Corrections to mass scale predictions in SO(10) grand unified theory with higher dimensional operators

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We calculate the two loop contributions to the predictions of the mass scales in an SO(10) grand unified theory. We consider the modified unification scale boundary conditions due to the nonrenormalizable higher dimensional terms arising from quantum gravity or spontaneous compactification of extra dimensions in a Kaluza-Klein-type theory. We find the range of these couplings which allows left-right symmetry to survive till very low energy (as low as $\sim$ TeV) and still be compatible with the latest values of $\sin^2\theta_W$ and α_s derived from CERN LEP. We consider both the situations when the left-right parity is broken and conserved. We consider both supersymmetric and nonsupersymmetric versions of the SO(10) theory. Taking the D-conserved non-SUSY case as an example, we calculate the effects of moderate threshold uncertainties at the heavy scale, due to the unknown Higgs boson masses, on the gravity induced couplings.

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There are many extensions of the standard model, which are suggested on various aesthetical grounds. But so far experiments could not find anything which is not predicted by the standard model. In other words, the standard model is consistent with all the experiments carried out so far, although there are appealing reasons to believe that there is physics beyond the standard model. In the standard model the $(V-A)$ nature of the theory is put in by hand, whereas in a left-right symmetric extension [1] of the standard model this comes about through spontaneous symmetry breaking.

In the left-right symmetric extension of the standard model, at higher energies the gauge group is extended to a left-right symmetric group $G_{LR} \equiv SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. When appropriate Higgs fields acquire vacuum expectation values (VEV's), this group breaks down to one of its subgroups $G_{\rm std} \equiv SU(2)_L \otimes U(1)_Y$. There will then be new scalar and gauge particles of mass of the order of this symmetry-breaking scale M_R. The mixing of these gauge bosons with the standard model gauge bosons puts a lower bound on this scale.

The K_L-K_S mass difference gives a lower bound [2] of about 1.6 TeV on M_R from the box diagram with both W_L and W_R exchanges. However, this constraint is subject to the assumption of manifest left-right symmetry, which is to assume that the Kobayashi-Maskawa matrices of the left- and the right-handed sectors are the same. In the absence of this artificial symmetry (which does not have any natural explanation) the bound [3] on M_R is relaxed to 300 GeV. From the direct search [4] at the Collider Detector at Fermilab (CDF) the lower bound on M_{W_R} is 520 GeV. This bound is not applicable to left-right symmetric models where the W_R couples only to the heavy neutrinos, which again decay very fast. The strongest bound on M_R comes from an analysis [5,6] of the precision measurement of the Z pole from the CERN e^+e^- collider LEP [7]. From a fit of the 1992 data and for the commonly chosen Higgs-triplet fields for the left-right symmetry breaking, the lower bound on M_R is of the order of TeV.

In the standard model the three gauge coupling constants are free parameters and are all different. This has a natural explanation in grand unified theories [8] in which the strong and the electroweak interaction are only low-energy manifestations of a single interaction. The GUT interaction is a gauge interaction based on a simple gauge group with only one gauge coupling constant. Through spontaneous symmetry breaking this breaks down to a low-energy symmetry group. Then the different coupling constants evolve in different ways to give the present day low-energy coupling constants. Some of the attractive features of GUT's were their natural explanation of the problem of baryogenesis and their unique prediction of proton decay. However, proton decay has not yet been observed and the question of baryogenesis took a completely different shape following the observation of large anomalous baryon number nonconservation at high temperatures in the presence of sphaleron fields. The main interest in GUT's remains its unification of coupling constants and charge quantization.

Recently, there has again been an upsurge of interest [9–13] in GUT's following the precision measurement of the three gauge coupling constants at LEP. The normalized gauge coupling constants for the groups $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$, as obtained [7] from analyzing the LEP data, are given by

$$\alpha_1(M_z) = 0.16887 \pm 0.000040,$$
$$\alpha_2(M_z) = 0.03322 \pm 0.00025,$$
$$\alpha_3(M_z) = 0.120 \pm 0.007, \qquad (1)$$

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respectively. With the minimal particle content, it is not possible to unify all three coupling constants at any energy. This apparently rules out [9] minimal SU(5) GUT and any GUT's without any intermediate scales and new particles unless the effect of gravity modifies the situation.

