

## BRIEF REPORTS

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## Limits on direct detection of gravitational waves

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We compute the energy density and strain induced by a primordial spectrum of gravitational waves on terrestrial- and space-based detectors (e.g., LIGO) as constrained by the COBE detection of microwave background anisotropy. For the case where the spectrum is created during inflation, we find new, stricter upper bounds on the induced strain, making detection unlikely. However, detectors might be useful for discovering (or ruling out) exotic, noninflationary sources.

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Events in the early Universe may have left a primordial spectrum of gravitational waves. Detecting these elusive remnants would not only establish the existence of this as-yet-unverified prediction of general relativity, but it would also provide a new critical test for all proposed scenarios of the evolution of the early Universe. In particular, a basic feature of the inflationary model of the Universe is the prediction of a relic spectrum of gravitational waves [1], whose detection would lend strong support to the theory.

Such a detection might occur in three possible ways. Gravitational waves distort the cosmic microwave background (CMB) through the Sachs-Wolfe effect [2], thus raising the possibility that some or even most of the temperature variations observed by the Cosmic Background Explorer (COBE) [3] are due to gravitational waves [4]. Another possibility for detecting tensor fluctuations indirectly is through their effect on the polarization of the microwave background [5,6]. In this paper, we consider the possibility of direct observation of the primordial gravitational background, in a laser interferometer gravitational wave detector [7] or through its effects on pulsar timing [8].

Gravitational waves may be produced by many sources. At the Planck time, quantum fluctuations in the metric are significant and they produce gravitons. Phase transitions in the Universe may lead to topological defects such as cosmic strings, which generate gravitational waves. A period of inflation leaves behind gravitational waves. We first consider the case of gravitational waves produced by inflation, and discuss revised predictions for the strain induced in direct detection. Independent analyses have been made in Refs. [9–12]. We indicate which assumptions and which results are essential specifically to the prediction of the gravitational wave spectrum. We

then broaden the discussion to consider more general spectra, and the limits that may be placed on their existence.

In inflation, gravitational waves are produced in conjunction with density fluctuations. Inflation is a proposed solution to the homogeneity, flatness, and monopole problems that are unexplained in the standard big bang model. The essential idea of inflation is that the Universe underwent a period of extraordinarily rapid expansion  $10^{-35}$  sec or so after the big bang [13]. Both density perturbations [14] and gravitational waves [1] are produced as a result of the rapid stretching of quantum fluctuations occurring during inflation, in the inflaton field and graviton field, respectively. After inflation, both fluctuations span a broadband spectrum, ranging from the scale of the present horizon down to microphysical scales. We assume that the variation of amplitude with scale can be approximated as a power law. The index for scalar (density) fluctuations we denote  $n_S$ , and that for tensor (gravitational wave) fluctuations  $n_T$ , where we choose a convention in which a strictly scale-invariant spectrum corresponds to  $n_S=1$  or  $n_T=0$  [4]. The broad spectrum makes it possible to detect gravitational waves using CMB anisotropy on cosmological scales, pulsar timing measurements on astrophysical scales, and laser interferometer detectors on terrestrial scales.

If quadrupole temperature fluctuations in the CMB consist of long wavelengths contributions of gravitational waves ( $C_2^T$ ), and energy-density perturbations ( $C_2^S$ ), then the COBE measurement of large-angular CMB anisotropy fixes the sum  $C_2^T + C_2^S$ . This leaves the ratio undetermined. Naive gravitational wave limits for inflation had assumed  $C_2^S=0$ , and scale invariance  $n_T=0$ . Using COBE this leads to the  $n_T=0$  prediction (dotted curve) shown in Fig. 1. Here we consider the implications of a recent refinement of the inflationary predictions [4,11,12]. Namely, inflation does not predict precisely scale-invariant spectra. Rather, models of inflation give various values of  $n_T$  and  $n_S$  but with the fraction of the gravi-

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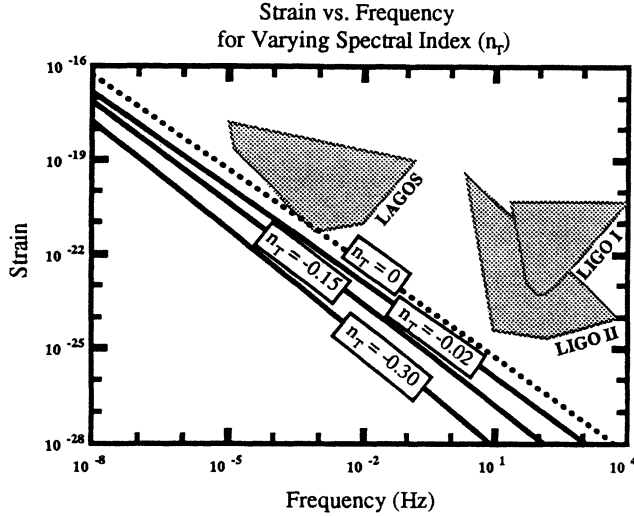


