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Green's functions in the color field of a large nucleus

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We compute the Green's functions for scalars, fermions, and vectors in the color field associated with the infinite momentum frame wave function of a large nucleus. Expectation values of this wave function can be computed by integrating over random orientations of the valence quark charge density. This relates the Green's functions to correlation functions of a two-dimensional, ultraviolet finite, field theory. We show how one can compute the sea quark distribution functions and explicitly compute them in the kinematic range of transverse momenta, $\alpha_s^2\mu^2 << k_t^2 << \mu^2$, where μ^2 is the average color charge squared per unit area. When $m_{\rm quark}^2 << \mu^2 \sim A^{1/3}$, the sea quark contribution to the infinite momentum frame wave function saturates at a value that is the same as that for massless sea quarks.

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I. INTRODUCTION

In two previous papers [1,2] we argued that the quark and gluon distribution functions for very large nuclei at small values of Bjorken x were computable in a weakly coupled many-body theory. We argued that when $x << A^{-1/3}$ and when a parameter proportional to the density of valence quarks per unit area,

$$\mu^2 = 1.1 A^{1/3}\ \mathrm{fm}^{-2}\,, \tag{1}$$

is large, the theory is governed by a weak coupling constant $\alpha_s(\mu)$. The valence quarks serve as sources of color charge and can be treated as static sources along the light cone. They follow straight line trajectories traveling at the speed of light. We further argued that as long as we were measuring parton distribution functions on transverse momentum scales which are $k_t^2 << \mu^2$, the sources of valence charge could be treated classically. The field is stochastic and we must average over the classical sources of charge ρ with a Gaussian weight

$$\int [d\rho]\ \exp\left(-\frac{1}{2\mu^2}\int d^2x_t\ \rho^2(x)\right). \tag{2}$$

To lowest order in α_s, the theory can be reduced to computations of the two-dimensional Euclidean correlation functions of an ultraviolet finite gauge theory.

These correlation functions are related to the solution of the classical equations of motion in the presence of an external current which is localized on the light cone:

$$J_a^\mu(x) = \delta^{\mu +}\delta(x^-)\rho_a(x^+, x_t)\,. \tag{3}$$

This source corresponds to a sheet of charge in the two transverse spatial dimensions propagating at the speed of light $x = t$. Here we use the light cone variables

$$x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3)\,. \tag{4}$$

In the light cone gauge, $A_- = -A^+ = 0$, a solution to the classical equations of motion for the gauge fields is $A_+ = -A^- = 0$ with the transverse components of the gauge field given by

$$A_j(x) = \theta(x^-)\alpha_j(x_t)\,. \tag{5}$$

Further, the field α_j may be inserted into the equations of motion to show that

$$F_t = 0\,, \tag{6}$$

where F_t are the transverse components of the field strength tensor, and

$$\nabla\cdot\alpha = g\rho\,. \tag{7}$$

These conditions are equivalent to α_j being a pure gauge transform of the vacuum for a two-dimensional Yang-Mills theory, with the gauge condition being the above equation. We may, therefore, write

$$\alpha_j = -\frac{1}{ig}\ U(x_t)\nabla_j U^\dagger(x_t)\,, \tag{8}$$

where the equation which determines U is

$$\nabla\cdot(U\nabla U^\dagger) = -ig^2\rho\,. \tag{9}$$

This solution has zero light cone Hamiltonian, $P^- = 0$.

Finally, the integration over all the color orientations of the external sheet of charge must be performed. This is equivalent to computing the expectation values of $\langle\alpha_i(x_t)\alpha_j(y_t)\rangle$ with the measure

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$$\int [d\alpha]\, \exp\left\{-\frac{1}{g^2\mu^2}\int d^2x_t (\nabla\cdot\alpha)^2\right\}\delta(F_t)\det(\nabla\cdot D)\,, \tag{10}$$

where D is the covariant derivative. This measure can also be expressed in terms of the compact fields U as

$$\int [dU]\, \exp\left(-\frac{1}{g^4\mu^2}\int d^2x_t \mathrm{Tr}\left[\nabla\cdot\left(U\frac{1}{i}\nabla U^\dagger\right)\right]^2\right) \times\det(\nabla\cdot \mathbf{D})\,. \tag{11}$$

The above form for the equations explicitly demonstrates a scaling behavior. Namely, the two-dimensional expectation values of the variables U are functions of $g^2\mu\, x_t$ or $g^2\mu/k_t$. The dependence on k^+ of the external field is particularly simple and has the form $1/k^+$. Thus to lowest order in α_s, and to all orders in $\alpha_s^2\mu^2$, the distribution function for gluons is flat in rapidity, and has the above simple dependence on transverse momentum scales. When $k_t \gg \alpha_s\mu$, we showed in our previous papers that the distribution functions were simply Wiezsäcker–Williams distributions.

Although the limitation that the computations we employ are only valid for $k_t^2 \ll \mu^2$ arose from requiring that we can treat the external sources classically, it would seem that one might easily extend the kinematic region where our results hold to larger values of transverse momentum. After all, in this region, the gluon field is essentially the perturbative Wiezsäcker–Williams field, with the only essential modification that the source is quantum mechanical. In the lowest order, the source squared averages to the same values as for the classical result, and the result is the same.