It was further pointed out that the scale of the intermediate symmetry breaking can be severely constrained by the present values of the gauge coupling constants. For the minimal supersymmetric GUT's, the supersymmetry-breaking scale $M_R \sim 1$ TeV gives a good fit [9] to evolve all the gauge couplings to a unification point. However, threshold effects and higher order corrections make this scale uncertain by orders of magnitude. This makes the threshold effects and higher order corrections very important in studying the evolution of the gauge coupling constants in light of the LEP data.

It was pointed out that if one studies any GUT's with left-right symmetric group G_{LR} as one of its intermediate symmetry group, then the present LEP data severely constrain [10] this symmetry-breaking scale M_R. For any GUT's and any number of new symmetries above M_R, one obtains a lower bound

$$M_R > 10^9 \;\; \text{GeV}.$$

This bound can be relaxed [14] if one breaks the left-right parity and the left-right symmetric group G_{LR} at different scales.

If new signatures of the right-handed gauge bosons are found in the next generation accelerators (since the experimental lower bound is only around a TeV), that will not, however, mean that there is inconsistency in GUT's. It was shown that in a very specific supersymmetric SO(10) GUT one can satisfy [13] the unification constraint with low M_R. The details of this deserve further study.

Since the GUT scale is very close to the Planck scale, the effects of gravity may not be negligible. It was shown that if effects of gravity are considered through higher dimensional operators, then even the minimal SU(5) GUT with no new particle content may be consistent [15] with the LEP data and proton decay.

We have studied [12] the effect of gravity to see if the constraints on M_R can be relaxed. We considered higher dimensional nonrenormalizable operators which may arise due to quantum gravity or spontaneous compactification of extra dimensions in the Kaluza-Klein-type theory and their effect in the SO(10) Lagrangian. The GUT scale boundary condition was found to be modified, and for certain choice of parameters low M_R could be made consistent with SO(10) GUT. In this paper we present details of our analysis. Here we include the threshold effects and study the two-loop evolution of the coupling constants, which are also very significant in these analyses. First we present the formalism and then present our analysis. At the end we summarize our results.

Higher dimensional operators were considered originally [16] to help solve some problems in fermion masses. The idea is to find out if the low-energy physics contains some signatures of gravity effects. In all these analyses the coupling constants in these nonrenormalizable terms are free parameters. Someday we may learn if such coupling constants may arise from gravity naturally.

In our analysis we consider dimension-five and dimension-six operators when the contribution from dimension-five operators vanish. We note that the effect of all operators higher than dimension six can be absorbed in the couplings of the dimension-six operators, and hence their inclusion does not increase the number of parameters. We therefore consider only dimension-five and dimension-six operators in our analysis.

The main objective of our study is to look for consistency of low M_R. For this purpose we consider the symmetry-breaking chain

$$\begin{aligned} \mathrm{SO}(10) &\xrightarrow{M_U} \mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \\ &\quad [\equiv G_{PS}]\,, \\ &\xrightarrow{M_I} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \mathrm{U}(1)_{(B-L)} \\ &\quad [\equiv G_{LR}]\,, \\ &\xrightarrow{M_R} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \\ &\quad [\equiv G_{\mathrm{std}}]\,, \\ &\xrightarrow{M_W} \mathrm{SU}(3)_c \times \mathrm{U}(1)_{\mathrm{em}}. \end{aligned} \tag{2}$$

Near the scale $M_U \sim 10^{16}$ GeV or higher the gravity effects are not negligible. But we assume that any theory beyond this scale respects the SO(10) symmetry. Then the Lagrangian will contain all the usual SO(10) invariant dimension-four interaction terms and, in addition, will contain SO(10) invariant higher-dimensional nonrenormalizable terms. These higher dimensional terms will be suppressed by the Planck scale (in theories [16] where these terms are induced by quantum gravity) or by the Kaluza-Klein compactification scale (in theories [17] where these terms are induced by spontaneous compactification of the extra dimensions in the Kaluza-Klein-type theories), which can even be two orders of magnitude below the Planck scale.

