Inhomogeneous chiral condensate in the Schwinger model at finite density

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The behavior of the chiral symmetry order parameter $\langle \bar{\psi}\psi \rangle$ in the (1+1)-dimensional Schwinger model at finite chemical potential is studied. We find an inhomogeneous chiral condensate.

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One of the most important concepts in our current understanding of particle physics is the spontaneous breakdown of the chiral symmetry [1]. It is believed that the order parameter $\langle \bar{\psi}\psi \rangle$ for chiral symmetry is nonvanishing in four-dimensional QCD (QCD_4) at sufficiently low fermion density and temperature. A lot of low energy particle physics can be understood if we treat pions and kaons as the Goldstone bosons for the broken chiral symmetry [2]. It is also believed that as we raise temperature and/or fermion density, we shall eventually reach the quark-gluon plasma phase where the chiral symmetry is recovered and $\langle \bar{\psi}\psi \rangle$ vanishes. Although results from lattice gauge theory support the above picture [2], we still lack an analytic and accurate confirmation of the idea directly from the QCD_4 Lagrangian. In this Brief Report, we shall examine $\langle \bar{\psi}\psi \rangle$ in the exactly solvable (1+1)-dimensional Schwinger model [3], a favorite theoretical laboratory for testing ideas in QCD_4. Because of instantons (vortices), $\langle \bar{\psi}\psi \rangle$ is nonvanishing [4] in the (one-flavor) Schwinger model and the chiral condensate disappears only at infinite temperature [5,6]. (It would be more interesting if we could find a finite critical temperature such as what we expect to happen in QCD_4.) We want to point out in this Brief Report that in the Schwinger model, when the chemical potential μ of the fermions is not zero, there exists an inhomogeneous chiral condensate $\langle \bar{\psi}\psi(x) \rangle$ [$x=(x_0,x_1)$, x_1 is the spatial coordinate] whose μ dependence shows up in an oscillatory factor $\cos(2\mu x_1)$. There has been no mention of this oscillatory behavior of $\langle \bar{\psi}\psi \rangle$ in the literature, although the work by Fischler, Kogut, and Susskind [7] on the nonuniform charge density in the massive Schwinger model may have anticipated what we have found. It is not completely clear yet that this peculiar behavior of $\langle \bar{\psi}\psi \rangle$ is not just due to the low dimensionality of the model, but we are somewhat surprised to find that Deryagin, Grigoriev, and Rubakov [8] had already argued for the existence of an inhomogeneous and anisotropic chiral condensate in QCD_4. Our calculation indicates that the proposal of Deryagin, Grigoriev, and Rubakov merits further study. If they are right, our view of the dense hadronic matter may have to be modified.

The Schwinger model at finite fermion density is defined by the following Lagrangian density in (1+1)-dimensional spacetime:

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}+\bar{\psi}(i\partial\!\!\!/-eA\!\!\!/-\mu\gamma_0)\psi+eA_0\rho_b \ , \qquad (1)$$

where μ is the chemical potential and the Dirac matrices γ_μ are 2×2 matrices obeying $\{\gamma_\mu,\gamma_\nu\}=2g_{\mu\nu}$ with $g_{00}=-g_{11}=1$. We have put in a uniform background charge density ρ_b for the purpose of canceling the infinite electromagnetic energy carried by the fermions, as the Schwinger model itself exhibits the quark confinement phenomenon. Using the well-known bosonization rules [9]

$$:\bar{\psi}i\partial\!\!\!/\psi:=\tfrac{1}{2}(\partial_\mu\phi)^2 \ , \qquad (2)$$

$$:\bar{\psi}\gamma_\mu\psi:=\frac{1}{\sqrt{\pi}}\varepsilon_{\mu\nu}\partial^\nu\phi \ , \qquad (3)$$

$$:\bar{\psi}\psi:=-cmN_m\cos(2\sqrt{\pi}\phi) \ , \qquad (4)$$

where $c=e^\gamma/2\pi$, γ is Euler's constant, and N_m denotes normal ordering with respect to mass m, we know that the Schwinger model in the Coulomb gauge is equivalent to a bosonic theory defined by the Hamiltonian

$$\mathcal{H}=\tfrac{1}{2}(\partial_0\phi)^2+\tfrac{1}{2}(\partial_1\phi)^2+\frac{e^2}{2\pi}(\phi+\phi_b)^2+\frac{\mu}{\sqrt{\pi}}\partial_1\phi \ . \qquad (5)$$

The effect of the background charge is represented by ϕ_b defined as $\rho_b=(1/\sqrt{\pi})\partial_1\phi_b$. For a constant ρ_b, ϕ_b is $\sqrt{\pi}\rho_b x_1$ and the model is no longer translationally invariant. As explained by Fischler, Kogut, and Susskind [7], a uniform background charge can be regarded as the infinite limit of a finite line of charge, and the center of the background charge is the source of the noninvariance under translation. To handle the chemical potential term, it is convenient to use a new field variable $\tilde{\phi}$ defined by $\phi=\tilde{\phi}-(\mu/\sqrt{\pi})x_1$ so that the bosonic Hamiltonian $\mathcal{H}$ becomes (up to a constant)

$$\mathcal{H}=\tfrac{1}{2}(\partial_0\tilde{\phi})^2+\tfrac{1}{2}(\partial_1\tilde{\phi})^2+\frac{e^2}{2\pi}\tilde{\phi}^2 \ , \qquad (6)$$

if we choose ρ_b to be μ/π. We now have a free massive theory in terms of the $\tilde{\phi}$ field, and it is easy to calculate the correlation functions.

