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Leptonic flavor violations in the presence of an extra Z

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Gauged extensions $[SU(2)_L\otimes U(1)_Y\otimes U(1)_X]$ of the standard $SU(2)_L\otimes U(1)_Y$ model obtained without extending the fermion content of the model are studied. Models that are possible when $U(1)_X$ is identified with some combination of the family lepton numbers are systematically classified. Most of these contain flavor violations in the leptonic sector. These flavor violations are correlated to the mixing of the $U(1)_X$ gauge boson Z' with the ordinary Z in the models considered here. Detailed phenomenological implications of a typical model are discussed. Constraints on the Z' mass and the Z-Z' mixing following from (i) the observations at CERN LEP, (ii) rare processes such as $Z\to e\tau$, and (iii) flavor-violating τ decays are presented. It is found that the constraints coming from the LEP allow rare processes such as $\tau\to eee$ at the level of the present limits on its branching ratio. Thus the future search could either improve on the existing LEP limits or would find such flavor-violating decays.

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I. INTRODUCTION

Lepton numbers L_i for each family $i=e,\mu,\tau$, are globally conserved in the standard $SU(2)_L\otimes U(1)_Y$ model. Within the minimal $SU(2)_L\otimes U(1)_Y$ theory, these symmetries have to be global since attempts to gauge them would introduce anomalies and would spoil the renormalizability of the theory. Nevertheless, it is possible to gauge some linear combinations of L_i. Specifically, it was shown in Ref. [1] that only three linear combinations of L_i, namely, $X=L_e-L_\mu$, L_e-L_τ, and $L_\mu-L_\tau$ are gaugeable and only one can be gauged along with $SU(2)_L\otimes U(1)_Y$ at a time. Thus the maximal permissible gauge group with the minimal content of fermions and Higgs field is $SU(2)_L\otimes U(1)_Y\otimes U(1)_X$. One could easily enlarge the gauge symmetry by adding more fermions as happens for example in the left-right symmetric or E_6 models [2]. But it is possible to enlarge the available choices of $U(1)_X$ by adding more Higgs doublets transforming nontrivially under it. The point is that the requirement of anomaly cancellations allow for more than the above-mentioned three possibilities for X. In the absence of Higgs doublets which are nontrivial under $U(1)_X$, these additional choices of $U(1)_X$ are physically indistinguishable from the above three choices. Otherwise, they are inequivalent and could lead to different predictions. $U(1)_X$ groups of this type fall in the category of horizontal symmetries characterized by enlargement of the gauge and Higgs sector. Many examples of such gauge symmetries have been proposed [3] and studied. Crucial tests of such symmetries are flavor violations associated with these symmetries. Most studies of these flavor violations were done before the results of the e^+e^- collider became available. These results can provide additional constraints on such theory. We wish to study here the simplest of such horizontal symmetries. We shall study various choices of $U(1)_X$ under the assumption that (a) the fermion sector of the standard model is not extended, (b) X is some linear combination of lepton family numbers, (c) the Higgs sector of the standard model (SM) is enlarged by adding one or more Higgs doublets transforming nontrivially under the gauge group.

It is possible to classify systematically all the allowed choices of $U(1)_X$ in the presence of an enlarged Higgs sector. We do such a classification. Different $U(1)_X$ groups studied here differ from groups of Ref. [1] in three important ways. The $U(1)_X$ current in the present case is nonvectorial. Second, the $U(1)_X$ current due to its horizontal nature leads to flavor violations in the leptonic sector. Third, the neutral gauge boson Z' associated with $U(1)_X$ necessarily mixes with Z in these models. The last two properties put significant constraints on parameters of the model. We systematically work them out. It follows from the analysis presented here that the observable flavor violations, e.g., in $\tau\to eee$ decay are possible within the models in spite of the severe constraints imposed by the observations at the CERN e^+e^- collider LEP.

We shall discuss in the next section all possible choices of $SU(2)_L\otimes U(1)_Y\otimes U(1)_X$ and general structure of the current coupled to Z and Z'. Then we discuss a specific model in Sec. III. Section IV contains a discussion on constraints on parameters taking the model of Sec. III as an illustration. A summary is contained in Sec. V.

