' F^{qg}_{\mathrm{I}}(s,x,y,\cos\theta_1',\theta_2')\ , \\ \sigma^{P,\mathrm{I}}_{qg,\mathrm{col}-} &= \frac{1}{8N_c^2C_F(1-\epsilon)}\frac{1}{2s}(4\pi)^{\epsilon-2}\frac{1}{\Gamma(1-\epsilon)}\frac{s^{-\epsilon}}{\pi}\frac{\pi}{\epsilon}\left[\frac{2}{y_0}\right]^\epsilon \\ &\quad\times\int_{\rho(s)}^1 dx\,\Phi(sx)(1-x)^{-2\epsilon}\int_{-1}^1 d\cos\theta_1'\sin^{-2\epsilon}\theta_1' F^{qg(\mathrm{col}-)}_{\mathrm{I}}(s,x,\cos\theta_1')\ ,\end{aligned} \tag{3.46}$$

and

$$F^{qg(\mathrm{col}-)}_{\mathrm{I}}(s,x,\cos\theta_1') = F^{qg}_{\mathrm{I}}(s,x,y=-1,\cos\theta_1',\theta_2')\ . \tag{3.47}$$

To compute the limit on the RHS of (3.47) we write, as we did in (3.16) and (3.17),

$$\begin{aligned}\sum|M^{qg}_{\mathrm{Ia}}|^2 = &\frac{g_S^2}{(2p_2\cdot k)^2}C_F R^{\alpha\alpha'}_{qg,\mathrm{I}} \\ &\times\sum M[q(p_1)\bar{q}(p_2-k,\alpha)\to W(Q_1)\gamma(Q_2)] \\ &\times M^*[q(p_1)\bar{q}(p_2-k,\alpha')\to W(Q_1)\gamma(Q_2)]\ ,\end{aligned} \tag{3.48}$$

where now, in the qg center of mass frame,

$$\begin{aligned}R^{\alpha\alpha'}_{qg,\mathrm{I}} = -4p_2\cdot k\Bigg[\gamma_0\Bigg\{\left[\frac{2-n}{2}+(1-x)\frac{(1-y)}{2}\right]\not{p}_2 \\ -[(1-x)(1-y)-1]\not{k} \\ -(1-x)(1+y)\not{p}_1\Bigg\}\Bigg]^{\alpha'\alpha}\ .\end{aligned} \tag{3.49}$$

From the above equation it is now clear that there will be no soft quark singularity coming from the type Ia squared matrix element. Using (3.49), (3.47), and (3.42) we obtain

$$F_{\rm I}^{qg({\rm col}-)}(s,x,\cos\theta_1')=2g_S^2C_F\frac{2x(1-x)-1+\epsilon}{x}\times\sum|M^{q\bar{q}\to W\gamma}(xs,b^-)|^2 \; . \qquad (3.50)$$

The latter expression contains an implicit factor of $C(Q_1,Q_2,(1-x)p_2)$ to account for experimental cuts. Using (3.50) in (3.46) we can rewrite the collinear contribution [neglecting terms of $O(\epsilon)$]

$$\sigma_{qg,{\rm col}-}^{P,{\rm I}}=\frac{\alpha_S}{2\pi\bar{\epsilon}}\int_{\rho(s)}^1 dx\left[x(1-x)-\tfrac{1}{2}+\epsilon\left\{\ln\left[\frac{2\mu^2}{sy_0}\right][x(1-x)-\tfrac{1}{2}]+\ln(1-x)+x(1-x)[1-2\ln(1-x)]\right\}\right]\sigma^{(0)}[q(p_1)\bar{q}(xp_2)\to W\gamma] \; . \qquad (3.51)$$

To treat the type II term on the RHS of (3.41) it is convenient to rotate the reference frame so that in the $W\gamma$ center of mass frame we have

$$\begin{aligned}
&p_1'=p_{1,0}'(1,0,\ldots,0,\sin\psi',\cos\psi') \; ,\\
&p_2'=p_{2,0}'(1,0,\ldots,0,-\sin\chi',\cos\chi') \; ,\\
&k'=k_0'(1,0,\ldots,0,0,1) \; ,\\
&Q_1'=|\mathbf{Q}_1'|\left[\frac{Q_{1,0}'}{|\mathbf{Q}_1'|},\ldots,-\sin\phi_1'\sin\phi_2'\cos\phi_3',-\sin\phi_1'\cos\phi_2',-\cos\phi_1'\right] \; ,\\
&Q_2'=|\mathbf{Q}_1'|(1,\ldots,\sin\phi_1'\sin\phi_2'\cos\phi_3',\sin\phi_1'\cos\phi_2',\cos\phi_1') \; .
\end{aligned} \qquad (3.52)$$

We can thus write, for the two- to three-body integral,

$$\begin{aligned}
\int DQ_1DQ_2\frac{D^2\sigma^{P,{\rm II}}}{DQ_1DQ_2}[q(p_1)g(p_2)\to W(Q_1)\gamma(Q_2)q]=&\frac{1}{8N_c^2C_f(1-\epsilon)}\frac{1}{2s}(4\pi)^{\epsilon-2}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\frac{s^{1-\epsilon}}{2\pi}\beta^{2-2\epsilon}(s)\\
&\times\int_0^1 dz\,\Phi(s[1+\beta(s)(z-1)])(1-z)^{1-2\epsilon}\int_{-1}^1 dy(1-y^2)^{-\epsilon}\\
&\times\int_{-1}^1 dv(1-v^2)^{-\epsilon}\int_0^\pi d\phi_2'\sin^{-2\epsilon}\phi_2'\sum|M_{\rm II}^{qg}(s,a,b,c,d)|^2 \; ,
\end{aligned} \qquad (3.53)$$

with

$$\begin{aligned}
&\cos\chi'=\frac{1-x-y(1+x)}{1+x-y(1-x)} \; ,\\
&v\equiv\cos\phi_1' \; ,\\
&z\equiv1+\frac{x-1}{\beta(s)} \; .
\end{aligned} \qquad (3.54)$$

The explicit form of the invariants in the squared matrix element in terms of the new integration angles is given as

$$\begin{aligned}
b&\equiv2p_1\cdot Q_2\\
&=2p_{1,0}'|\mathbf{Q}_1'|(1-\sin\psi'\sin\phi_1'\cos\phi_2'-\cos\psi'\cos\phi_1') \; ,
\end{aligned}$$

$$c\equiv2k\cdot Q_2=2k_0'|\mathbf{Q}_1'|(1-\cos\phi_1') \; , \qquad (3.55)$$

$$\begin{aligned}
d&\equiv2p_2\cdot Q_2\\
&=2p_{2,0}'|\mathbf{Q}_1'|(1+\sin\chi'\sin\phi_1'\cos\phi_2'-\cos\chi'\cos\phi_1') \; .
\end{aligned}$$

The rest of the invariants and variables remain as defined in (3.11)–(3.13). To isolate the singularity in the type II squared matrix element it is enough to define

$$F_{\rm II}^{qg}(s,z,y,v,\phi_2')\equiv-2k\cdot Q_2\sum|M_{\rm II}^{qg}|^2 \; , \qquad (3.56)$$

so that now the second term on the RHS of (3.41) may be written

$$\begin{aligned}
\sigma^{P,{\rm II}}[q(p_1)g(p_2)\to W\gamma q]=&-\frac{1}{8N_c^2C_F(1-\epsilon)}\frac{1}{2s}(4\pi)^{\epsilon-2}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\frac{s^{-\epsilon}}{\pi}\frac{2^{2\epsilon}}{\Gamma(1-\epsilon)}\frac{1}{16\pi}\beta^{1-4\epsilon}(s)\left[\frac{4\pi}{s}\right]^{\epsilon}\\
&\times\int_0^1 dz[z(1-z)]^{-2\epsilon}[1+(z-1)\beta(s)]^{\epsilon}\int_{-1}^1 dy(1-y^2)^{-\epsilon}\\
&\times\int_{-1}^1 dv(1+v)^{-\epsilon}(1-v)^{-1-\epsilon}\int_0^\pi d\phi_2'\sin^{-2\epsilon}\phi_2' F_{\rm II}^{qg}(s,z,y,v,\phi_2') \; .
\end{aligned} \qquad (3.57)$$