FIG. 1. The dimensionless strain of gravitational waves from inflation (solid curves) vs the present physical frequency, for  $n_T=0, -0.02, -0.15,$  and  $-0.30$ . The scale-invariant curve assuming  $x_T=1$  (dotted curve) is also shown. The projected sensitivities of the LIGO I, LIGO II, and LAGOS detectors are shown (shading) for comparison.

tational waves contribution to the CMB quadrupole anisotropy predicted:

$$r = \frac{C_2^T}{C_2^S} \approx -7n_T \quad (1)$$

(Refs. [15,16]).

The fraction  $x_T = C_2^T / (C_2^T + C_2^S)$  determines the gravitational wave amplitude at long [ $O(H_0^{-1})$ ] wavelengths and  $n_T$  determines the relative amplitude on smaller wavelengths. Equation (1) modifies the predictions for gravitational wave detectors. A strictly scale-invariant spectrum ( $n_T=0$ ) is now forced towards  $x_T=0$ . To obtain an appreciable amplitude,  $x_T$  must be  $>0$ ; however, this only occurs if  $n_T < 0$ , reducing the relative amplitude on smaller scales below that of the scale-invariant case with the same  $x_T$ . Hence, Eq. (1) reduces the expected strain in direct detectors for gravitational waves coming from inflation.

It is also possible that there is a noninflationary energy density spectrum of gravitational waves from another source, with some index  $n_T$ . Regardless of the source, if such a spectrum contributed significantly to the COBE anisotropy (long wavelengths), then a high-enough  $n_T$  would make the shorter wavelengths directly detectable by gravitational wave detectors. Such a detection would indeed cause excitement, since there is no established mechanism that generates such a spectrum. Conversely, a lack of detection at the sensitivities of proposed detectors would serve to place an upper limit on such exotic spectra.

Fluctuations in the metric produce temperature anisotropies in the CMB through the Sachs-Wolfe effect. These temperature fluctuations  $\Delta T/T$  can be written in terms of spherical harmonics. If

$$\frac{\Delta T}{T} = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi), \quad (2)$$

then the quadrupole is

$$C_2 = \langle |a_{2m}|^2 \rangle. \quad (3)$$

The rms quadrupole amplitude measured by COBE [3] is

$$Q_T^2 = \frac{5}{4\pi} C_2. \quad (4)$$

Inflation predicts that the tensor and scalar contributions are independent,  $C_2 = C_2^T + C_2^S$ .

The quadrupole  $C_2^T$  is produced by graviton modes of long wavelengths  $\lambda$ . Here  $\lambda$  denotes the comoving wavelength, which is the same as the present physical wavelength  $\lambda_{\text{phys}}^0$  if we set the present scale factor  $R_0=1$ . We also assume a flat  $\Omega=1$  universe and set  $h=0.75$ , where  $H_0=100h \text{ km sec}^{-1} \text{ Mpc}^{-1}$ . For a gravitational wave spectrum with index  $n_T$ , the energy density of a given mode outside the horizon is proportional to  $\nu^{n_T}$ , expressed in terms of the frequency  $\nu=c/\lambda$ . Horizon crossing for a given mode at time  $t$  is defined by  $\lambda_{\text{phys}}^t = 2cH_t^{-1}$  (in particular  $\nu_0=H_0/2$ ). At the horizon crossing,  $\Omega_g(\nu) = (1/\epsilon_{\text{cr}})\nu(d\epsilon_g/d\nu) \sim \nu^{n_T}$  ( $\epsilon_g$  is the energy density of the gravitons,  $\epsilon_{\text{cr}}$  is the critical value of the energy density) [9]. Once the mode crosses inside, its energy density redshifts  $\propto R^{-4}$ , as for a relativistic species. During the radiation-dominated epoch,  $\epsilon_{\text{cr}}$  is also  $\propto R^{-4}$  and therefore  $\Omega_g(\nu)$  stays constant. During the matter-dominated epoch  $\epsilon_{\text{cr}} \propto R^{-3}$  and so  $\Omega_g(\nu) \propto R^{-1}$ . A more careful treatment of the transition from radiation to matter domination yields, in terms of the quadrupole anisotropy, a spectrum at the present of [12,9,10]

$$\Omega_g(\nu) = \frac{x_T C_2}{15g(n_T)} T(\nu) \left( \frac{\nu}{\nu_0} \right)^{n_T-2}, \quad (5)$$

where  $T(\nu) \approx 1 + 3.49R_{\text{eq}}^{1/2}(\nu/\nu_0) + 16.9R_{\text{eq}}(\nu/\nu_0)^2$ ,  $g(n_T) \approx e^{1.3n_T}$  [1], and  $R_{\text{eq}} = 4.18 \times 10^{-5} h^{-2}$  [15].