The Lagrangian can be written as,

$$L = L_R + L_{\mathrm{NR}} \tag{3}$$

where the first part of the Lagrangian contains all the renormalizable dimension-four terms including the SO(10) gauge-invariant term

$$L = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}), \tag{4}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu],$$

$$A_\mu = A_\mu^i \frac{\lambda_i}{2},$$

with

$$\mathrm{Tr}(\lambda_i \lambda_j) = \frac{1}{2}\delta_{ij},$$

where the λ's are the SO(10) generators. The nonrenormalizable part of the Lagrangian contains all the higher-dimensional SO(10) invariant terms. We are presently interested in only dimension-five and -six terms, which are given as

$$L = \sum_{n=5} L^{(n)},$$

$$L^{(5)} = -\frac{1}{2}\frac{\eta^{(1)}}{M_{\rm Pl}}\mathrm{Tr}(F_{\mu\nu}\phi F^{\mu\nu}), \tag{5}$$

$$\begin{aligned} L^{(6)} = -\frac{1}{2}\frac{1}{M_{\rm Pl}^2}\{&\eta_a^{(2)}[\mathrm{Tr}(F_{\mu\nu}\phi^2F^{\mu\nu}) + \mathrm{Tr}(F_{\mu\nu}\phi F^{\mu\nu}\phi)] \\ &+\eta_b^{(2)}\mathrm{Tr}(\phi^2)\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) \\ &+\eta_c^{(2)}\mathrm{Tr}(F^{\mu\nu}\phi)\mathrm{Tr}(F_{\mu\nu}\phi)\}, \end{aligned} \tag{6}$$

where $\eta^{(n)}$ are dimensional couplings of the higher dimensional operators. When any Higgs scalar ϕ acquires a VEV ϕ_0, these operators induce effective dimension-four terms, which modify the boundary conditions at the scale ϕ_0.

Let us consider the symmetry-breaking chain [2] at the scale M_U. We shall first consider the case when this symmetry breaking is mediated by the VEV of a 54-plet of a Higgs field. In this case the left-right parity is broken at M_R only when $SU(2)_R$ is broken, and the gauge coupling constants g_L and g_R corresponding to the groups $SU(2)_L$ and $SU(2)_R$, respectively, evolve similarly between M_U and M_R so that $g_L(M_R) = g_R(M_R)$. In the second case, we shall consider the symmetry breaking at the scale M_U by a 210-plet of a Higgs field. This breaks the discrete left-right parity symmetry [14] D, so that g_L and g_R evolve in a different way below M_U and, as a result, one obtains $g_L(M_R) \neq g_R(M_R)$.

In the D-conserving case, the symmetry breaking at M_U takes place when the 54-plet Higgs field Σ of SO(10) acquires a VEV:

$$\langle\Sigma\rangle = \frac{1}{\sqrt{30}}\ \Sigma_0\ \mathrm{diag}(1,1,1,1,1,1,-\tfrac{3}{2},-\tfrac{3}{2},-\tfrac{3}{2},-\tfrac{3}{2}), \tag{7}$$

where $\Sigma_0 = \sqrt{\frac{6}{5\pi\alpha_G}}M_U$ and $\alpha_G = g_0^2/4\pi$ is the GUT coupling constant. We now introduce the parameters

$$\epsilon^{(1)} = \left[\left(\frac{1}{25\pi\alpha_G}\right)^{\frac{1}{2}}\frac{M_U}{M_{\rm Pl}}\right]\eta^{(1)}, \tag{8}$$

and

$$\epsilon_i^{(2)} = \left[\left(\frac{1}{25\pi\alpha_G}\right)^{\frac{1}{2}}\frac{M_U}{M_{\rm Pl}}\right]^2\eta_i^{(2)}. \tag{9}$$