II. POSSIBLE EXTENSIONS

We shall confine ourselves to the minimal fermionic content as in the standard model but would consider a general gauge group $SU(2)_L\otimes U(1)_Y\otimes U(1)_X$, where X is taken to be a linear combination of three lepton numbers. In general X need not act vectorially on the weak interaction basis $e'_{i_{L,R}}$ $(i=1,2,3)$ although the X assignments of members of a given $SU(2)_L$ doublet have to be identical. For notational convenience let us write X charges in terms of diagonal matrices in the generation space:

$$X_{L,R}=\mathrm{diag}(\alpha_1,\alpha_2,\alpha_3)_{L,R}\ .$$

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X_L determine the $U(1)_X$ assignment of the leptonic doublet, while X_R that of the charged right-handed leptons. The possible choices of α_{iL} and α_{iR} are restricted due to anomaly cancellation which require

$$\sum_i \alpha_{iL} = \sum_i \alpha_{iR} = 0 \; , \qquad \sum_i \alpha_{iL}^2 = \sum_i \alpha_{iR}^2 \; , \qquad 2\sum_i \alpha_{iL}^3 = \sum_i \alpha_{iR}^3 \; . \tag{1}$$

These constraints can be satisfied by taking any two of α_{iL} and α_{iR} to be ± 1 and the third to be zero. This can be done in a variety of ways, but a particularly simple choice results when one takes $X_L = X_R$. In this case the allowed X is restricted [1] either to $L_e - L_\mu$, $L_e - L_\tau$, or $L_\tau - L_\mu$. The current coupled to $U(1)_X$ boson Z' is vectorial when expressed in the weak basis in this case. Since the initial choice of basis is arbitrary one could always redefine the right-handed fields e'_{iR} to obtain the choice $X_L = X_R$. But the structure of physical current coupled to mass eigenstates of fermions depends upon the choice of Higgs fields. In the event of only one Higgs doublet neutral under $U(1)_X$, the charged leptonic mass matrix is diagonal and the physical current coupled to Z' is vectorial. When one introduces more Higgs fields transforming nontrivially under $U(1)_X$, the $U(1)_X$ no longer remains vectorial. The possible choices of $X \equiv X_L = X_R$ are severely limited due to the anomaly constraints, Eq. (1). In particular only three choices are possible which, respectively, correspond to

$$L_\mu - L_\tau, \quad X = \mathrm{diag}(0,1,-1) \; , \qquad L_e - L_\tau, \quad X = \mathrm{diag}(1,0-1) \; , \qquad L_e - L_\mu, \quad X = \mathrm{diag}(1,-1,0) \; . \tag{2}$$

The structure of the current associated with the new Z' can be written as

$$\mathcal{L}_{Z'} = \frac{g'}{\cos\theta}\{\bar{e}'_L X \gamma_\mu e'_L + \bar{e}'_R X \gamma_\mu e'_R\} Z'^\mu \; , \tag{3}$$

where e'_{LR} are column vectors in generation space and θ is the weak mixing angle introduced here purely for notational convenience. The coupling of the physical (i.e., mass eigenstate) fermions to Z' depends upon the structure of the mass matrix M_l for the charged leptons. This is dictated by the charge matrix Q whose (i,j)th element correspond to the X charge of bilinear $\bar{e}'_{iL} e'_{jR}$. For example, we have, in the case of $L_e - L_\tau$,

$$Q = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} . \tag{4}$$

The possible structures of mass matrices follow from that of Q. In particular, a Higgs field with charge $-Q_{ij}$ would contribute to the (i,j)th element of the mass matrix M_l. Note that the two different fields contribute to the $(M_l)_{ij}$ and $(M_l)_{ji}$. Hence M_l is necessarily non-Hermitian except when it is diagonal with only one Higgs doublet carrying zero charge under $U(1)_X$. In this case, the weak basis $e'_{L,R}$ coincides with the mass basis $e_{L,R}$, and Z' couples to a vector current corresponding to X. When one introduces one or more additional doublets transforming nontrivially under $U(1)_X$ then M_l is necessarily non-Hermitian and can be diagonalized by a biunitary transformation:

$$U_L M_l U_R^\dagger = \mathrm{diag}(m_e, m_\mu, m_\tau) \; , \tag{5}$$

$$e_{L,R} = U_{L,R}\, e'_{L,R} \; . \tag{6}$$

$\mathcal{L}'_Z$ then assumes the following form in terms of the mass eigenstates:

$$\mathcal{L}_{Z'} = \frac{g'}{\cos\theta}(\kappa_{Lij}\bar{e}_{iL}\gamma_\mu e_{jL} + \kappa_{Rij}\bar{e}_{iR}\gamma_\mu e_{jR}) Z'^\mu \tag{7}$$

where

$$\kappa_a \equiv U_a X U^\dagger{}_a, \quad a = L, R \; . \tag{8}$$

Equation (7) represents the general form of the Z' interactions in all the $SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ models under study. Different models are specified by the choice of X and the Higgs fields which determine M_l and hence $U_{L,R}$. The following two important properties are enforced by the structure of X.