We rewrite the factor $(1-v)^{-1-\epsilon}$ as a distribution

$$(1-v)^{-1-\epsilon} \sim -\frac{v_0^{-\epsilon}}{\epsilon}\delta(1-v) + \left\{\frac{1}{1-v}\right\}_{v_0} + O(\epsilon) \tag{3.58}$$

with $0 < v_0 \le 2$, so that (3.57) may be rewritten as

$$\sigma^{P,\mathrm{II}(1)}[q(p_1)g(p_2) \to W\gamma q] = \sigma^{P,\mathrm{II}}_{qg,\mathrm{finite}} + \sigma^{P,\mathrm{II}}_{qg,\mathrm{col}+} + O(\epsilon) , \tag{3.59}$$

where

$$\begin{aligned}\sigma^{P,\mathrm{II}}_{qg,\mathrm{finite}} &= -\frac{1}{8N_c^2 C_F}\frac{1}{2s}(4\pi)^{-4}\beta(s)\int_0^1 dz \int_{-1}^1 dy \int_{-1}^1 dv \left\{\frac{1}{1-v}\right\}_{v_0}\int_0^\pi d\phi_2' F^{qg}_{\mathrm{II}}(s,z,y,v,\phi_2') ,\\ \sigma^{P,\mathrm{II}}_{qg,\mathrm{col}+} &= \frac{1}{8N_c^2 C_F(1-\epsilon)}\frac{1}{2s}(4\pi)^{\epsilon-2}\frac{1}{\Gamma(1-\epsilon)}\frac{s^{-\epsilon}}{\pi}\frac{\pi}{\epsilon}\left[\frac{2}{v_0}\right]^{\epsilon}\Phi(s)\beta^{-2\epsilon}(s)\\ &\quad\times\int_0^1 dz[z(1-z)]^{-2\epsilon}[1+(z-1)\beta(s)]^{\epsilon}\int_{-1}^1 dy(1-y^2)^{-\epsilon}F^{qg(\mathrm{col}+)}_{\mathrm{II}}(s,z,y) ,\end{aligned} \tag{3.60}$$

and

$$F^{qg(\mathrm{col}+)}_{\mathrm{II}}(s,z,y) = F^{qg}_{\mathrm{II}}(s,z,y,v=1,\phi_2') . \tag{3.61}$$

The type II squared matrix element is given by

$$\begin{aligned}\sum |M^{qg}_{\mathrm{II}}|^2 = \frac{e_q^2}{(2k\cdot Q_2)^2}R^{\alpha\alpha'}_{qg,\mathrm{II}}\\ \times\sum M[q(p_1)g(p_2)\to W(Q_1)q(Q_2+k,\alpha)]\\ \times M^*[q(p_1)g(p_2)\to W(Q_1)q(Q_2+k,\alpha')] ,\end{aligned} \tag{3.62}$$

with e_q the charge of the outgoing quark. In the qg center of mass frame we have

$$\begin{aligned}R^{\alpha\alpha'}_{qg,\mathrm{II}} = -4k\cdot Q_2\Bigg[\gamma_0\Bigg\{\left[\frac{2-n}{2} - \frac{(1-z)(1+v)}{2z}\right]\not{Q}_2\\ -\left[1+\frac{(1-z)(1+v)}{z}\right]\not{k}\\ +\left[\frac{(1-z)(1-v)}{2z}\right]\bar{\not{Q}}_2\Bigg\}\Bigg]^{\alpha'\alpha} ,\end{aligned} \tag{3.63}$$

where we have summed over physical polarizations of the outgoing photon in the covariant gauge. Again we note that there will be no singularities in the soft quark limit, that is, when $z \to 1$. For the collinear limit of F^{qg}_{II} we obtain

$$\begin{aligned}F^{qg(\mathrm{col}+)}_{\mathrm{II}}(s,z,y) = -2e_q^2\frac{1+(1-z)^2-\epsilon z^2}{z}\\ \times\sum\left|M^{qg\to Wq}\left[s,\frac{b^\dagger_{\mathrm{II}}}{z}\right]\right|^2 ,\end{aligned} \tag{3.64}$$

with

$$b^\dagger_{\mathrm{II}} \equiv b(s,z,y,v=1) = \frac{s}{2}\beta(s)z(1-y) . \tag{3.65}$$

Remember that (3.64) has an implicit factor of $C(Q_1,Q_2,(1-z)Q_2/z)$. Using (3.64) in (3.60) we can rewrite the collinear contribution [neglecting terms of $O(\epsilon)$]

$$\begin{aligned}\sigma^{P,\mathrm{II}}_{qg,\mathrm{col}+} = -\frac{\alpha}{2\pi\bar{\epsilon}}\int_0^1 dz\left[\hat{e}_q^2\frac{1+(1-z)^2}{z} + \epsilon\hat{e}_q^2\left\{\frac{1+(1-z)^2}{z}\left[\ln\left[\frac{2\mu^2}{sv_0}\right] + \ln\left[\frac{1+(z-1)\beta(s)}{z^2(1-z)^2\beta^2(s)}\right]\right] - z\right\}\right]\\ \times\int DQ_1 DQ_2\frac{D^2\sigma^{(1)}}{DQ_1 DQ_2}\left[q(p_1)g(p_2)\to W(Q_1)q\left[\frac{Q_2}{z}\right]\right] .\end{aligned} \tag{3.66}$$

In (3.66) we made the replacement $e^2 = 4\pi\alpha\mu^{2\epsilon}$.

Using (3.41), (3.45), (3.46), (3.51), (3.59), (3.60), and (3.66) in (2.26) we obtain, for the $O(\alpha_S)$ hard scattering cross section,

$$\sigma^{(1)}[q(p_1)g(p_2)\to W\gamma q]=\sigma^{P,\mathrm{I}}_{qg,\mathrm{finite}}+\frac{\alpha_S}{2\pi}\int_{\rho(s)}^1 dx\left[\frac{1}{2}+[x^2+(1-x)^2]\left\{\ln(1-x)-\frac{1}{2}\left[1+\ln\left[\frac{2\mu^2}{sy_0}\right]\right]\right\}\right]$$

$$\times\sigma^{(0)}[q(p_1)\bar{q}(xp_2)\to W\gamma]+\sigma^{P,\mathrm{II}}_{qg,\mathrm{finite}}$$

$$+\frac{\alpha}{2\pi}\hat{e}_q^2\int_0^1 dz\left\{z-\left[\frac{1+(1-z)^2}{z}\right]\left[\ln\left[\frac{2\mu^2}{sv_0}\right]+\ln\left[\frac{1+(z-1)\beta(s)}{z^2(1-z)^2\beta^2(s)}\right]\right]\right\}$$

$$\times\int DQ_1Dq_2\frac{D^2\sigma^{(1)}}{DQ_1Dq_2}[q(p_1)g(p_2)\to W(Q_1)q(q_2)]$$

$$+\tilde{\sigma}^P[q(p_1)g(p_2)\to W\gamma q] \tag{3.67}$$

with $q_2=Q_2/\tau$. We have thus canceled all singularities and we can now summarize for all the contributions in the incoming qg partonic channel:

$$\int DQ_1DQ_2\sum_X\left[\frac{D^2\sigma^H}{DQ_1DQ_2}\right]^{qg}[p(P_1)\bar{p}(P_2)\to W(Q_1)\gamma(Q_2)X]$$

$$=\int_0^1 d\tau_1\int_0^1 d\tau_2\left\{f_{qp}(\tau_1)f_{g\bar{p}}(\tau_2)\left[\sigma^{(1)}[q(p_1)g(p_2)\to W\gamma q]+\int_0^1 d\tau f_{\gamma q}(\tau)\right.\right.$$

$$\left.\left.\times\int DQ_1Dq_2\frac{D^2\sigma^{(1)}}{DQ_1Dq_2}[q(p_1)g(p_2)\to W(Q_1)q(q_2)]\right]+(p\leftrightarrow\bar{p},\tau_1\leftrightarrow\tau_2,p_1\leftrightarrow p_2)\right\}, \tag{3.68}$$

with $p_1=\tau_1P_1,p_2=\tau_2P_2$. In (3.67) and (3.68) we need the squared matrix element

$$\sum|M^{qg\to Wq}(s,\hat{b})|^{2(1)}=2^6\pi N_cC_FC_W^2\alpha_S\left[\frac{s^2+(s-M_W^2-\hat{b})^2-2\hat{b}M_W^2}{s(s-M_W^2-\hat{b})}\right], \tag{3.69}$$

where $\hat{b}=b/\tau$. (3.69) carries an implicit factor of $C(Q_1,\tau q_2,(1-\tau)q_2)$. The pieces of squared matrix element needed in the last term in (3.67) are too long to be presented here but they may be obtained upon request.