Then the G_{PS} invariant effective Lagrangian will be modified by these higher dimensional operators as follows:

$$\begin{aligned} &-\frac{1}{2}(1+\epsilon_4)\ \mathrm{Tr}[F_{\mu\nu}^{(4)}\ F^{(4)\mu\nu}] \\ &-\frac{1}{2}(1+\epsilon_2)\ \mathrm{Tr}[F_{\mu\nu}^{(2L)}\ F^{(2L)\mu\nu}] \\ &-\frac{1}{2}(1+\epsilon_2)\ \mathrm{Tr}[F_{\mu\nu}^{(2R)}\ F^{(2R)\mu\nu}], \end{aligned} \tag{10}$$

where

$$\epsilon_4 = \epsilon^{(1)} + \epsilon_a^{(2)} + \frac{1}{2}\epsilon_b^{(2)}$$

and

$$\epsilon_2 = -\frac{3}{2}\epsilon^{(1)} + \frac{9}{4}\epsilon_a^{(2)} + \frac{1}{2}\epsilon_b^{(2)}.$$

Then the usual G_{PS} Lagrangian can be recovered with the modified coupling constants

$$\begin{aligned} g_4^2(M_U) &= \bar{g}_4^2(M_U)(1+\epsilon_4)^{-1}, \\ g_{2L}^2(M_U) &= \bar{g}_{2L}^2(M_U)(1+\epsilon_2)^{-1}, \\ g_{2R}^2(M_U) &= \bar{g}_{2R}^2(M_U)(1+\epsilon_2)^{-1}, \end{aligned} \tag{11}$$

where $\bar{g}_i$ are the coupling constants in the absence of the nonrenormalizable terms and g_i are the physical coupling constants that evolve below M_U. Similarly, the physical gauge fields are defined as $A_i' = A_i\sqrt{1+\epsilon_i}$.

The VEV of Σ leaves unbroken a larger symmetry group than G_{PS}, which is O(6)⊗O(4). The D parity is thus unbroken and hence $SU(2)_L$ and $SU(2)_R$ always receive equal contributions. Furthermore, since overall contributions to all the gauge groups cannot change the predictions of $\sin^2\theta_W$ and α_s, the VEV of Σ can only contribute to one combination of the couplings, i.e., the relative couplings of SU(4) and SU(2). For this reason, no matter how many higher dimensional terms we consider, what contributes to the low-energy predictions of $\sin^2\theta_W$ and α_s is only the combination

$$\epsilon = \epsilon_4 - \epsilon_2. \tag{12}$$

If we now assume that the dimension-six terms $\epsilon_i^{(2)}$ are negligible compared to the dimension-five terms $\epsilon^{(1)}$, then we further get

$$\epsilon_4 = \frac{2}{5}\epsilon\ , \quad \text{and} \quad \epsilon_2 = -\frac{3}{5}\epsilon.$$

As we argued earlier, this does not reduce the number of parameters in the theory. If we include the higher

TABLE I. Higgs spectrum at various mass scales for the D-conserved and the D-nonconserved chain.

Group G_i	Higgs content
$(2_L2_R4_CP)$	$(2,2,1)_{10}$ $(1,3,\overline{10})_{126}$ $(3,1,10)_{126}$ $(1,1,15)_{45}$
$(2_L2_R4_C)$	$(2,2,1)_{10}$ $(1,3,\overline{10})_{126}$ $(1,1,15)_{45}$
$(2_L2_R1_{B-L}3_CP)$	$(2,2,0,1)_{10}$ $(1,3,2,1)_{126}$ $(3,1,2,1)_{126}$
$(2_L2_R1_{B-L}3_C)$	$(2,2,0,1)_{10}$ $(1,3,2,1)_{126}$

dimensional terms, then the allowed region in ϵ will be shared by the other $\epsilon^{(n)}$'s.

It was pointed out in Ref. [12] that for any choice of the parameter ϵ it was not possible to have a consistent theory with low M_R. It was necessary to make the symmetry-breaking scale M_I very close to M_U, so that higher dimensional operators can introduce another parameter, which can then allow low M_R.