(i) The current coupled to Z' is nonvectorial except in a specified case $U_L = U_R = I$. This follows since M_l is necessarily non-Hermitian when it is not diagonal as already discussed. Hence $U_L \neq U_R$. Moreover, for $U_L \neq U_R$, $U_L X U_L^\dagger$ and $U_R X U_R^\dagger$ cannot be identical[1] leading to a nonvector current.

(ii) The current coupled to Z' would violate leptonic flavor; i.e., κ_{aij} are nonzero for $i \neq j$, if M_l is not diagonal. In this case, U_L and/or U_R are different from unity. To see this, consider $X \equiv L_e - L_\tau$. Because of the form of X given in Eq. (2), it is easy to see that $U_L X U_L^\dagger (U_R X U_R^\dagger)$ will have nonzero off diagonal couplings unless mixing between $e'_L(e'_R)$ and $\tau'_L(\tau'_R)$ is forbidden. Since such couplings would invariably occur in models with extended Higgs structure, one expects the flavor changing Z' couplings in these cases. This occurrence of the flavor changing current is a well-known phenomena [4] which arises when fermions of the same charge and helicity transform differently under a gauge group, $U(1)_X$ in the present case.

Since the structure of the Z' current is fixed by X and M_l, it is easy to classify all models that are possible within the present scheme. One has basically three types of models.

(i) Models with only one Higgs doublet neutral under $U(1)_X$. In these, Eq. (4) requires M_l to be diagonal. Hence, one has vector currents and no flavor violation.

[1]To prove this explicitly, we write $U_R = U_L V$, V being a unitary matrix different from I. Then $U_L X U_L^\dagger = U_R X U_R^\dagger$ only if $VX = XV$. This is not possible because of the restricted structure of X.

There are three models in this category studied in Ref. [1].

(ii) Models with two Higgs doublets carrying $U(1)_X$ charge 0, and ± 1 or ± 2. In this case only one nondiagonal entry is possible in M_l [see Eq. (10)]. In these types of models, one of the leptons remains unmixed while the other two mix with some mixing angles $\theta_{L,R}$.

(iii) The third category of the models follows when one introduces two or more additional Higgs fields carrying $U(1)_X$ charge ± 1 or ± 2. These represent the general class of models with mixing among all three generations.

We shall study in the next section detailed phenomenology of a model in category (ii).

III. $SU(2)_L \otimes U(1)_Y \otimes U(1)_{L_e - L_\tau}$ MODEL

We consider $SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ model containing standard fermions, two Higgs doublets $\phi_{1,2}$, and an $SU(2)_L \otimes U(1)_Y$ singlet η. X is chosen to be $L_e - L_\tau$ charge from Eq. (2). X charges of ϕ_1 and ϕ_2 are chosen to be 0 and $+2$, respectively. The field η is assumed to carry some nonzero charge under $U(1)_X$, and it is solely introduced to provide a different mass scale characteristic of the $U(1)_X$ breaking.

The quark sector of the model remains the same as in the SM while lepton couplings to the neutral Higgs fields are given by

$$-\mathcal{L}_Y = h_{ii}\bar{e}'_{iL} e'_{iR}\phi_1^0 + h_{13}\bar{e}'_{1L} e'_{3R}\phi_2^0$$
$$\equiv \frac{m_i}{\langle \phi_1^0 \rangle}\bar{e}'_{iL} e'_{iR}\phi_1^0 + \frac{\delta}{\langle \phi_2^0 \rangle}\bar{e}'_{1L} e'_{3R}\phi_2^0 + \text{H.c.} \tag{9}$$

This leads to the following mass matrix M_l:

$$M_l = \begin{bmatrix} m_1 & 0 & \delta \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} . \tag{10}$$

Let $U_{L,R}$ diagonalize M_l, i.e.,

$$U_L M_l U_R^\dagger = \text{diag}(m_e, m_\mu, m_\tau) .$$

where

$$m_\mu^2 = m_2^2 ,$$
$$m_e^2 = \tfrac{1}{2}\{m_1^2 + m_3^2 + \delta^2 + [(m_1^2 - m_3^2)^2 + 2\delta^2(m_1^2 + m_3^2) + \delta^4]^{1/2}\} ,$$
$$m_\tau^2 = \tfrac{1}{2}\{m_1^2 + m_3^2 + \delta^2 - [(m_1^2 - m_3^2)^2 + 2\delta^2(m_1^2 + m_3^2) + \delta^4]^{1/2}\} ,$$

$$U_{L,R} = \begin{bmatrix} \cos\theta_{L,R} & 0 & \sin\theta_{L,R} \\ 0 & 1 & 0 \\ -\sin\theta_{L,R} & 0 & \cos\theta_{L,R} \end{bmatrix} . \tag{11}$$