D. The $g\bar{q}$ channel

The treatment of this channel is analogous to the qg channel. We can again decompose the partonic cross section as

$$\sigma^P[g(p_1)\bar{q}(p_2)\to W\gamma\bar{q}]=\sigma^{P,\mathrm{I}}[g(p_1)\bar{q}(p_2)\to W\gamma\bar{q}]+\sigma^{P,\mathrm{II}}[g(p_1)\bar{q}(p_2)\to W\gamma\bar{q}]+\tilde{\sigma}^P[g(p_1)\bar{q}(p_2)\to W\gamma\bar{q}], \tag{3.70}$$

where the nonsingular term $\tilde{\sigma}^P[g(p_1)\bar{q}(p_2)\to W\gamma\bar{q}]$ contains all interference pieces of the squared matrix element and also $|M_{\mathrm{III}}^{g\bar{q}}|^2$. The other terms in (3.70) are decomposed as follows:

$$\sigma^{P,\mathrm{I}(1)}[g(p_1)\bar{q}(p_2)\to W\gamma\bar{q}]=\sigma^{P,\mathrm{I}}_{g\bar{q},\mathrm{finite}}+\sigma^{P,\mathrm{I}}_{g\bar{q},\mathrm{col}+}+O(\epsilon), \tag{3.71}$$

$$\sigma^{P,\mathrm{II}(1)}[g(p_1)\bar{q}(p_2)\to W\gamma\bar{q}]=\sigma^{P,\mathrm{II}}_{g\bar{q},\mathrm{finite}}+\sigma^{P,\mathrm{II}}_{g\bar{q},\mathrm{col}+}+O(\epsilon),$$

with

$$\sigma^{P,\mathrm{I}}_{g\bar{q},\mathrm{finite}} = -\frac{1}{8N_c^2C_F}\frac{1}{2s}(4\pi)^{-4}\int_{\rho(s)}^1 dx\,\beta(sx)\int_{-1}^1 dy\left\{\frac{1}{1-y}\right\}_{y_0}\int_{-1}^1 d\cos\theta_1'\int_0^\pi d\theta_2' F_{\mathrm{I}}^{g\bar{q}}(s,x,y,\cos\theta_1',\theta_2')\,,$$

$$\sigma^{P,\mathrm{I}}_{g\bar{q},\mathrm{col}+} = \frac{\alpha_S}{2\pi\bar{\epsilon}}\int_{\rho(s)}^1 dx\left[x(1-x)-\tfrac{1}{2}+\epsilon\left\{\ln\left[\frac{2\mu^2}{sy_0}\right][x(1-x)-\tfrac{1}{2}]+\ln(1-x)+x(1-x)[1-2\ln(1-x)]\right\}\right]$$
$$\times\sigma^{(0)}[q(xp_1)\bar{q}(p_2)\to W\gamma]\,, \tag{3.72}$$
$$\sigma^{P,\mathrm{II}}_{g\bar{q},\mathrm{finite}} = -\frac{1}{8N_c^2C_F}\frac{1}{2s}(4\pi)^{-4}\beta(s)\int_0^1 dz\int_{-1}^1 dy\int_{-1}^1 dv\left\{\frac{1}{1-v}\right\}_{v_0}\int_0^\pi d\phi_2' F_{\mathrm{II}}^{g\bar{q}}(s,z,y,v,\phi_2')\,,$$

$$\sigma^{P,\mathrm{II}}_{g\bar{q},\mathrm{col}+} = -\frac{\alpha}{2\pi\bar{\epsilon}}\int_0^1 dz\left[\hat{e}_{\bar{q}}^2\frac{1+(1-z)^2}{z}+\epsilon\hat{e}_{\bar{q}}^2\left\{\frac{1+(1-z)^2}{z}\left[\ln\left[\frac{2\mu^2}{sv_0}\right]+\ln\left[\frac{1+(z-1)\beta(s)}{z^2(1-z)^2\beta^2(s)}\right]\right]-z\right\}\right]$$
$$\times\int DQ_1DQ_2\frac{D^2\sigma^{(1)}}{DQ_1DQ_2}\left[g(p_1)\bar{q}(p_2)\to W(Q_1)\bar{q}\left[\frac{Q_2}{z}\right]\right].$$

In (3.72) we used the nonsingular functions

$$F_{\mathrm{I}}^{g\bar{q}}(s,x,y,\cos\theta_1',\theta_2') \equiv -2p_1\cdot k\sum|M_{\mathrm{Ia}}^{g\bar{q}}|^2\,,$$
$$F_{\mathrm{II}}^{g\bar{q}}(s,z,y,v,\phi_2') \equiv -2k\cdot Q_2\sum|M_{\mathrm{II}}^{g\bar{q}}|^2\,. \tag{3.73}$$

Using (3.70)–(3.72) in (2.30) we obtain the cancellation of all singularities in this channel:

$$\sigma^{(1)}[g(p_1)\bar{q}(p_2)\to W\gamma\bar{q}]$$
$$= \sigma^{P,\mathrm{I}}_{g\bar{q},\mathrm{finite}}+\frac{\alpha_S}{2\pi}\int_{\rho(s)}^1 dx\left[\tfrac{1}{2}+[x^2+(1-x)^2]\left\{\ln(1-x)-\frac{1}{2}\left[1+\ln\left[\frac{2\mu^2}{sy_0}\right]\right]\right\}\right]$$
$$\times\sigma^{(0)}[q(xp_1)\bar{q}(p_2)\to W\gamma]+\sigma^{P,\mathrm{II}}_{g\bar{q},\mathrm{finite}}$$
$$+\frac{\alpha}{2\pi}\hat{e}_{\bar{q}}^2\int_0^1 dz\left\{z-\left[\frac{1+(1-z)^2}{z}\right]\left[\ln\left[\frac{2\mu^2}{sv_0}\right]+\ln\left[\frac{1+(z-1)\beta(s)}{z^2(1-z)^2\beta^2(s)}\right]\right]\right\}$$
$$\times\int DQ_1Dq_2\frac{D^2\sigma^{(1)}}{DQ_1Dq_2}[g(p_1)\bar{q}(p_2)\to W(Q_1)\bar{q}(q_2)]+\bar{\sigma}^P[g(p_1)\bar{q}(p_2)\to W\gamma\bar{q}]\,. \tag{3.74}$$

We can now summarize all contributions in the incoming $g\bar{q}$ partonic channel:

$$\int DQ_1DQ_2\sum_X\left[\frac{D^2\sigma^H}{DQ_1DQ_2}\right]^{g\bar{q}}[p(P_1)\bar{p}(P_2)\to W(Q_1)\gamma(Q_2)X]$$
$$=\int_0^1 d\tau_1\int_0^1 d\tau_2\left\{f_{gp}(\tau_1)f_{\bar{q}\bar{p}}(\tau_2)\left[\sigma^{(1)}[g(p_1)\bar{q}(p_2)\to W\gamma\bar{q}]\right.\right.$$
$$\left.+\int_0^1 d\tau f_{\gamma\bar{q}}(\tau)\int DQ_1Dq_2\frac{D^2\sigma^{(1)}}{DQ_1Dq_2}[g(p_1)\bar{q}(p_2)\to W(Q_1)\bar{q}(q_2)]\right]$$
$$\left.+(p\leftrightarrow\bar{p},\tau_1\leftrightarrow\tau_2,p_1\leftrightarrow p_2)\right\}, \tag{3.75}$$

with $p_1=\tau_1 P_1$, $p_2=\tau_2 P_2$, and $q_2=Q_2/\tau$. In (3.74) and (3.75) we still need the squared matrix element:

$$\sum |M^{g\bar{q}\to W\bar{q}}(s,\hat{b})|^{2(1)} = 2^6\pi N_c C_F C_W^2 \alpha_S \left| \frac{s^2+\hat{b}^2-2(s-M_W^2-\hat{b})M_W^2}{s\hat{b}} \right| \tag{3.76}$$

with $\hat{b}=b/\tau$. The comments after (3.69) apply here too.