The problem is that there will be big quantum corrections to our lowest order result. These will arise as factors involving $\alpha_s \ln(k_t/\mu)$ and can be large. This is a consequence of the fact that the background field is becoming weak, of the order of $A_k^2 \sim \alpha_s\mu^2/k_t^2$ and when $k_t \sim \mu$, the background field is of the order of the quantum corrections.

Eventually it may be possible to extend the range of validity of our method to larger values of k_t. Indeed, the corrections seem to be similar to corrections to bremsstrahlung radiation which are understood to some degree. At this time we do not know how to make these corrections, and our results are restricted to transverse momenta $k_t \ll \mu$. This is a nontrivial restriction because the bremsstrahlung spectrum is hard, and as we shall see for quarks with masses $m_{\mathrm{quark}} \gg \mu$, the dominant contribution to the sea quark spectrum comes from this region.

The central issue we will address in this paper is that of computing the Green's functions for scalars, fermions, and vectors in the presence of the above background field. We will consider scalars and fermions in the fundamental representation of the gauge group. The vectors will be the gluons. We shall first compute the Green's function before averaging over all values of the valence quark charge, and later average over all values to obtain our final result. We will get explicit expressions for the Green's functions in terms of the background field, and will thus be able to determine the scaling properties of the distribution functions associated with these Green's functions.

It is not too surprising that the Green's functions can be explicitly computed. In the region $x^- <, 0$ and in the region $x^- > 0$, the background field is a pure gauge. Only the step function at $x^- = 0$ prevents the field from being entirely a gauge transform of the vacuum configuration.

There are at least two uses towards which the Green's function computation can be applied. The first is to compute, to lowest order in α_s, the contribution of sea quarks to the wave function of a nucleus. This is of interest for heavy quarks because the enhancement of the momentum scale arising from the typically higher density of partons in a nucleus leads to a correspondingly enhanced contribution of the strange and charm quarks to the nuclear wave function. In fact we will see that if $m_{\mathrm{quark}} \ll \mu$, then the contribution of sea quarks has saturated at a value which is the same as that of the massless sea quarks.

The Green's functions may also be applied to determine the higher order corrections in α_s to the quark and gluon distribution functions. This will be the subject of a later analysis, where we will explicitly compute the first-order corrections. It will be necessary to understand the pattern of such corrections if the Lipatov enhancement [3] is to be understood. Hopefully, all of this will be feasible.

For the problems we wish to study in this paper, it is useful to know the relation between the Green's functions and the distribution functions for quarks and gluons. We explicitly derive this relationship in Sec. II.

In Sec. III we compute, in the fundamental representation, the scalar particle Green's function in the presence of the background field. We show how averaging over the different orientations of the sources of charge simplifies the result.

In Sec. IV we generalize our results for the Green's function to fermions in the fundamental representation and to gluons.

In Sec. V we compute the sea quark contribution to the nuclear wave function. We compute the ratio of the contributions of light mass quarks to glue. For heavy quarks, we compute the mass dependence of our results. Unfortunately, for heavy quarks with $m_{\mathrm{quark}} \geq \mu$, the dominant contribution to the integrated spectrum comes for values of $k_t \gg \mu$.

We will summarize our results in Sec. VI. In the Appendix we obtain an expression for the quantum fluctuations in the constrained vector fields. These fields, which are not dynamical fields, are nevertheless necessary for computations of some pieces in the gluon Green's functions.

II. PROPAGATORS AND DISTRIBUTION FUNCTIONS

We want to be able to relate the distribution functions for various species of particles to the propagators for these particles. Let us first consider the example of a scalar field in the fundamental representation of the gauge group. The scalar field may be written in terms of creation and annihilation operators as

$$\phi^\alpha(x^+,\mathbf{x}) = \int_{k^+>0} \frac{d^3k}{\sqrt{2k^+}(2\pi)^3} \times \left\{e^{ikx}a^\alpha(x^+,\mathbf{k}) + e^{-ikx}b^{\alpha\dagger}(x^+,\mathbf{k})\right\} . \quad (12)$$

In this equation, $\mathbf{k}$ denotes the set $k^+,\mathbf{k}_t$. The equal (light cone) time commutation relations for the a and b fields are

$$[a^\alpha(x^+,\mathbf{k}), a^{\beta\dagger}(x^+,\mathbf{q})] = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{q})\delta^{\alpha\beta} \quad (13)$$

and

$$[b^\alpha(x^+,\mathbf{k}), b^{\beta\dagger}(x^+,\mathbf{q})] = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{q})\delta^{\alpha\beta} , \quad (14)$$

with all other commutators vanishing. The operator $a^\dagger$ creates a scalar particle and the operator $b^\dagger$ creates its antiparticle charge conjugated partner. If we define

$$b^{\alpha\dagger}(x^+,\mathbf{k}) = a^\alpha(x^+,-\mathbf{k}) , \quad (15)$$

we then have

$$\phi^\alpha(x) = \int \frac{d^3k}{\sqrt{2\mid k^+\mid}(2\pi)^3}\, e^{ikx}a^\alpha(x^+,\mathbf{k}) . \quad (16)$$