In Fig. 1, the spectrum of gravitational waves from inflation has been calculated as a function of  $n_T$  using Eqs. (1), (4), and (5). We describe the predictions in terms of the dimensionless strain  $h_g(\nu)$ , where  $h_g^2(\nu) = 24\Omega_g(\nu)/(\nu/\nu_0)^2$ . A comparison of the strain predictions (solid curves) with the projected sensitivities of LIGO [17] (the Laser Interferometer Gravitational Wave Observatory) and LAGOS [18] (the Laser Gravitational Wave Observatory in Space), and current experimental limits from pulsar timing [7] shows that the inflationary maximum ( $n_T = -0.02$ ) lies 0.5 of an order of magnitude below the current estimate of the sensitivity of LAGOS and 1.5 orders of magnitude below the estimate for LIGO advanced detectors. Note that the maximal inflationary prediction, based on Eq. (1), lies a factor of 4 below the naive limit for  $n_T=0$  assuming  $x_T=1$  (dotted curve).

For a general, noninflationary spectrum, we must replace Eq. (1) by some other assumption. If we assume  $x_T=1$  then we find that LIGO I can be used to detect spectra with  $n_T > 0.3$  ( $n_T > 0.1$  for LIGO II). LAGOS

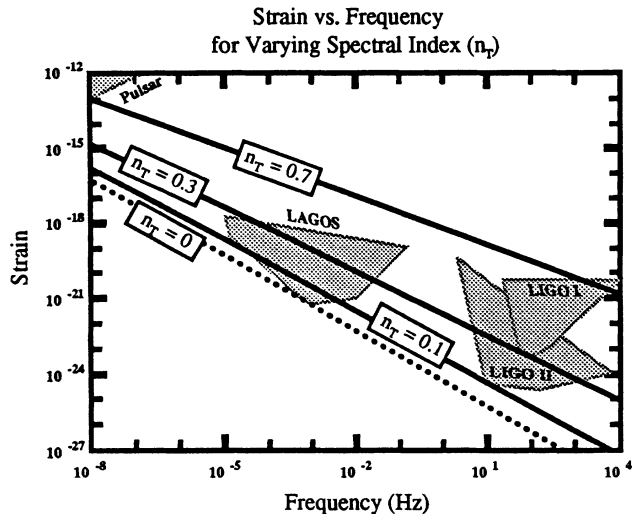


FIG. 2. The dimensionless strain of noninflationary gravitational waves all assuming  $x_T=1$  (solid curves) vs the present physical frequency, for  $n_T=0.1, 0.3,$  and  $0.7$ . The scale-invariant curve assuming  $x_T=1$  (dotted curve) is also shown. The projected sensitivities of the LIGO I, LIGO II, and LAGOS detectors as well as current limits from pulsar timing measurements are shown (shading) for comparison.

would be sensitive to spectra down to the scale-invariant  $n_T=0$ . (See Fig. 2.) If we assume  $x_T=1\%$  instead we obtain  $n_T > 0.4$  for LIGO I,  $n_T > 0.2$  for LIGO II, and  $n_T > 0.1$  for LAGOS. The best experimental limits that we can currently achieve are derived from pulsar timing measurements. For  $x_T=1$ , we get  $n_T < 0.7$ , and for  $x_T=1\%$ , we get  $n_T < 0.9$ .