The VEV of a 45-plet field H can break the symmetry group G_{PS} to G_{LR}:

$$\langle H\rangle = \frac{1}{\sqrt{12}\,i}\; H_0 \begin{pmatrix} 0_{33} & 1_{33} & 0_{34} \\ -1_{33} & 0_{33} & 0_{34} \\ 0_{43} & 0_{43} & 0_{44} \end{pmatrix}, \qquad (13)$$

where 0_{mn} is an $m \times n$ null matrix and 1_{mm} is an $m \times m$ unit matrix. The antisymmetry of the matrix H will imply that to dimension-five operators there is no contribution from this Higgs field. The lowest-order contribution comes from the dimension-six operators (in Ref. 12 the dimension-five operator was taken to give the lowest-order contribution; this is incorrect, since, due to the antisymmetry of the **45** representation, the dimension-five operator is zero):

$$L'^{(2)} = -\frac{1}{2}\frac{1}{M_{\rm Pl}^2}\Big[\eta_a'^{(2)}{\rm Tr}(F_{\mu\nu}H^2F^{\mu\nu}) + \eta_b'^{(2)}{\rm Tr}(H^2){\rm Tr}(F_{\mu\nu}F^{\mu\nu}) + \eta_c'^{(2)}{\rm Tr}(F^{\mu\nu}H){\rm Tr}(F_{\mu\nu}H)\Big]. \qquad (14)$$

The VEV of H does not modify the SU(2) couplings. The SU(4) invariant effective Lagrangian will only contain a new contribution:

$$L''^{(2)} = -\frac{1}{2}\frac{1}{M_{\rm Pl}^2}\Big[\eta_a'^{(2)}{\rm Tr}(F_{\mu\nu}\phi_{15}^2F^{\mu\nu}) + \eta_b'^{(2)}{\rm Tr}(\phi_{15}^2){\rm Tr}(F_{\mu\nu}F^{\mu\nu}) + \eta_c'^{(2)}{\rm Tr}(F^{\mu\nu}\phi_{15}){\rm Tr}(F_{\mu\nu}\phi_{15})\Big], \qquad (15)$$

where ϕ_{15} transforms as $(15,1,1)$ of G_{PS}. At M_I the symmetry group $SU(4)_c$ breaks down to $SU(3)_c \otimes U(1)_{B-L}$ when the field ϕ_{15} acquires a VEV:

$$\phi_{15} = \frac{1}{\sqrt{24}}\phi_0{\rm diag}[1,1,1,-3], \qquad (16)$$

with $\phi_0 = \sqrt{6/5\pi\alpha_4}M_I$. We now define

$$\epsilon_i'^{(2)} = \frac{\eta_i'^{(2)}\phi_0^2}{24M_{\rm Pl}^2} = \left[\frac{1}{20\pi\alpha_4}\left(\frac{M_I}{M_{\rm Pl}}\right)^2\right]\eta_i'^{(2)}, \qquad (17)$$

TABLE II. The Higgs bosons at M_U.

SO(10) multiplet	$G_{2,2,4}$ multiplet
54	$S_1(1,1,1)$,$S_2(1,1,20')$,$S_3(3,3,1)$
45	$\phi_1(3,11)$,$\phi_2(2,2,6)$,$\phi_3(1,3,1)$
126	$\Sigma_1(2,2,15)$,$\Sigma_2(1,1,6)$

where $i = a,b,c$. The $SU(3)_c \otimes U(1)_{B-L}$ invariant kinetic energy term for the gauge bosons will then be given by

$$-\frac{1}{2}(1+\epsilon_3')\,{\rm Tr}[F_{\mu\nu}^{(3)}\,F^{(3)\mu\nu}] - \frac{1}{2}(1+\epsilon_1')\,{\rm Tr}[F_{\mu\nu}^{(1)}\,F^{(1)\mu\nu}], \qquad (18)$$

where

$$\epsilon_3' = \epsilon_a'^{(2)} + 12\epsilon_b'^{(2)}$$

and

$$\epsilon_1' = 7\epsilon_a'^{(2)} + 12\epsilon_b'^{(2)} + 12\epsilon_c'^{(2)}.$$