The mixing angles $\theta_{L,R}$ are given by

$$\sin 2\theta_L = -\frac{2\delta m_3}{m_\tau^2 - m_e^2}, \quad \sin 2\theta_R = -\frac{2\delta m_1}{m_\tau^2 - m_e^2} .$$

As we will soon see, the $\theta_{L,R}$ are constrained to be quite small. It is therefore appropriate to work in the approximation $\delta < m_1, m_3$. In this limit,

$$\sin 2\theta_R \approx -\frac{2\delta m_e}{m_\tau^2}, \quad \sin 2\theta_L \approx -\frac{2\delta}{m_\tau} . \tag{12}$$

The parameters $\kappa_{aij}(a = L,R)$ determining the couplings of Z' to leptons through Eq. (8) are explicitly given in the present case by

$$\kappa_{a11} = \cos 2\theta_a = -\kappa_{a33} ,$$
$$\kappa_{a13} = -\sin 2\theta_a , \tag{13}$$
$$\kappa_{a2i} = 0, \quad i = 1,2,3 .$$

Since one of the doublets carry nonzero $U(1)_X$ charge, the Z' will mix with the conventional Z boson to produce two mass eigenstates $Z_{1,2}$:

$$Z = \cos\phi Z_1 + \sin\phi Z_2 ,$$
$$Z' = -\sin\phi Z_1 + \cos\phi Z_2 . \tag{14}$$

The couplings of the neutral gauge boson $Z_{1,2}$ to the leptons are now given by

$$\mathcal{L}_Z = \frac{g}{\cos\theta}\left\{ \sum_{m=1,2} F_{Lmij}\bar{e}_{iL}\gamma_\mu e_{jL} Z_m^\mu + L \leftrightarrow R \right\} , \tag{15}$$

where

$$F_{L1ij} = \cos\phi \left[-\frac{1}{2} + \sin^2\theta \right]\delta_{ij} - \sin\phi \frac{g'}{g}\kappa_{Lij} ,$$
$$F_{R1ij} = \cos\phi \sin^2\theta\delta_{ij} - \sin\phi \frac{g'}{g}\kappa_{Rij} ,$$
$$F_{L2ij} = \sin\phi \left[-\frac{1}{2} + \sin^2\theta \right]\delta_{ij} + \cos\phi \frac{g'}{g}\kappa_{Lij} ,$$
$$F_{R2ij} = \sin\phi \sin^2\theta\delta_{ij} + \cos\phi \frac{g'}{g}\kappa_{Rij} .$$

As would be expected, Eqs. (10) and (13) show that the muon number is exactly conserved in the model. This is a consequence of the fact that both the Z' interactions as well as the mass matrix, Eq. (10), respect this symmetry. When $\delta \ll m_\tau$, the flavor violations and departure from vectorial symmetry are very small. Moreover, these departures are more suppressed in the right-handed sector compared to the left-handed sector.

The generalization to other models in this category is obvious. One could construct another model with additional Higgs doublets carrying $L_e - L_\tau$ charge -2 instead of $+2$. In this case $(M_l)_{31}$ will be nonzero instead of $(M_l)_{13}$ as in Eq. (10). All the couplings of this model are then obtained by interchange of $\theta_L \leftrightarrow \theta_R$ in Eq. (13). In addition to these two models with $L_e - L_\tau$ symmetry, one could construct a pair of models each with symmetry $L_e - L_\mu$ and $L_\mu - L_\tau$. These are, respectively, characterized by an unbroken L_τ and L_e.

In addition to the flavor violations induced by Z', there exist other flavor violations associated with the Higgs fields. These arise in a well-known [5] manner whenever the fermions with the same charge obtain their masses from two different Higgs fields as in Eq. (9). Using Eqs. (9)–(11) it follows that

$$-\mathcal{L}_{Y(\mathrm{FCNC})}=\delta\left[\frac{\phi_1^0}{\langle\phi_1^0\rangle}-\frac{\phi_2^0}{\langle\phi_2^0\rangle}\right]\times\{\cos\theta_L\cos\theta_R\bar{e}_L\tau_R-\sin\theta_L\sin\theta_R\bar{\tau}_L e_R\}+\mathrm{H.c.}$$

It follows from Eq. (12) that these flavor violations are of $O(\delta/\langle\phi_{1,2}^0\rangle)$, and hence would be suppressed compared to Z' induced flavor violations unless the associated Higgs field is much lighter than Z'. We shall therefore concentrate on the Z' induced flavor violations in the next section.