The distribution function for the scalars is

$$\frac{dN}{d^3k} = \frac{1}{(2\pi)^3}\sum_\alpha \langle a^{\alpha\dagger}(x^+,\mathbf{k})a^\alpha(x^+,\mathbf{k}) + b^{\alpha\dagger}(x^+,\mathbf{k})b^\alpha(x^+,\mathbf{k})\rangle . \quad (17)$$

The sum in the above equation goes over both particles and antiparticles. For the systems we consider, the sum will be symmetric under interchange of particles and antiparticles. We therefore have

$$\frac{dN}{d^3k} = \frac{2}{(2\pi)^3}\sum_\alpha \langle a^{\alpha\dagger}(x^+,\mathbf{k})a^\alpha(x^+,\mathbf{k})\rangle . \quad (18)$$

Now on the other hand, we also have that

$$a^\alpha(x^+,\mathbf{k}) = \sqrt{2\mid k^+\mid}\int d^3x\, e^{-ikx}\phi^\alpha(x) , \quad (19)$$

so that

$$\frac{dN}{d^3k} = 2i\,\frac{(2k^+)}{(2\pi)^3}\sum_\alpha D^{\alpha\alpha}(x^+,\mathbf{k},x^+,\mathbf{k}) . \quad (20)$$

The propagator in the above equation is defined as

$$D^{\alpha\beta}(x,y) = -i\langle\phi^\alpha(x)\overline{\phi}^\beta(y)\rangle , \quad (21)$$

with

$$D(x^+,\mathbf{k},y^+,\mathbf{q}) = \int d^3x d^3y\, e^{-ikx+iqy}D(x,y) . \quad (22)$$

Finally, we can write the distribution function as the following integral over the fully Fourier transformed propagator:

$$\frac{dN}{d^3k} = 2i\,\frac{2k^+}{(2\pi)^3}\sum_\alpha\int\frac{dk^-}{2\pi}\frac{dq^-}{2\pi}\,D^{\alpha\alpha}(k^-,\mathbf{k},q^-,\mathbf{k}) . \quad (23)$$

In general, our propagator will have the structure

$$D^{\alpha\beta}(k,q) = \delta^{\alpha\beta}(2\pi)^3\delta(k^- - q^-)\delta^{(2)}(\mathbf{k}_t - \mathbf{q}_t) \times\Delta(k^+,q^+,k^-,\mathbf{k}_t) . \quad (24)$$

This form follows because, after the averaging over sources, the propagator must obey translational invariance in the transverse spatial directions and color invariance. The external field does not depend on the light cone time x^+—hence one also obtains a delta function of $k^- - q^-$. Since the background field depends on x^- and y^- even after summing over the charges of the valence quarks, there is a nontrivial dependence of the propagator on both k^+ and q^+.

Combining terms together, we finally see that

$$\frac{dN^{\rm scalar}}{d^3k} = \frac{2in_{\rm scalar}(2k^+)}{(2\pi)^3}\,\pi R^2 \times\int\frac{dk^-}{2\pi}\Delta^{\rm scalar}(k^+,k^+,k^-,\mathbf{k}_t) . \quad (25)$$

In exactly the same way, we derive, for gluons,

$$\frac{dN^{\rm gluon}}{d^3k} = \frac{in_{\rm gluon}(2k^+)}{(2\pi)^3}\,\pi R^2 \times\int\frac{dk^-}{2\pi}\sum_i\Delta^{\rm gluon}_{ii}(k^+,k^+,k^-,\mathbf{k}_t) , \quad (26)$$

where the sum over i is a sum over transverse gluon polarizations.

For fermions we get

$$\frac{dN^{\rm fermion}}{d^3k} = \frac{2in_{\rm fermion}(2k^+)}{(2\pi)^3}\,\pi R^2 \times\int\frac{dk^-}{2\pi}\sum_s\Delta^{\rm fermion}_{ss}(k^+,k^+,k^-,\mathbf{k}_t) . \quad (27)$$

Here the sum over s is a sum over fermion spin degrees of freedom. In the above, $n_{\rm scalar}$, $n_{\rm gluon}$, and $n_{\rm fermion}$ are the respective color degeneracies of the scalars, gluons, and fermions. In the fundamental representation, this factor is N_c and in the adjoint representation it is $N_c^2 - 1$. A factor of 2 for the degeneracy of particles and antiparticles is included in the above expressions. Further, the sum over spins and polarizations will reproduce the spin degeneracy factors.

III. THE PROPAGATOR FOR SCALAR BOSONS

We shall now compute the propagator for a scalar field in the fundamental representation of the gauge group propagating in the background gauge field:

$$A^a_+ = 0$$
$$A^a_- = 0$$
$$\tau\cdot A_i = \theta(x^-)\alpha_i(x_t) , \quad (28)$$

where

$$\alpha_i(x_t) = -\frac{1}{ig} U(x_t) \nabla_i U^\dagger(x_t) \,. \qquad (29)$$

To do this, we first solve the Klein–Gordon equation

$$\left\{-(\nabla_t - ig\alpha)^2 + 2\partial^+\partial^- + M^2\right\} \phi_\lambda(x) = \lambda \phi_\lambda(x) \qquad (30)$$

for the scalar field ϕ and the corresponding equation for $\overline{\phi}(x)$, the complex conjugate of the scalar field.