In general, ϵ_1' and ϵ_3' may be treated as two free parameters. But we shall assume $\epsilon_b'^{(2)} = \epsilon_a'^{(2)} = \epsilon_c'^{(2)}$ and, hence,

$$\epsilon_3' = 0.42, \quad \epsilon_1' = \epsilon'(\text{say}). \qquad (19)$$

Thus the parameter space in ϵ' and ϵ we present here may be further relaxed to some extent. However, the number of parameters in the $\sin^2\theta_W$ and α_s is not changed and we cannot expect any change in low-energy predictions. In our analysis we shall present the parameter space of ϵ' and ϵ, which allows low M_R. For the D-nonconserved case, a 210-plet Higgs field is used to break SO(10) to the group G_{PS} without D-parity conservation. The VEV of the Higgs field is given by

$$\langle H_{210}\rangle = \frac{1}{\sqrt{32}}\; H_0\;{\rm diag}(1_{44},1_{44},-1_{44},-1_{44}), \qquad (20)$$

where H_0 is related to the vector boson mass M_X by $\sqrt{\frac{2}{\pi\alpha_G}}M_X = H_0$. Keeping only the dimension-five operator we get

$$\epsilon_4 = 0,$$

$$\epsilon_{2L} = -\epsilon_{2R} = 8\epsilon^{(1)} = \epsilon$$

where

$$\epsilon^{(1)} = \sqrt{\frac{2}{32\pi\alpha_G}}\left(\frac{M_X}{M_{\rm Pl}}\right).$$

TABLE III. The Higgs bosons at M_I.

SO(10) multiplet	$G_{2_L,2_R,1_{B-L},3_C}$ multiplet
45	$Y_1(1,1,1,0)$,$Y_2(1,1,0,8)$,$\bar{X}(1,1,\frac{4}{3},\bar{3})$,$X(1,1,-\frac{4}{3},3)$
126	$\xi_1(3,1,-\frac{2}{3},6)$,$\xi_2(1,3,\frac{2}{3},\bar{6})$, $\xi_3(3,1,\frac{2}{3},3)$,$\xi_4(1,3,-\frac{2}{3},\bar{3})$

TABLE IV. Allowed ranges for ϵ and ϵ' for the D-broken non-SUSY case.

ϵ' (10^{-3})	ϵ (10^{-3})	M_I	M_U	$\frac{\alpha_{2L}(M_R)}{\alpha_{2R}(M_R)}$
4.92–10.38	−10–−5.43	10^{16}	10^{17}	1.4
11.69–12.46	−9.72–−1.032	10^{16}	10^{18}	1.3
6.46–8.46	−7.09–−5.32	10^{17}	10^{17}	1.3
11.08–13.23	−9.59–−7.91	10^{17}	10^{18}	1.2
9.23–10.92	−9.18–−7.75	10^{17}	10^{18}	1.3
8.23–10.4	−7.33–−5.50	10^{18}	10^{18}	1.2

For the evolution of the coupling constants we use the two-loop renormalization group equations [9,11,18]

$$\mu\frac{\partial}{\partial\mu}\alpha_i(\mu) = \frac{2}{4\pi}\left[b_i + \sum_j \frac{b_{ij}}{4\pi}\alpha_j(\mu)\right]\alpha_i^2(\mu) + \frac{2b_{ij}}{(4\pi)^2}\alpha_i^3, \tag{21}$$

where the i,j index represents the different subgroups at the energy scale μ and $\alpha_i = \frac{1}{4\pi}g_i^2$. The various β functions with supersymmetry (SUSY) and without SUSY are given in Ref. [19]. We use the survival hypothesis [20] to find the Higgs content at the various mass scales for any given chain. In Table I we show the Higgs bosons that live at different mass scales.

An approximate solution of the evolution equation can be written as

$$\begin{aligned}\alpha_i^{-1}(\mu') &= \beta_0 \ln\frac{\mu'}{\mu} + \frac{1}{\alpha_i(\mu)} + \frac{\beta_1}{\beta_0}\ln\left[\frac{\alpha^{-1}(\mu') + \frac{\beta_1}{\beta_0}}{\alpha^{-1}(\mu) + \frac{\beta_1}{\beta_0}}\right],\\ \beta_0 &= \frac{1}{2\pi}\left[b_i + \sum b_{ij}\alpha_j(\mu)\right],\\ \beta_1 &= -\frac{2b_{ii}}{(4\pi)^2}.\end{aligned} \tag{22}$$