We close this section with a brief mention of the Z-Z' mixing in these models and quote well-known formulas [5] to be used later on. The neutral gauge boson mass matrix M_0^2 in the Z-Z' basis is given by

$$M_0^2=\begin{bmatrix} M_Z^2 & \delta M^2 \\ \delta M^2 & M_{Z'}^2\end{bmatrix},$$

where

$$M_Z^2=\tfrac{1}{4}g^2[\langle\phi_1\rangle^2+\langle\phi_2\rangle^2],\quad M_{Z'}^2=g'^2[\langle\phi_2\rangle^2+\langle\eta\rangle^2],$$

$$\frac{\delta M^2}{M_Z^2}=4(g'/g)\sin^2\beta,\quad \text{where } \sin^2\beta=\frac{\langle\phi_2\rangle^2}{\langle\phi_1\rangle^2+\langle\phi_2\rangle^2}. \tag{16}$$

The mixing angle ϕ appearing in Eq. (14) is then given by

$$\tan^2\phi=\frac{M_Z^2-M_1^2}{M_2^2-M_Z^2}. \tag{17}$$

In addition, one has

$$M_1^2\cos^2\phi+M_2^2\sin^2\phi=\frac{M_W^2}{\cos^2\theta}$$

and (18)

$$\sin\phi\cos\phi=\frac{\delta M^2}{M_2^2-M_1^2},$$

$\theta(M_W)$ being the Weinberg angle (W mass) at the tree level.

IV. PHENOMENOLOGY OF $SU(2)_L\otimes U(1)_Y\otimes U(1)_{L_e-L_\tau}$

We shall now explore the phenomenological consequences of the $SU(2)_L\times U(1)_Y\times U(1)_X$ models. The extra Z boson associated with $U(1)_X$ change the phenomenology of the SM in two ways. The extra Z' contribute to the known processes induced by the Z boson. In addition, in the present case, Z' induce new flavor-violating processes. The detailed phenomenology will depend upon the model. We shall take the model presented in the last section as an illustrative example and work out consequences within that model.

In the absence of additional Higgs bosons, the Z' induced flavor violation disappears. Moreover, the Z' does not mix with the ordinary Z. In this case Z' makes its effect felt by contributing to known processes like $e^+e^-\to\mu^+\mu^-$ scattering. The detailed restrictions on the relevant parameters by LEP results have been worked out in Ref. [1] for this case. These restrictions continue to hold in the present case. But additionally one gets more stringent restrictions due to flavor violations and Z-Z' mixing. We shall concentrate on these in the following.

The phenomenology of models with extra Z boson is extensively discussed in the literature [5,6]. The present class of models have characteristic differences arising due to the fact that Z' couples only to leptons. In other models, an important restriction on the Z' mass arises from the direct experimental observations at the hadronic colliders. These restrictions though model dependent strongly constrain the Z' mass. For example in the left-right symmetric model [7], the search in $\bar{p}p$ collisions imply [8] $M_{Z_{LR}}>310$ GeV. Similar restrictions are not applicable here since Z' couples only to leptons. Its production at the hadronic colliders arise only through mixing with the ordinary Z and is therefore highly suppressed. The Z' mass as well as its mixing with Z is constrained in the present case by (a) the observations at LEP and (b) the observed limits on the leptonic flavor violations. We discuss them in turn.

A. Constraints from the LEP data

We closely follow the analysis of Ref. [5] in deriving constraints on the relevant parameters from observations at LEP. These constraints have been derived in two different ways. The observations of the ratio M_W/M_1 and the Z mass M_1, at the Collider Detector at Fermilab (CDF) and LEP, respectively, constrain the ρ parameter and lead to restrictions on M_2 and $\tan\phi$. Another method is to use the fact that the extra Z induce changes in the observables such as width to fermions, peak cross section in e^+e^- collisions etc. One could then make a detailed fit to the LEP data and derive constraints on M_2 and ϕ.

The mixing between Z and Z' change the tree-level relation between the W and the Z mass. Specifically,

$$\frac{M_W^2}{\rho_M M_1^2}=\cos^2\theta,$$

θ being the tree-level weak mixing angle. The parameter ρ_M can be read off from the mixing matrix between Z and Z' [see Eq. (18)]:

$$\rho_M=\frac{1+\tan^2\phi M_2^2/M_1^2}{1+\tan^2\phi}. \tag{19}$$

One could eliminate $\cos^2\theta$ in favor of G_F, α, and M_1 to obtain

$$\frac{M_W^2}{\rho_M M_1^2}=\left[\frac{1}{2}+\left[\frac{1}{4}-\frac{\mu^2}{\rho_M M_1^2}\right]^{1/2}\right], \tag{20}$$