We will soon see that an eigenstate of the above equation is labeled by its four momentum p and its color label s for a color spinor with index β. The Green's function is given in terms of the above eigenfunctions as

$$G^{\beta\delta}(x,x') = \int \frac{d\lambda}{\lambda - i\epsilon} \int \frac{d^4p}{(2\pi)^4}\, \delta(\lambda - p^2 - M^2) \times \sum_s \phi^{\beta s}_{\lambda p}(x) \overline{\phi}^{\,\delta s}_{\lambda p}(x') \,. \qquad (31)$$

The solutions are normalized so that

$$\int d^4x\, \overline{\phi}^{\,s'}_{\lambda' p'}(x) \phi^s_{\lambda p}(x) \;=\; \delta^{ss'} \delta^{(4)}(p' - p) \,. \qquad (32)$$

For $x^- < 0$, the external field vanishes. The solutions are therefore plane waves:

$$\phi^{\alpha s}_{\lambda p}(x) = \exp\left(ip_t\, x_t - ip^- x^+ - \frac{p_t^2 + M^2 - \lambda}{2p^-} x^-\right) u^\alpha_s \,. \qquad (33)$$

Here u is the elementary color spinor, such that

$$u^\dagger_{s'} u_s = \delta_{s's} \,. \qquad (34)$$

For $x^- > 0$, the field is a gauge transformation of the vacuum field configuration. The solutions for fixed p are therefore

$$\phi^{\alpha s}_{\lambda p}(x) = [U(x_t) u_s]^\alpha \exp\left(ip_t\, x_t - ip^- x^+ - \frac{p_t^2 + M^2 - \lambda}{2p^-} x^-\right) \,. \qquad (35)$$

We must now construct the solution to the equations of motion which is continuous across the discontinuity in x^-. The solution is

$$\phi^{\alpha s}_{\lambda p}(x) = e^{ip_t x_t - ip^- x^+} \Bigg\{ \theta(-x^-) u^\alpha_s \exp\left(-i\frac{p_t^2 + M^2 - \lambda}{2p^-} x^-\right) + \theta(x^-) \int \frac{d^2q_t}{(2\pi)^2}\, e^{iq_t x_t} \exp\left(-i\frac{(p_t + q_t)^2 + M^2 - \lambda}{2p^-} x^-\right) \times \left(U(x_t) V^\dagger(q_t) u_s\right)^\alpha \Bigg\} \,, \qquad (36)$$

where we have defined

$$V^\dagger(p_t) = \int d^2x_t\, e^{-ip_t x_t} U^\dagger(x_t) \,. \qquad (37)$$

The above solution for ϕ is properly normalized as may be easily verified. Upon defining

$$p^+ = \frac{p_t^2 + M^2 - \lambda}{2p^-} \,, \qquad (38)$$

ϕ may alternatively be written as

$$\phi^{\alpha s}_{\lambda p}(x) = e^{ipx} \Bigg\{ \theta(-x^-) u^\alpha_s + \theta(x^-) \int \frac{d^2q_t}{(2\pi)^2}\, e^{iq_t x_t} \exp\left(-i\frac{(p_t + q_t)^2 - p_t^2}{2p^-} x^-\right) \left(U(x_t) V^\dagger(q_t) u_s\right)^\alpha \Bigg\} \,. \qquad (39)$$

With the above solutions in hand, we can now construct the Green's function for the scalar field in an arbitrary background field of the type described above. It is important to note that this expression makes no assumption about the color averaging over the color labels of the external sources corresponding to the valence quarks. This averaging will be done later. (The expression we quote before averaging is the quantity which will be useful in loop graph computations.) The result of a considerable amount of straightforward but tedious algebra is

$$G^{\alpha\beta}(x,y) = \theta(-x^-)\theta(-y^-) G_0^{\alpha\beta}(x - y) + \theta(x^-)\theta(y^-) \left(U(x_t) G_0(x - y) U^\dagger(y_t)\right)^{\alpha\beta} + \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 + M^2 - i\epsilon} e^{ip(x-y)} \times \int \frac{d^2q_t}{(2\pi)^2} d^2z_t \Bigg\{ \theta(x^-)\theta(-y^-) e^{i(p_t - q_t)(z_t - x_t)} e^{\left[-i\frac{(q_t^2 - p_t^2)}{2p^-} x^-\right]} \left(U(x_t) U^\dagger(z_t)\right)^{\alpha\beta} + \theta(-x^-)\theta(y^-) e^{-i(p_t - q_t)(z_t - y_t)} \exp\left[i\frac{(q_t^2 - p_t^2)}{2p^-} y^-\right] \left(U(z_t) U^\dagger(y_t)\right)^{\alpha\beta} \Bigg\} \,, \qquad (40)$$

where the free particle Green's function

$$G_0^{\alpha\beta}(x-y) = \delta^{\alpha\beta} \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip\cdot(x-y)}}{p^2+M^2-i\epsilon}\,. \tag{41}$$