IV. THE THREE SCENARIOS AND THE EXPERIMENTAL CUTS

A. Two-body inclusive production of W^+ and γ

In this scenario ("two-body inclusive scenario") one does not tag the outgoing jet, so it will include events with zero and one outgoing jet. We may define this scenario requiring the following conditions for the outgoing particles:

$$\begin{aligned} &|\cos\theta_\gamma|, |\cos\theta_W| < \cos(\theta^-) , \\ &P_{t\gamma}, P_{tW} > P_t^- , \\ &R_{W,\gamma} > R^- , \\ &(R_{\text{jet},\gamma} < R^-) \Longrightarrow (s_{(\text{jet},\gamma)} < s^-) , \\ &(R_{\text{jet},W} < R^-) \Longrightarrow (s_{(\text{jet},W)} < s^-) , \end{aligned} \tag{4.1}$$

where we call θ_i the angle between the incoming proton axis and the axis of the outgoing particle i; P_{ti} is the transverse momentum of particle i. $R_{i,j}$ is the cone size between a pair of outgoing particles: $R_{i,j} = \sqrt{(\Delta_{i,j}\eta^*)^2+(\Delta_{i,j}\phi)^2}$ with the pseudorapidity

$$\eta^* \equiv \tfrac{1}{2}\ln[(1+\cos\theta)/(1-\cos\theta)]$$

and ϕ the azimuthal angle; $s_{(\text{jet},W)}=E_{\text{jet}}/E_W$ is the "shadowing ratio" between the untagged jet and the W boson. The last two conditions in (4.1) discard events where the jet is too close to the W or the photon is at the same time of comparable energy so that it would "shadow" one of the two tagged particles, making it undetectable. For this purpose we check the cone size $R_{\text{jet},\gamma}$ ($R_{\text{jet},W}$) and if this is less than R^- we keep the event only when $s_{(\text{jet},\gamma)}$ ($s_{(\text{jet},W)}$) is less than s^-, setting the differential cross section to zero otherwise. The quantities θ^-, Pt^-, R^-, and s^- are constants related to the acceptance and resolution of the detector. All the quantities are defined in the proton-antiproton center of mass frame.

B. Production of W^+ and γ with one jet

Here one detects three outgoing particles, namely W^+, γ, and one jet. We call this the "one-jet scenario" and we define it by imposing the conditions

$$\begin{aligned} &|\cos\theta_\gamma|, |\cos\theta_W|, |\cos\theta_{\text{jet}}| < \cos(\theta^-) , \\ &P_{t\gamma}, P_{tW}, P_{t\,\text{jet}} > P_t^- , \\ &R_{W,\gamma} > R^- , \\ &R_{\text{jet},\gamma} > R^- \\ &R_{\text{jet},W} > R^- . \end{aligned} \tag{4.2}$$

C. Production of W^+ and γ with zero jets

In this scenario ("zero-jet scenario") we select events where the W^+ and γ are detected but no outgoing jet is detected. This includes two- to two-body events and two- to three-body events where the outgoing jet has a small angle with respect to the beam, a small transverse momentum, or it is "shadowed" by the photon or the W so that it remains undetected. We may define this scenario requiring the following conditions for the outgoing particles:

$$\begin{aligned} &|\cos\theta_\gamma|, |\cos\theta_W| < \cos(\theta^-) , \\ &P_{t\gamma}, P_{tW} > P_t^- , \\ &R_{W,\gamma} > R^- , \\ &(R_{\text{jet},\gamma} < R^-) \Longrightarrow (s_{(\text{jet},\gamma)} < s^-) , \\ &(R_{\text{jet},W} < R^-) \Longrightarrow (s_{(\text{jet},W)} < s^-) , \\ &[\,|\cos\theta_{\text{jet}}| > \cos(\theta^-)] \text{ or } (P_{t\,\text{jet}} < P_t^-) . \end{aligned} \tag{4.3}$$

D. General remarks

We note that the second and third scenarios are complementary, in the sense that an event in the first scenario falls in either of the last two. In other words, we may obtain the histograms of the zero-jet production scenario by subtracting the histograms of the one-jet scenario from the corresponding histograms for the two-body inclusive scenario.

To implement the three experimental cut functions $C(Q_1,Q_2,k)$ which define each of the scenarios in A, B, and C all quantities involved in the above conditions have to be defined in terms of the partonic invariants that are used in the integrands of the corresponding cross section formulas. In the proton-antiproton center of mass frame we have

$$\begin{aligned} E_\gamma &= \frac{P_1\cdot Q_2+P_2\cdot Q_2}{\sqrt{S}} , \\ \cos\theta_\gamma &= -\frac{P_1\cdot Q_2-P_2\cdot Q_2}{P_1\cdot Q_2+P_2\cdot Q_2} , \\ E_W &= \frac{P_1\cdot Q_1+P_2\cdot Q_1}{\sqrt{S}} , \\ \cos\theta_W &= \frac{P_1\cdot Q_1-P_2\cdot Q_1}{\sqrt{(P_1\cdot Q_1+P_2\cdot Q_1)^2-SM_W^2}} , \\ E_{\text{jet}} &= \frac{P_1\cdot k+P_2\cdot k}{\sqrt{S}} , \\ \cos\theta_{\text{jet}} &= -\frac{P_1\cdot k-P_2\cdot k}{P_1\cdot k+P_2\cdot k} . \end{aligned} \tag{4.4}$$

P_1 and P_2 represent the proton and antiproton momenta, respectively; they must be appropriately expressed in terms of the incoming parton momenta p_1, p_2 and their momentum fractions τ_1, τ_2 in all the cross section formulas. $\sqrt{S} = \sqrt{2P_1 \cdot P_2}$ is the proton-antiproton center of mass energy. Q_1, Q_2, and k are the momenta of the W boson, the photon, and the jet, respectively. The rest of the quantities needed can be computed using the ones in (4.4).

When we replaced the divergent factors $(1-x)^{-1-2\epsilon}$, $(1\pm y)^{-1-\epsilon}$, and $(1-v)^{-1-\epsilon}$ in Sec. III with distributions, the resulting equations remained valid as long as the variables x, y, and v were integrated over their whole range. The energy of the outgoing jet in the incoming parton-parton center of mass frame is linearly related to the variable x [see (3.10)], so it is in principle not a physical quantity unless $x < x_0$, in which case the symbol $\sim$ can be replaced by $=$ in the corresponding distribution in (3.22). Similarly, the angle between the outgoing jet and the beam in the parton-parton frame is related to y and the angle between the outgoing jet and the photon is related to v, so these quantities are not physical either, unless the variables y and v fall inside the range where we can replace $\sim$ with $=$ in the corresponding distributions.

According to the above observations we should not have any trouble in the two-body inclusive and in the zero-jet scenarios, since in these cases the outgoing jet is not being tagged so the unphysical variables are not "observed," but they are rather integrated over their whole range. However, in the one-jet scenario, the energy and angles of the jet are observed and these are directly related to the variables x, y, and v. According to the way we defined the one-jet scenario in (4.2), the outgoing jet is never allowed to be soft or collinear to the beams or the outgoing photon so the subtraction of divergences will never be active. With this in mind we can easily choose the parameters x_0, y_0, and v_0 in Monte Carl simulation in such a way that the sampled ranges of x, y, and v always fall inside the regions where $\sim$ may be replaced with $=$ in (3.22), (3.44), and (3.58). To accomplish this we can just take $x_0 = 1$ and $y_0 = v_0 = 0$. The experimental cut function $C(Q_1, Q_2, k)$ will accordingly set to zero all the terms containing ill defined logarithms.

V. THE NUMERICAL IMPLEMENTATION

When numerically implementing the "generalized plus" distributions defined in (3.23) to compute total cross sections, the second terms on the RHS of (3.23) are finite when the soft or collinear limits are approached. However, when we produce histograms of single or double differential cross sections it is necessary to split the second terms on the RHS of these definitions into two parts, as we will explain next. For the case of the x integration we have

$$\int_{x_0}^{1} dx \frac{f(x)-f(1)}{1-x} = \int_{x_0}^{1} dx \frac{f(x)}{1-x} - \int_{x_0}^{1} dx \frac{f(1)}{1-x} \ . \tag{5.1}$$

The first term on the RHS of (5.1) is naturally made into a histogram using two- to three-body kinematics. The soft pieces that resulted from the expansion in (3.22) were added to other two- to two-body contributions in order to cancel singularities, so the remaining pieces are naturally made into histograms using two- to two-body kinematics. This means that in order to keep consistency in our computation we have to make the second term on the RHS of (5.1) a histogram, which is the term that compensates for the soft singular terms in (3.22), using two- to two-body kinematics as well. It is thus clear that a consistent way of making histograms cannot be achieved in a simple way without splitting the LHS of (5.1). In doing so we introduce logarithmic singularities in each of the terms on the RHS of (5.1) that cancel each other only after summing both contributions bin by bin. To control the numerical cancellations we introduce small adimensional cuts Δ_x, Δ_y, and Δ_v in the lower or upper limits of the corresponding integrals. A first order estimate of the error introduced by the cuts along with the requirement of good numerical convergence will help us find the best values for these parameters.