An upper bound on  $n_T$  for  $n_T > 0$  can presently be obtained by considering the contribution of the gravitational waves to the total  $\Omega$  of the Universe, and using the constraint  $\Omega < 2$ . If the spectrum is produced at time  $t$  with corresponding  $\nu_t$  then

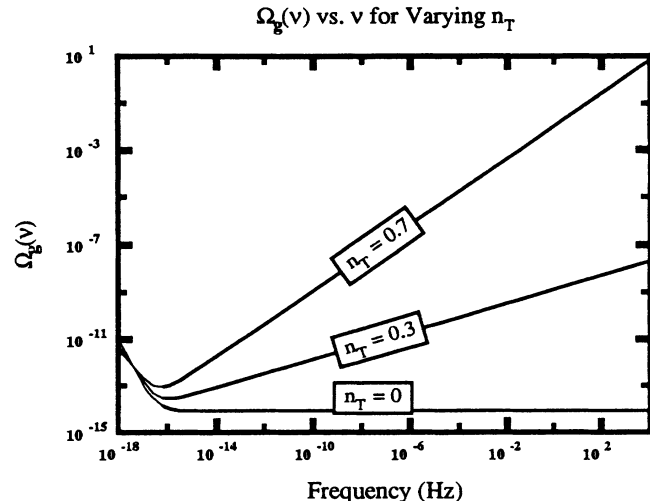


FIG. 3. The spectral energy density (in units of the critical energy density) of noninflationary gravitational waves assuming  $x_T=1$  vs the present physical frequency, for  $n_T=0, 0.3,$  and  $0.7$ .

$$\Omega = \frac{\epsilon_g}{\epsilon_{cr}} = \int_{\nu_0}^{\nu_t} \frac{\Omega_g(\nu)}{\nu} d\nu. \quad (6)$$

Assuming  $\nu_t > 10^4$  Hz (corresponds to LIGO) we obtain the following limits (see Fig. 3):  $n_T < 0.7$  for  $x_T=1$ ,  $n_T < 0.8$  for  $x_T=1\%$ . These limits on gravitational waves are the best now available, but it appears that projected experiments will soon yield much stricter limits. Note that our limits do not apply for spectra which are not of power-law form. For example, gravitational waves produced by bubble collisions at the end of inflation are peaked over a narrow range of frequencies [19].

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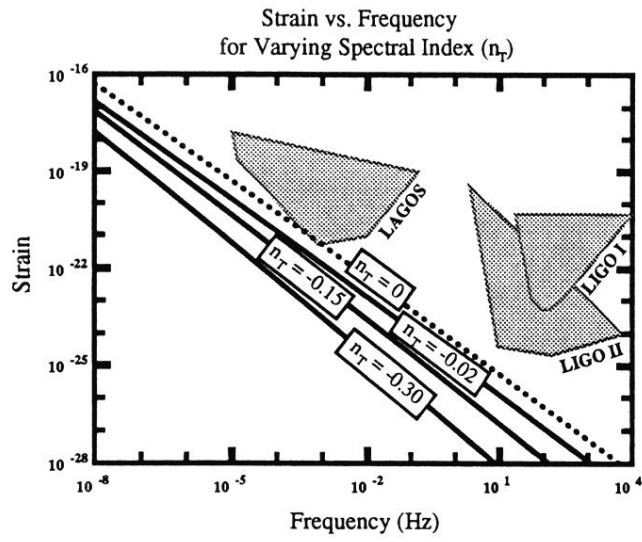


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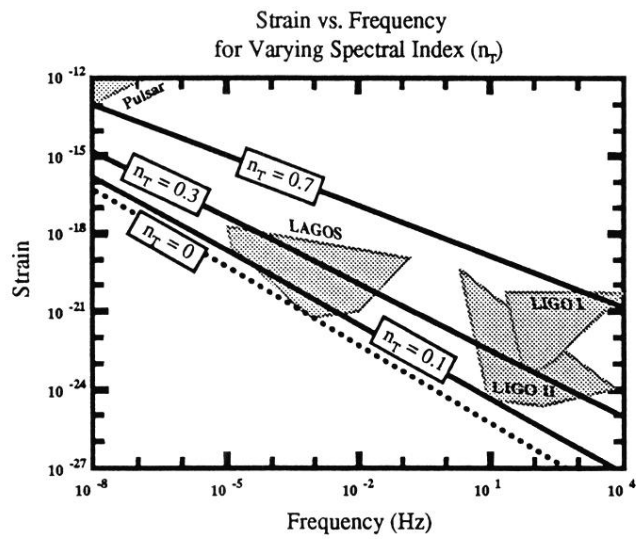


FIG. 2. The dimensionless strain of noninflationary gravitational waves all assuming  $x_T=1$  (solid curves) vs the present physical frequency, for  $n_T=0.1, 0.3,$  and  $0.7$ . The scale-invariant curve assuming  $x_T=1$  (dotted curve) is also shown. The projected sensitivities of the LIGO I, LIGO II, and LAGOS detectors as well as current limits from pulsar timing measurements are shown (shading) for comparison.