Now if we average the above Green's function over all the possible values of the color labels corresponding to the valence quarks, the result simplifies even further. Defining

$$\langle \left(U(x_t)U^\dagger(y_t)\right)^{\alpha\beta} \rangle = \delta^{\alpha\beta}\,\Gamma(x_t-y_t)\,, \tag{42}$$

we see that

$$\Gamma(0)=1\,, \tag{43}$$

which follows from the unitarity of the matrices U. Defining the Fourier transform of Γ,

$$\gamma(p_t) = \int d^2x_t\, e^{-ip_tx_t}\Gamma(x_t)\,, \tag{44}$$

we have the sum rule

$$\int \frac{d^2p_t}{(2\pi)^2}\,\gamma(p_t) = 1\,. \tag{45}$$

This expression can now be inserted into the definition of the propagator. The result can be expressed most simply in terms of the Fourier transform of the Green's function—the propagator $D(p,q)$. Letting $\delta D(p,q) = D(p,q) - D_0(p,q)$ where D_0 is the free propagator, and writing

$$\delta D^{\alpha\beta}(p,q) = i\delta^{\alpha\beta}(2\pi)^3\delta(p^- - q^-)\delta^{(2)}(\mathbf{p}_t - \mathbf{q}_t)\delta\Delta(p,q)\,, \tag{46}$$

it is straightforward to compute $\delta\Delta$. As a result of considerable algebra and using the above sum rule for the integral of $\gamma(p_t)$, we finally find that

$$\begin{aligned}\delta\Delta(p,q) = {}&\theta(p^-)\left(\frac{1}{p^+-q^++i\epsilon} - 2p^-\Delta_0(q)\right)\int \frac{d^2l_t}{(2\pi)^2}\,\Delta_0(p-l_t)\left[\gamma(l_t)-(2\pi)^2\delta^{(2)}(l_t)\right]\\ &+\theta(-p^-)\left(\frac{1}{p^+-q^++i\epsilon} - 2p^-\Delta_0(p)\right)\int \frac{d^2l_t}{(2\pi)^2}\,\Delta_0(q-l_t)\left[\gamma(l_t)-(2\pi)^2\delta^{(2)}(l_t)\right]\,.\end{aligned} \tag{47}$$

In the above, Δ_0 represents the usual scalar propagator:

$$\Delta_0(p) = \frac{1}{p^2+M^2-i\epsilon}\,. \tag{48}$$

IV. PROPAGATOR FOR FERMIONS AND GLUONS

It is easy to show that each component of the free fermion wave function obeys the Klein-Gordon equation. Hence the Green's function defined in Eq. (31) for the fermions is simply related to the scalar Green's function by a relative normalization factor of $2k^+$. This relation also holds for their Fourier transforms—the propagators. Explicitly,

$$\delta\Delta(p,q)^{\text{fermion}}_{ss} = 2p^+\delta\Delta(p,q)\,, \tag{49}$$

where $\delta\Delta(p,q)$ is given in Eq. (47).

The solutions of the small fluctuation equations for the gluons in the background field are plane waves in the adjoint representation for $x^- < 0$ and gauge transforms of plane waves in the adjoint representation for $x^- > 0$. Defining $p^+ = (p_t^2 - \lambda)/2p^-$, and matching the fields across the discontinuity at $x^- = 0$, we obtain for the transverse components of the gauge field the relation

$$\begin{aligned}A_t^{\alpha\beta} = e^{ipx}\eta_t\Bigg\{&\theta(-x^-)\tau^{\alpha\beta} + \theta(x^-)\int \frac{d^2q_t}{(2\pi)^2}e^{iq_tx_t}\exp\left(-i\left[\frac{2p_tq_t+q_t^2}{2p^-}x^-\right]\right)\\ &\times\int d^2\bar{x}_t e^{-iq_t\bar{x}_t}\left(U(x_t)U^\dagger(\bar{x}_t)\tau U(\bar{x}_t)U^\dagger(x_t)\right)^{\alpha\beta}\Bigg\}\,,\end{aligned} \tag{50}$$

where η is a unit vector and the τ's correspond to the usual SU(3) matrices. The computation of the Green's functions for the vector case is exactly analogous to that of the scalar Green's functions outlined in Sec. III. To obtain our final result, though, we have to make use of the Fierz identity

$$(\tau)^{\alpha\delta}(\tau)^{\gamma\beta} = \frac{1}{2}\left(\delta^{\alpha\beta}\delta^{\gamma\delta} - \frac{1}{3}\delta^{\alpha\delta}\delta^{\gamma\beta}\right)\,. \tag{51}$$

Our result for the Green's function is then

$$G^{\alpha\beta;\alpha'\beta'}(x,y) = \int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2-i\epsilon}\Bigg\{\theta(-x^-)\theta(-y^-)(\tau)^{\alpha\beta}(\tau)^{\alpha'\beta'} + \theta(x^-)\theta(y^-)\big(U(x_t)\tau U^\dagger(x_t)\big)^{\alpha\beta}\big(U(y_t)\tau U^\dagger(y_t)\big)^{\alpha'\beta'}$$
$$+\int\frac{d^2q_t}{(2\pi)^2}d^2\bar{x}_t[\theta(x^-)\theta(-y^-)e^{i(p_t-q_t)(\bar{x}_t-x_t)}e^{-i(q_t^2-p_t^2)x^-/2p^-}\big(U(x_t)\tau U^\dagger(x_t)\big)^{\alpha\beta}\big(U(\bar{x}_t)\tau U^\dagger(\bar{x}_t)\big)^{\alpha'\beta'}$$
$$+\theta(-x^-)\theta(y^-)e^{i(p_t-q_t)(y_t-\bar{x}_t)}e^{i(q_t^2-p_t^2)y^-/2p^-}\big(U(\bar{x}_t)\tau U^\dagger(\bar{x}_t)\big)^{\alpha\beta}\big(U(y_t)\tau U^\dagger(y_t)\big)^{\alpha'\beta'}]\Bigg\}. \quad (52)$$