In what follows we will rewrite the partonic hard scattering cross sections for each channel taking into account the Δ parameters introduced above. The contribution of each of these terms to the hadronic cross section is obtained after multiplying by the corresponding experimental cut function, convoluting with parton densities (see Sec. II), and adding the corresponding "inverted channels" (i.e., the ones obtained by interchange of the incoming partons.) Numerical results for each of these hadronic contributions are presented in the following paper [7].

For the $q\bar{q}$ hard channel cross section needed in (3.39) we have

$$\sigma_{q\bar{q}} = \sigma_{q\bar{q}}^{\text{Born}} + \sigma_{q\bar{q}(\text{SV})}^{P} + \sigma_{\text{Ia}} + \sigma_{\text{Ib}} + \sigma_{\text{I},4} + \sigma_{q\bar{q}(\text{finite})}^{P} + \sigma_{q\bar{q}(\text{brems})} + \sigma_{q\bar{q}(\text{error})} \ , \tag{5.2}$$

where

$$\begin{aligned}
\sigma_{\text{Ia}} &= \sigma_{\text{Ia},1} + \sigma_{\text{Ia},2} + \sigma_{\text{Ia},3} \ , \\
\sigma_{\text{Ib}} &= \sigma_{\text{Ib},1} + \sigma_{\text{Ib},2} + \sigma_{\text{Ib},3} \ , \\
\sigma_{q\bar{q}(\text{finite})}^{P} &= \sigma_{f,1,1,\text{a}} + \sigma_{f,1,2,\text{a}} + \sigma_{f,1,3,\text{a}} + \sigma_{f,1,1,\text{b}} + \sigma_{f,1,2,\text{b}} + \sigma_{f,1,3,\text{b}} \\
&\quad + \sigma_{f,2,1,\text{a}} + \sigma_{f,2,2,\text{a}} + \sigma_{f,2,3,\text{a}} + \sigma_{f,2,1,\text{b}} + \sigma_{f,2,2,\text{b}} + \sigma_{f,2,3,\text{b}} + \sigma_{f,3} \ , \\
\sigma_{q\bar{q}(\text{error})} &= \sigma_{\text{Ia,error}} + \sigma_{\text{Ib,error}} + \sigma_{f,1,\text{error,a}} + \sigma_{f,1,\text{error,b}} + \sigma_{f,2,\text{error,a}} + \sigma_{f,2,\text{error,b}} + \sigma_{f,\text{error}} \ ,
\end{aligned} \tag{5.3}$$

and

$$\sigma_{q\bar q}^{\rm Born}\equiv\sigma^{(0)}[q(p_1)\bar q(p_2)\to W\gamma]$$
$$=C_{q\bar q,3}\beta(s)\int_0^1 d\cos\theta_1\sum|M^{q\bar q\to W\gamma}(s,b)|^{2(0)}\,,$$

$$\sigma_{\rm Ia,1}\equiv\frac{C_{q\bar q,1}}{2}\int_{\rho(s)}^1 dx\frac{\beta(sx)}{x}(1-x)\int_{-1}^1 d\cos\theta_1'\sum|M^{q\bar q\to W\gamma}(xs,xb^+)|^{2(0)}\,,$$

$$\sigma_{\rm Ia,2}\equiv-\frac{C_{q\bar q,1}}{2}\int_{\rho(s)}^{x_0}dx\frac{\beta(sx)}{x}\left[\frac{1+x^2}{1-x}\right]\left[\ln\left[\frac{2\mu^2}{sy_0}\right]-2\ln(1-x)\right]\int_{-1}^1 d\cos\theta_1'\sum|M^{q\bar q\to W\gamma}(xs,xb^+)|^{2(0)}\,,$$

$$\sigma_{\rm Ia,3}\equiv-\frac{C_{q\bar q,1}}{2}\int_{x_0}^{1-\Delta x}dx\frac{\beta(sx)}{x}\left[\frac{1+x^2}{1-x}\right]\left[\ln\left[\frac{2\mu^2}{sy_0}\right]-2\ln(1-x)\right]\int_{-1}^1 d\cos\theta_1'\sum|M^{q\bar q\to W\gamma}(xs,xb^+)|^{2(0)}\,,$$

$$\sigma_{\rm Ia,error}\equiv\frac{C_{q\bar q,1}}{2}\Delta_x\left[\ln\left[\frac{2\mu^2}{sy_0}\right]+2-2\ln\Delta_x\right]$$
$$\times\frac{\partial}{\partial x}\left[\frac{\beta(sx)}{x}(1+x^2)\int_{-1}^1 d\cos\theta_1'\sum|M^{q\bar q\to W\gamma}(xs,xb^+)|^{2(0)}\right]\Bigg|_{x=1}+O(\Delta_x^2\ln\Delta_x)\,,$$

$$\sigma_{\rm Ib,1}\equiv\frac{C_{q\bar q,1}}{2}\int_{\rho(s)}^1 dx\frac{\beta(sx)}{x}(1-x)\int_{-1}^1 d\cos\theta_1'\sum|M^{q\bar q\to W\gamma}(xs,b^-)|^{2(0)}\,,$$

$$\sigma_{\rm Ib,2}\equiv-\frac{C_{q\bar q,1}}{2}\int_{\rho(s)}^{x_0}dx\frac{\beta(sx)}{x}\left[\frac{1+x^2}{1-x}\right]\left[\ln\left[\frac{2\mu^2}{sy_0}\right]-2\ln(1-x)\right]\int_{-1}^1 d\cos\theta_1'\sum|M^{q\bar q\to W\gamma}(xs,b^-)|^{2(0)}\,,$$

$$\sigma_{\rm Ib,3}\equiv-\frac{C_{q\bar q,1}}{2}\int_{x_0}^{1-\Delta x}dx\frac{\beta(sx)}{x}\left[\frac{1+x^2}{1-x}\right]\left[\ln\left[\frac{2\mu^2}{sy_0}\right]-2\ln(1-x)\right]\int_{-1}^1 d\cos\theta_1'\sum|M^{q\bar q\to W\gamma}(xs,b^-)|^{2(0)}\,,$$

$$\sigma_{\rm Ib,error}\equiv\frac{C_{q\bar q,1}}{2}\Delta_x\left[\ln\left[\frac{2\mu^2}{sy_0}\right]+2-2\ln\Delta_x\right]$$
$$\times\frac{\partial}{\partial x}\left[\frac{\beta(sx)}{x}(1+x^2)\int_{-1}^1 d\cos\theta_1'\sum|M^{q\bar q\to W\gamma}(xs,b^-)|^{2(0)}\right]\Bigg|_{x=1}+O(\Delta_x^2\ln\Delta_x)\,,$$

$$\sigma_{\rm I,4}\equiv2C_{q\bar q,1}\beta(s)\ln\left[\frac{1-x_0}{\Delta_x}\right]\left[\ln\left[\frac{2\mu^2}{sy_0}\right]-\ln[(1-x_0)\Delta_x]\right]\int_{-1}^1 d\cos\theta_1'\sum|M^{q\bar q\to W\gamma}(s,b^{\rm soft})|^{2(0)}\,,$$

$$\sigma_{f,1,1,\rm a}\equiv\frac{C_{q\bar q,2}}{s}\int_{\rho(s)}^{x_0}dx\frac{\beta(sx)}{1-x}\int_{-1}^{1-y_0}dy\frac{1}{1-y}\int_{-1}^1 d\cos\theta_1'\int_0^\pi d\theta_2'F^{q\bar q}(s,x,y,\cos\theta_1',\theta_2')\,,$$

$$\sigma_{f,1,2,\rm a}\equiv\frac{C_{q\bar q,2}}{s}\int_{\rho(s)}^{x_0}dx\frac{\beta(sx)}{1-x}\int_{1-y_0}^{1-\Delta_y}dy\frac{1}{1-y}\int_{-1}^1 d\cos\theta_1'\int_0^\pi d\theta_2'F^{q\bar q}(s,x,y,\cos\theta_1',\theta_2')\,,$$