At each symmetry-breaking threshold we use the following matching conditions for the couplings when the group G breaks to the group G_i [21]:

$$\alpha_i^{-1}(\mu) = \alpha_G^{-1} - \frac{\lambda_i}{12\pi}, \tag{23}$$

where

$$\lambda_i = C_G - C_{Gi} + \mathrm{Tr}(\theta_i^H)^2 \ln\frac{M_H}{\mu}.$$

θ_i^H are the generators of G_i for the representation in which the Higgs bosons M_H appear. C_G and C_{Gi} are the quadratic Casimir invariants for the group G and the group G_i, while μ is the symmetry-breaking scale. Gravity-induced corrections change $\alpha_i^{-1}(\mu)$ to $\alpha_i^{-1}(\mu)(1+\epsilon_i)^{-1}$ as in Eq. (11). In our analysis we identify μ, the unification scale, with the vector boson mass. Threshold corrections will occur due to the nondegeneracy of the Higgs boson masses with the vector boson mass. Using the D-conserved non-SUSY case as an example, we have calculated the effect of threshold corrections at the heavy scales M_I and M_U. The Higgs boson masses are assumed to vary between $\frac{1}{5}$ and 5 times the vector boson mass. The threshold corrections enter through the factors λ_i appearing in Eq. 24. The Higgs bosons that live around the mass scale M_U and M_I are given in Table III and Table IV.

Defining $\eta_H = \ln\frac{M_H}{M_X}$, one can write, at M_U,

$$\begin{aligned}\lambda_4 &= (4 + 16\eta_{S_2} + 8\eta_{\phi_2} + 32\eta_{\Sigma_1} + 2\eta_{\Sigma_2} + 2\eta_{H'}),\\ \lambda_2 &= (6 + 12\eta_{S_3} + 12\eta_{\phi_2} + 30\eta_{\Sigma_1} + 4\eta_{\phi_1}).\end{aligned} \tag{24}$$

At the scale M_I one has

$$\begin{aligned}\lambda_3 &= (1 + 6\eta_{Y_2} + 15\eta_{\xi_1} + 15\eta_{\xi_2} + 3\eta_{\xi_3} + 3\eta_{\xi_4}),\\ \lambda_{B-L} &= (4 + 6\eta_{\xi_1} + 6\eta_{\xi_2} + 3\eta_{\xi_3} + 3\eta_{\xi_4}),\\ \lambda_{2L} &= (24\eta_{\xi_1} + 12\eta_{\xi_3}),\\ \lambda_{2R} &= (24\eta_{\xi_2} + 12\eta_{\xi_4}).\end{aligned} \tag{25}$$

The quantities $\lambda_4 - \lambda_2$ and $\lambda_3 - \lambda_{B-L}$ appear in the solution for M_U and M_I, respectively. We consider two cases where the Higgs boson masses are chosen such that the above quantities are at their extreme values. We further make the assumption that the Higgs bosons at a given scale coming from the same SO(10) multiplet have the same masses. In the first case, we choose M_Σ,M_S,$M_{H'}$ to be $(\frac{1}{5})M_U$, while M_ϕ to be $5M_U$. At M_I we choose M_Y and M_ξ to be $\frac{1}{5}M_I$. For the second case, we just flip

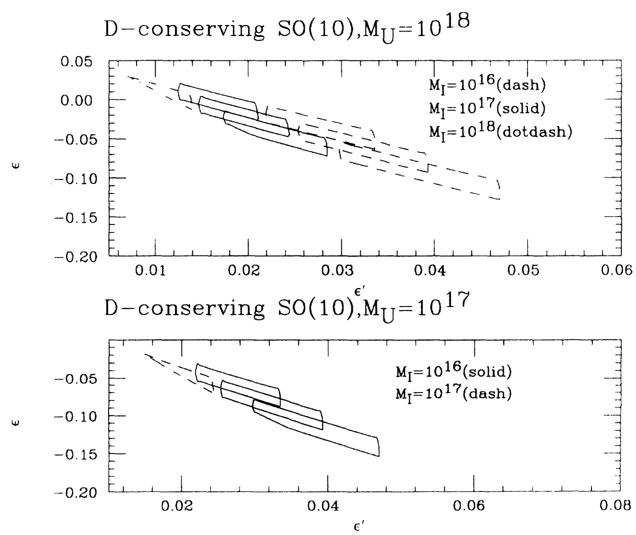

FIG. 1. The allowed regions in ϵ and ϵ' space for D-conserved non-SUSY SO(10) for pairs of M_I and M_U. For M_I not equal to M_U, the upper and the lower regions correspond to case (a) and case (b), the two cases considered for threshold corrections.