where

$$\mu=\left[\frac{\pi\alpha}{\sqrt{2}G_F}\right]^{1/2}=(37.280\ \text{GeV})\ .$$

These restrictions are valid at the tree level. Since the extra Z induced effects are comparable to the radiative corrections in the standard model, one must incorporate the latter. This has been done in Ref. [5], assuming that the radiative corrections induced by Z_2 are negligible. The radiative corrections of SM are included using the improved Born approximation which changes Eq. (20) to

$$\frac{M_W^2}{\rho M_1^2}=\left[\frac{1}{2}+\left[\frac{1}{4}-\frac{\mu^2}{\rho M_1^2(1-\Delta\alpha)}\right]^{1/2}\right]\ , \tag{21}$$

where the ρ parameter is now

$$\rho=\frac{\rho_M}{1-\Delta\rho_T}\ ,$$

with

$$\Delta\rho_T\simeq\frac{G_F m_t^2}{8\pi^2\sqrt{2}}$$

and

$$\Delta\alpha=0.0602+\frac{40}{9}\frac{\alpha}{\pi}\ln\frac{M_1(\text{GeV})}{92}\pm0.0009\ .$$

The CDF result on $M_W/M_1=0.779$ together with the LEP result on[2] the Z mass M_1 can be used to obtain $\rho=1.005\pm0.003$ in Eq. (21). This implies, at 1σ,

$$\Delta\rho_M=\rho_M-1\le0.008-0.003\left[\frac{m_t(\text{GeV})}{100}\right]^2\ . \tag{22}$$

In addition to this restriction, $\Delta\rho_M$ can also be constrained [5,6] by the other observables at LEP. Specifically, the presence of Z' would change the three leptonic widths $\Gamma_{e\mu\tau}$ as well as the hadronic width Γ_h of the Z_1. These changes can be parametrized [5] in terms of $\Delta\rho_M$ and mixing angle ϕ:

$$d\Gamma_i=A_i\Delta\rho_M+B_i\phi\ . \tag{23}$$

In our case,

$$A_i=4N_c\rho_f\left[(T_{3Li}-\sin^2\theta_f Q_i)^2+T_{3Li}^2+\frac{4\sin^2\theta_f\cos^2\theta_f}{\cos2\theta_f}Q_i(T_{3Li}-\sin^2\theta_f Q_i)\right]\ ,$$

$$B_i=8N_c\rho_f[(T_{3Li}-\sin^2\theta_f Q_i)g'_{Vi}-T_{3Li}g'_{Ai}]\ ,$$

where

$$g'_{Vi}=\frac{g'}{g}(\kappa_{Lii}+\kappa_{Rii});\quad g'_{Ai}=\frac{g'}{g}(\kappa_{Lii}-\kappa_{Rii})\ ;$$

$$\rho_f\equiv\frac{\rho}{\rho_M};\quad \sin^2\theta_f=\frac{1}{2}-\left[\frac{1}{4}-\frac{\mu^2}{\rho_f M_1^2(1-\Delta^{\alpha})}\right]^{1/2}\ .$$

$N_c=3[1+(\alpha_s/\pi)]$ for quarks and 1 for leptons. The fermionic widths Γ_i of Z_1 have been extracted from the LEP data in a model-independent way. We use the values derived in Ref. [9] to constrain $\Delta\rho_M$ and ϕ. Specifically,

$$\Gamma_e=82.6\pm0.7\ \text{MeV}\ ,$$
$$\Gamma_\mu=83.6\pm1.1\ \text{MeV}\ ,$$
$$\Gamma_\tau=83.1\pm1.2\ \text{MeV}\ ,$$
$$\Gamma_h=1.741\pm0.015\ \text{MeV}\ .$$

We use these values and determine the best values for $\Delta\rho_M$ and ϕ appearing in Eq. (23) through a least squares fit. This gives (for $m_t=150$ GeV), at 1σ,

$$\Delta\rho_M=-0.0018\pm0.004,\quad \phi=0.0094\pm0.012\ . \tag{24}$$

The value of $\Delta\rho_M$ as determined by Eq. (24) is less stringent than following from Eq. (22) derived on the basis of the CDF result on M_W/M_1. We shall therefore use the values given by Eq. (22) for $\Delta\rho_M$ in the next section to constrain the parameters of the model.