Calculating $\langle \bar{\psi}\psi(x) \rangle$ is straightforward with m equal to $e/\sqrt{\pi}$:

$$\begin{aligned}\langle \bar{\psi}\psi(x)\rangle &= \langle -cmN_m\cos[\sqrt{4\pi}\phi(x)]\rangle \\ &= -\frac{cm}{2}\langle N_m(e^{i\sqrt{4\pi}\phi(x)}+e^{-i\sqrt{4\pi}\phi(x)})\rangle \\ &= -\frac{cm}{2}\langle N_m(e^{-2i\mu x_1}e^{i\sqrt{4\pi}\tilde{\phi}(x)}+e^{2i\mu x_1}e^{-i\sqrt{4\pi}\tilde{\phi}(x)})\rangle \\ &= -\frac{cm}{2}(e^{-2i\mu x_1}+e^{2i\mu x_1})=\cos(2\mu x_1)\langle \bar{\psi}\psi\rangle_{\mu=0} \;. \end{aligned} \tag{7}$$

At finite temperature $T=1/\beta$, it is easy to see that $\langle \bar{\psi}\psi(x)\rangle$ is still equal to $\cos(2\mu x_1)\langle \bar{\psi}\psi\rangle_{\mu=0}$, except that $\langle \bar{\psi}\psi\rangle_{\mu=0}$, instead of being $-cm$, is now equal to [5,6]

$$-cm\exp\left[-\int_{-\infty}^{+\infty}\frac{dk}{\sqrt{k^2+m^2}(e^{\beta\sqrt{k^2+k^2}}-1)}\right] . \tag{8}$$

The demonstration of the occurrence of the oscillatory factor $\cos(2\mu x_1)$ in $\langle \bar{\psi}\psi(x)\rangle$, though very simple, may still be surprising to some. But, as explained above, we understand the reason for the lack of translational invariance in the model. One may suspect that an inhomogeneous chiral condensate exists only in the Schwinger model. However, by solving the Schwinger-Dyson equation, Deryagin, Grigoriev, and Rubakov [8] found that the chiral condensate in QCD_4 at zero temperature and finite chemical potential in the limit $N_c\to\infty$ and fixed $g^2N_c\ll 1$ is of the form

$$\langle \bar{\psi}\psi(\mathbf{x})\rangle=\cos(2\mathbf{p}\cdot\mathbf{x})F(g,\mu,N_c) \;, \tag{9}$$

where the vector $\mathbf{p}$ has no specified direction and $|\mathbf{p}|=\mu$. It remains to be seen if their claim receives more confirmation in further studies of QCD_4. Three final remarks: (1) We have also computed $\lim_{|x_1-y_1|\to\infty}\langle \bar{\psi}\psi(x)\bar{\psi}\psi(y)\rangle$ following the procedure of Refs. [4,6] without using bosonization. The result is that

$$\lim_{|x_1-y_1|\to\infty}\langle \bar{\psi}\psi(x)\bar{\psi}\psi(y)\rangle = \lim_{|x_1-y_1|\to\infty}\langle \bar{\psi}\psi(x)\bar{\psi}\psi(y)\rangle_{\mu=0}\cos[2\mu(x_1-y_1)] \;.$$

Such an oscillatory behavior in $\langle \bar{\psi}\psi(x)\bar{\psi}\psi(y)\rangle$ has already been observed by Wiegmann [10] in the low temperature phase of certain one-dimensional Fermi systems. But there, unlike the case in the Schwinger model, $\langle \bar{\psi}\psi(x)\bar{\psi}\psi(y)\rangle$ decays to zero as $|x_1-y_1|\to\infty$. Hence Wiegmann found no oscillatory condensate. (2) $\langle \psi^\dagger\psi(x)\rangle$ is still a constant, displaying no coordinate dependence. Therefore, we do not have a Wigner crystal as found by Schulz [11] in the one-dimensional electron gas with long-range Coulomb interaction and by Fischler, Kogut, and Susskind [7] in the massive Schwinger model. (3) The oscillatory behavior in Eqs. (7) and (9) is reminiscent of the Friedel oscillator [12] arising from the sharp Fermi surface in many-body systems. This hints that the oscillatory behavior may be robust.

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