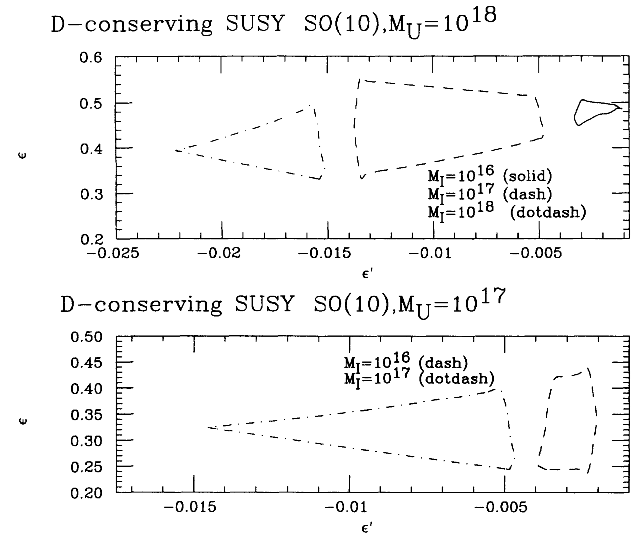

FIG. 2. The allowed regions in ϵ and ϵ' space for D-conserved SO(10).

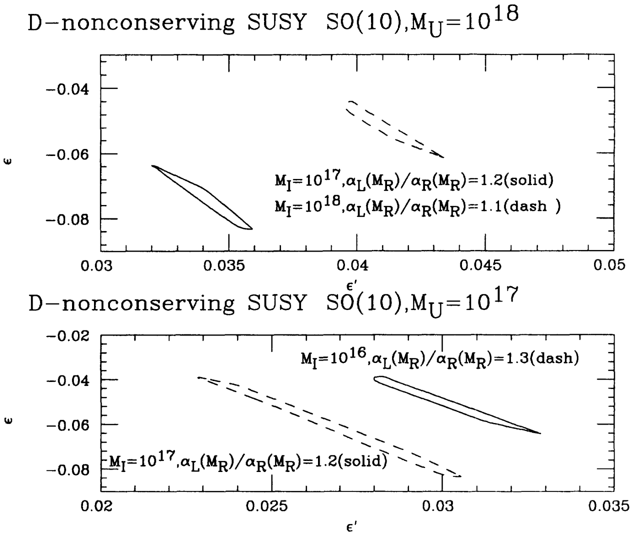

FIG. 3. The allowed regions in ϵ and ϵ' space for D-nonconserved SO(10).

the Higgs bosons around at the two scales. We refer to these two cases as case (a) and case (b). We have only considered the cases when M_I is not equal to M_U.

RESULTS

Using the values of the standard model couplings at M_Z [Eq. (1)], the evolution equations, and the matching conditions [Eqs. (24) and (25)] we find regions in the ϵ, ϵ' space which allow a low M_R for various values of the intermediate scale M_I and the unification scale M_U. In Fig. 1 the allowed regions for the D-conserved non-SUSY case are shown. For M_U not equal to M_I the effects of threshold corrections have been included. In Figs. 2 and 3 the allowed regions for the D-conserved and D-nonconserved SUSY case are shown. For the D-broken non-SUSY case the width of the allowed regions are too small to be shown graphically and therefore we present the results for this case in Table IV. For the supersymmetric version the allowed regions are larger, but no solution was found for the case $M_I = 10^{16}$, $M_U = 10^{18}$. Even though we have not carried out a full analysis of the threshold effects, from the examples considered, we do not expect moderate threshold effects to alter the regions in the parameter space drastically. In conclusion, we have shown that both for the D-conserved and D-nonconserved case we can find regions in the parameter space of gravity-induced couplings that allow M_R in the TeV range.

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