B. Constraints from the rare processes

As already discussed, the model of the last section contains flavor violations involving τ and e. The muon number is exactly conserved in the model. As a consequence one expects the following rare processes to occur in the model: $Z_{1,2}\to e\tau$; $\tau\to eee$; $\tau\to e\mu\mu$. The branching ratios for these processes can be easily worked out and are given by

$$\frac{\Gamma(\tau\to eee)}{\Gamma(\tau\to\nu_\tau\nu_e e)}=16M_1^4\{(g_{LL}^e)^2+(g_{RR}^e)^2+\tfrac{1}{2}[(g_{LR}^e)^2+(g_{RL}^e)^2]\}\ ,$$

$$\frac{\Gamma(\tau\to e\mu\mu)}{\Gamma(\tau\to\nu_\tau\nu_e e)}=4M_1^4[(g_{LL}^\mu)^2+(g_{RR}^\mu)^2+(g_{LR}^\mu)^2+(g_{RL}^\mu)^2]\ ,$$

$$\Gamma(Z\to\tau e)=\frac{G_F M_1^3}{3\sqrt{2}\pi}[(F_{L1}^{\tau e})^2+(F_{R1}^{\tau e})^2]\ ,$$

where

$$g_{LL}^m=\frac{F_{L1}^{\tau e}F_{L1}^{mm}}{M_1^2}+\frac{F_{L2}^{\tau e}F_{L2}^{mm}}{M_2^2},\quad g_{LR}^m=\frac{F_{L1}^{\tau e}F_{R1}^{mm}}{M_1^2}+\frac{F_{L2}^{\tau e}F_{R2}^{mm}}{M_2^2}\ .$$

$m=e,\mu$. g_{RR} and g_{RL} are obtained by $L\leftrightarrow R$ interchange in the above equation. The difference in the rates for the $\tau\to eee$ and $\tau\to e\mu\mu$ arise due to both the s and t channel $Z_{1,2}$ exchanges contributing to the former. In addition to constraints from the LEP discussed earlier the rare decays also provide important constraints on the model. The specific constraints are [8] given by

[2]Note that unlike the fermionic width, the determination of M_1 from the data is fairly insensitive to the presence of Z'.

$$B(Z \to e^+\mu^-) < 2.4\times10^{-5}\ ,$$
$$B(Z \to e^+\tau^-) < 3.4\times10^{-5}\ ,$$
$$B(Z \to \mu^+\tau^-) < 4.8\times10^{-5}\ ,$$
$$B(\tau \to eee) < 2.7\times10^{-5}\ ,$$
$$B(\tau \to e\mu\mu) < 2.7\times10^{-5}\ ,$$
$$B(\tau \to \mu\mu\mu) < 1.7\times10^{-5}\ .$$

The basic parameters of models are mixing angles $\theta_{L,R}$, Z_2 mass M_2, Z-Z' mixing angle ϕ, and the $U(1)_X$ gauge coupling g'. Both the Z-Z' mixing and the flavor violation arise in the model from the presence of the additional doublet ϕ_2. Thus both are related to the parameter $\tan\beta = \langle\phi_2\rangle/\langle\phi_1\rangle$. The relation between ϕ and β follows from Eqs. (16) and (18):

$$\sin\phi \sim 4C\left[\frac{M_1}{M_2}\right]^2 \sin^2\beta\ , \qquad (25)$$

where

$$C \approx \frac{g'}{g}\left[1-\frac{M_1^2}{M_2^2}\right]^{-1} \sim 1\ .$$

The $\theta_{L,R}$ also goes to zero when $\beta \to 0$. If one assumes that the flavor-violating Yukawa coupling h_{13} in Eq. (9) is of the same order as flavor-conserving one (namely h_{33}) then $\delta \approx m_\tau \tan\beta$ and hence, from Eq. (3),

$$\sin 2\theta_L \approx -2\tan\beta\ . \qquad (26)$$

The existing limits on the $B(\tau\to eee)$ as well as $Z\to e\tau$ imply restrictions on the parameters β and M_2. These are displayed in Fig. 1 assuming $h_{13}=h_{33}$. Analogous constraints also follow from the process $\tau\to e\mu\mu$. This process is comparatively suppressed in the present case and hence imply much weaker constraints. This is not displayed in the figure for simplicity. The same parameters are also constrained by $\Delta\rho_M$ and ϕ [see Eqs. (19) and (25)].

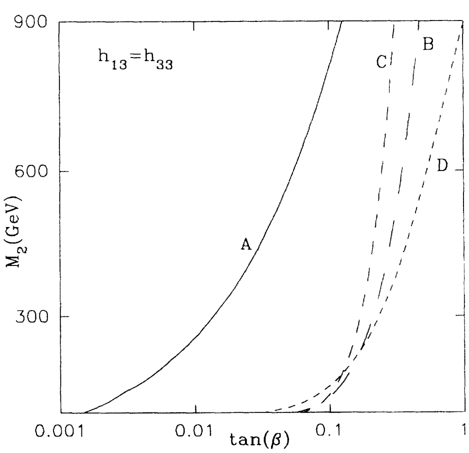

FIG. 1. The allowed region in the $M_2-\tan\beta$ plane implied by various constraints: Curve (A) is a contour for $B(\tau\to eee)=2.7\times10^{-5}$; (B) for $B(Z\to e\tau)=3.4\times10^{-5}$; (C) for $\Delta\rho_M=0.001\,25$; and (D) for $\phi=0.021$. These curves are for $h_{13}=h_{33}$ (see text). Region to the left of the curves is allowed.