Again, as in the case of scalars, we define the expectation value

$$\langle\big(U(x_t)\tau U^\dagger(x_t)\big)^{\alpha\beta}\big(U(y_t)\tau U^\dagger(y_t)\big)^{\alpha'\beta'}\rangle = \frac{1}{2}\left(\delta^{\alpha\beta'}\delta^{\alpha'\beta}-\frac{1}{3}\delta^{\alpha\beta}\delta^{\alpha'\beta'}\right)\Gamma(x_t-y_t). \quad (53)$$

The rest of the discussion is identical to that in Sec. III and the change in the gluon propagator $\delta D^{\alpha\beta;\alpha'\beta'}(p,q)$ may be expressed as

$$\delta D^{\alpha\beta;\alpha'\beta'}(p,q) = \frac{1}{2}i\left(\delta^{\alpha\beta'}\delta^{\alpha'\beta}-\frac{1}{3}\delta^{\alpha\beta}\delta^{\alpha'\beta'}\right)(2\pi)^3\delta(p^- - q^-)\delta^{(2)}(\mathbf{p}_t-\mathbf{q}_t)\delta\Delta(p,q), \quad (54)$$

where $\delta\Delta(p,q)$ has a form identical to the result obtained in Eq. (47).

V. SEA QUARK DISTRIBUTION FUNCTIONS

Now that we have computed the fermion propagator in the classical background field in Sec. IV, we are in a position to calculate, to the lowest order in α_s, the sea quark distributions in this background field. The relation between propagators and the corresponding distribution functions has been discussed in Sec. II. However, due to the singular nature of the propagators in the background field, the actual computation of the distributions is somewhat subtle. This computation will be outlined below. In the rest of this section, we compute the ratio of sea quarks to glue in the background field and study the mass dependence of our results—whether one obtains an enhanced contribution from strange and charm quarks to the nuclear wave function.

From Eqs. (23) and (49) we can show that the sea quark distribution function can be written in terms of the scalar propagator as

$$\frac{1}{\pi R^2}\frac{dN^{\rm fermion}}{d^3k} = \frac{4N_c(2k^+)}{(2\pi)^3}i \times\int dk^{+\prime}dk^{-\prime}\delta D(k_t,k^{-\prime},k^+;k^{+\prime}) \times\delta(k^{+\prime}-k^+). \quad (55)$$

Note that we have implicitly included a factor of 2 from the two light cone spin degrees of freedom in the above. Typically, the above equation would reduce to

$$\frac{1}{\pi R^2}\frac{dN^{\rm fermion}}{d^3k} = \frac{4N_c(2k^+)}{(2\pi)^3}i\int\frac{dk^-}{2\pi}\delta D(k_t,k^-,k^+;k^+). \quad (56)$$

However, this cannot be done in this case due to the singularity in $\delta D(k_t,k^{-\prime},k^+;k^{+\prime})$ as $k^{+\prime}\to k^+$ [see Eq. (47)]. The limit must be taken only at the end, after performing the integrations in Eq. (55).

Substituting Eq. (47) in Eq. (55), we obtain

$$\frac{1}{\pi R^2}\frac{dN^{\rm fermion}}{d^3k} = \frac{4N_c(2k^+)}{(2\pi)^3}\lim_{k^{+\prime}\to k^+}\int\frac{d^2p_t}{(2\pi)^2}[(2\pi)^2\delta^{(2)}(p_t)-\gamma(p_t)]$$
$$\times\Bigg\{\int_{-\infty}^{\infty}\frac{dk^{-\prime}}{2\pi}(-2k^{-\prime})\Delta_0(k)\Delta_0(k'+p_t) + \Bigg[\int_{-\infty}^{0}\frac{dk^{-\prime}}{2\pi}\frac{\Delta_0(k+p_t)}{k^{+\prime}-k^++i\epsilon}$$
$$+\int_0^{\infty}\frac{dk^{-\prime}}{2\pi}\frac{\Delta_0(k'+p_t)}{k^{+\prime}-k^++i\epsilon}\Bigg]\Bigg\}. \quad (57)$$

The first term within the curly brackets vanishes because both poles in the contour integral lie on the same side of the contour. The remaining two integrals are apparently logarithmically divergent. It can also be shown that the pieces singular in the limit $k^{+\prime}\to k^+$ either vanish or cancel out. The sum of the two integrals in the brackets may then be written as

$$\lim_{Z\to\infty} \frac{1}{2} \frac{\{\ln\left[(k_t+p_t)^2+M^2\right]-\ln Z\}}{k^{+2}}, \tag{58}$$

where Z is a constant corresponding to the upper limit of the divergent integrals.