$$\sigma_{f,1,3,\rm a}\equiv-\frac{C_{q\bar q,1}}{4}\ln\left[\frac{y_0}{\Delta_y}\right]\int_{\rho(s)}^{x_0}dx\frac{\beta(sx)}{1-x}\left[\frac{1+x^2}{x}\right]\int_{-1}^1 d\cos\theta_1'\sum|M^{q\bar q\to W\gamma}(xs,xb^+)|^{2(0)}\,,$$

$$\sigma_{f,1,\rm error,a}\equiv-\frac{C_{q\bar q,2}}{s}\Delta_y\int_{\rho(s)}^{x_0}dx\frac{\beta(sx)}{1-x}\int_{-1}^1 d\cos\theta_1'\int_0^\pi d\theta_2'\left[\frac{\partial F^{q\bar q}(s,x,y,\cos\theta_1',\theta_2')}{\partial y}\right]\Bigg|_{y=1}+O(\Delta_y^2)\,,$$

$$\sigma_{f,1,1,\rm b}\equiv\frac{C_{q\bar q,2}}{s}\int_{\rho(s)}^{x_0}dx\frac{\beta(sx)}{1-x}\int_{-1+y_0}^{1}dy\frac{1}{1+y}\int_{-1}^1 d\cos\theta_1'\int_0^\pi d\theta_2'F^{q\bar q}(s,x,y,\cos\theta_1',\theta_2')\,,$$

$$\sigma_{f,1,2,\rm b}\equiv\frac{C_{q\bar q,2}}{s}\int_{\rho(s)}^{x_0}dx\frac{\beta(sx)}{1-x}\int_{-1+\Delta_y}^{-1+y_0}dy\frac{1}{1+y}\int_{-1}^1 d\cos\theta_1'\int_0^\pi F^{q\bar q}(s,x,y,\cos\theta_1',\theta_2')\,,$$

$$\sigma_{f,1,3,\rm b}\equiv-\frac{C_{q\bar q,1}}{4}\ln\left[\frac{y_0}{\Delta_y}\right]\int_{\rho(s)}^{x_0}dx\frac{\beta(sx)}{1-x}\left[\frac{1+x^2}{x}\right]\int_{-1}^1 d\cos\theta_1'\sum|M^{q\bar q\to W\gamma}(xs,b^-)|^{2(0)}\,,$$

$$\sigma_{f,1,\rm error,b}\equiv\frac{C_{q\bar q,2}}{s}\Delta_y\int_{\rho(s)}^{x_0}dx\frac{\beta(sx)}{1-x}\int_{-1}^1 d\cos\theta_1'\int_0^\pi d\theta_2'\left[\frac{\partial F^{q\bar q}(s,x,y,\cos\theta_1',\theta_2')}{\partial y}\right]\Bigg|_{y=-1}+O(\Delta_y^2)\,,$$

$$\sigma_{f,2,1,\mathrm{a}}\equiv\frac{C_{q\bar q,2}}{s}\int_{x_0}^{1-\Delta_x}dx\frac{\beta(sx)}{1-x}\int_{-1}^{1-y_0}dy\frac{1}{1-y}\int_{-1}^{1}d\cos\theta_1'\int_0^{\pi}d\theta_2' F^{q\bar q}(s,x,y,\cos\theta_1',\theta_2')\ ,$$

$$\sigma_{f,2,2,\mathrm{a}}\equiv\frac{C_{q\bar q,2}}{s}\int_{x_0}^{1-\Delta_x}dx\frac{\beta(sx)}{1-x}\int_{1-y_0}^{1-\Delta_y}dy\frac{1}{1-y}\int_{-1}^{1}d\cos\theta_1'\int_0^{\pi}d\theta_2' F^{q\bar q}(s,x,y,\cos\theta_1',\theta_2')\ ,$$

$$\sigma_{f,2,3,\mathrm{a}}\equiv-\frac{C_{q\bar q,1}}{4}\ln\left[\frac{y_0}{\Delta_y}\right]\int_{x_0}^{1-\Delta_x}dx\frac{\beta(sx)}{1-x}\left[\frac{1+x^2}{x}\right]\int_{-1}^{1}d\cos\theta_1'\sum|M^{q\bar q\to W\gamma}(xs,xb^{+})|^{2(0)}\ ,$$

$$\sigma_{f,2,\mathrm{error,a}}\equiv-\frac{C_{q\bar q,2}}{s}\Delta_y\int_{x_0}^{1-\Delta_x}dx\frac{\beta(sx)}{1-x}\int_{-1}^{1}d\cos\theta_1'\int_0^{\pi}d\theta_2'\left[\frac{\partial F^{q\bar q}(s,x,y,\cos\theta_1',\theta_2')}{\partial y}\right]\Bigg|_{y=1}+O(\Delta_y^2)\ ,$$

$$\sigma_{f,2,1,\mathrm{b}}\equiv\frac{C_{q\bar q,2}}{s}\int_{x_0}^{1-\Delta_x}dx\frac{\beta(sx)}{1-x}\int_{-1+y_0}^{1}dy\frac{1}{1+y}\int_{-1}^{1}d\cos\theta_1'\int_0^{\pi}d\theta_2' F^{q\bar q}(s,x,y,\cos\theta_1',\theta_2')\ ,$$

$$\sigma_{f,2,2,\mathrm{b}}\equiv\frac{C_{q\bar q,2}}{s}\int_{x_0}^{1-\Delta_x}dx\frac{\beta(sx)}{1-x}\int_{-1+\Delta_y}^{-1+y_0}dy\frac{1}{1+y}\int_{-1}^{1}d\cos\theta_1'\int_0^{\pi}d\theta_2' F^{q\bar q}(s,x,y,\cos\theta_1',\theta_2')\ ,$$

$$\sigma_{f,2,3,\mathrm{b}}\equiv-\frac{C_{q\bar q,1}}{4}\ln\left[\frac{y_0}{\Delta_y}\right]\int_{x_0}^{1-\Delta_x}dx\frac{\beta(sx)}{1-x}\left[\frac{1+x^2}{x}\right]\int_{-1}^{1}d\cos\theta_1'\sum|M^{q\bar q\to W\gamma}(xs,b^{-})|^{2(0)}\ ,$$

$$\sigma_{f,2,\mathrm{error,b}}\equiv\frac{C_{q\bar q,2}}{s}\Delta_y\int_{x_0}^{1-\Delta_x}dx\frac{\beta(sx)}{1-x}\int_{-1}^{1}d\cos\theta_1'\int_0^{\pi}d\theta_2'\left[\frac{\partial F^{q\bar q}(s,x,y,\cos\theta_1',\theta_2')}{\partial y}\right]\Bigg|_{y=-1}+O(\Delta_y^2)\ ,$$

$$\sigma_{f,3}\equiv-2C_{q\bar q,1}\beta(s)\ln\left[\frac{1-x_0}{\Delta_x}\right]\ln\left[\frac{2}{y_0}\right]\int_{-1}^{1}d\cos\theta_1'\sum|M^{q\bar q\to W\gamma}(s,b^{\mathrm{soft}})|^{2(0)}\ ,$$

$$\sigma_{f,\mathrm{error}}\equiv-\frac{C_{q\bar q,2}}{s}\Delta_x\int_{-1}^{1}dy\left[\left\{\frac{1}{1-y}\right\}_{y_0}+\left\{\frac{1}{1+y}\right\}_{y_0}\right]$$
$$\times\int_{-1}^{1}d\cos\theta_1'\int_0^{\pi}d\theta_2'\frac{\partial}{\partial x}[\beta(sx)F^{q\bar q}(s,x,y,\cos\theta_1',\theta_2')]\Bigg|_{x=1}+O(\Delta_x^2)\ ,$$

$$\sigma_{q\bar q(\mathrm{brems})}\equiv\int_0^1 d\tau f_{\gamma g}(\tau)\int DQ_1Dq_2\frac{D^2\sigma^{(1)}}{DQ_1Dq_2}[q(p_1)\bar q(p_2)\to W(Q_1)g(q_2)]$$
$$=C_{q\bar q,3}\beta(s)\int_0^1 d\tau f_{\gamma g}(\tau)\int_0^1 d\cos\theta_1\sum\left|M^{q\bar q\to Wg}\left[s,\frac{b}{\tau}\right]\right|^{2(1)}\ . \tag{5.4}$$