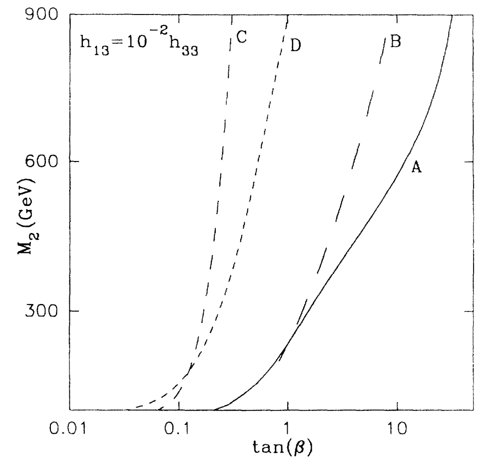

FIG. 2. Same as Fig. 1 except that $h_{13}=10^{-2}h_{33}$.

It follows that the strongest constraints on the parameters are implied by the rare decay $\tau\to eee$. Hence the process $\tau\to eee$ is allowed by the LEP data to occur at a rate consistent with the present experimental precision. Improvement in the limits for this process would either imply more stringent restrictions on β and M_2 or one should be able to see this decay in future. Figure 1 was based on the assumption of equal Yukawa couplings, $h_{13}=h_{33}$, in Eq. (9). For comparison we also display in Fig. 2 limits on β and M_2 in case of $h_{13}=10^{-2}h_{33}$. Reduction in the value of h_{13} strongly suppresses the flavor-violating couplings of τ. $\Delta\rho_M$ and ϕ remain unchanged. As a result, now the LEP data imply stronger restrictions on $\tan\beta$ and M_2. In this case, the LEP observations already rule out the possibility of seeing flavor violation in future experiments which are expected to provide improved limits on $\tau\to eee$.

It is clear from Figs. 1 and 2 that as long as $M_2 \lesssim 1$ TeV, $\tan\beta$ is restricted to be $\lesssim 0.1-0.5$. Hence the vacuum expectation value of the field ϕ_2 responsible for flavor violations is strongly constrained in the model. Likewise, low values of M_2 (e.g., 400 GeV) are possible only if $\tan\beta$ is chosen small (0.03 in case of $h_{13}=h_{33}$, and 0.3 in case of $h_{13}=10^{-2}h_{33}$).

Although we restricted ourselves to the L_e-L_τ model, the analogous constraints would follow in models with $X=L_e-L_\mu$ or $L_\mu-L_\tau$. In particular, one would expect very severe constraint if L_e-L_μ is gauged since $\mu\to eee$ is much severely constrained experimentally.

V. SUMMARY

We have studied in this paper a specific class of extended gauge models of the form $SU(2)_L\otimes U(1)_Y\otimes U(1)_X$. All these extensions are characterized by the fact that it is possible to gauge $U(1)_X$ without extending the fermionic sector of the standard model. Thus models studied here are the simplest gauge extensions of the SM. These models are prototypes of more general horizontal symmetries [6]. We have concentrated here (a) on a systematic classification of $U(1)_X$ models and (b) on deriving constraints on parameters of a prototype model using the LEP results. In the specific case of $U(1)_X$ coupling to leptons, we have categorized all possible choice of $U(1)_X$. In general $U(1)_X$ provide important restrictions on the mixing matrices. Moreover, they also give rise to interesting flavor violations thus providing a window into the existence of such symmetry. The mixing of the $U(1)_X$ gauge boson Z' with the ordinary Z is correlated in these models to the flavor violation. In fact both these features originate from the existence of a Higgs doublet carrying nonzero $U(1)_X$ charge. As a result the observations at LEP could indirectly provide important constraints on flavor violations. The detailed study presented here shows that under reasonable assumptions on relevant Yukawa couplings, the LEP observations do allow sizable flavor violations, and it is possible to obtain a rate for $\tau \rightarrow eee$ near its present experimental limit. In contrast, the lepton flavor-violating decays of Z are considerably suppressed in these models.

We mainly studied models in which $U(1)_X$ acts only on leptons. Models with $U(1)_X$ acting on quarks [3] or both can be analogously studied. A systematic study of these horizontal models and restrictions on flavor violations in these models in the light of LEP observations would be interesting in its own right.

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