Substituting the above result in Eq. (57), we obtain

$$\frac{1}{\pi R^2}\frac{dN^{\text{fermion}}}{d^3k} = 4\frac{N_c(2k^+)}{(2\pi)^4}\int \frac{d^2p_t}{(2\pi)^2}[\gamma(p_t)-(2\pi)^2\delta^{(2)}(p_t)]\frac{1}{2}\lim_{Z\to\infty}\frac{\{\ln\left[(k_t+p_t)^2+M^2\right]-\ln Z\}}{k^{+2}}. \tag{59}$$

However, the Z-dependent piece of the above integral vanishes due to the sum rule in Eq. (45):

$$\int \frac{d^2p_t}{(2\pi)^2}[\gamma(p_t)-(2\pi)^2\delta^{(2)}(p_t)] = 0. \tag{60}$$

Our result for the sea quark distribution can therefore be written as

$$\frac{1}{\pi R^2}\frac{dN^{\text{fermion}}}{d^3k} = \frac{4N_c}{(2\pi)^4k^+} \times \int \frac{d^2p_t}{(2\pi)^2}\gamma(p_t)\ln\left[\frac{(k_t+p_t)^2+M^2}{k_t^2+M^2}\right]. \tag{61}$$

A key feature of the above result is that the sea quarks demonstrate the same $1/x$ scaling behavior as that obeyed by the gluons.

The sea quark distribution in rapidity per unit rapidity per unit transverse area can be represented as

$$\frac{1}{\pi R^2}\frac{dN}{dy} = \frac{4N_c}{(2\pi)^4}\int\frac{d^2p_t}{(2\pi)^2}\gamma(p_t) \times \int d^2k_t \ln\left[\frac{(k_t+p_t)^2+M^2}{k_t^2+M^2}\right]. \tag{62}$$

It turns out that the integration over k_t can be performed analytically and one obtains the simple result πp_t^2. This result implicitly assumes that the range of k_t and quark mass is $<< \mu$. If otherwise, for reasons stated in the introduction, we expect large contributions due to radiative corrections. The stated range of transverse momentum is the dominant one if we require that the mass be $m_{\text{quark}} << \mu$. This is because this range of the sea quark transverse momentum corresponds to the range $p_t \le \mu$ of the gluon distribution—the contribution from the large $k_t \ge m_{\text{quark}}$ is suppressed in this range of the gluon transverse momentum.

Now, in Ref. [2], we argued that the dominant contribution from the averaging over the color charge distribution [see Eq. (11)] came from the range $\alpha_s^2\mu^2 << p_t^2$. In this range, the leading order contribution to $\gamma(p_t)$ from the color averaging is simply

$$\gamma(p_t) = \frac{2(4\pi)^2(N_c^2-1)}{2N_c}\frac{\alpha_s^2\mu^2}{p_t^4}. \tag{63}$$

Therefore, in the dominant range of integration, $\alpha_s\mu << p_t << \mu$, we obtain the astonishingly simple result for the sea quark rapidity distribution:

$$\frac{1}{\pi R^2}\frac{dN_{\text{sea}}}{dy} = \frac{2(N_c^2-1)}{\pi^2}\mu^2\alpha_s^2\ln\left(\frac{1}{\alpha_s}\right). \tag{64}$$

This result is the main result of our paper. Recall that the lowest order result for the gluon distribution arising from the same range of momenta was just the Wiezsäcker–Williams result scaled by μ^2:

$$\frac{1}{\pi R^2}\frac{dN_{\text{gluon}}}{dy} = \frac{2\alpha_s\ln(\frac{1}{\alpha_s})\mu^2(N_c^2-1)}{\pi}. \tag{65}$$

The ratio of intrinsic quarks to glue in the nuclear wave function is therefore suppressed by a factor α_s/π. Note also that the sea quark distribution saturates; it is not dependent on the sea quark mass. This result is true for all quark masses which are $m_{\text{quark}} << \mu$. The dominant contribution for heavier quarks comes from large transverse momentum where our results cannot yet be extended.

VI. SUMMARY

We have given in this paper explicit expressions for the quark and gluon propagators in the background field generated by a heavy nucleus in the infinite momentum frame. These expressions will allow us to compute the first radiative corrections to the distribution functions. If there is a Lipatov enhancement, it will appear in the first-order radiative corrections. If such an enhancement does occur, once understood, a technique must be developed for including its effects to all orders. This issue will be the subject of later analyses.

We have estimated the contributions of light to intermediate mass sea quarks to the distribution functions in the region of momentum where our results are reliable. To get a reliable estimate for larger masses, one must understand the kinematic range when $k_t \ge \mu$. This region is basically a weakly coupled region, but is not yet understood within our framework. The region where the distribution function is cutoff, $k_t \sim \alpha_s\mu$, must also be properly understood if one wants to go beyond an approximation which is accurate to the leading logarithm of α_s. Finally, the modifications due to a possible Lipatov enhancement must be understood.

There is also some hope that the result for heavy nuclei might be extended to hadrons. This would occur if there is a Lipatov enhancement, since in this case the typical parton separation might become smaller than the hadronic size. We have made the first step toward computing this possible enhancement in the context of this theory.