The invariant b in the unprimed frame was defined in (3.6). In (5.4) appropriate experimental cut functions are implicit in each of the corresponding integrands. We have introduced the constants

$$C_{q\bar q,1}\equiv\frac{1}{4N_c^2}\frac{1}{2s}\frac{1}{16\pi^2}\alpha_S C_F\ ,$$
$$C_{q\bar q,2}\equiv\frac{1}{4N_c^2}\frac{1}{2s}\frac{1}{2^{10}\pi^4}\ , \tag{5.5}$$
$$C_{q,\bar q,3}\equiv\frac{1}{4N_c^2}\frac{1}{2s}\frac{1}{16\pi}\ .$$

Now we rewrite the hard scattering cross section for the qg channel needed in (3.68):

$$\sigma_{qg}=\sigma^{P,\mathrm{I}}_{qg,\mathrm{finite}}+\sigma^{\mathrm{I,col}}_{qg}+\sigma^{P,\mathrm{II}}_{qg,\mathrm{finite}}+\sigma^{\mathrm{II,col}}_{qg}+\tilde\sigma^{P}_{qg}+\sigma_{qg(\mathrm{brems})}+\sigma_{qg(\mathrm{error})} \tag{5.6}$$

where

$$\sigma^{P,\mathrm{I}}_{qg,\mathrm{finite}}=\sigma^{\mathrm{I}}_{qg,f,1}+\sigma^{\mathrm{I}}_{qg,f,2}+\sigma^{\mathrm{I}}_{qg,f,3}\ ,$$
$$\sigma^{P,\mathrm{II}}_{qg,\mathrm{finite}}=\sigma^{\mathrm{II}}_{qg,f,1}+\sigma^{\mathrm{II}}_{qg,f,2}+\sigma^{\mathrm{II}}_{qg,f,3}\ , \tag{5.7}$$
$$\sigma_{qg(\mathrm{error})}=\sigma^{\mathrm{I}}_{qg,\mathrm{error}}+\sigma^{\mathrm{II}}_{qg,\mathrm{error}}\ ,$$

and

$$\begin{aligned}
\sigma^{\mathrm{I}}_{qg,f,1} &\equiv -C_{qg,3}\int_{\rho(s)}^{1} dx\, \beta(sx)\int_{-1+y_0}^{1} dy\frac{1}{1+y}\int_{-1}^{1} d\cos\theta_1'\int_0^{\pi} d\theta_2' F_{\mathrm{I}}^{qg}(s,x,y,\cos\theta_1',\theta_2')\ ,\\
\sigma^{\mathrm{I}}_{qg,f,2} &\equiv -C_{qg,3}\int_{\rho(s)}^{1} dx\, \beta(xs)\int_{-1+\Delta_y}^{-1+y_0} dy\frac{1}{1+y}\int_{-1}^{1} d\cos\theta_1'\int_0^{\pi} d\theta_2' F_{\mathrm{I}}^{qg}(s,x,y,\cos\theta_1',\theta_2')\ ,\\
\sigma^{\mathrm{I}}_{qg,f,3} &\equiv -\frac{C_{qg,1}}{2}\ln\left[\frac{y_0}{\Delta_y}\right]\int_{\rho(s)}^{1} dx\frac{\beta(sx)}{x}[x^2+(1-x)^2]\int_{-1}^{1} d\cos\theta_1' \sum |M^{q\bar q\to W\gamma}(xs,b^-)|^{2(0)}\ ,\\
\sigma^{\mathrm{I}}_{qg,\mathrm{error}} &\equiv C_{qg,3}\Delta_y\int_{\rho(s)}^{1} dx\,\beta(sx)\int_{-1}^{1} d\cos\theta_1'\int_0^{\pi} d\theta_2'\left[\frac{\partial F_{\mathrm{I}}^{qg}(s,x,y,\cos\theta_1',\theta_2')}{\partial y}\right]\Bigg|_{y=-1}+O(\Delta_y^2)\ ,\\
\sigma^{\mathrm{I,col}}_{qg} &\equiv C_{qg,1}\int_{\rho(s)}^{1} dx\frac{\beta(sx)}{x}\left\{\tfrac{1}{2}+[x^2+(1-x)^2]\left[\ln(1-x)-\tfrac{1}{2}-\tfrac{1}{2}\ln\left[\frac{2\mu^2}{sy_0}\right]\right]\right\}\\
&\qquad\times\int_{-1}^{1} d\cos\theta_1'\sum |M^{q\bar q\to W\gamma}(xs,b^-)|^{2(0)}\ ,\\
\sigma^{\mathrm{II}}_{qg,f,1} &\equiv -C_{qg,3}\beta(s)\int_0^1 dz\int_{-1}^{1} dy\int_{-1}^{1-v_0} dv\frac{1}{1-v}\int_0^{\pi} d\phi_2' F_{\mathrm{II}}^{qg}(s,z,y,v,\phi_2')\ ,\\
\sigma^{\mathrm{II}}_{qg,f,2} &\equiv -C_{qg,3}\beta(s)\int_0^1 dz\int_{-1}^{1} dy\int_{1-v_0}^{1-\Delta_v} dv\frac{1}{1-v}\int_0^{\pi} d\phi_2' F_{\mathrm{II}}^{qg}(s,z,y,v,\phi_2')\ ,\\
\sigma^{\mathrm{II}}_{qg,f,3} &\equiv -C_{qg,2}\hat e_q^2\ln\left[\frac{v_0}{\Delta_v}\right]\beta(s)\int_0^1 dz\left[\frac{1+(1-z)^2}{z}\right]\int_{-1}^{1} dy\sum\left|M^{qg\to Wq}\left[s,\frac{b_{\mathrm{II}}^{\dagger}}{z}\right]\right|^{2(1)}\ ,\\
\sigma^{\mathrm{II}}_{qg,\mathrm{error}} &\equiv C_{qg,3}\Delta_v\beta(s)\int_0^1 dz\int_{-1}^{1} dy\int_0^{\pi} d\phi_2'\left[\frac{\partial F_{\mathrm{II}}^{qg}(s,z,y,v,\phi_2')}{\partial v}\right]\Bigg|_{v=1}+O(\Delta_v^2)\ ,\\
\sigma^{\mathrm{II,col}}_{q,g} &\equiv C_{qg,2}\hat e_q^2\beta(s)\int_0^1 dz\left\{z-\left[\frac{1+(1-z)^2}{z}\right]\left[\ln\left[\frac{2\mu^2}{sv_0}\right]+\ln\left[\frac{1+(z-1)\beta(s)}{z^2(1-z)^2\beta^2(s)}\right]\right]\right\}\\
&\qquad\times\int_{-1}^{1} dy\sum\left|M^{qg\to Wq}\left[s,\frac{b_{\mathrm{II}}^{\dagger}}{z}\right]\right|^{2(1)}\ ,\\
\tilde\sigma^{P}_{qg} &\equiv \tilde\sigma^{P}[q(p_1)g(p_2)\to W\gamma q]\ ,\\
\sigma_{qg(\mathrm{brems})} &\equiv \int_0^1 d\tau f_{\gamma q}(\tau)\int DQ_1 Dq_2\frac{D^2\sigma^{(1)}}{DQ_1 Dq_2}[q(p_1)g(p_2)\to W(Q_1)q(q_2)]\\
&= C_{qg,4}\beta(s)\int_0^1 d\tau f_{\gamma q}(\tau)\int_0^1 d\cos\theta_1\sum\left|M^{qg\to Wq}\left[s,\frac{b}{\tau}\right]\right|^{2(1)}\ .
\end{aligned}\tag{5.8}$$

The comments after (5.4) are also valid here. In (5.8) we have introduced the constants

$$\begin{aligned}
C_{qg,1} &\equiv \frac{1}{8N_c^2}\frac{1}{2s}\frac{1}{16\pi^2}\alpha_S\ ,\\
C_{qg,2} &\equiv \frac{1}{8N_c^2C_F}\frac{1}{2s}\frac{1}{32\pi^2}\alpha\ ,\\
C_{qg,3} &\equiv \frac{1}{8N_c^2C_F}\frac{1}{2s}\frac{1}{2^8\pi^4}\ ,\\
C_{qg,4} &\equiv \frac{1}{8N_c^2C_F}\frac{1}{2s}\frac{1}{16\pi}\ .
\end{aligned}\tag{5.9}$$