There is finally the issue of how to relate these computations to experimental measurements of structure functions. This can only be done by studying the dependence on the Q^2 of the probe by using an Altarelli-Parisi analysis. There are two important weak coupling regions. In the first, $Q^2 >> \mu^2$, presumably, the ordinary Altarelli–Parisi analysis goes through. The other region is $\mu^2 >> Q^2 >> \Lambda^2_{\rm QCD}$. In this region the coupling is weak, but the intrinsic transverse momentum of the quarks and glue is important. It is this region which must be studied carefully since this region provides the greatest potential for measuring the intrinsic properties of the hadronic wave function.

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APPENDIX

We have derived in this paper expressions for the Green's functions for the scalar, vector, and fermion *independent* fields. One can also compute Green's functions for the *dependent* fields by making use of the constraint conditions on the light cone. These constraints are obtained from the equations of motion on the light cone (see discussion in Ref. [1]).

In this brief appendix, we obtain an expression for the dependent vector field A^- in terms of the transverse vector field A_t. The Green's function for the constrained fields may then be obtained following a procedure identical to that followed in Sec. IV.

The general constraint condition for the A^- vector field is obtained from the equations of motion to be

$$-\partial_-^2 A^- = J^+ + D_t E^t \,. \qquad \text{(A1)}$$

In the above equation, J^+ is the component of the light cone current defined by Eq. (3), D_t are the transverse components of the covariant derivative and $E^t = \partial_- A^t$ are the transverse electric fields.

If we define $A^\mu = A^\mu_{\rm cl} + A^\mu_{qu}$, then keeping only terms up to order $O(A^\mu_{qu})$,

$$-\partial_-^2 A^-_{qu} = D_t^{(0)}(\partial_- A^t_{qu}) - ig[A^t_{qu}, (\partial_- A^t_{\rm cl})] \,. \qquad \text{(A2)}$$

Here $D_t^{(0)} = (\partial_t - igA_{{\rm cl},t})$.

In Eq. (50), we had obtained an expression for $A^{\alpha\beta}_{qu,t}$. This expression can be rewritten in the compact form

$$A^{\alpha\beta}_{qu,t} = e^{-ip^-x^+}\left\{\theta(-x^-)e^{-ip^+x^-}f^{\alpha\beta}_{1t} + \theta(x^-)e^{-ip^+x^-}\int\frac{d^2q_t}{(2\pi)^2}e^{i(q_t+p_t)x_t}\exp\left(-i\left[\frac{2p_tq_t+q_t^2}{2p^-}x^-\right]\right)f^{\alpha\beta}_{2t}(q_t)\right\}, \qquad \text{(A3)}$$

where

$$f^{\alpha\beta}_{1t} = \eta_t e^{ip_tx_t}\tau^{\alpha\beta}\,,$$

$$f^{\alpha\beta}_{2t} = \eta_t\int d^2\bar{x}_t e^{-q_t\bar{x}_t}\left(U(x_t)U^\dagger(\bar{x}_t)\tau U(\bar{x}_t)U^\dagger(x_t)\right)^{\alpha\beta}\,, \qquad \text{(A4)}$$

where η_t is a unit vector. Further, $A^t_{\rm cl} = \theta(x^-)\alpha^t_{\rm cl}$.

To obtain an expression for A_+, we integrate both sides of Eq. (A2). We choose the boundary condition $A^-(x^- = 0) = 0$. This particular boundary condition is convenient because it implies that the source current J^+ has a constant value for all light cone times x^+. Though convenient, this boundary condition is not sufficient to ensure that the fields vanish at infinity as implied by the original constraint equation. We shall return to this tricky issue in a later work.

Our final result for the quantum correction to the constrained field A_+ is

$$(A^{qu}_+)^{\alpha\beta} = e^{-ip^+x^- - ip^-x^+ + ip_tx_t}\Bigg\{-\theta(-x^-)(1-e^{ip^+x^-})\frac{p_t}{p^+}\eta_t\tau^{\alpha\beta} + \theta(x^-)\frac{i}{p^+}\int\frac{d^2q_t}{(2\pi)^2}\frac{e^{iq_tx_t}}{\left[1+\frac{(2p_tq_t+q_t^2)}{2p^+p^-}\right]}$$
$$\times\left[\exp\left(-ix^-\frac{(2p_tq_t+q_t^2)}{2p^-}\right) - e^{ip^+x^-}\right][i(q_t+p_t)f^{\alpha\beta}_{2t}]$$
$$+igx^-e^{ip^+x^-}\theta(x^-)[\eta_t\tau,\alpha_t]^{\alpha\beta}\Bigg\}\,. \qquad \text{(A5)}$$

Note that this solution for A_+^{qu} is defined only up to a term Bx^-, where B is an arbitrary function which does not depend on x^-. The Green's functions for the constrained fields can now be computed in the manner discussed in previous sections.

An essential feature of the above result is that it has terms infrared singular in p^+. There is an extensive literature on how to regularize such singularities which are endemic in light cone quantization [4–7]. A discussion of light cone regularization schemes is outside the scope of this paper. However, this issue will be important when one considers one-loop corrections to our results and the problem of regularization will be addressed more fully at that time.

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