Finally, the hard scattering cross section in the $g\bar q$ channel needed in (3.75) may be rewritten

$$\sigma_{g\bar q} = \sigma^{P,\mathrm{I}}_{g\bar q,\mathrm{finite}}+\sigma^{\mathrm{I,col}}_{g\bar q}+\sigma^{P,\mathrm{II}}_{g\bar q,\mathrm{finite}}+\sigma^{\mathrm{II,col}}_{g\bar q}+\tilde\sigma^{P}_{g\bar q}+\sigma_{g\bar q(\mathrm{brems})}+\sigma_{g\bar q(\mathrm{error})}\ ,\tag{5.10}$$

where

$$
\begin{aligned}
&\sigma^{P,\mathrm{I}}_{g\bar q,\mathrm{finite}}=\sigma^{\mathrm{I}}_{g\bar q,f,1}+\sigma^{\mathrm{I}}_{g\bar q,f,2}+\sigma^{\mathrm{I}}_{g\bar q,f,3}\ ,\\
&\sigma^{P,\mathrm{II}}_{g\bar q,\mathrm{finite}}=\sigma^{\mathrm{II}}_{g\bar q,f,1}+\sigma^{\mathrm{II}}_{g\bar q,f,2}+\sigma^{\mathrm{II}}_{g\bar q,f,3}\ ,\\
&\sigma_{g\bar q(\mathrm{error})}=\sigma^{\mathrm{I}}_{g\bar q,\mathrm{error}}+\sigma^{\mathrm{II}}_{g\bar q,\mathrm{error}}\ ,
\end{aligned}
\tag{5.11}
$$

and

$$
\begin{aligned}
&\sigma^{\mathrm{I}}_{g\bar q,f,1}\equiv-C_{qg,3}\int_{\rho(s)}^{1}dx\,\beta(sx)\int_{-1}^{1-y_0}dy\frac{1}{1-y}\int_{-1}^{1}d\cos\theta_1'\int_0^{\pi}d\theta_2'F_{\mathrm{I}}^{g\bar q}(s,x,y,\cos\theta_1',\theta_2')\ ,\\
&\sigma^{\mathrm{I}}_{g\bar q,f,2}\equiv-C_{qg,3}\int_{\rho(s)}^{1}dx\,\beta(sx)\int_{1-y_0}^{1-\Delta_y}dy\frac{1}{1-y}\int_{-1}^{1}d\cos\theta_1'\int_0^{\pi}d\theta_2'F_{\mathrm{I}}^{g\bar q}(s,x,y,\cos\theta_1',\theta_2')\ ,\\
&\sigma^{\mathrm{I}}_{g\bar q,f,3}\equiv-\frac{C_{qg,1}}{2}\ln\left[\frac{y_0}{\Delta_y}\right]\int_{\rho(s)}^{1}dx\frac{\beta(sx)}{x}[x^2+(1-x)^2]\int_{-1}^{1}d\cos\theta_1'\sum|M^{q\bar q\to W\gamma}(xs,xb^{+})|^{2(0)}\ ,\\
&\sigma^{\mathrm{I}}_{g\bar q,\mathrm{error}}\equiv C_{qg,3}\Delta_y\int_{\rho(s)}^{1}dx\,\beta(sx)\int_{-1}^{1}d\cos\theta_1'\int_0^{\pi}d\theta_2'\left.\left[\frac{\partial F_{\mathrm{I}}^{g\bar q}(s,x,y,\cos\theta_1',\theta_2')}{\partial y}\right]\right|_{y=1}+O(\Delta_y^2)\ ,\\
&\sigma^{\mathrm{I,col}}_{g\bar q}\equiv C_{qg,1}\int_{\rho(s)}^{1}dx\frac{\beta(sx)}{x}\left\{\tfrac12+[x^2+(1-x)^2]\left[\ln(1-x)-\tfrac12-\frac12\ln\left[\frac{2\mu^2}{sy_0}\right]\right]\right\}\\
&\qquad\times\int_{-1}^{1}d\cos\theta_1'\sum|M^{q\bar q\to W\gamma}(xs,xb^{+})|^{2(0)}\ ,\\
&\sigma^{\mathrm{II}}_{g\bar q,f,1}\equiv-C_{qg,3}\beta(s)\int_0^1dz\int_{-1}^{1}dy\int_{-1}^{1-v_0}dv\frac{1}{1-v}\int_0^{\pi}d\phi_2'F_{\mathrm{II}}^{g\bar q}(s,z,y,v,\phi_2')\ ,\\
&\sigma^{\mathrm{II}}_{g\bar q,f,2}\equiv-C_{qg,3}\beta(s)\int_0^1dz\int_{-1}^{1}dy\int_{1-v_0}^{1-\Delta_v}dv\frac{1}{1-v}\int_0^{\pi}d\phi_2'F_{\mathrm{II}}^{g\bar q}(s,z,y,v,\phi_2')\ ,\\
&\sigma^{\mathrm{II}}_{g\bar q,f,3}\equiv-C_{gq,2}\hat e_{\bar q}^2\ln\left[\frac{v_0}{\Delta_v}\right]\beta(s)\int_0^1dz\left[\frac{1+(1-z)^2}{z}\right]\int_{-1}^{1}dy\sum\left|M^{g\bar q\to Wq}\left[s,\frac{b_{\mathrm{II}}^{\dagger}}{z}\right]\right|^{2(1)}\ ,\\
&\sigma^{\mathrm{II}}_{g\bar q,\mathrm{error}}\equiv C_{qg,3}\Delta_v\beta(s)\int_0^1dz\int_{-1}^{1}dy\int_0^{\pi}d\phi_2'\left.\left[\frac{\partial F_{\mathrm{II}}^{g\bar q}(s,z,y,v,\phi_2')}{\partial v}\right]\right|_{v=1}+O(\Delta_v^2)\ ,\\
&\sigma^{\mathrm{II,col}}_{g\bar q}\equiv C_{qg,2}\hat e_{\bar q}^2\beta(s)\int_0^1dz\left\{z-\left[\frac{1+(1-z)^2}{z}\right]\left[\ln\left[\frac{2\mu^2}{sv_0}\right]+\ln\left[\frac{1+(z-1)\beta(s)}{z^2(1-z)^2\beta^2(s)}\right]\right]\right\}\int_{-1}^{1}dy\sum\left|M^{g\bar q\to W\bar q}\left[s,\frac{b_{\mathrm{II}}^{\dagger}}{z}\right]\right|^{2(1)}\ ,\\
&\tilde\sigma^{P}_{g\bar q}\equiv\tilde\sigma^{P}[g(p_1)\bar q(p_2)\to W\gamma\bar q]\ ,\\
&\sigma_{g\bar q(\mathrm{brems})}\equiv\int_0^1d\tau f_{\gamma\bar q}(\tau)\int DQ_1Dq_2\frac{D^2\sigma^{(1)}}{DQ_1Dq_2}[g(p_1)\bar q(p_2)\to W(Q_1)\bar q(q_2)]\\
&\qquad=C_{qg,4}\beta(s)\int_0^1d\tau f_{\gamma\bar q}(\tau)\int_0^1d\cos\theta_1\sum\left|M^{g\bar q\to W\bar q}\left[s,\frac{b}{\tau}\right]\right|^{2(1)}\ .
\end{aligned}
\tag{5.12}
$$

The comments after (5.4) are also valid here.

All the above terms will contribute in the two-body inclusive scenario and in the zero-jet scenario. However, in the one-jet scenario, as we mentioned in Sec. IV D, by setting $x_0=1$ and $y_0=v_0=0$ we are left only with the contributions

$$
\sigma_{q\bar q}=\sigma_{f,1,1,\mathrm{a}}+\sigma_{f,1,1,\mathrm{b}}\ ,\quad \sigma_{qg}=\sigma^{\mathrm{I}}_{qg,f,1}+\sigma^{\mathrm{II}}_{qg,f,1}+\tilde\sigma^{P}_{qg}\ ,\quad \sigma_{g\bar q}=\sigma^{\mathrm{I}}_{g\bar q,f,1}+\sigma^{\mathrm{II}}_{g\bar q,f,1}+\tilde\sigma^{P}_{g\bar q}\ . \tag{5.13}
$$

ACKNOWLEDGMENTS

S.M. would like to thank Professor D. Soper for some clarifying discussions during the CTEQ summer school. The work in this paper was supported in part by the contract NSF 9309888.

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